my Quantum Mechanics cheat sheet

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Table 1: QM cheat sheet

	Position Operator X	Momentum operator P	Hamiltonian operator H
Eigenvalue eigenvector relation	$X\|x\rangle = x\|x\rangle$ where x is the eigenvalue (size) of the $\|x\rangle$ which is the position vector associated with x measured.	$P\|\phi_p\rangle = p\|\phi_p\rangle$ where p is the momentum of the particle.	$H\|\Psi_{E_i}\rangle = E_i\|\Psi_{E_i}\rangle$ where E_i is the energy level of the particle.
Normalization relation	$\int_{-\infty}^{\infty} \|x\rangle\langle x\| \ dx = 1$	$\int_{-\infty}^{\infty} \|\phi_p\rangle\langle\phi_p\| \ dp = 1$	$\int_{-\infty}^{\infty} \|\Psi_{E_i}\rangle\langle\Psi_{E_i}\| \ dE = 1$
orthogonality	$\langle x\|x'\rangle = \delta(x-x')$	$\langle\phi_p\|\phi_{p'}\rangle = \delta(p-p')$	$\langle\Psi_{E_i}\|\Psi_{E_j}\rangle = \delta(E_i - E_j)$
Matrix element of operator	$\langle x \mid X \mid x'\rangle = x'\delta(x-x')$. Operator X is diagonal matrix.	$\langle x\|P\|x'\rangle = -i\hbar\delta(x-x')\frac{d}{dx'}$ where momentum operator P is expressed in position operator $\|x\rangle$ basis. Note that operator P is not a diagonal matrix.	$\langle x \mid H \mid x'\rangle =?$
Function form of the state function $\|\Psi\rangle$	N/A ?	$\begin{aligned} P\|\phi_p\rangle &= p\|\phi_p\rangle \\ \int P\|x'\rangle\langle x'\|\phi_p\rangle \ dx &= p\int \|x'\rangle\langle x'\|\phi_p\rangle \ dx \\ \int \langle x\|P\|x'\rangle\langle x'\|\phi_p\rangle \ dx &= p\int \langle x\|x'\rangle\langle x'\|\phi_p\rangle \ dx \\ \int -i\hbar\delta(x-x')\frac{d}{dx'}\phi_p(x') \, dx &= p\int \delta(x-x')\phi_p(x') \, dx \\ &= \frac{2}{L}\frac{L}{2} \\ &= 1 \end{aligned}$	$\begin{aligned} \langle\Psi\|\Psi\rangle &= \int_{-\infty}^{\infty} \langle\Psi\|x\rangle\langle x\|\Psi\rangle \ dx \\ &= \int_{-\infty}^{\infty} \langle x\|\Psi\rangle\langle x\|\Psi\rangle \ dx \\ &= \int_{-\infty}^{\infty} \Psi^*(x)\Psi(x) \, dx \\ &= \int_0^L \left(\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}\right)^2 \ dx \\ &= \frac{2}{L}\frac{L}{2} \\ &= 1 \end{aligned}$

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	Position Operator X	Momentum operator Φ_p	Hamiltonian operator
Vector form to function form	$\langle x \mid \Psi \rangle = \Psi(x)$	$\langle x \mid \phi_p \rangle = \phi_p(x)$	$\langle x \mid \Psi_E \rangle = \Psi_E(x)$
Expansion of state vector $\lvert\Psi\rangle$	$\lvert\Psi\rangle = \int_{-\infty}^{\infty} \lvert x\rangle \langle x\lvert\Psi\rangle \; dx$	$\lvert\Psi\rangle = \int_{-\infty}^{\infty} \lvert\phi_p\rangle \langle \phi_p\lvert\Psi\rangle \; dp$	$\lvert\Psi\rangle = \int_{-\infty}^{\infty} \lvert E_i\rangle \langle E_i\lvert\Psi\rangle \; di$
State function $\lvert\Psi\rangle$ For infinite potential deep well of width $x < 0 < L$	todo	todo	todo
Probability of measurement	1. Probability of measuring x given system is in state $\lvert\Psi\rangle$ is $\lvert\langle\Psi\lvert\Psi\rangle\rvert^2$. For infinite potential deep well of width $x < 0 < L$ this becomes $$\begin{aligned} \langle\Psi\lvert\Psi\rangle &= \int_{-\infty}^{\infty} \langle\Psi\lvert x\rangle \langle x\lvert\Psi\rangle \; dx \\ &= \int_{-\infty}^{\infty} \langle x\lvert\Psi\rangle \langle x\lvert\Psi\rangle \; dx \\ &= \int_{-\infty}^{\infty} \Psi^*(x)\Psi(x) \, dx \\ &= \int_0^L \left(\sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L}\right)^2 \; dx \\ &= \frac{2}{L}\frac{L}{2} \\ &= 1 \end{aligned}$$ 2. Probability of measuring x given system is in state ϕ_p	$\lvert\Psi\rangle = \int_{-\infty}^{\infty} \lvert\phi_p\rangle \langle \phi_p\lvert\Psi\rangle \; dp$	$\lvert\Psi\rangle = \int_{-\infty}^{\infty} \lvert E_i\rangle \langle E_i\lvert\Psi\rangle \; di$