

**Theorem 1 (Residue Theorem)** Let  $f$  be analytic in the region  $G$  except for the isolated singularities  $a_1, a_2, \dots, a_m$ . If  $\gamma$  is a closed rectifiable curve in  $G$  which does not pass through any of the points  $a_k$  and if  $\gamma \approx 0$  in  $G$ , then

$$\frac{1}{2\pi i} \int_{\gamma} f = \sum_{k=1}^m n(\gamma; a_k) \operatorname{Res}(f; a_k).$$

$$\begin{pmatrix} D_1 t & -a_{12} t_2 & \dots & -a_{1n} t_n \\ -a_{21} t_1 & D_2 t & \dots & -a_{2n} t_n \\ \dots & \dots & \dots & \dots \\ -a_{n1} t_1 & -a_{n2} t_2 & \dots & D_n t \end{pmatrix}$$

$$\left( \sum_{i \in n} a_i x_i \right) \det \mathbf{K}(t = 1, x_1, \dots, x_n; l|l)$$

$$= \left( \prod_{i \in n} \hat{x}_i \right) \sum_{I \subseteq n - \{l\}} (-1)^{|I|} \mathbf{A}^{(\lambda)}(I|I) \det \mathbf{A}^{(\lambda)}(\bar{I} \cup \{l\} | \bar{I} \cup \{l\}). \quad (1)$$

$$v_i^k = \begin{cases} 1 & \text{if } i \in \Lambda_k, \\ 0 & \text{otherwise.} \end{cases}$$

$$\frac{\partial x}{\partial y} \Big| \frac{\partial y}{\partial z}$$

$$\sum_{\substack{i < B \\ i \text{ odd}}} \prod_{\kappa} \kappa F(r_i) \quad \sum_{\substack{i < B \\ i \text{ odd}}} \prod_{\kappa} \kappa(r_i)$$

$$\overrightarrow{\psi_{\delta}(t) E_t h} = \underline{\psi_{\delta}(t) E_t h}$$

$$\overleftarrow{\psi_{\delta}(t) E_t h} = \overleftarrow{\psi_{\delta}(t) E_t h}$$

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Then we have the series  $A_1, A_2, \dots$ , the regional sum  $A_1 + A_2 + \dots$ , the orthogonal product  $A_1 A_2 \dots$ , and the infinite integral

$$\int_{A_1} \int_{A_2} \dots$$

$$\hat{H} \quad \check{C} \quad \tilde{T} \quad \acute{A} \quad \grave{G} \quad \dot{D} \quad \ddot{D} \quad \breve{B} \quad \bar{B} \quad \vec{V}$$

$$\ddot{Q} \quad \ddot{R}$$

$$\sqrt{\beta k}$$

$$\boxed{W_t - F \subseteq V(P_i) \subseteq W_t}$$

$$0 \xleftarrow[\zeta]{\alpha} F \times \Delta[n-1] \xrightarrow{\partial_0 \alpha(b)} E^{\partial_0 b}$$

$$* \prod_k^* \sum_{0 \leq i \leq m}^{\prime} E_i \beta x$$

$$\mathbf{y} = \mathbf{y}' \quad \text{if and only if} \quad y'_k = \delta_k y_{\tau(k)}$$

$$\|f\|_{\infty} = \operatorname{ess\,sup}_{x \in R^n} |f(x)|$$

$$\operatorname{meas}_1 \{u \in R_+^1 : f^*(u) > \alpha\} = \operatorname{meas}_n \{x \in R^n : |f(x)| \geq \alpha\} \quad \forall \alpha > 0.$$

$$\overline{\lim}_{n \rightarrow \infty} Q(u_n, u_n - u^{\#}) \leq 0 \quad (2)$$

$$\underline{\lim}_{n \rightarrow \infty} |a_{n+1}| / |a_n| = 0 \quad (3)$$

$$\underline{\lim}_{\rightarrow} (m_i^{\lambda})^* \leq 0 \quad (4)$$

$$\underline{\lim}_{p \in S(A)} A_p \leq 0 \quad (5)$$

$$x \equiv y + 1 \pmod{m^2} \quad (6)$$

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$$x \equiv y + 1 \pmod{m^2} \quad (8)$$

$$\begin{aligned} \sum_{\gamma \in \Gamma_C} I_{\gamma} &= 2^k - \binom{k}{1} 2^{k-1} + \binom{k}{2} 2^{k-2} \\ &+ \cdots + (-1)^l \binom{k}{l} 2^{k-l} + \cdots + (-1)^k \\ &= (2-1)^k = 1 \end{aligned} \quad (9)$$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} \right)$$

$$u(r, \theta, 0) = 0$$

$$u_t(r, \theta, 0) = \begin{cases} \frac{1}{\pi \epsilon^2} & \text{if } r \leq \epsilon \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{1}{\sqrt{2} + \frac{1}{\sqrt{2} + \frac{1}{\sqrt{2} + \frac{1}{\sqrt{2} + \frac{1}{\sqrt{2} + \frac{1}{\sqrt{2} + \cdots}}}}}}$$

$$P_{r-j} = \begin{cases} 0 & \text{if } r-j \text{ is odd,} \\ r!(-1)^{(r-j)/2} & \text{if } r-j \text{ is even.} \end{cases} \quad (10)$$

$$\begin{array}{c} \vartheta \quad \varrho \\ \varphi \quad \bar{\omega} \end{array} \left( \begin{array}{c} \vartheta \quad \varrho \\ \varphi \quad \bar{\omega} \end{array} \right) \left[ \begin{array}{c} \vartheta \quad \varrho \\ \varphi \quad \bar{\omega} \end{array} \right] \left\{ \begin{array}{c} \vartheta \quad \varrho \\ \varphi \quad \bar{\omega} \end{array} \right\} \left| \begin{array}{c} \vartheta \quad \varrho \\ \varphi \quad \bar{\omega} \end{array} \right| \left\| \begin{array}{c} \vartheta \quad \varrho \\ \varphi \quad \bar{\omega} \end{array} \right\|$$

This is a small matrix  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$W(\Phi) = \left\| \begin{array}{cccc} \frac{\varphi}{(\varphi_1, \varepsilon_1)} & 0 & \dots & 0 \\ \frac{\varphi k_{n2}}{(\varphi_2, \varepsilon_1)} & \frac{\varphi}{(\varphi_2, \varepsilon_2)} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ \frac{\varphi k_{n1}}{(\varphi_n, \varepsilon_1)} & \frac{\varphi k_{n2}}{(\varphi_n, \varepsilon_2)} & \dots & \frac{\varphi k_{nn-1}}{(\varphi_n, \varepsilon_{n-1})} \frac{\varphi}{(\varphi_n, \varepsilon_n)} \end{array} \right\|$$

$$\sum_{\substack{0 \leq i \leq m \\ 0 < j < n}} P(i, j)$$

$$\left( \mathbf{E}_y \int_0^{t_\varepsilon} L_{x, y^x(s)} \varphi(x) ds \right)$$

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$$f_{h, \varepsilon}(x, y) = \varepsilon \mathbf{E}_{x, y} \int_0^{t_\varepsilon} L_{x, y_\varepsilon(\varepsilon u)} \varphi(x) du$$

$$= h \int L_{x, z} \varphi(x) \rho_x(dz)$$

$$+ h \left[ \frac{1}{t_\varepsilon} \left( \mathbf{E}_y \int_0^{t_\varepsilon} L_{x, y^x(s)} \varphi(x) ds - t_\varepsilon \int L_{x, z} \varphi(x) \rho_x(dz) \right) \right] \quad (11)$$

$$+ \frac{1}{t_\varepsilon} \left( \mathbf{E}_y \int_0^{t_\varepsilon} L_{x, y^x(s)} \varphi(x) ds - \mathbf{E}_{x, y} \int_0^{t_\varepsilon} L_{x, y_\varepsilon(\varepsilon s)} \varphi(x) ds \right) \Big]$$

$$= h \widehat{L}_x \varphi(x) + h \theta_\varepsilon(x, y),$$

$$\begin{aligned}
|I_1| &= \left| \int_{\Omega} gRu \, d\Omega \right| \\
&\leq C_3 \left[ \int_{\Omega} \left( \int_a^x g(\xi, t) \, d\xi \right)^2 \, d\Omega \right]^{1/2} \\
&\quad \times \left[ \int_{\Omega} \left\{ u_x^2 + \frac{1}{k} \left( \int_a^x cu_t \, d\xi \right)^2 \right\} c\Omega \right]^{1/2} \\
&\leq C_4 \left\| f \right\|_{\tilde{S}_{a,-}^{-1,0} W_2(\Omega, \Gamma_l)} \left\| |u| \overset{\circ}{\rightarrow} W_2^{\tilde{A}}(\Omega; \Gamma_r, T) \right\|.
\end{aligned} \tag{12}$$

$$\begin{aligned}
|I_2| &= \left| \int_0^T \psi(t) \left\{ u(a, t) - \int_{\gamma(t)}^a \frac{d\theta}{k(\theta, t)} \int_a^{\theta} c(\xi) u_t(\xi, t) \, d\xi \right\} dt \right| \\
&\leq C_6 \left\| f \right\|_{\tilde{S}_{a,-}^{-1,0} W_2(\Omega, \Gamma_l)} \left\| |u| \overset{\circ}{\rightarrow} W_2^{\tilde{A}}(\Omega; \Gamma_r, T) \right\|.
\end{aligned} \tag{13}$$

$$\begin{aligned}
&\int_a^b \left\{ \int_a^b [f(x)^2 g(y)^2 + f(y)^2 g(x)^2] - 2f(x)g(x)f(y)g(y) \, dx \right\} dy \\
&= \int_a^b \left\{ g(y)^2 \int_a^b f^2 + f(y)^2 \int_a^b g^2 - 2f(y)g(y) \int_a^b fg \right\} dy \tag{14}
\end{aligned}$$

$$D(a, r) \equiv \{z \in \mathbf{C}: |z - a| < r\}, \tag{15}$$

$$\text{seg}(a, r) \equiv \{z \in \mathbf{C}: \Im z = \Im a, |z - a| < r\},$$

$$c(e, \theta, r) \equiv \{(x, y) \in \mathbf{C}: |x - e| < y \tan \theta, 0 < y < r\}, \tag{16}$$

$$C(E, \theta, r) \equiv \bigcup_{e \in E} c(e, \theta, r). \tag{17}$$

$$\gamma_x(t) = (\cos tu + \sin tx, v), \tag{18}$$

$$\gamma_y(t) = (u, \cos tv + \sin ty), \tag{19}$$

$$\gamma_z(t) = \left( \cos tu + \frac{\alpha}{\beta} \sin tv, -\frac{\beta}{\alpha} \sin tu + \cos tv \right). \tag{20}$$

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$$\gamma_y(t) = (u, \cos tv + \sin ty),$$

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$$x = y \quad \text{by eq:C} \tag{21}$$

$$x' = y' \quad \text{by eq:D} \tag{22}$$

$$x + x' = y + y' \quad \text{by Axiom 1.} \tag{23}$$

$$\begin{aligned}\varphi(x, z) &= z - \gamma_{10}x - \gamma_{mn}x^m z^n \\ &= z - Mr^{-1}x - Mr^{-(m+n)}x^m z^n\end{aligned}\quad (24)$$

$$\begin{aligned}\zeta^0 &= (\xi^0)^2, \\ \zeta^1 &= \xi^0 \xi^1, \\ \zeta^2 &= (\xi^1)^2,\end{aligned}$$

$$\begin{aligned}\varphi(x, z) &= z - \gamma_{10}x - \gamma_{mn}x^m z^n \\ &= z - Mr^{-1}x - Mr^{-(m+n)}x^m z^n\end{aligned}\quad (25)$$

$$\zeta^0 = (\xi^0)^2, \quad (25)$$

$$\zeta^1 = \xi^0 \xi^1, \quad (26)$$

$$\zeta^2 = (\xi^1)^2, \quad (27)$$

$$V_i = v_i - q_i v_j, \quad X_i = x_i - q_i x_j, \quad U_i = u_i, \quad \text{for } i \neq j; \quad (28)$$

$$V_j = v_j, \quad X_j = x_j, \quad U_j u_j + \sum_{i \neq j} q_i u_i. \quad (29)$$

$$u_{\theta\theta} + \frac{v^2}{1 - \frac{v^2}{c^2}} u_{vv} + v u_v = 0$$

$$u(\theta, v) = \frac{e^{\frac{-4\sqrt{-c_1}\theta c^2 + v^2}{4c^2}} \left( \text{WhittakerW} \left( -\frac{c_1}{2} + \frac{1}{2}, \frac{i\sqrt{-c_1}}{2}, \frac{v^2}{2c^2} \right) c_4 + \text{WhittakerM} \left( -\frac{c_1}{2} + \frac{1}{2}, \frac{i\sqrt{-c_1}}{2}, \frac{v^2}{2c^2} \right) c_3 \right) (c_1)}{v}$$

$$u(x, t) = \sum_{n=1}^{\infty} \left( D_n \cos \left( c \frac{n\pi}{L} t \right) + E_n \sin \left( c \frac{n\pi}{L} t \right) \right) \Phi_n(x) \quad (1)$$

$$\begin{aligned}\int_0^L f(x) \Phi_n(x) dx &= D_n \int_0^L \Phi_n^2(x) dx \\ &= \frac{L}{2} D_n\end{aligned}$$

$$\begin{aligned}\int_0^L g(x) \Phi_n(x) dx &= E_n c \frac{n\pi}{L} \int_0^L \Phi_n^2(x) dx \\ &= \frac{L}{2} E_n c \frac{n\pi}{L} \\ &= \frac{1}{2} E_n c n \pi\end{aligned}$$

$$\Delta F_0 = \sqrt{\sum_{i=1}^n \left( \frac{\delta F_0}{\delta x_i} \Delta x_i \right)^2} \quad (30)$$

$$\Delta F_0 = \sqrt{6.044 \cdot 10^{-6} \text{m}^2} \quad (31)$$

$$a = b + c$$

$$d = e + f + g + r + c + e + f + g + r + c + e + f + g + r \quad (1)$$

$$h + i = j$$

$$\frac{du}{dt} = \frac{\partial u}{\partial x} \frac{dx}{dt} + \frac{\partial u}{\partial t} \quad (2)$$

Integrating the above w.r.t  $t$  gives

$$\int (x'x'' + 6x'x^5) dt = 0$$

$$\frac{x'^2}{2} + x^6 = c_1$$

$$\int (1-x)^{20} x^4 dx = -\frac{1}{21}(1-x)^{21} + \frac{2}{11}(1-x)^{22} - \frac{6}{23}(1-x)^{23} + \frac{1}{6}(1-x)^{24} - \frac{1}{25}(1-x)^{25}$$

And

$$\int \frac{(-15 + x^2) \log(x^2) + (180 + 24x - 12x^2) \log(\log(x^2)) + (-45 - (225 - 240x + 94x^2 - 16x^3 + x^4) \log(x^2) + (1350 - 540x - 96x^2 + 60x^3 - 6x^4) \log(x^2) \log^2(\log(x^2)))}{(225 - 240x + 94x^2 - 16x^3 + x^4) \log(x^2) + (1350 - 540x - 96x^2 + 60x^3 - 6x^4) \log(x^2) \log^2(\log(x^2))} dx$$

$$\frac{d}{dx} \phi(x, y) = 0$$

Hence

$$\frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y} \frac{dy}{dx} = 0 \quad (B)$$

Integrating (1) w.r.t.  $x$  gives

$$\int \frac{\partial \phi}{\partial x} dx = \int M dx$$

$$\int \frac{\partial \phi}{\partial x} dx = \int -2x - 1 dx$$

$$\phi = -x^2 - x + f(y) \quad (3)$$

Where  $f(y)$  is used for the constant of integration since  $\phi$  is a function of both  $x$  and  $y$ . Taking derivative of equation (3) w.r.t  $y$  gives

$$\frac{\partial \phi}{\partial y} = 0 + f'(y) \quad (4)$$

But since  $\phi$  itself is a constant function, then let  $\phi = c_2$  where  $c_2$  is new constant and combining  $c_1$  and  $c_2$  constants into the constant  $c_1$  gives the solution as

$$c_1 = -x^2 - x + y$$