My control systems cheat sheet

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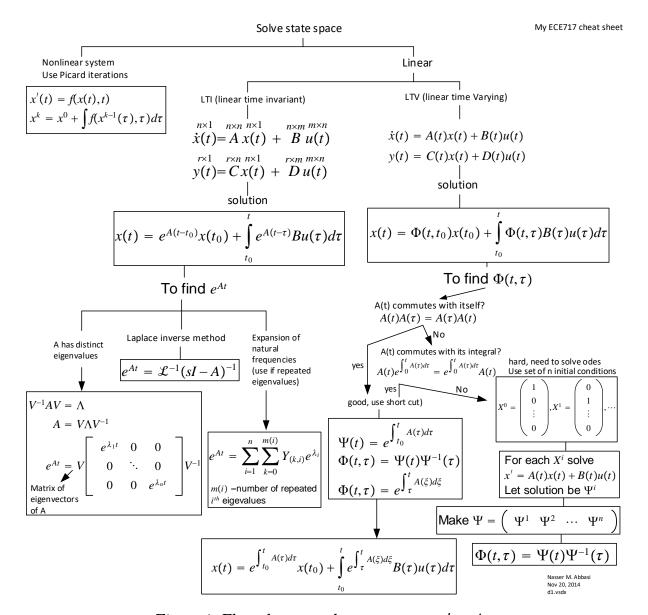
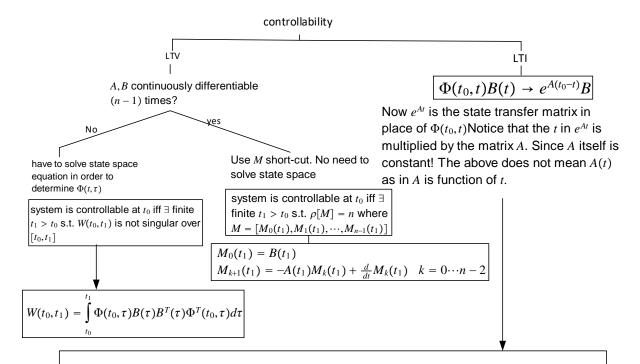


Figure 1: Flow chart to solve state space x' = Ax



If LTI is controllable, then it is controllable for any t. LTI is controllable if any one of the following is true

1.
$$W(t) = \int_{0}^{t} e^{A\tau} BB^{T} e^{(A^{T})\tau} d\tau$$
 is not singular for $t > 0$

2. Let $f = -e^{At}B$. Since analytic, set up $F = [f, f', f'', \cdots]$. Use Cayley Hamilton to reduce the above to $F = [f, f', f^{(n-1)}]$ then this simplifies to controllability matrix $\mathbb{C} = [B, AB, A^2B, \cdots, A^{n-1}B]$. Then the criteria becomes $\rho[\mathbb{C}] = n$ for controllable.

3. If all rows of $e^{At}B$ are linearly independent on $[0,\infty)$

4. if all rows of $(sI - A)^{-1}B$ are linearly independent on $[0,\infty)$

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Figure 2: Controllability flow chart

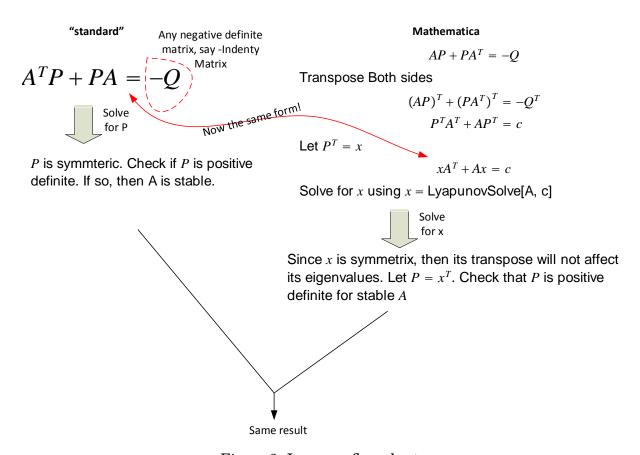


Figure 3: Lyupnov flow chart

Lyapunov stability. All what this says, is that if we start with any initial conditions $x_0(t_0)$ at time t_0 near the origin, then if solution x(t) is always bounded from above for any future time t, then we say that the origin is stable equilibrium point.

To make this more mathematically precise, we say that for any $||x_0|| \leq \delta(t_0,)$ we can find $\epsilon(\delta)$ such that $||x(t)|| \leq \epsilon$ for any $t \geq t_0$.

In this both δ and ϵ are some positive quantities. And ϵ depends on choice of δ

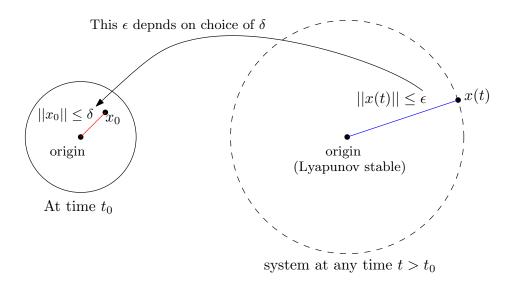


Figure 4: Lyupnov stability definition

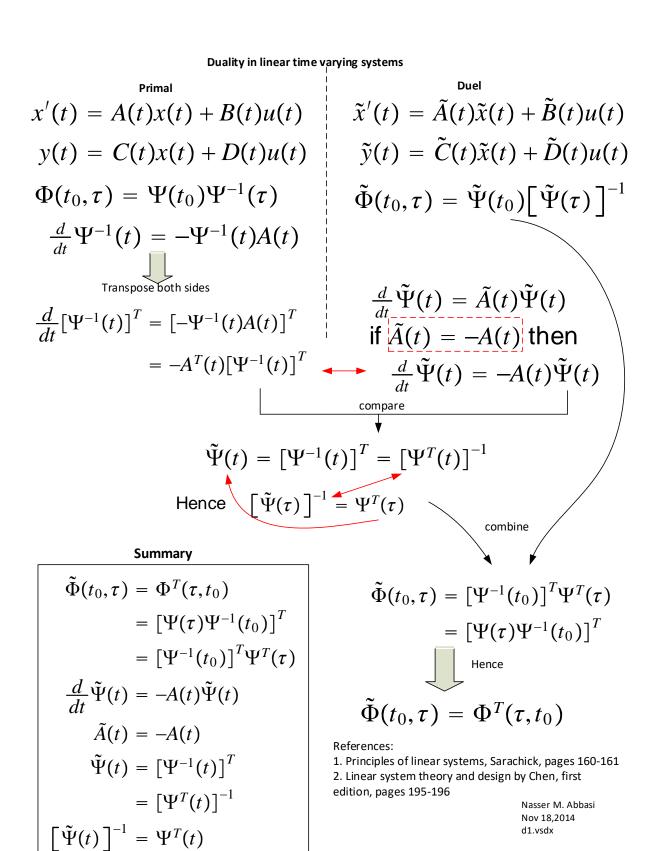


Figure 5: Duality in Linear time varying

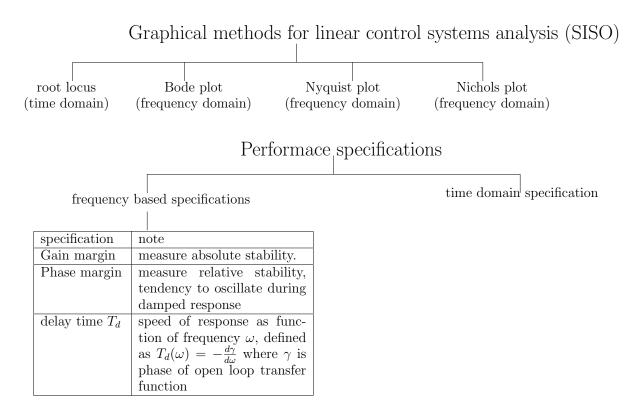


Figure 6: Graphical methods for linear control system analysis