

# CAS integration tests regression report

## Sympy 1.10.1 under Python 3.10.4 vs. Sympy 1.8 under Python 3.8.8

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September 21, 2022

Compiled on September 21, 2022 at 5:01pm

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# 1 Summary of regression test table

Table 1: Summary table of regression tests

#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
1	9	235	0 (not solved)	1 (pass)
2	9	237	0 (not solved)	1 (pass)
3	9	250	-2 (exception) Exception raised: AttributeError >> ' NoneType' object has no attribute 'primitive'	1 (pass)
4	10	216	-1 (time out)	1 (pass)
5	13	324	-1 (time out)	1 (pass)
6	13	1075	-1 (time out)	1 (pass)
7	13	1089	-1 (time out)	1 (pass)
8	13	1090	-1 (time out)	1 (pass)
9	13	1287	-1 (time out)	1 (pass)
10	13	1288	-1 (time out)	1 (pass)
11	13	1315	-1 (time out)	1 (pass)
12	13	1366	-1 (time out)	1 (pass)
13	13	1374	-1 (time out)	1 (pass)
14	14	162	-1 (time out)	1 (pass)
15	14	163	-1 (time out)	1 (pass)
16	14	164	-1 (time out)	1 (pass)
17	14	165	-1 (time out)	1 (pass)
18	14	166	-1 (time out)	1 (pass)
19	14	291	-1 (time out)	1 (pass)
20	14	292	-1 (time out)	1 (pass)
21	14	293	-1 (time out)	1 (pass)
22	14	490	-1 (time out)	1 (pass)
23	14	502	-1 (time out)	1 (pass)

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#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
24	14	514	-1 (time out)	1 (pass)
25	14	836	-1 (time out)	1 (pass)
26	14	850	-1 (time out)	1 (pass)
27	14	908	-1 (time out)	1 (pass)
28	14	1034	-1 (time out)	1 (pass)
29	14	1046	-1 (time out)	1 (pass)
30	14	1047	-1 (time out)	1 (pass)
31	14	1063	-1 (time out)	1 (pass)
32	15	149	-1 (time out)	1 (pass)
33	15	150	-1 (time out)	1 (pass)
34	15	153	-1 (time out)	1 (pass)
35	15	154	-1 (time out)	1 (pass)
36	15	155	-1 (time out)	1 (pass)
37	15	158	-1 (time out)	1 (pass)
38	15	159	-1 (time out)	1 (pass)
39	17	10	-1 (time out)	1 (pass)
40	17	11	-1 (time out)	1 (pass)
41	17	15	-1 (time out)	1 (pass)
42	17	16	-1 (time out)	1 (pass)
43	17	19	-1 (time out)	1 (pass)
44	17	30	-1 (time out)	1 (pass)
45	17	34	-1 (time out)	1 (pass)
46	17	35	-1 (time out)	1 (pass)
47	17	38	-1 (time out)	1 (pass)
48	17	39	-1 (time out)	1 (pass)
49	19	424	-1 (time out)	1 (pass)
50	19	425	-1 (time out)	1 (pass)
51	19	426	-1 (time out)	1 (pass)
52	20	62	-1 (time out)	1 (pass)

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#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
53	20	63	-1 (time out)	1 (pass)
54	21	46	-1 (time out)	1 (pass)
55	21	48	-1 (time out)	1 (pass)
56	21	49	-1 (time out)	1 (pass)
57	21	50	-1 (time out)	1 (pass)
58	21	51	-1 (time out)	1 (pass)
59	21	53	-1 (time out)	1 (pass)
60	21	228	-1 (time out)	1 (pass)
61	21	237	-1 (time out)	1 (pass)
62	21	335	-1 (time out)	1 (pass)
63	21	340	-1 (time out)	1 (pass)
64	21	539	-1 (time out)	1 (pass)
65	21	613	-1 (time out)	1 (pass)
66	21	626	-1 (time out)	1 (pass)
67	21	1084	0 (not solved)	1 (pass)
68	22	15	-1 (time out)	1 (pass)
69	24	131	-1 (time out)	1 (pass)
70	24	139	-1 (time out)	1 (pass)
71	24	140	-1 (time out)	1 (pass)
72	24	151	-1 (time out)	1 (pass)
73	25	1309	-2 (exception) Exception raised: SystemError >> excessive stack use: stack is 3655 deep	1 (pass)
74	25	1314	-2 (exception) Exception raised: SystemError >> excessive stack use: stack is 3655 deep	1 (pass)

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#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
75	25	1760	-2 (exception) Exception raised: SystemError >> excessive stack use: stack is 3277 deep	1 (pass)
76	25	2357	-1 (time out)	1 (pass)
77	25	2366	-1 (time out)	1 (pass)
78	26	18	-1 (time out)	1 (pass)
79	26	167	-1 (time out)	1 (pass)
80	26	290	-1 (time out)	1 (pass)
81	26	352	-1 (time out)	1 (pass)
82	26	355	-1 (time out)	1 (pass)
83	26	362	-1 (time out)	1 (pass)
84	26	365	-1 (time out)	1 (pass)
85	26	369	-1 (time out)	1 (pass)
86	26	371	-1 (time out)	1 (pass)
87	26	372	-1 (time out)	1 (pass)
88	26	373	-1 (time out)	1 (pass)
89	26	374	-1 (time out)	1 (pass)
90	27	46	-1 (time out)	1 (pass)
91	27	47	-1 (time out)	1 (pass)
92	27	48	-1 (time out)	1 (pass)
93	27	49	-1 (time out)	1 (pass)
94	27	111	-1 (time out)	1 (pass)
95	27	115	-1 (time out)	1 (pass)
96	27	769	-1 (time out)	1 (pass)
97	27	774	-1 (time out)	1 (pass)
98	27	777	-1 (time out)	1 (pass)
99	27	1013	-1 (time out)	1 (pass)

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#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
100	29	72	-1 (time out)	1 (pass)
101	29	129	-1 (time out)	1 (pass)
102	29	130	-1 (time out)	1 (pass)
103	29	131	-1 (time out)	1 (pass)
104	29	132	-1 (time out)	1 (pass)
105	29	150	-1 (time out)	1 (pass)
106	29	229	-1 (time out)	1 (pass)
107	29	245	-1 (time out)	1 (pass)
108	29	255	-1 (time out)	1 (pass)
109	29	261	-1 (time out)	1 (pass)
110	29	263	-1 (time out)	1 (pass)
111	29	265	-1 (time out)	1 (pass)
112	29	268	-1 (time out)	1 (pass)
113	29	269	-1 (time out)	1 (pass)
114	29	403	-1 (time out)	1 (pass)
115	29	404	-1 (time out)	1 (pass)
116	29	405	-1 (time out)	1 (pass)
117	29	406	-1 (time out)	1 (pass)
118	29	407	-1 (time out)	1 (pass)
119	29	491	-1 (time out)	1 (pass)
120	33	255	-1 (time out)	1 (pass)
121	33	256	-1 (time out)	1 (pass)
122	33	275	-1 (time out)	1 (pass)
123	33	486	-1 (time out)	1 (pass)
124	33	487	-1 (time out)	1 (pass)
125	33	1288	-1 (time out)	1 (pass)
126	33	1478	-1 (time out)	1 (pass)
127	33	1479	-1 (time out)	1 (pass)
128	33	1480	-1 (time out)	1 (pass)

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#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
129	33	1495	-1 (time out)	1 (pass)
130	33	1496	-1 (time out)	1 (pass)
131	33	1526	-1 (time out)	1 (pass)
132	33	1527	-1 (time out)	1 (pass)
133	33	1798	-1 (time out)	1 (pass)
134	33	1863	-1 (time out)	1 (pass)
135	33	1864	-1 (time out)	1 (pass)
136	33	1896	-1 (time out)	1 (pass)
137	33	1897	-1 (time out)	1 (pass)
138	33	2014	-1 (time out)	1 (pass)
139	33	2129	-1 (time out)	1 (pass)
140	33	2130	-1 (time out)	1 (pass)
141	33	2140	-1 (time out)	1 (pass)
142	33	2154	-1 (time out)	1 (pass)
143	33	2201	-1 (time out)	1 (pass)
144	33	2214	-1 (time out)	1 (pass)
145	33	2291	-1 (time out)	1 (pass)
146	34	877	-1 (time out)	1 (pass)
147	34	878	-1 (time out)	1 (pass)
148	34	1123	-1 (time out)	1 (pass)
149	34	1124	-1 (time out)	1 (pass)
150	34	1155	-1 (time out)	1 (pass)
151	34	1308	-1 (time out)	1 (pass)
152	34	1309	-1 (time out)	1 (pass)
153	34	1323	-1 (time out)	1 (pass)
154	34	1346	-1 (time out)	1 (pass)
155	34	1347	-1 (time out)	1 (pass)
156	34	1513	-1 (time out)	1 (pass)
157	34	1522	-1 (time out)	1 (pass)

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#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
158	34	1524	-1 (time out)	1 (pass)
159	34	1540	-1 (time out)	1 (pass)
160	34	1672	-1 (time out)	1 (pass)
161	34	1685	-1 (time out)	1 (pass)
162	35	65	-1 (time out)	1 (pass)
163	35	66	-1 (time out)	1 (pass)
164	35	67	-1 (time out)	1 (pass)
165	35	267	-1 (time out)	1 (pass)
166	35	529	-1 (time out)	1 (pass)
167	35	604	-1 (time out)	1 (pass)
168	35	794	-1 (time out)	1 (pass)
169	37	3	-1 (time out)	1 (pass)
170	38	58	-1 (time out)	1 (pass)
171	38	67	-1 (time out)	1 (pass)
172	39	853	-1 (time out)	1 (pass)
173	39	861	-1 (time out)	1 (pass)
174	39	868	-1 (time out)	1 (pass)
175	39	870	-1 (time out)	1 (pass)
176	39	883	-1 (time out)	1 (pass)
177	39	892	-1 (time out)	1 (pass)
178	40	259	-1 (time out)	1 (pass)
179	40	266	-1 (time out)	1 (pass)
180	40	273	-1 (time out)	1 (pass)
181	41	102	-1 (time out)	1 (pass)
182	41	109	-1 (time out)	1 (pass)
183	41	126	-1 (time out)	1 (pass)
184	41	127	-1 (time out)	1 (pass)
185	42	27	-1 (time out)	1 (pass)
186	42	43	-1 (time out)	1 (pass)

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#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
187	43	64	-1 (time out)	1 (pass)
188	43	136	-1 (time out)	1 (pass)
189	43	141	-1 (time out)	1 (pass)
190	46	142	-1 (time out)	1 (pass)
191	46	150	-1 (time out)	1 (pass)
192	46	321	-1 (time out)	1 (pass)
193	46	324	-1 (time out)	1 (pass)
194	46	502	-1 (time out)	1 (pass)
195	46	503	-1 (time out)	1 (pass)
196	46	619	-1 (time out)	1 (pass)
197	46	623	-1 (time out)	1 (pass)
198	46	644	-1 (time out)	1 (pass)
199	46	648	-1 (time out)	1 (pass)
200	48	9	-1 (time out)	1 (pass)
201	50	88	-1 (time out)	1 (pass)
202	50	89	-1 (time out)	1 (pass)
203	50	91	-1 (time out)	1 (pass)
204	50	92	-1 (time out)	1 (pass)
205	50	96	-1 (time out)	1 (pass)
206	50	98	-1 (time out)	1 (pass)
207	51	38	-1 (time out)	1 (pass)
208	52	289	0 (not solved)	1 (pass)
209	52	290	0 (not solved)	1 (pass)
210	52	573	0 (not solved)	1 (pass)
211	52	577	0 (not solved)	1 (pass)
212	52	580	0 (not solved)	1 (pass)
213	52	915	0 (not solved)	1 (pass)
214	57	120	-1 (time out)	1 (pass)
215	57	343	-1 (time out)	1 (pass)

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#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
216	57	344	-1 (time out)	1 (pass)
217	59	3	-1 (time out)	1 (pass)
218	59	31	-1 (time out)	1 (pass)
219	59	59	-1 (time out)	1 (pass)
220	60	45	-1 (time out)	1 (pass)
221	60	52	-1 (time out)	1 (pass)
222	60	106	-1 (time out)	1 (pass)
223	60	110	-1 (time out)	1 (pass)
224	60	120	-1 (time out)	1 (pass)
225	60	245	-1 (time out)	1 (pass)
226	61	74	-2 (exception)  Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
227	61	75	-2 (exception)  Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
228	64	26	0 (not solved)	1 (pass)
229	106	216	-1 (time out)	1 (pass)
230	113	13	0 (not solved)	1 (pass)
231	141	591	0 (not solved)	1 (pass)
232	144	474	0 (not solved)	1 (pass)
233	173	206	0 (not solved)	1 (pass)
234	196	201	0 (not solved)	1 (pass)
235	199	256	0 (not solved)	1 (pass)
236	199	326	0 (not solved)	1 (pass)
237	199	338	0 (not solved)	1 (pass)

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#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
238	210	86	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
239	210	89	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
240	210	152	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
241	210	222	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
242	210	255	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
243	210	284	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)

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#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
244	210	307	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
245	210	330	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
246	210	473	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
247	210	598	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
248	210	675	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
249	210	700	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)

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#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
250	210	778	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
251	210	835	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
252	210	917	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
253	210	1003	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
254	210	1043	-2 (exception) Exception raised: SystemError >> excessive stack use: stack is 3005 deep	1 (pass)
255	210	1049	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)

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#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
256	210	1259	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
257	210	1276	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
258	210	1331	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
259	210	1387	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
260	210	1455	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
261	210	1587	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)

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#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
262	210	1657	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
263	210	1661	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
264	210	1705	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
265	210	1820	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
266	210	2019	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
267	210	2117	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)

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#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
268	210	2240	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
269	210	2277	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
270	210	2318	-1 (time out)	1 (pass)
271	210	2344	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
272	210	2398	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
273	210	2405	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
274	210	2423	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)

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#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
275	210	2508	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
276	210	2569	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
277	210	2570	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
278	210	2671	-1 (time out)	1 (pass)
279	210	2849	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
280	210	2893	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
281	210	2897	-2 (exception) Exception raised: SystemError >> excessive stack use: stack is 3006 deep	1 (pass)

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#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
282	210	2907	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
283	210	3063	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
284	210	3087	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
285	210	3089	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
286	210	3156	-1 (time out)	1 (pass)
287	210	3383	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
288	210	3596	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
289	210	3706	-1 (time out)	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
290	210	3715	-1 (time out)	1 (pass)
291	210	3737	-2 (exception) Exception raised: SystemError >> excessive stack use: stack is 6437 deep	1 (pass)
292	210	3939	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of 'Poly' and 'int'	1 (pass)
293	210	4051	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of 'Poly' and 'int'	1 (pass)
294	210	4056	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of 'Poly' and 'int'	1 (pass)
295	210	4088	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of 'Poly' and 'int'	1 (pass)
296	210	4109	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of 'Poly' and 'int'	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
297	210	4134	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
298	210	4177	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
299	210	4204	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
300	210	4469	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
301	210	4629	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
302	210	4683	-1 (time out)	1 (pass)
303	210	4944	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
304	210	5196	-1 (time out)	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
305	210	5357	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
306	210	5388	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
307	210	5539	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
308	210	5630	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
309	210	5647	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
310	210	5655	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
311	210	5656	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
312	210	5688	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
313	210	5709	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
314	210	5746	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
315	210	5929	-1 (time out)	1 (pass)
316	210	5942	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
317	210	5958	-1 (time out)	1 (pass)
318	210	5964	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
319	210	6025	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
320	210	6249	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
321	210	6406	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
322	210	6477	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
323	210	6591	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
324	210	6634	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
325	210	6685	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
326	210	6772	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
327	210	6869	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
328	210	7164	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
329	210	7254	-1 (time out)	1 (pass)
330	210	7265	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
331	210	7310	-1 (time out)	1 (pass)
332	210	7313	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
333	210	7350	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
334	210	7422	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
335	210	7508	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
336	210	7742	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
337	210	7758	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
338	210	7785	-2 (exception) Exception raised: SystemError >> excessive stack use: stack is 6438 deep	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
339	210	7935	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
340	210	8068	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
341	210	8168	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
342	210	8246	-1 (time out)	1 (pass)
343	210	8247	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
344	210	8305	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
345	210	8314	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
346	210	8325	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
347	210	8348	-1 (time out)	1 (pass)
348	210	8392	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
349	210	8435	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
350	210	8519	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
351	210	8523	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
352	210	8610	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
353	210	8617	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
354	210	8620	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
355	210	8793	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
356	210	8800	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
357	210	8869	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
358	210	8893	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
359	210	8900	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
360	210	9001	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
361	210	9011	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
362	210	9095	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
363	210	9101	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
364	210	9111	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
365	210	9117	-1 (time out)	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
366	210	9277	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
367	210	9310	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
368	210	9541	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
369	210	9749	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
370	210	9775	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
371	210	9947	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
372	210	10034	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
373	210	10123	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
374	210	10148	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
375	210	10161	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
376	210	10182	-1 (time out)	1 (pass)
377	210	10263	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)
378	210	10318	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Sympy 1.10.1 under Python 3.10.4	Sympy 1.8 under Python 3.8.8
379	210	10323	-2 (exception) Exception raised: TypeError >> '>' not supported between instances of ' Poly' and 'int'	1 (pass)

## 2 Test file number 9

Test folder name:

test\_cases/0\_Independent\_test\_suites/9\_Stewart\_Problems

### 2.1 Problem number 235

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}} + \sqrt{x}} dx$$

Optimal antiderivative

$$\begin{aligned} & 6x^{\frac{1}{6}} + x + \frac{6 \ln\left(1 - x^{\frac{1}{6}}\right)}{5} - \frac{3 \ln\left(2 + x^{\frac{1}{6}} + 2x^{\frac{1}{3}} - x^{\frac{1}{6}}\sqrt{5}\right) \left(-\sqrt{5} + 1\right)}{10} \\ & - \frac{3 \ln\left(2 + x^{\frac{1}{6}} + 2x^{\frac{1}{3}} + x^{\frac{1}{6}}\sqrt{5}\right) \left(\sqrt{5} + 1\right)}{10} \\ & - \frac{3 \arctan\left(\frac{\left(1+4x^{\frac{1}{6}}+\sqrt{5}\right)\sqrt{50+10\sqrt{5}}}{20}\right) \sqrt{10-2\sqrt{5}}}{5} \\ & - \frac{3 \arctan\left(\frac{1+4x^{\frac{1}{6}}-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right) \sqrt{10+2\sqrt{5}}}{5} \end{aligned}$$

command

```
integrate(x**(1/2)/(-1/x**(1/3)+x**(1/2)),x)
```

Sympy 1.10.1 under Python 3.10.4 output

$$\int \frac{x^{\frac{5}{6}}}{\left(\sqrt[6]{x} - 1\right) \left(\sqrt[6]{x} + x^{\frac{2}{3}} + \sqrt[3]{x} + \sqrt{x} + 1\right)} dx$$

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& 6\sqrt[6]{x} + x + \frac{6 \log(\sqrt[6]{x} - 1)}{5} - \frac{3\sqrt{5} \log(8\sqrt[6]{x} + 8\sqrt{5}\sqrt[6]{x} + 16\sqrt[3]{x} + 16)}{10} \\
& - \frac{3 \log(8\sqrt[6]{x} + 8\sqrt{5}\sqrt[6]{x} + 16\sqrt[3]{x} + 16)}{10} - \frac{3 \log(-8\sqrt{5}\sqrt[6]{x} + 8\sqrt[6]{x} + 16\sqrt[3]{x} + 16)}{10} \\
& + \frac{3\sqrt{5} \log(-8\sqrt{5}\sqrt[6]{x} + 8\sqrt[6]{x} + 16\sqrt[3]{x} + 16)}{10} \\
& - \frac{3\sqrt{2} \sqrt{\sqrt{5} - \sqrt{5}} \operatorname{atan}\left(\frac{2\sqrt{2}\sqrt[6]{x}}{\sqrt{\sqrt{5} - \sqrt{5}}} + \frac{\sqrt{2}}{2\sqrt{\sqrt{5} - \sqrt{5}}} + \frac{\sqrt{10}}{2\sqrt{\sqrt{5} - \sqrt{5}}}\right)}{5} \\
& - \frac{3\sqrt{2} \sqrt{\sqrt{5} + 5} \operatorname{atan}\left(\frac{2\sqrt{2}\sqrt[6]{x}}{\sqrt{\sqrt{5} + 5}} - \frac{\sqrt{10}}{2\sqrt{\sqrt{5} + 5}} + \frac{\sqrt{2}}{2\sqrt{\sqrt{5} + 5}}\right)}{5}
\end{aligned}$$

## 2.2 Problem number 237

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}} + \frac{1}{\sqrt[4]{x}}} dx$$

Optimal antiderivative

$$\begin{aligned}
& 12x^{\frac{1}{12}} - 6x^{\frac{1}{6}} + 4x^{\frac{1}{4}} - 3x^{\frac{1}{3}} + \frac{12x^{\frac{5}{12}}}{5} + \frac{12x^{\frac{7}{12}}}{7} - \frac{3x^{\frac{2}{3}}}{2} + \frac{4x^{\frac{3}{4}}}{3} - \frac{6x^{\frac{5}{6}}}{5} \\
& + \frac{12x^{\frac{11}{12}}}{11} - x + \frac{12x^{\frac{13}{12}}}{13} - \frac{6x^{\frac{7}{6}}}{7} + \frac{4x^{\frac{5}{4}}}{5} - 12 \ln(1 + x^{\frac{1}{12}}) - 2\sqrt{x}
\end{aligned}$$

command

`integrate(1/(1/x**(1/3)+1/x**(1/4)),x)`

Sympy 1.10.1 under Python 3.10.4 output

$$\int \frac{x^{\frac{7}{12}}}{\sqrt[4]{x} + \sqrt[3]{x}} dx$$

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{12x^{\frac{13}{12}}}{13} + \frac{12x^{\frac{11}{12}}}{11} + \frac{12x^{\frac{7}{12}}}{7} + \frac{12x^{\frac{5}{12}}}{5} + 12\sqrt[12]{x} - \frac{6x^{\frac{7}{6}}}{7} - \frac{6x^{\frac{5}{6}}}{5} - 6\sqrt[6]{x} \\
& + \frac{4x^{\frac{5}{4}}}{5} + \frac{4x^{\frac{3}{4}}}{3} + 4\sqrt[4]{x} - \frac{3x^{\frac{2}{3}}}{2} - 3\sqrt[3]{x} - 2\sqrt{x} - x - 12 \log(\sqrt[12]{x} + 1)
\end{aligned}$$

### 2.3 Problem number 250

$$\int \frac{1}{b \cos(x) + a \sin(x)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{a \cos(x) - b \sin(x)}{\sqrt{a^2 + b^2}}\right)}{\sqrt{a^2 + b^2}}$$

command

```
integrate(1/(b*cos(x)+a*sin(x)),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: AttributeError

Sympy 1.8 under Python 3.8.8 output

$$\left\{ \begin{array}{ll} \infty(-\log(\tan(\frac{x}{2}) - 1) + \log(\tan(\frac{x}{2}) + 1)) & \text{for } a = 0 \wedge b = 0 \\ \frac{\log(\tan(\frac{x}{2}))}{a} & \text{for } b = 0 \\ \frac{1}{b \sin(x) + i\sqrt{b^2} \cos(x)} & \text{for } a = -\sqrt{-b^2} \\ \frac{1}{b \sin(x) - i\sqrt{b^2} \cos(x)} & \text{for } a = \sqrt{-b^2} \\ -\frac{\log\left(-\frac{a}{b} + \tan\left(\frac{x}{2}\right) - \frac{\sqrt{a^2 + b^2}}{b}\right)}{\sqrt{a^2 + b^2}} + \frac{\log\left(-\frac{a}{b} + \tan\left(\frac{x}{2}\right) + \frac{\sqrt{a^2 + b^2}}{b}\right)}{\sqrt{a^2 + b^2}} & \text{otherwise} \end{array} \right.$$

## 3 Test file number 10

Test folder name:

test\_cases/0\_Independent\_test\_suites/10\_Timofeev\_Problems

### 3.1 Problem number 216

$$\int \frac{1}{(1-x)^{7/2} x^5} dx$$

Optimal antiderivative

$$\frac{3003}{320(1-x)^{\frac{5}{2}}} + \frac{1001}{64(1-x)^{\frac{3}{2}}} - \frac{1}{4(1-x)^{\frac{5}{2}}x^4} - \frac{13}{24(1-x)^{\frac{5}{2}}x^3} - \frac{143}{96(1-x)^{\frac{5}{2}}x^2} - \frac{429}{64(1-x)^{\frac{5}{2}}x} - \frac{3003 \operatorname{arctanh}(\sqrt{1-x})}{64} + \frac{3003}{64\sqrt{1-x}}$$

command

```
integrate(1/(1-x)**(7/2)/x**5,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\left\{ \begin{array}{l} -\frac{45045ix^7 \operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)}{-960x^7+2880x^6-2880x^5+960x^4} + \frac{45045ix^6\sqrt{x-1}}{-960x^7+2880x^6-2880x^5+960x^4} + \frac{135135ix^6 \operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)}{-960x^7+2880x^6-2880x^5+960x^4} - \frac{105105ix^5\sqrt{x-1}}{-960x^7+2880x^6-2880x^5+960x^4} \\ -\frac{45045x^7 \log(x)}{-1920x^7+5760x^6-5760x^5+1920x^4} + \frac{90090x^7 \log(\sqrt{1-x}+1)}{-1920x^7+5760x^6-5760x^5+1920x^4} - \frac{45045i\pi x^7}{-1920x^7+5760x^6-5760x^5+1920x^4} + \frac{90090x^6\sqrt{1-x}}{-1920x^7+5760x^6-5760x^5+1920x^4} \end{array} \right.$$

## 4 Test file number 13

Test folder name:

```
test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.1_Linear/13_1.1.1.2-a+b_x-  
^m-c+d_x-^n
```

### 4.1 Problem number 324

$$\int \frac{(a+bx)^{9/2}}{x^8} dx$$

Optimal antiderivative

$$\begin{aligned} &-\frac{3b^3(bx+a)^{\frac{3}{2}}}{64x^4} - \frac{3b^2(bx+a)^{\frac{5}{2}}}{40x^5} - \frac{3b(bx+a)^{\frac{7}{2}}}{28x^6} - \frac{(bx+a)^{\frac{9}{2}}}{7x^7} \\ &-\frac{9b^7 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{1024a^{\frac{5}{2}}} - \frac{3b^4\sqrt{bx+a}}{128x^3} - \frac{3b^5\sqrt{bx+a}}{512ax^2} + \frac{9b^6\sqrt{bx+a}}{1024a^2x} \end{aligned}$$

command

```
integrate((b*x+a)**(9/2)/x**8,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& -\frac{a^5}{7\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{23a^4\sqrt{b}}{28x^{\frac{13}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{541a^3b^{\frac{3}{2}}}{280x^{\frac{11}{2}}\sqrt{\frac{a}{bx}+1}} \\
& - \frac{5249a^2b^{\frac{5}{2}}}{2240x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{6653ab^{\frac{7}{2}}}{4480x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{1027b^{\frac{9}{2}}}{2560x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} \\
& + \frac{3b^{\frac{11}{2}}}{1024ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{9b^{\frac{13}{2}}}{1024a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}} - \frac{9b^7 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{1024a^{\frac{5}{2}}}
\end{aligned}$$

## 4.2 Problem number 1075

$$\int (1-x)^{9/2}(1+x)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{3(1-x)^{\frac{3}{2}}x(1+x)^{\frac{3}{2}}}{8} + \frac{3(1-x)^{\frac{5}{2}}(1+x)^{\frac{5}{2}}}{10} + \frac{3(1-x)^{\frac{7}{2}}(1+x)^{\frac{5}{2}}}{14} \\
& + \frac{(1-x)^{\frac{9}{2}}(1+x)^{\frac{5}{2}}}{7} + \frac{9\arcsin(x)}{16} + \frac{9x\sqrt{1-x}\sqrt{1+x}}{16}
\end{aligned}$$

command

`integrate((1-x)**(9/2)*(1+x)**(3/2), x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\left\{ \begin{aligned}
& -\frac{9i \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} + \frac{i(x+1)^{\frac{15}{2}}}{7\sqrt{x-1}} - \frac{23i(x+1)^{\frac{13}{2}}}{14\sqrt{x-1}} + \frac{541i(x+1)^{\frac{11}{2}}}{70\sqrt{x-1}} - \frac{5249i(x+1)^{\frac{9}{2}}}{280\sqrt{x-1}} + \frac{6653i(x+1)^{\frac{7}{2}}}{280\sqrt{x-1}} - \frac{1027i(x+1)^{\frac{5}{2}}}{80\sqrt{x-1}} - \frac{3i(x+1)^{\frac{3}{2}}}{16\sqrt{x-1}} \\
& \frac{9 \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} - \frac{(x+1)^{\frac{15}{2}}}{7\sqrt{1-x}} + \frac{23(x+1)^{\frac{13}{2}}}{14\sqrt{1-x}} - \frac{541(x+1)^{\frac{11}{2}}}{70\sqrt{1-x}} + \frac{5249(x+1)^{\frac{9}{2}}}{280\sqrt{1-x}} - \frac{6653(x+1)^{\frac{7}{2}}}{280\sqrt{1-x}} + \frac{1027(x+1)^{\frac{5}{2}}}{80\sqrt{1-x}} + \frac{3(x+1)^{\frac{3}{2}}}{16\sqrt{1-x}}
\end{aligned} \right.$$

### 4.3 Problem number 1089

$$\int (1-x)^{9/2}(1+x)^{5/2} dx$$

Optimal antiderivative

$$\frac{15(1-x)^{\frac{3}{2}}x(1+x)^{\frac{3}{2}}}{64} + \frac{3(1-x)^{\frac{5}{2}}x(1+x)^{\frac{5}{2}}}{16} + \frac{9(1-x)^{\frac{7}{2}}(1+x)^{\frac{7}{2}}}{56} \\ + \frac{(1-x)^{\frac{9}{2}}(1+x)^{\frac{7}{2}}}{8} + \frac{45 \arcsin(x)}{128} + \frac{45x\sqrt{1-x}\sqrt{1+x}}{128}$$

command

```
integrate((1-x)**(9/2)*(1+x)**(5/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\left\{ \begin{array}{l} -\frac{45i \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{64} + \frac{i(x+1)^{\frac{17}{2}}}{8\sqrt{x-1}} - \frac{79i(x+1)^{\frac{15}{2}}}{56\sqrt{x-1}} + \frac{725i(x+1)^{\frac{13}{2}}}{112\sqrt{x-1}} - \frac{1699i(x+1)^{\frac{11}{2}}}{112\sqrt{x-1}} + \frac{8191i(x+1)^{\frac{9}{2}}}{448\sqrt{x-1}} - \frac{4099i(x+1)^{\frac{7}{2}}}{448\sqrt{x-1}} - \dots \\ \frac{45 \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{64} - \frac{(x+1)^{\frac{17}{2}}}{8\sqrt{1-x}} + \frac{79(x+1)^{\frac{15}{2}}}{56\sqrt{1-x}} - \frac{725(x+1)^{\frac{13}{2}}}{112\sqrt{1-x}} + \frac{1699(x+1)^{\frac{11}{2}}}{112\sqrt{1-x}} - \frac{8191(x+1)^{\frac{9}{2}}}{448\sqrt{1-x}} + \frac{4099(x+1)^{\frac{7}{2}}}{448\sqrt{1-x}} + \frac{3(x+1)^{\frac{5}{2}}}{128\sqrt{1-x}} + \dots \end{array} \right.$$

### 4.4 Problem number 1090

$$\int (1-x)^{7/2}(1+x)^{5/2} dx$$

Optimal antiderivative

$$\frac{5(1-x)^{\frac{3}{2}}x(1+x)^{\frac{3}{2}}}{24} + \frac{(1-x)^{\frac{5}{2}}x(1+x)^{\frac{5}{2}}}{6} + \frac{(1-x)^{\frac{7}{2}}(1+x)^{\frac{7}{2}}}{7} + \frac{5 \arcsin(x)}{16} + \frac{5x\sqrt{1-x}\sqrt{1+x}}{16}$$

command

```
integrate((1-x)**(7/2)*(1+x)**(5/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\left\{ \begin{array}{l} \frac{5i \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} - \frac{i(x+1)^{\frac{15}{2}}}{7\sqrt{x-1}} + \frac{55i(x+1)^{\frac{13}{2}}}{42\sqrt{x-1}} - \frac{193i(x+1)^{\frac{11}{2}}}{42\sqrt{x-1}} + \frac{1237i(x+1)^{\frac{9}{2}}}{168\sqrt{x-1}} - \frac{769i(x+1)^{\frac{7}{2}}}{168\sqrt{x-1}} - \frac{i(x+1)^{\frac{5}{2}}}{48\sqrt{x-1}} - \frac{5i}{48\sqrt{x-1}} \\ \frac{5 \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} + \frac{(x+1)^{\frac{15}{2}}}{7\sqrt{1-x}} - \frac{55(x+1)^{\frac{13}{2}}}{42\sqrt{1-x}} + \frac{193(x+1)^{\frac{11}{2}}}{42\sqrt{1-x}} - \frac{1237(x+1)^{\frac{9}{2}}}{168\sqrt{1-x}} + \frac{769(x+1)^{\frac{7}{2}}}{168\sqrt{1-x}} + \frac{(x+1)^{\frac{5}{2}}}{48\sqrt{1-x}} + \frac{5(x+1)}{48\sqrt{1-x}} \end{array} \right.$$

#### 4.5 Problem number 1287

$$\int \frac{(c+dx)^7}{(a+bx)^5} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{21d^5(-ad+bc)^2 x}{b^7} - \frac{(-ad+bc)^7}{4b^8 (bx+a)^4} - \frac{7d(-ad+bc)^6}{3b^8 (bx+a)^3} - \frac{21d^2(-ad+bc)^5}{2b^8 (bx+a)^2} \\ & - \frac{35d^3(-ad+bc)^4}{b^8 (bx+a)} + \frac{7d^6(-ad+bc)(bx+a)^2}{2b^8} + \frac{d^7(bx+a)^3}{3b^8} + \frac{35d^4(-ad+bc)^3 \ln(bx+a)}{b^8} \end{aligned}$$

command

```
integrate((d*x+c)**7/(b*x+a)**5,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & x^2 \left( -\frac{5ad^7}{2b^6} + \frac{7cd^6}{2b^5} \right) + x \left( \frac{15a^2d^7}{b^7} - \frac{35acd^6}{b^6} + \frac{21c^2d^5}{b^5} \right) \\ & + \frac{-319a^7d^7 + 1197a^6bcd^6 - 1617a^5b^2c^2d^5 + 875a^4b^3c^3d^4 - 105a^3b^4c^4d^3 - 21a^2b^5c^5d^2 - 7ab^6c^6d - 3b^7c^7 + x^3(-42d^7 - 35d^6c - 105d^5c^2 - 105d^4c^3 - 35d^3c^4 - 7d^2c^5 - 7dc^6 - c^7)}{b^8} \\ & + \frac{d^7x^3}{3b^5} - \frac{35d^4(ad-bc)^3 \log(a+bx)}{b^8} \end{aligned}$$



#### 4.6 Problem number 1288

$$\int \frac{(c + dx)^7}{(a + bx)^6} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^6(-6ad + 7bc)x}{b^7} + \frac{d^7x^2}{2b^6} - \frac{(-ad + bc)^7}{5b^8(bx + a)^5} - \frac{7d(-ad + bc)^6}{4b^8(bx + a)^4} - \frac{7d^2(-ad + bc)^5}{b^8(bx + a)^3} \\ & - \frac{35d^3(-ad + bc)^4}{2b^8(bx + a)^2} - \frac{35d^4(-ad + bc)^3}{b^8(bx + a)} + \frac{21d^5(-ad + bc)^2 \ln(bx + a)}{b^8} \end{aligned}$$

command

```
integrate((d*x+c)**7/(b*x+a)**6,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & x \left( -\frac{6ad^7}{b^7} + \frac{7cd^6}{b^6} \right) \\ & + \frac{459a^7d^7 - 1218a^6bcd^6 + 959a^5b^2c^2d^5 - 140a^4b^3c^3d^4 - 35a^3b^4c^4d^3 - 14a^2b^5c^5d^2 - 7ab^6c^6d - 4b^7c^7 + x^4(700a^3b^4c^7)}{b^{11}} \\ & + \frac{d^7x^2}{2b^6} + \frac{21d^5(ad - bc)^2 \log(a + bx)}{b^8} \end{aligned}$$

#### 4.7 Problem number 1315

$$\int \frac{(c + dx)^{10}}{(a + bx)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{210d^4(-ad + bc)^6x}{b^{10}} - \frac{(-ad + bc)^{10}}{3b^{11}(bx + a)^3} - \frac{5d(-ad + bc)^9}{b^{11}(bx + a)^2} - \frac{45d^2(-ad + bc)^8}{b^{11}(bx + a)} \\ & + \frac{126d^5(-ad + bc)^5(bx + a)^2}{b^{11}} + \frac{70d^6(-ad + bc)^4(bx + a)^3}{b^{11}} \\ & + \frac{30d^7(-ad + bc)^3(bx + a)^4}{b^{11}} + \frac{9d^8(-ad + bc)^2(bx + a)^5}{b^{11}} \\ & + \frac{5d^9(-ad + bc)(bx + a)^6}{3b^{11}} + \frac{d^{10}(bx + a)^7}{7b^{11}} + \frac{120d^3(-ad + bc)^7 \ln(bx + a)}{b^{11}} \end{aligned}$$

command

```
integrate((d*x+c)**10/(b*x+a)**4,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
 & x^6 \left( -\frac{2ad^{10}}{3b^5} + \frac{5cd^9}{3b^4} \right) + x^5 \left( \frac{2a^2d^{10}}{b^6} - \frac{8acd^9}{b^5} + \frac{9c^2d^8}{b^4} \right) + x^4 \left( -\frac{5a^3d^{10}}{b^7} + \frac{25a^2cd^9}{b^6} - \frac{45ac^2d^8}{b^5} + \frac{30c^3d^7}{b^4} \right) \\
 & + x^3 \left( \frac{35a^4d^{10}}{3b^8} - \frac{200a^3cd^9}{3b^7} + \frac{150a^2c^2d^8}{b^6} - \frac{160ac^3d^7}{b^5} + \frac{70c^4d^6}{b^4} \right) \\
 & + x^2 \left( -\frac{28a^5d^{10}}{b^9} + \frac{175a^4cd^9}{b^8} - \frac{450a^3c^2d^8}{b^7} + \frac{600a^2c^3d^7}{b^6} - \frac{420ac^4d^6}{b^5} + \frac{126c^5d^5}{b^4} \right) \\
 & + x \left( \frac{84a^6d^{10}}{b^{10}} - \frac{560a^5cd^9}{b^9} + \frac{1575a^4c^2d^8}{b^8} - \frac{2400a^3c^3d^7}{b^7} + \frac{2100a^2c^4d^6}{b^6} - \frac{1008ac^5d^5}{b^5} + \frac{210c^6d^4}{b^4} \right) \\
 & + \frac{-121a^{10}d^{10} + 955a^9bcd^9 - 3285a^8b^2c^2d^8 + 6420a^7b^3c^3d^7 - 7770a^6b^4c^4d^6 + 5922a^5b^5c^5d^5 - 2730a^4b^6c^6d^4 + 660a^3b^7c^7d^3 - 120d^3(ad-bc)^7 \log(a+bx)}{7b^4} - \frac{120d^3(ad-bc)^7 \log(a+bx)}{b^{11}}
 \end{aligned}$$

#### 4.8 Problem number 1366

$$\int \frac{(a+bx)^5}{(c+dx)^8} dx$$

Optimal antiderivative

$$\frac{(bx+a)^6}{7(-ad+bc)(dx+c)^7} + \frac{b(bx+a)^6}{42(-ad+bc)^2(dx+c)^6}$$

command

```
integrate((b*x+a)**5/(d*x+c)**8,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-6a^5d^5 - 5a^4bcd^4 - 4a^3b^2c^2d^3 - 3a^2b^3c^3d^2 - 2ab^4c^4d - b^5c^5 - 21b^5d^5x^5 + x^4(-70ab^4d^5 - 35b^5cd^4) + x^3(-105a^2b^4d^5 - 105a^2b^5cd^4) + x^2(-105a^3b^4d^5 - 105a^3b^5cd^4) + x(-105a^4b^4d^5 - 105a^4b^5cd^4) + 105a^5b^4d^5 + 105a^5b^5cd^4}{42c^7d^6 + 294c^6d^7x + 882c^5d^8x^2}$$

#### 4.9 Problem number 1374

$$\int \frac{1}{(a+bx)^3(c+dx)^8} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b^7}{2(-ad+bc)^8(bx+a)^2} + \frac{8b^7d}{(-ad+bc)^9(bx+a)} + \frac{d^2}{7(-ad+bc)^3(dx+c)^7} \\ & + \frac{bd^2}{2(-ad+bc)^4(dx+c)^6} + \frac{6b^2d^2}{5(-ad+bc)^5(dx+c)^5} + \frac{5b^3d^2}{2(-ad+bc)^6(dx+c)^4} \\ & + \frac{5b^4d^2}{(-ad+bc)^7(dx+c)^3} + \frac{21b^5d^2}{2(-ad+bc)^8(dx+c)^2} \\ & + \frac{28b^6d^2}{(-ad+bc)^9(dx+c)} + \frac{36b^7d^2 \ln(bx+a)}{(-ad+bc)^{10}} - \frac{36b^7d^2 \ln(dx+c)}{(-ad+bc)^{10}} \end{aligned}$$

command

```
integrate(1/(b*x+a)**3/(d*x+c)**8,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

## 5 Test file number 14

Test folder name:

```
test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.1_Linear/14_1.1.1.3-a+b_x-  
^m-c+d_x-^n-e+f_x-^p
```

### 5.1 Problem number 162

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{15}} dx$$

Optimal antiderivative

$$-\frac{A(bx+a)^{11}}{14ax^{14}} + \frac{(3Ab-14aB)(bx+a)^{11}}{182a^2x^{13}} - \frac{b(3Ab-14aB)(bx+a)^{11}}{1092a^3x^{12}} + \frac{b^2(3Ab-14aB)(bx+a)^{11}}{12012a^4x^{11}}$$

command

```
integrate((b*x+a)**10*(B*x+A)/x**15,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-858Aa^{10} - 4004Bb^{10}x^{11} + x^{10}(-3003Ab^{10} - 30030Bab^9) + x^9(-24024Aab^9 - 108108Ba^2b^8) + x^8(-90090Aa^2b^8 - 1081080Aab^7 - 10810800A^2b^6)}{x^{15}}$$

## 5.2 Problem number 163

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{16}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{A(bx+a)^{11}}{15ax^{15}} + \frac{(4Ab-15aB)(bx+a)^{11}}{210a^2x^{14}} - \frac{b(4Ab-15aB)(bx+a)^{11}}{910a^3x^{13}} \\ & + \frac{b^2(4Ab-15aB)(bx+a)^{11}}{5460a^4x^{12}} - \frac{b^3(4Ab-15aB)(bx+a)^{11}}{60060a^5x^{11}} \end{aligned}$$

command

```
integrate((b*x+a)**10*(B*x+A)/x**16,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-4004Aa^{10} - 15015Bb^{10}x^{11} + x^{10}(-12012Ab^{10} - 120120Bab^9) + x^9(-100100Aab^9 - 450450Ba^2b^8) + x^8(-386100Aa^2b^8 - 1081080Aab^7 - 10810800A^2b^6)}{x^{16}}$$

## 5.3 Problem number 164

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{17}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{A(bx+a)^{11}}{16ax^{16}} + \frac{(5Ab-16aB)(bx+a)^{11}}{240a^2x^{15}} - \frac{b(5Ab-16aB)(bx+a)^{11}}{840a^3x^{14}} \\ & + \frac{b^2(5Ab-16aB)(bx+a)^{11}}{3640a^4x^{13}} - \frac{b^3(5Ab-16aB)(bx+a)^{11}}{21840a^5x^{12}} + \frac{b^4(5Ab-16aB)(bx+a)^{11}}{240240a^6x^{11}} \end{aligned}$$

command

```
integrate((b*x+a)**10*(B*x+A)/x**17,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-15015Aa^{10} - 48048Bb^{10}x^{11} + x^{10}(-40040Ab^{10} - 400400Bab^9) + x^9(-343200Aab^9 - 1544400Ba^2b^8) + x^8(-13}$$

## 5.4 Problem number 165

$$\int \frac{(a+bx)^{10}(A+Bx)}{x^{18}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{A(bx+a)^{11}}{17ax^{17}} + \frac{(6Ab-17aB)(bx+a)^{11}}{272a^2x^{16}} - \frac{b(6Ab-17aB)(bx+a)^{11}}{816a^3x^{15}} \\ & + \frac{b^2(6Ab-17aB)(bx+a)^{11}}{2856a^4x^{14}} - \frac{b^3(6Ab-17aB)(bx+a)^{11}}{12376a^5x^{13}} \\ & + \frac{b^4(6Ab-17aB)(bx+a)^{11}}{74256a^6x^{12}} - \frac{b^5(6Ab-17aB)(bx+a)^{11}}{816816a^7x^{11}} \end{aligned}$$

command

```
integrate((b*x+a)**10*(B*x+A)/x**18,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-48048Aa^{10} - 136136Bb^{10}x^{11} + x^{10}(-116688Ab^{10} - 1166880Bab^9) + x^9(-1021020Aab^9 - 4594590Ba^2b^8) + x^8(-13}$$

## 5.5 Problem number 166

$$\int \frac{(a + bx)^{10}(A + Bx)}{x^{19}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{a^{10}A}{18x^{18}} - \frac{a^9(10Ab + aB)}{17x^{17}} - \frac{5a^8b(9Ab + 2aB)}{16x^{16}} - \frac{a^7b^2(8Ab + 3aB)}{x^{15}} \\ & - \frac{15a^6b^3(7Ab + 4aB)}{7x^{14}} - \frac{42a^5b^4(6Ab + 5aB)}{13x^{13}} - \frac{7a^4b^5(5Ab + 6aB)}{2x^{12}} - \frac{30a^3b^6(4Ab + 7aB)}{11x^{11}} \\ & - \frac{3a^2b^7(3Ab + 8aB)}{2x^{10}} - \frac{5ab^8(2Ab + 9aB)}{9x^9} - \frac{b^9(Ab + 10aB)}{8x^8} - \frac{b^{10}B}{7x^7} \end{aligned}$$

command

```
integrate((b*x+a)**10*(B*x+A)/x**19, x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-136136Aa^{10} - 350064Bb^{10}x^{11} + x^{10}(-306306Ab^{10} - 3063060Bab^9) + x^9(-2722720Aab^9 - 12252240Ba^2b^8) + \dots}{\dots}$$

## 5.6 Problem number 291

$$\int \frac{x^6}{(a + bx)^2(c + dx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(2ad + 3bc)x}{b^3d^4} + \frac{x^2}{2b^2d^3} - \frac{a^6}{b^4(-ad + bc)^3(bx + a)} - \frac{c^6}{2d^5(-ad + bc)^2(dx + c)^2} \\ & + \frac{2c^5(-3ad + 2bc)}{d^5(-ad + bc)^3(dx + c)} - \frac{3a^5(-ad + 2bc)\ln(bx + a)}{b^4(-ad + bc)^4} + \frac{3c^4(5a^2d^2 - 6abcd + 2b^2c^2)\ln(dx + c)}{d^5(-ad + bc)^4} \end{aligned}$$

command

```
integrate(x**6/(b*x+a)**2/(d*x+c)**3, x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
 & 3a^5(ad - 2bc) \log \left( x + \frac{3a^{10}d^9(ad-2bc)}{b(ad-bc)^4} - \frac{15a^9cd^8(ad-2bc)}{(ad-bc)^4} + \frac{30a^8bc^2d^7(ad-2bc)}{(ad-bc)^4} - \frac{30a^7b^2c^3d^6(ad-2bc)}{(ad-bc)^4} + \frac{15a^6b^3c^4d^5(ad-2bc)}{(ad-bc)^4} + 3a^6cd^5 - \frac{3a^5b^4c^5d^4}{(ad-bc)^4} \right) \\
 & + \frac{b^4(ad-bc)^4}{3a^6d^6 - 6a^5bcd^5 - 15a^4b^4c^4d^2 + 18ab^5c^5d - 6b^6c^6} \\
 & + \frac{3c^4(5a^2d^2 - 6abcd + 2b^2c^2) \log \left( x + \frac{3a^6cd^5 + \frac{3a^5b^3c^4d^4(5a^2d^2 - 6abcd + 2b^2c^2)}{(ad-bc)^4}}{(ad-bc)^4} - 6a^5bc^2d^4 - \frac{15a^4b^4c^5d^3(5a^2d^2 - 6abcd + 2b^2c^2)}{(ad-bc)^4} + \frac{30a^3b^5c^6d^2}{(ad-bc)^4} \right)}{d^5} \\
 & + x \left( -\frac{2a}{b^3d^3} - \frac{3c}{b^2d^4} \right) \\
 & + \frac{2a^6c^2d^5 + 11a^2b^4c^6d - 7ab^5c^7 + x^2(2a^6d^7 + 12ab^5c^5d^2 - 2a^4b^4c^2d^8 - 6a^3b^5c^3d^7 + 6a^2b^6c^4d^6 - 2ab^7c^5d^5 + x^3(2a^3b^5d^{10} - 6a^2b^6cd^9 + 6ab^7c^2d^8 - 2b^8c^3d^7) + x^2(2a^4b^4d^{10} - 2a^3b^5cd^9 + 2a^2b^6c^2d^8 - 2ab^7c^3d^7) + x(2a^4b^4d^9 - 2a^3b^5cd^8 + 2a^2b^6c^2d^7 - 2ab^7c^3d^6) + 2a^4b^4d^8 - 2a^3b^5cd^7 + 2a^2b^6c^2d^6 - 2ab^7c^3d^5 + 2a^4b^4d^7 - 2a^3b^5cd^6 + 2a^2b^6c^2d^5 - 2ab^7c^3d^4 + 2a^4b^4d^6 - 2a^3b^5cd^5 + 2a^2b^6c^2d^4 - 2ab^7c^3d^3 + 2a^4b^4d^5 - 2a^3b^5cd^4 + 2a^2b^6c^2d^3 - 2ab^7c^3d^2 + 2a^4b^4d^4 - 2a^3b^5cd^3 + 2a^2b^6c^2d^2 - 2ab^7c^3d + 2a^4b^4d^3 - 2a^3b^5cd^2 + 2a^2b^6c^2d - 2ab^7c^3}{2b^2d^3}
 \end{aligned}$$

## 5.7 Problem number 292

$$\int \frac{x^5}{(a+bx)^2(c+dx)^3} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{x}{b^2d^3} + \frac{a^5}{b^3(-ad+bc)^3(bx+a)} + \frac{c^5}{2d^4(-ad+bc)^2(dx+c)^2} - \frac{c^4(-5ad+3bc)}{d^4(-ad+bc)^3(dx+c)} \\
 & + \frac{a^4(-2ad+5bc)\ln(bx+a)}{b^3(-ad+bc)^4} - \frac{c^3(10a^2d^2-10abcd+3b^2c^2)\ln(dx+c)}{d^4(-ad+bc)^4}
 \end{aligned}$$

command

```
integrate(x**5/(b*x+a)**2/(d*x+c)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& a^4(2ad - 5bc) \log \left( x + \frac{\frac{a^9 d^8 (2ad - 5bc)}{b(ad - bc)^4} - \frac{5a^8 cd^7 (2ad - 5bc)}{(ad - bc)^4} + \frac{10a^7 bc^2 d^6 (2ad - 5bc)}{(ad - bc)^4} - \frac{10a^6 b^2 c^3 d^5 (2ad - 5bc)}{(ad - bc)^4} + \frac{5a^5 b^3 c^4 d^4 (2ad - 5bc)}{(ad - bc)^4} + 2a^5 cd^4 - \frac{a^4 b^4 c^5}{(ad - bc)^4} \right) \\
& - \frac{b^3 (ad - bc)^4}{c^3 (10a^2 d^2 - 10abcd + 3b^2 c^2) \log \left( x + \frac{\frac{a^5 b^2 c^3 d^4 (10a^2 d^2 - 10abcd + 3b^2 c^2)}{(ad - bc)^4} + 2a^5 cd^4 - \frac{5a^4 b^3 c^4 d^3 (10a^2 d^2 - 10abcd + 3b^2 c^2)}{(ad - bc)^4} - 5a^4 bc^2 d^3 + \frac{10a^3 b^4 c^3 d^2}{(ad - bc)^4} \right)} \\
& + \frac{-2a^5 c^2 d^4 - 9a^2 b^3 c^5 d + 5ab^4 c^6 + x^2 (-2a^5 d^6 - 10ab^4 c^4 d^2 + 2a^4 b^3 c^2 d^7 - 6a^3 b^4 c^3 d^6 + 6a^2 b^5 c^4 d^5 - 2ab^6 c^5 d^4 + x^3 (2a^3 b^4 d^9 - 6a^2 b^5 cd^8 + 6ab^6 c^2 d^7 - 2b^7 c^3 d^6) + x^2 (2a^4 b^3 d^9 - 2a^3 b^4 cd^8 + 2a^2 b^5 c^2 d^7 - 2ab^6 c^3 d^6) + x (-2a^5 c^2 d^4 - 9a^2 b^3 c^5 d + 5ab^4 c^6) + x^2 (-2a^5 d^6 - 10ab^4 c^4 d^2 + 2a^4 b^3 c^2 d^7 - 6a^3 b^4 c^3 d^6 + 6a^2 b^5 c^4 d^5 - 2ab^6 c^5 d^4 + x^3 (2a^3 b^4 d^9 - 6a^2 b^5 cd^8 + 6ab^6 c^2 d^7 - 2b^7 c^3 d^6) + x^2 (2a^4 b^3 d^9 - 2a^3 b^4 cd^8 + 2a^2 b^5 c^2 d^7 - 2ab^6 c^3 d^6) + x (-2a^5 c^2 d^4 - 9a^2 b^3 c^5 d + 5ab^4 c^6) + x^2 (-2a^5 d^6 - 10ab^4 c^4 d^2 + 2a^4 b^3 c^2 d^7 - 6a^3 b^4 c^3 d^6 + 6a^2 b^5 c^4 d^5 - 2ab^6 c^5 d^4)}{b^2 d^3}
\end{aligned}$$

## 5.8 Problem number 293

$$\int \frac{x^4}{(a + bx)^2 (c + dx)^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{a^4}{b^2 (-ad + bc)^3 (bx + a)} - \frac{c^4}{2d^3 (-ad + bc)^2 (dx + c)^2} + \frac{2c^3 (-2ad + bc)}{d^3 (-ad + bc)^3 (dx + c)} \\
& - \frac{a^3 (-ad + 4bc) \ln(bx + a)}{b^2 (-ad + bc)^4} + \frac{c^2 (6a^2 d^2 - 4abcd + b^2 c^2) \ln(dx + c)}{d^3 (-ad + bc)^4}
\end{aligned}$$

command

```
integrate(x**4/(b*x+a)**2/(d*x+c)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& a^3(ad - 4bc) \log \left( x + \frac{\frac{a^8 d^7 (ad - 4bc)}{b(ad - bc)^4} - \frac{5a^7 cd^6 (ad - 4bc)}{(ad - bc)^4} + \frac{10a^6 bc^2 d^5 (ad - 4bc)}{(ad - bc)^4} - \frac{10a^5 b^2 c^3 d^4 (ad - 4bc)}{(ad - bc)^4} + \frac{5a^4 b^3 c^4 d^3 (ad - 4bc)}{(ad - bc)^4} + a^4 cd^3 - \frac{a^3 b^4 c^5 d^2 (ad - 4bc)}{(ad - bc)^4} \right) \\
& - \frac{b^2 (ad - bc)^4}{c^2 (6a^2 d^2 - 4abcd + b^2 c^2) \log \left( x + \frac{\frac{a^5 bc^2 d^4 (6a^2 d^2 - 4abcd + b^2 c^2)}{(ad - bc)^4} - \frac{5a^4 b^2 c^3 d^3 (6a^2 d^2 - 4abcd + b^2 c^2)}{(ad - bc)^4} + a^4 cd^3 + \frac{10a^3 b^3 c^4 d^2 (6a^2 d^2 - 4abcd + b^2 c^2)}{(ad - bc)^4} \right)} \\
& + \frac{d^3 (ad - bc)^4}{2a^4 b^2 c^2 d^6 - 6a^3 b^3 c^3 d^5 + 6a^2 b^4 c^4 d^4 - 2ab^5 c^5 d^3 + x^3 (2a^3 b^3 d^8 - 6a^2 b^4 cd^7 + 6ab^5 c^2 d^6 - 2b^6 c^3 d^5) + x^2 (2a^4 b^2 d^8 - 2a^3 b^3 cd^7 + 2a^2 b^4 c^2 d^6 - 2ab^5 c^3 d^5) + x (-2a^5 c^2 d^4 - 9a^2 b^3 c^5 d + 5ab^4 c^6) + x^2 (-2a^5 d^6 - 10ab^4 c^4 d^2 + 2a^4 b^3 c^2 d^7 - 6a^3 b^4 c^3 d^6 + 6a^2 b^5 c^4 d^5 - 2ab^6 c^5 d^4)}{b^2 d^3}
\end{aligned}$$



### 5.9 Problem number 490

$$\int x^{5/2}(a+bx)^{3/2}(A+Bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(12Ab - 7aB)x^{7/2}(bx+a)^{3/2}}{60b} + \frac{Bx^{7/2}(bx+a)^{5/2}}{6b} - \frac{a^5(12Ab - 7aB) \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx+a}}\right)}{512b^{9/2}} \\ & - \frac{a^3(12Ab - 7aB)x^{3/2}\sqrt{bx+a}}{768b^3} + \frac{a^2(12Ab - 7aB)x^{5/2}\sqrt{bx+a}}{960b^2} \\ & + \frac{a(12Ab - 7aB)x^{7/2}\sqrt{bx+a}}{160b} + \frac{a^4(12Ab - 7aB)\sqrt{x}\sqrt{bx+a}}{512b^4} \end{aligned}$$

command

```
integrate(x**(5/2)*(b*x+a)**(3/2)*(B*x+A), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

### 5.10 Problem number 502

$$\int x^{3/2}(a+bx)^{5/2}(A+Bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a(12Ab - 5aB)x^{5/2}(bx+a)^{3/2}}{96b} + \frac{(12Ab - 5aB)x^{5/2}(bx+a)^{5/2}}{60b} + \frac{Bx^{5/2}(bx+a)^{7/2}}{6b} \\ & + \frac{a^5(12Ab - 5aB) \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx+a}}\right)}{512b^{7/2}} + \frac{a^3(12Ab - 5aB)x^{3/2}\sqrt{bx+a}}{768b^2} \\ & + \frac{a^2(12Ab - 5aB)x^{5/2}\sqrt{bx+a}}{192b} - \frac{a^4(12Ab - 5aB)\sqrt{x}\sqrt{bx+a}}{512b^3} \end{aligned}$$

command

```
integrate(x**(3/2)*(b*x+a)**(5/2)*(B*x+A), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

## 5.11 Problem number 514

$$\int \frac{x^{7/2}(A+Bx)}{\sqrt{a+bx}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7a^4(10Ab - 9aB) \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx+a}}\right)}{128b^{\frac{11}{2}}} + \frac{7a^2(10Ab - 9aB)x^{\frac{3}{2}}\sqrt{bx+a}}{192b^4} \\ & - \frac{7a(10Ab - 9aB)x^{\frac{5}{2}}\sqrt{bx+a}}{240b^3} + \frac{(10Ab - 9aB)x^{\frac{7}{2}}\sqrt{bx+a}}{40b^2} \\ & + \frac{Bx^{\frac{9}{2}}\sqrt{bx+a}}{5b} - \frac{7a^3(10Ab - 9aB)\sqrt{x}\sqrt{bx+a}}{128b^5} \end{aligned}$$

command

`integrate(x**(7/2)*(B*x+A)/(b*x+a)**(1/2),x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & -\frac{35Aa^{\frac{7}{2}}\sqrt{x}}{64b^4\sqrt{1+\frac{bx}{a}}} - \frac{35Aa^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^3\sqrt{1+\frac{bx}{a}}} + \frac{7Aa^{\frac{3}{2}}x^{\frac{5}{2}}}{96b^2\sqrt{1+\frac{bx}{a}}} - \frac{A\sqrt{a}x^{\frac{7}{2}}}{24b\sqrt{1+\frac{bx}{a}}} \\ & + \frac{35Aa^4 \operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{9}{2}}} + \frac{Ax^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1+\frac{bx}{a}}} + \frac{63Ba^{\frac{9}{2}}\sqrt{x}}{128b^5\sqrt{1+\frac{bx}{a}}} \\ & + \frac{21Ba^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^4\sqrt{1+\frac{bx}{a}}} - \frac{21Ba^{\frac{5}{2}}x^{\frac{5}{2}}}{320b^3\sqrt{1+\frac{bx}{a}}} + \frac{3Ba^{\frac{3}{2}}x^{\frac{7}{2}}}{80b^2\sqrt{1+\frac{bx}{a}}} \\ & - \frac{B\sqrt{a}x^{\frac{9}{2}}}{40b\sqrt{1+\frac{bx}{a}}} - \frac{63Ba^5 \operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{11}{2}}} + \frac{Bx^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1+\frac{bx}{a}}} \end{aligned}$$

### 5.12 Problem number 836

$$\int \frac{x^2}{\sqrt{-1+x} \sqrt{1+x}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arccosh}(x)}{2} + \frac{x\sqrt{-1+x} \sqrt{1+x}}{2}$$

command

`integrate(x**2/(-1+x)**(1/2)/(1+x)**(1/2), x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{G_{6,6}^{6,2} \left( \begin{matrix} -\frac{3}{4}, -\frac{1}{4} & -\frac{1}{2}, -\frac{1}{2}, 0, 1 \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{matrix} \middle| \frac{1}{x^2} \right)}{4\pi^{\frac{3}{2}}} - \frac{iG_{6,6}^{2,6} \left( \begin{matrix} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} & -\frac{3}{2}, -1, -1, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{x^2} \right)}{4\pi^{\frac{3}{2}}}$$

### 5.13 Problem number 850

$$\int x^3(2+3x)^{3/2}\sqrt{1+4x} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(4103 - 7968x)(2+3x)^{\frac{5}{2}}(1+4x)^{\frac{3}{2}}}{829440} + \frac{x^2(2+3x)^{\frac{5}{2}}(1+4x)^{\frac{3}{2}}}{72} \\ & + \frac{1067875 \operatorname{arcsinh}\left(\frac{\sqrt{15}\sqrt{1+4x}}{5}\right)\sqrt{3}}{254803968} + \frac{42715(2+3x)^{\frac{3}{2}}\sqrt{1+4x}}{15925248} \\ & - \frac{8543(2+3x)^{\frac{5}{2}}\sqrt{1+4x}}{995328} + \frac{213575\sqrt{2+3x}\sqrt{1+4x}}{42467328} \end{aligned}$$

command

`integrate(x**3*(2+3*x)**(3/2)*(1+4*x)**(1/2), x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{3(4x+1)^{\frac{13}{2}}}{4096\sqrt{12x+8}} + \frac{7(4x+1)^{\frac{11}{2}}}{40960\sqrt{12x+8}} - \frac{869(4x+1)^{\frac{9}{2}}}{196608\sqrt{12x+8}} \\ & + \frac{2027(4x+1)^{\frac{7}{2}}}{1179648\sqrt{12x+8}} + \frac{119135(4x+1)^{\frac{5}{2}}}{14155776\sqrt{12x+8}} - \frac{904775(4x+1)^{\frac{3}{2}}}{84934656\sqrt{12x+8}} \\ & - \frac{1067875\sqrt{4x+1}}{84934656\sqrt{12x+8}} + \frac{1067875\sqrt{3} \operatorname{asinh}\left(\frac{\sqrt{15}\sqrt{4x+1}}{5}\right)}{254803968} \end{aligned}$$

### 5.14 Problem number 908

$$\int \frac{(ex)^{3/2}}{\sqrt[4]{1-x}\sqrt[4]{1+x}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{e^{\frac{3}{2}} \arctan\left(1 - \frac{\sqrt{2}\sqrt{ex}}{(-x^2+1)^{\frac{1}{4}}\sqrt{e}}\right) \sqrt{2}}{8} + \frac{e^{\frac{3}{2}} \arctan\left(1 + \frac{\sqrt{2}\sqrt{ex}}{(-x^2+1)^{\frac{1}{4}}\sqrt{e}}\right) \sqrt{2}}{8} \\ & - \frac{e^{\frac{3}{2}} \ln\left(\sqrt{e} - \frac{\sqrt{2}\sqrt{ex}}{(-x^2+1)^{\frac{1}{4}}} + \frac{x\sqrt{e}}{\sqrt{-x^2+1}}\right) \sqrt{2}}{16} \\ & + \frac{e^{\frac{3}{2}} \ln\left(\sqrt{e} + \frac{\sqrt{2}\sqrt{ex}}{(-x^2+1)^{\frac{1}{4}}} + \frac{x\sqrt{e}}{\sqrt{-x^2+1}}\right) \sqrt{2}}{16} - \frac{e(-x^2+1)^{\frac{3}{4}}\sqrt{ex}}{2} \end{aligned}$$

command

`integrate((e*x)**(3/2)/(1-x)**(1/4)/(1+x)**(1/4), x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{ie^{\frac{3}{2}} G_{6,6}^{6,2} \left( \begin{matrix} -\frac{5}{8}, -\frac{1}{8} & -\frac{1}{2}, -\frac{1}{4}, 0, 1 \\ -1, -\frac{5}{8}, -\frac{1}{2}, -\frac{1}{8}, 0, 0 \end{matrix} \middle| \frac{e^{-2i\pi}}{x^2} \right) e^{\frac{i\pi}{4}}}{4\pi\Gamma\left(\frac{1}{4}\right)} \\ & - \frac{e^{\frac{3}{2}} G_{6,6}^{2,6} \left( \begin{matrix} -\frac{5}{4}, -\frac{9}{8}, -\frac{3}{4}, -\frac{5}{8}, -\frac{1}{4}, 1 \\ -\frac{9}{8}, -\frac{5}{8} & -\frac{5}{4}, -1, -\frac{3}{4}, 0 \end{matrix} \middle| \frac{1}{x^2} \right)}{4\pi\Gamma\left(\frac{1}{4}\right)} \end{aligned}$$

### 5.15 Problem number 1034

$$\int \frac{(a + bx)^2(A + Bx)}{(d + ex)^8} dx$$

Optimal antiderivative

$$\frac{(-ae + bd)^2(-Ae + Bd)}{7e^4(ex + d)^7} - \frac{(-ae + bd)(-2Abe - Bae + 3Bbd)}{6e^4(ex + d)^6} + \frac{b(-Abe - 2Bae + 3Bbd)}{5e^4(ex + d)^5} - \frac{b^2B}{4e^4(ex + d)^4}$$

command

```
integrate((b*x+a)**2*(B*x+A)/(e*x+d)**8,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-60Aa^2e^3 - 20Aabde^2 - 4Ab^2d^2e - 10Ba^2de^2 - 8Babd^2e - 3Bb^2d^3 - 105Bb^2e^3x^3 + x^2(-84Ab^2e^3 - 168Babe^3 - 88Bbd^2e^3 - 168Bbd^2e^3x - 88Bbd^2e^3x^2 - 88Bbd^2e^3x^3 - 88Bbd^2e^3x^4 - 88Bbd^2e^3x^5 - 88Bbd^2e^3x^6 - 88Bbd^2e^3x^7 - 88Bbd^2e^3x^8 - 88Bbd^2e^3x^9 - 88Bbd^2e^3x^{10})}{420d^7e^4 + 2940d^6e^5x + 8820d^5e^6x^2 + 14700d^4e^7x^3 + 14700d^3e^8x^4 + 8820d^2e^9x^5 + 8820d^2e^9x^6 + 8820d^2e^9x^7 + 8820d^2e^9x^8 + 8820d^2e^9x^9 + 8820d^2e^9x^{10}}$$

### 5.16 Problem number 1046

$$\int \frac{(a + bx)^3(A + Bx)}{(d + ex)^6} dx$$

Optimal antiderivative

$$-\frac{(-Ae + Bd)(bx + a)^4}{5e(-ae + bd)(ex + d)^5} + \frac{(Abe - 5Bae + 4Bbd)(bx + a)^4}{20e(-ae + bd)^2(ex + d)^4}$$

command

```
integrate((b*x+a)**3*(B*x+A)/(e*x+d)**6,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-4Aa^3e^4 - 3Aa^2bde^3 - 2Aab^2d^2e^2 - Ab^3d^3e - Ba^3de^3 - 2Ba^2bd^2e^2 - 3Bab^2d^3e - 4Bb^3d^4 - 20Bb^3e^4x^4 + x^3(-10Bb^3e^4x - 10Bb^3e^4x^2 - 10Bb^3e^4x^3 - 10Bb^3e^4x^4 - 10Bb^3e^4x^5 - 10Bb^3e^4x^6 - 10Bb^3e^4x^7 - 10Bb^3e^4x^8 - 10Bb^3e^4x^9 - 10Bb^3e^4x^{10})}{100d^5e^4 + 300d^4e^5x + 300d^3e^6x^2 + 150d^2e^7x^3 + 150d^2e^7x^4 + 150d^2e^7x^5 + 150d^2e^7x^6 + 150d^2e^7x^7 + 150d^2e^7x^8 + 150d^2e^7x^9 + 150d^2e^7x^{10}}$$

### 5.17 Problem number 1047

$$\int \frac{(a + bx)^3(A + Bx)}{(d + ex)^7} dx$$

Optimal antiderivative

$$-\frac{(-Ae + Bd)(bx + a)^4}{6e(-ae + bd)(ex + d)^6} + \frac{(Abe - 3Bae + 2Bbd)(bx + a)^4}{15e(-ae + bd)^2(ex + d)^5} + \frac{b(Abe - 3Bae + 2Bbd)(bx + a)^4}{60e(-ae + bd)^3(ex + d)^4}$$

command

```
integrate((b*x+a)**3*(B*x+A)/(e*x+d)**7, x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-10Aa^3e^4 - 6Aa^2bde^3 - 3Aab^2d^2e^2 - Ab^3d^3e - 2Ba^3de^3 - 3Ba^2bd^2e^2 - 3Bab^2d^3e - 2Bb^3d^4 - 30Bb^3e^4x^4 + x^3(-$$

### 5.18 Problem number 1063

$$\int \frac{(a + bx)^6(A + Bx)}{(d + ex)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5b^3(-ae + bd)^2(-3Abe - 4Bae + 7Bbd)x}{e^7} + \frac{(-ae + bd)^6(-Ae + Bd)}{3e^8(ex + d)^3} \\ & -\frac{(-ae + bd)^5(-6Abe - Bae + 7Bbd)}{2e^8(ex + d)^2} + \frac{3b(-ae + bd)^4(-5Abe - 2Bae + 7Bbd)}{e^8(ex + d)} \\ & + \frac{3b^4(-ae + bd)(-2Abe - 5Bae + 7Bbd)(ex + d)^2}{2e^8} - \frac{b^5(-Abe - 6Bae + 7Bbd)(ex + d)^3}{3e^8} \\ & + \frac{b^6B(ex + d)^4}{4e^8} + \frac{5b^2(-ae + bd)^3(-4Abe - 3Bae + 7Bbd)\ln(ex + d)}{e^8} \end{aligned}$$

command

```
integrate((b*x+a)**6*(B*x+A)/(e*x+d)**4, x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{Bb^6x^4}{4e^4} + \frac{5b^2(ae - bd)^3(4Abe + 3Bae - 7Bbd) \log(d + ex)}{e^8} \\ & + x^3 \left( \frac{Ab^6}{3e^4} + \frac{2Bab^5}{e^4} - \frac{4Bb^6d}{3e^5} \right) + x^2 \left( \frac{3Aab^5}{e^4} - \frac{2Ab^6d}{e^5} + \frac{15Ba^2b^4}{2e^4} - \frac{12Bab^5d}{e^5} + \frac{5Bb^6d^2}{e^6} \right) \\ & + x \left( \frac{15Aa^2b^4}{e^4} - \frac{24Aab^5d}{e^5} + \frac{10Ab^6d^2}{e^6} + \frac{20Ba^3b^3}{e^4} - \frac{60Ba^2b^4d}{e^5} + \frac{60Bab^5d^2}{e^6} - \frac{20Bb^6d^3}{e^7} \right) \\ & + \frac{-2Aa^6e^7 - 6Aa^5bde^6 - 30Aa^4b^2d^2e^5 + 220Aa^3b^3d^3e^4 - 390Aa^2b^4d^4e^3 + 282Aab^5d^5e^2 - 74Ab^6d^6e - Ba^6de^6 - 1}{e^8} \end{aligned}$$

## 6 Test file number 15

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.1\_Linear/15\_1.1.1.4-a+b\_x-  
^m-c+d\_x-^n-e+f\_x-^p-g+h\_x-^q

### 6.1 Problem number 149

$$\int \frac{x(a + bx + cx^2)}{\sqrt{1 - dx} \sqrt{1 + dx}} dx$$

Optimal antiderivative

$$\frac{b \arcsin(dx)}{2d^3} - \frac{cx^2 \sqrt{-d^2x^2 + 1}}{3d^2} - \frac{(3bd^2x + 6ad^2 + 4c) \sqrt{-d^2x^2 + 1}}{6d^4}$$

command

```
integrate(x*(c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{iaG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{1}{4}, \frac{1}{4} \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^2} \\
& - \frac{aG_{6,6}^{2,6} \left( \begin{array}{c} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 \\ -\frac{3}{4}, -\frac{1}{4} \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^2} \\
& - \frac{ibG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{3}{4}, -\frac{1}{4} \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3} \\
& + \frac{bG_{6,6}^{2,6} \left( \begin{array}{c} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3} \\
& - \frac{icG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{5}{4}, -\frac{3}{4} \\ -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^4} \\
& - \frac{cG_{6,6}^{2,6} \left( \begin{array}{c} -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, 1 \\ -\frac{7}{4}, -\frac{5}{4} \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^4}
\end{aligned}$$

## 6.2 Problem number 150

$$\int \frac{a + bx + cx^2}{\sqrt{1 - dx} \sqrt{1 + dx}} dx$$

Optimal antiderivative

$$\frac{(2a d^2 + c) \arcsin(dx)}{2d^3} - \frac{b\sqrt{-d^2 x^2 + 1}}{d^2} - \frac{cx\sqrt{-d^2 x^2 + 1}}{2d^2}$$

command

```
integrate((c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output



$$\begin{aligned}
& \frac{iaG_{6,6}^{6,2} \left( \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d} + \frac{aG_{6,6}^{2,6} \left( \begin{array}{c} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 \\ -\frac{1}{4}, \frac{1}{4} \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d} \\
& - \frac{ibG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{1}{4}, \frac{1}{4} \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^2} \\
& - \frac{bG_{6,6}^{2,6} \left( \begin{array}{c} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 \\ -\frac{3}{4}, -\frac{1}{4} \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^2} \\
& - \frac{icG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{3}{4}, -\frac{1}{4} \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3} \\
& + \frac{cG_{6,6}^{2,6} \left( \begin{array}{c} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3}
\end{aligned}$$

### 6.3 Problem number 153

$$\int \frac{a + bx + cx^2}{x^3 \sqrt{1 - dx} \sqrt{1 + dx}} dx$$

Optimal antiderivative

$$\frac{(a d^2 + 2c) \operatorname{arctanh}(\sqrt{-d^2 x^2 + 1})}{2} - \frac{a \sqrt{-d^2 x^2 + 1}}{2x^2} - \frac{b \sqrt{-d^2 x^2 + 1}}{x}$$

command

```
integrate((c*x**2+b*x+a)/x**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{iad^2 G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{7}{4}, \frac{9}{4}, 1 & 2, 2, \frac{5}{2} \\ \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2} & 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}} - \frac{ad^2 G_{6,6}^{2,6} \left( \begin{array}{c|c} 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 1 & \\ \frac{5}{4}, \frac{7}{4} & 1, \frac{3}{2}, \frac{3}{2}, 0 \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}} \\
& + \frac{ibd G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{5}{4}, \frac{7}{4}, 1 & \frac{3}{2}, \frac{3}{2}, 2 \\ 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2 & 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}} + \frac{bd G_{6,6}^{2,6} \left( \begin{array}{c|c} \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 1 & \\ \frac{3}{4}, \frac{5}{4} & \frac{1}{2}, 1, 1, 0 \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}} \\
& + \frac{ic G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{3}{4}, \frac{5}{4}, 1 & 1, 1, \frac{3}{2} \\ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2} & 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}} - \frac{c G_{6,6}^{2,6} \left( \begin{array}{c|c} 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1 & \\ \frac{1}{4}, \frac{3}{4} & 0, \frac{1}{2}, \frac{1}{2}, 0 \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}}
\end{aligned}$$

#### 6.4 Problem number 154

$$\int \frac{x(a + bx + cx^2)}{\sqrt{-1 + dx} \sqrt{1 + dx}} dx$$

Optimal antiderivative

$$\frac{b \operatorname{arccosh}(dx)}{2d^3} + \frac{c x^2 \sqrt{dx - 1} \sqrt{dx + 1}}{3d^2} + \frac{(3b d^2 x + 6a d^2 + 4c) \sqrt{dx - 1} \sqrt{dx + 1}}{6d^4}$$

command

`integrate(x*(c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2), x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{aG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{1}{4}, \frac{1}{4} \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 \end{array} \middle| \begin{array}{c} 0, 0, \frac{1}{2}, 1 \\ \frac{1}{d^2 x^2} \end{array} \right)}{4\pi^{\frac{3}{2}} d^2} \\
& + \frac{iaG_{6,6}^{2,6} \left( \begin{array}{c} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 \\ -\frac{3}{4}, -\frac{1}{4} \end{array} \middle| \begin{array}{c} -1, -\frac{1}{2}, -\frac{1}{2}, 0 \\ \frac{e^{2i\pi}}{d^2 x^2} \end{array} \right)}{4\pi^{\frac{3}{2}} d^2} \\
& + \frac{bG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{3}{4}, -\frac{1}{4} \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{array} \middle| \begin{array}{c} -\frac{1}{2}, -\frac{1}{2}, 0, 1 \\ \frac{1}{d^2 x^2} \end{array} \right)}{4\pi^{\frac{3}{2}} d^3} \\
& - \frac{ibG_{6,6}^{2,6} \left( \begin{array}{c} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \end{array} \middle| \begin{array}{c} -\frac{3}{2}, -1, -1, 0 \\ \frac{e^{2i\pi}}{d^2 x^2} \end{array} \right)}{4\pi^{\frac{3}{2}} d^3} \\
& + \frac{cG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{5}{4}, -\frac{3}{4} \\ -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 0 \end{array} \middle| \begin{array}{c} -1, -1, -\frac{1}{2}, 1 \\ \frac{1}{d^2 x^2} \end{array} \right)}{4\pi^{\frac{3}{2}} d^4} \\
& + \frac{icG_{6,6}^{2,6} \left( \begin{array}{c} -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, 1 \\ -\frac{7}{4}, -\frac{5}{4} \end{array} \middle| \begin{array}{c} -2, -\frac{3}{2}, -\frac{3}{2}, 0 \\ \frac{e^{2i\pi}}{d^2 x^2} \end{array} \right)}{4\pi^{\frac{3}{2}} d^4}
\end{aligned}$$

## 6.5 Problem number 155

$$\int \frac{a + bx + cx^2}{\sqrt{-1 + dx} \sqrt{1 + dx}} dx$$

Optimal antiderivative

$$\frac{(2a d^2 + c) \operatorname{arccosh}(dx)}{2d^3} + \frac{(cx + 2b) \sqrt{dx - 1} \sqrt{dx + 1}}{2d^2}$$

command

```
integrate((c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{aG_{6,6}^{6,2} \left( \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right) - iaG_{6,6}^{2,6} \left( \begin{array}{c} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 \\ -\frac{1}{4}, \frac{1}{4} \end{array} \middle| \frac{e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d} \\
& + \frac{bG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{1}{4}, \frac{1}{4} \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^2} \\
& + \frac{ibG_{6,6}^{2,6} \left( \begin{array}{c} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 \\ -\frac{3}{4}, -\frac{1}{4} \end{array} \middle| \frac{e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^2} \\
& + \frac{cG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{3}{4}, -\frac{1}{4} \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3} \\
& - \frac{icG_{6,6}^{2,6} \left( \begin{array}{c} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \end{array} \middle| \frac{e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3}
\end{aligned}$$

## 6.6 Problem number 158

$$\int \frac{a + bx + cx^2}{x^3 \sqrt{-1 + dx} \sqrt{1 + dx}} dx$$

Optimal antiderivative

$$\frac{(a d^2 + 2c) \arctan\left(\sqrt{dx - 1} \sqrt{dx + 1}\right)}{2} + \frac{a \sqrt{dx - 1} \sqrt{dx + 1}}{2x^2} + \frac{b \sqrt{dx - 1} \sqrt{dx + 1}}{x}$$

command

```
integrate((c*x**2+b*x+a)/x**3/(d*x-1)**(1/2)/(d*x+1)**(1/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{array}{r}
\frac{ad^2 G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{7}{4}, \frac{9}{4}, 1 & 2, 2, \frac{5}{2} \\ \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2} & 0 \end{array} \left| \frac{1}{d^2 x^2} \right. \right)}{4\pi^{\frac{3}{2}}} + \frac{iad^2 G_{6,6}^{2,6} \left( \begin{array}{c|c} 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 1 & \\ \frac{5}{4}, \frac{7}{4} & 1, \frac{3}{2}, \frac{3}{2}, 0 \end{array} \left| \frac{e^{2i\pi}}{d^2 x^2} \right. \right)}{4\pi^{\frac{3}{2}}} \\
\frac{bd G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{5}{4}, \frac{7}{4}, 1 & \frac{3}{2}, \frac{3}{2}, 2 \\ 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2 & 0 \end{array} \left| \frac{1}{d^2 x^2} \right. \right)}{4\pi^{\frac{3}{2}}} - \frac{ibd G_{6,6}^{2,6} \left( \begin{array}{c|c} \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 1 & \\ \frac{3}{4}, \frac{5}{4} & \frac{1}{2}, 1, 1, 0 \end{array} \left| \frac{e^{2i\pi}}{d^2 x^2} \right. \right)}{4\pi^{\frac{3}{2}}} \\
\frac{cG_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{3}{4}, \frac{5}{4}, 1 & 1, 1, \frac{3}{2} \\ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2} & 0 \end{array} \left| \frac{1}{d^2 x^2} \right. \right)}{4\pi^{\frac{3}{2}}} + \frac{icG_{6,6}^{2,6} \left( \begin{array}{c|c} 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1 & \\ \frac{1}{4}, \frac{3}{4} & 0, \frac{1}{2}, \frac{1}{2}, 0 \end{array} \left| \frac{e^{2i\pi}}{d^2 x^2} \right. \right)}{4\pi^{\frac{3}{2}}}
\end{array}$$

## 6.7 Problem number 159

$$\int \frac{a + bx + cx^2}{x^4 \sqrt{-1 + dx} \sqrt{1 + dx}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{bd^2 \arctan\left(\sqrt{dx-1} \sqrt{dx+1}\right)}{2} + \frac{a\sqrt{dx-1} \sqrt{dx+1}}{3x^3} \\
& + \frac{b\sqrt{dx-1} \sqrt{dx+1}}{2x^2} + \frac{(2ad^2 + 3c)\sqrt{dx-1} \sqrt{dx+1}}{3x}
\end{aligned}$$

command

`integrate((c*x**2+b*x+a)/x**4/(d*x-1)**(1/2)/(d*x+1)**(1/2), x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{array}{r}
\frac{ad^3 G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{9}{4}, \frac{11}{4}, 1 & \frac{5}{2}, \frac{5}{2}, 3 \\ 2, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}, 3 & 0 \end{array} \left| \frac{1}{d^2 x^2} \right. \right)}{4\pi^{\frac{3}{2}}} - \frac{iad^3 G_{6,6}^{2,6} \left( \begin{array}{c|c} \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2}, 1 & \\ \frac{7}{4}, \frac{9}{4} & \frac{3}{2}, 2, 2, 0 \end{array} \left| \frac{e^{2i\pi}}{d^2 x^2} \right. \right)}{4\pi^{\frac{3}{2}}} \\
\frac{bd^2 G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{7}{4}, \frac{9}{4}, 1 & 2, 2, \frac{5}{2} \\ \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2} & 0 \end{array} \left| \frac{1}{d^2 x^2} \right. \right)}{4\pi^{\frac{3}{2}}} + \frac{ibd^2 G_{6,6}^{2,6} \left( \begin{array}{c|c} 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 1 & \\ \frac{5}{4}, \frac{7}{4} & 1, \frac{3}{2}, \frac{3}{2}, 0 \end{array} \left| \frac{e^{2i\pi}}{d^2 x^2} \right. \right)}{4\pi^{\frac{3}{2}}} \\
\frac{cd G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{5}{4}, \frac{7}{4}, 1 & \frac{3}{2}, \frac{3}{2}, 2 \\ 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2 & 0 \end{array} \left| \frac{1}{d^2 x^2} \right. \right)}{4\pi^{\frac{3}{2}}} - \frac{icd G_{6,6}^{2,6} \left( \begin{array}{c|c} \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 1 & \\ \frac{3}{4}, \frac{5}{4} & \frac{1}{2}, 1, 1, 0 \end{array} \left| \frac{e^{2i\pi}}{d^2 x^2} \right. \right)}{4\pi^{\frac{3}{2}}}
\end{array}$$

## 7 Test file number 17

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.1\_Linear/17\_1.1.1.6\_P-x-a+b\_x-<sup>m</sup>-c+d\_x-<sup>n</sup>-e+f\_x-<sup>p</sup>

### 7.1 Problem number 10

$$\int \frac{(e + fx)(A + Bx + Cx^2)}{\sqrt{1 - dx} \sqrt{1 + dx}} dx$$

Optimal antiderivative

$$\frac{(2Ad^2e + Bf + Ce) \arcsin(dx)}{2d^3} - \frac{C(fx + e)^2 \sqrt{-d^2x^2 + 1}}{3d^2f} - \frac{(6d^2f(Af + Be) - 2C(d^2e^2 - 2f^2) - d^2f(-3Bf + Ce)x) \sqrt{-d^2x^2 + 1}}{6d^4f}$$

command

```
integrate((f*x+e)*(C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

### 7.2 Problem number 11

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx} \sqrt{1 + dx}} dx$$

Optimal antiderivative

$$\frac{(2Ad^2 + C) \arcsin(dx)}{2d^3} - \frac{B \sqrt{-d^2x^2 + 1}}{d^2} - \frac{Cx \sqrt{-d^2x^2 + 1}}{2d^2}$$

command

```
integrate((C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{iAG_{6,6}^{6,2} \left( \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d} + \frac{AG_{6,6}^{2,6} \left( \begin{array}{c} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 \\ -\frac{1}{4}, \frac{1}{4} \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d} \\
& - \frac{iBG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{1}{4}, \frac{1}{4} \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^2} \\
& - \frac{BG_{6,6}^{2,6} \left( \begin{array}{c} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 \\ -\frac{3}{4}, -\frac{1}{4} \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^2} \\
& - \frac{iCG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{3}{4}, -\frac{1}{4} \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3} \\
& + \frac{CG_{6,6}^{2,6} \left( \begin{array}{c} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3}
\end{aligned}$$

**7.3 Problem number 15**

$$\int \frac{x(a + bx + cx^2)}{\sqrt{1 - dx} \sqrt{1 + dx}} dx$$

Optimal antiderivative

$$\frac{b \arcsin(dx)}{2d^3} - \frac{cx^2 \sqrt{-d^2 x^2 + 1}}{3d^2} - \frac{(3bd^2 x + 6ad^2 + 4c) \sqrt{-d^2 x^2 + 1}}{6d^4}$$

command

```
integrate(x*(c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{iaG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{1}{4}, \frac{1}{4} \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^2} \\
& - \frac{aG_{6,6}^{2,6} \left( \begin{array}{c} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 \\ -\frac{3}{4}, -\frac{1}{4} \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^2} \\
& - \frac{ibG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{3}{4}, -\frac{1}{4} \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3} \\
& + \frac{bG_{6,6}^{2,6} \left( \begin{array}{c} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3} \\
& - \frac{icG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{5}{4}, -\frac{3}{4} \\ -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^4} \\
& - \frac{cG_{6,6}^{2,6} \left( \begin{array}{c} -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, 1 \\ -\frac{7}{4}, -\frac{5}{4} \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^4}
\end{aligned}$$

#### 7.4 Problem number 16

$$\int \frac{a + bx + cx^2}{\sqrt{1 - dx} \sqrt{1 + dx}} dx$$

Optimal antiderivative

$$\frac{(2a d^2 + c) \arcsin(dx)}{2d^3} - \frac{b\sqrt{-d^2 x^2 + 1}}{d^2} - \frac{cx\sqrt{-d^2 x^2 + 1}}{2d^2}$$

command

```
integrate((c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output



$$\begin{aligned}
& \frac{iaG_{6,6}^{6,2} \left( \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d} + \frac{aG_{6,6}^{2,6} \left( \begin{array}{c} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 \\ -\frac{1}{4}, \frac{1}{4} \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d} \\
& - \frac{ibG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{1}{4}, \frac{1}{4} \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^2} \\
& - \frac{bG_{6,6}^{2,6} \left( \begin{array}{c} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 \\ -\frac{3}{4}, -\frac{1}{4} \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^2} \\
& - \frac{icG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{3}{4}, -\frac{1}{4} \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3} \\
& + \frac{cG_{6,6}^{2,6} \left( \begin{array}{c} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3}
\end{aligned}$$

## 7.5 Problem number 19

$$\int \frac{a + bx + cx^2}{x^3 \sqrt{1 - dx} \sqrt{1 + dx}} dx$$

Optimal antiderivative

$$-\frac{(a d^2 + 2c) \operatorname{arctanh}\left(\sqrt{-d^2 x^2 + 1}\right)}{2} - \frac{a \sqrt{-d^2 x^2 + 1}}{2x^2} - \frac{b \sqrt{-d^2 x^2 + 1}}{x}$$

command

```
integrate((c*x**2+b*x+a)/x**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{iad^2 G_{6,6}^{5,3} \left( \begin{array}{c} \frac{7}{4}, \frac{9}{4}, 1 \\ \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2} \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}} - \frac{ad^2 G_{6,6}^{2,6} \left( \begin{array}{c} 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 1 \\ \frac{5}{4}, \frac{7}{4} \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}} \\
& + \frac{ibd G_{6,6}^{5,3} \left( \begin{array}{c} \frac{5}{4}, \frac{7}{4}, 1 \\ 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}} + \frac{bd G_{6,6}^{2,6} \left( \begin{array}{c} \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 1 \\ \frac{3}{4}, \frac{5}{4} \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}} \\
& + \frac{ic G_{6,6}^{5,3} \left( \begin{array}{c} \frac{3}{4}, \frac{5}{4}, 1 \\ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2} \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}} - \frac{c G_{6,6}^{2,6} \left( \begin{array}{c} 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1 \\ \frac{1}{4}, \frac{3}{4} \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}}
\end{aligned}$$

## 7.6 Problem number 30

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx} \sqrt{ac - bcx}} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{B(-b^2x^2 + a^2)}{b^2\sqrt{bx+a}\sqrt{-bcx+ac}} - \frac{Cx(-b^2x^2 + a^2)}{2b^2\sqrt{bx+a}\sqrt{-bcx+ac}} \\
& + \frac{(2Ab^2 + a^2C) \arctan\left(\frac{bx\sqrt{c}}{\sqrt{-b^2cx^2 + a^2c}}\right)}{2b^3\sqrt{c}\sqrt{bx+a}\sqrt{-bcx+ac}}
\end{aligned}$$

command

```
integrate((C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{iAG_{6,6}^{6,2} \left( \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 \end{array} \middle| \frac{a^2}{b^2 x^2} \right)}{4\pi^{\frac{3}{2}} b \sqrt{c}} \\
& + \frac{AG_{6,6}^{2,6} \left( \begin{array}{c} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 \\ -\frac{1}{4}, \frac{1}{4} \end{array} \middle| \frac{a^2 e^{-2i\pi}}{b^2 x^2} \right)}{4\pi^{\frac{3}{2}} b \sqrt{c}} \\
& - \frac{iBaG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{1}{4}, \frac{1}{4} \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 \end{array} \middle| \frac{a^2}{b^2 x^2} \right)}{4\pi^{\frac{3}{2}} b^2 \sqrt{c}} \\
& - \frac{BaG_{6,6}^{2,6} \left( \begin{array}{c} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 \\ -\frac{3}{4}, -\frac{1}{4} \end{array} \middle| \frac{a^2 e^{-2i\pi}}{b^2 x^2} \right)}{4\pi^{\frac{3}{2}} b^2 \sqrt{c}} \\
& - \frac{iCa^2G_{6,6}^{6,2} \left( \begin{array}{c} -\frac{3}{4}, -\frac{1}{4} \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{array} \middle| \frac{a^2}{b^2 x^2} \right)}{4\pi^{\frac{3}{2}} b^3 \sqrt{c}} \\
& + \frac{Ca^2G_{6,6}^{2,6} \left( \begin{array}{c} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \end{array} \middle| \frac{a^2 e^{-2i\pi}}{b^2 x^2} \right)}{4\pi^{\frac{3}{2}} b^3 \sqrt{c}}
\end{aligned}$$

## 7.7 Problem number 34

$$\int \frac{x(a + bx + cx^2)}{\sqrt{-1 + dx} \sqrt{1 + dx}} dx$$

Optimal antiderivative

$$\frac{b \operatorname{arccosh}(dx)}{2d^3} + \frac{cx^2 \sqrt{dx-1} \sqrt{dx+1}}{3d^2} + \frac{(3bd^2x + 6ad^2 + 4c) \sqrt{dx-1} \sqrt{dx+1}}{6d^4}$$

command

`integrate(x*(c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2), x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{aG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{1}{4}, \frac{1}{4} \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 \end{array} \middle| \begin{array}{c} 0, 0, \frac{1}{2}, 1 \\ \frac{1}{d^2 x^2} \end{array} \right)}{4\pi^{\frac{3}{2}} d^2} \\
& + \frac{iaG_{6,6}^{2,6} \left( \begin{array}{c} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 \\ -\frac{3}{4}, -\frac{1}{4} \end{array} \middle| \begin{array}{c} -1, -\frac{1}{2}, -\frac{1}{2}, 0 \\ \frac{e^{2i\pi}}{d^2 x^2} \end{array} \right)}{4\pi^{\frac{3}{2}} d^2} \\
& + \frac{bG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{3}{4}, -\frac{1}{4} \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{array} \middle| \begin{array}{c} -\frac{1}{2}, -\frac{1}{2}, 0, 1 \\ \frac{1}{d^2 x^2} \end{array} \right)}{4\pi^{\frac{3}{2}} d^3} \\
& - \frac{ibG_{6,6}^{2,6} \left( \begin{array}{c} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \end{array} \middle| \begin{array}{c} -\frac{3}{2}, -1, -1, 0 \\ \frac{e^{2i\pi}}{d^2 x^2} \end{array} \right)}{4\pi^{\frac{3}{2}} d^3} \\
& + \frac{cG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{5}{4}, -\frac{3}{4} \\ -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 0 \end{array} \middle| \begin{array}{c} -1, -1, -\frac{1}{2}, 1 \\ \frac{1}{d^2 x^2} \end{array} \right)}{4\pi^{\frac{3}{2}} d^4} \\
& + \frac{icG_{6,6}^{2,6} \left( \begin{array}{c} -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, 1 \\ -\frac{7}{4}, -\frac{5}{4} \end{array} \middle| \begin{array}{c} -2, -\frac{3}{2}, -\frac{3}{2}, 0 \\ \frac{e^{2i\pi}}{d^2 x^2} \end{array} \right)}{4\pi^{\frac{3}{2}} d^4}
\end{aligned}$$

## 7.8 Problem number 35

$$\int \frac{a + bx + cx^2}{\sqrt{-1 + dx} \sqrt{1 + dx}} dx$$

Optimal antiderivative

$$\frac{(2a d^2 + c) \operatorname{arccosh}(dx)}{2d^3} + \frac{(cx + 2b) \sqrt{dx - 1} \sqrt{dx + 1}}{2d^2}$$

command

```
integrate((c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{aG_{6,6}^{6,2} \left( \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right) - iaG_{6,6}^{2,6} \left( \begin{array}{c} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 \\ -\frac{1}{4}, \frac{1}{4} \end{array} \middle| \frac{e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d} \\
& + \frac{bG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{1}{4}, \frac{1}{4} \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^2} \\
& + \frac{ibG_{6,6}^{2,6} \left( \begin{array}{c} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 \\ -\frac{3}{4}, -\frac{1}{4} \end{array} \middle| \frac{e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^2} \\
& + \frac{cG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{3}{4}, -\frac{1}{4} \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3} \\
& - \frac{icG_{6,6}^{2,6} \left( \begin{array}{c} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \end{array} \middle| \frac{e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3}
\end{aligned}$$

## 7.9 Problem number 38

$$\int \frac{a + bx + cx^2}{x^3 \sqrt{-1 + dx} \sqrt{1 + dx}} dx$$

Optimal antiderivative

$$\frac{(a d^2 + 2c) \arctan\left(\sqrt{dx - 1} \sqrt{dx + 1}\right)}{2} + \frac{a \sqrt{dx - 1} \sqrt{dx + 1}}{2x^2} + \frac{b \sqrt{dx - 1} \sqrt{dx + 1}}{x}$$

command

```
integrate((c*x**2+b*x+a)/x**3/(d*x-1)**(1/2)/(d*x+1)**(1/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{ad^2 G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{7}{4}, \frac{9}{4}, 1 & 2, 2, \frac{5}{2} \\ \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2} & 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}} + \frac{iad^2 G_{6,6}^{2,6} \left( \begin{array}{c|c} 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 1 & \\ \frac{5}{4}, \frac{7}{4} & 1, \frac{3}{2}, \frac{3}{2}, 0 \end{array} \middle| \frac{e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}} \\
& \frac{bd G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{5}{4}, \frac{7}{4}, 1 & \frac{3}{2}, \frac{3}{2}, 2 \\ 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2 & 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}} - \frac{ibd G_{6,6}^{2,6} \left( \begin{array}{c|c} \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 1 & \\ \frac{3}{4}, \frac{5}{4} & \frac{1}{2}, 1, 1, 0 \end{array} \middle| \frac{e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}} \\
& \frac{cG_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{3}{4}, \frac{5}{4}, 1 & 1, 1, \frac{3}{2} \\ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2} & 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}} + \frac{icG_{6,6}^{2,6} \left( \begin{array}{c|c} 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1 & \\ \frac{1}{4}, \frac{3}{4} & 0, \frac{1}{2}, \frac{1}{2}, 0 \end{array} \middle| \frac{e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}}
\end{aligned}$$

### 7.10 Problem number 39

$$\int \frac{a + bx + cx^2}{x^4 \sqrt{-1 + dx} \sqrt{1 + dx}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{bd^2 \arctan(\sqrt{dx-1} \sqrt{dx+1})}{2} + \frac{a\sqrt{dx-1} \sqrt{dx+1}}{3x^3} \\
& + \frac{b\sqrt{dx-1} \sqrt{dx+1}}{2x^2} + \frac{(2ad^2 + 3c)\sqrt{dx-1} \sqrt{dx+1}}{3x}
\end{aligned}$$

command

`integrate((c*x**2+b*x+a)/x**4/(d*x-1)**(1/2)/(d*x+1)**(1/2), x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{ad^3 G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{9}{4}, \frac{11}{4}, 1 & \frac{5}{2}, \frac{5}{2}, 3 \\ 2, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}, 3 & 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}} - \frac{iad^3 G_{6,6}^{2,6} \left( \begin{array}{c|c} \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2}, 1 & \\ \frac{7}{4}, \frac{9}{4} & \frac{3}{2}, 2, 2, 0 \end{array} \middle| \frac{e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}} \\
& \frac{bd^2 G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{7}{4}, \frac{9}{4}, 1 & 2, 2, \frac{5}{2} \\ \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2} & 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}} + \frac{ibd^2 G_{6,6}^{2,6} \left( \begin{array}{c|c} 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 1 & \\ \frac{5}{4}, \frac{7}{4} & 1, \frac{3}{2}, \frac{3}{2}, 0 \end{array} \middle| \frac{e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}} \\
& \frac{cd G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{5}{4}, \frac{7}{4}, 1 & \frac{3}{2}, \frac{3}{2}, 2 \\ 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2 & 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}} - \frac{icd G_{6,6}^{2,6} \left( \begin{array}{c|c} \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 1 & \\ \frac{3}{4}, \frac{5}{4} & \frac{1}{2}, 1, 1, 0 \end{array} \middle| \frac{e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}}}
\end{aligned}$$

## 8 Test file number 19

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.2\_Quadratic/19\_1.1.2.2-c\_x<sup>m</sup>-a+b\_x<sup>2</sup>-<sup>p</sup>

### 8.1 Problem number 424

$$\int \frac{(a + bx^2)^{9/2}}{x^{15}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3b^3(bx^2 + a)^{\frac{3}{2}}}{128x^8} - \frac{3b^2(bx^2 + a)^{\frac{5}{2}}}{80x^{10}} - \frac{3b(bx^2 + a)^{\frac{7}{2}}}{56x^{12}} - \frac{(bx^2 + a)^{\frac{9}{2}}}{14x^{14}} \\ & - \frac{9b^7 \operatorname{arctanh}\left(\frac{\sqrt{bx^2 + a}}{\sqrt{a}}\right)}{2048a^{\frac{5}{2}}} - \frac{3b^4\sqrt{bx^2 + a}}{256x^6} - \frac{3b^5\sqrt{bx^2 + a}}{1024ax^4} + \frac{9b^6\sqrt{bx^2 + a}}{2048a^2x^2} \end{aligned}$$

command

```
integrate((b*x**2+a)**(9/2)/x**15,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & -\frac{a^5}{14\sqrt{b}x^{15}\sqrt{\frac{a}{bx^2} + 1}} - \frac{23a^4\sqrt{b}}{56x^{13}\sqrt{\frac{a}{bx^2} + 1}} - \frac{541a^3b^{\frac{3}{2}}}{560x^{11}\sqrt{\frac{a}{bx^2} + 1}} \\ & - \frac{5249a^2b^{\frac{5}{2}}}{4480x^9\sqrt{\frac{a}{bx^2} + 1}} - \frac{6653ab^{\frac{7}{2}}}{8960x^7\sqrt{\frac{a}{bx^2} + 1}} - \frac{1027b^{\frac{9}{2}}}{5120x^5\sqrt{\frac{a}{bx^2} + 1}} \\ & + \frac{3b^{\frac{11}{2}}}{2048ax^3\sqrt{\frac{a}{bx^2} + 1}} + \frac{9b^{\frac{13}{2}}}{2048a^2x\sqrt{\frac{a}{bx^2} + 1}} - \frac{9b^7 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2048a^{\frac{5}{2}}} \end{aligned}$$

## 8.2 Problem number 425

$$\int x^6 (a + bx^2)^{9/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3a^3 x^7 (bx^2 + a)^{\frac{3}{2}}}{256} + \frac{3a^2 x^7 (bx^2 + a)^{\frac{5}{2}}}{128} + \frac{9a x^7 (bx^2 + a)^{\frac{7}{2}}}{224} \\ & + \frac{x^7 (bx^2 + a)^{\frac{9}{2}}}{16} - \frac{45a^8 \operatorname{arctanh}\left(\frac{x\sqrt{b}}{\sqrt{bx^2 + a}}\right)}{32768b^{\frac{7}{2}}} + \frac{45a^7 x \sqrt{bx^2 + a}}{32768b^3} \\ & - \frac{15a^6 x^3 \sqrt{bx^2 + a}}{16384b^2} + \frac{3a^5 x^5 \sqrt{bx^2 + a}}{4096b} + \frac{9a^4 x^7 \sqrt{bx^2 + a}}{2048} \end{aligned}$$

command

```
integrate(x**6*(b*x**2+a)**(9/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{45a^{\frac{15}{2}} x}{32768b^3 \sqrt{1 + \frac{bx^2}{a}}} + \frac{15a^{\frac{13}{2}} x^3}{32768b^2 \sqrt{1 + \frac{bx^2}{a}}} - \frac{3a^{\frac{11}{2}} x^5}{16384b \sqrt{1 + \frac{bx^2}{a}}} \\ & + \frac{4099a^{\frac{9}{2}} x^7}{28672 \sqrt{1 + \frac{bx^2}{a}}} + \frac{8191a^{\frac{7}{2}} bx^9}{14336 \sqrt{1 + \frac{bx^2}{a}}} + \frac{1699a^{\frac{5}{2}} b^2 x^{11}}{1792 \sqrt{1 + \frac{bx^2}{a}}} + \frac{725a^{\frac{3}{2}} b^3 x^{13}}{896 \sqrt{1 + \frac{bx^2}{a}}} \\ & + \frac{79\sqrt{a} b^4 x^{15}}{224 \sqrt{1 + \frac{bx^2}{a}}} - \frac{45a^8 \operatorname{asinh}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{32768b^{\frac{7}{2}}} + \frac{b^5 x^{17}}{16\sqrt{a} \sqrt{1 + \frac{bx^2}{a}}} \end{aligned}$$

## 8.3 Problem number 426

$$\int x^4 (a + bx^2)^{9/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3a^3 x^5 (bx^2 + a)^{\frac{3}{2}}}{128} + \frac{3a^2 x^5 (bx^2 + a)^{\frac{5}{2}}}{80} + \frac{3a x^5 (bx^2 + a)^{\frac{7}{2}}}{56} + \frac{x^5 (bx^2 + a)^{\frac{9}{2}}}{14} \\ & + \frac{9a^7 \operatorname{arctanh}\left(\frac{x\sqrt{b}}{\sqrt{bx^2 + a}}\right)}{2048b^{\frac{5}{2}}} - \frac{9a^6 x \sqrt{bx^2 + a}}{2048b^2} + \frac{3a^5 x^3 \sqrt{bx^2 + a}}{1024b} + \frac{3a^4 x^5 \sqrt{bx^2 + a}}{256} \end{aligned}$$



command

```
integrate(x**4*(b*x**2+a)**(9/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & -\frac{9a^{\frac{13}{2}}x}{2048b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a^{\frac{11}{2}}x^3}{2048b\sqrt{1+\frac{bx^2}{a}}} + \frac{1027a^{\frac{9}{2}}x^5}{5120\sqrt{1+\frac{bx^2}{a}}} + \frac{6653a^{\frac{7}{2}}bx^7}{8960\sqrt{1+\frac{bx^2}{a}}} \\ & + \frac{5249a^{\frac{5}{2}}b^2x^9}{4480\sqrt{1+\frac{bx^2}{a}}} + \frac{541a^{\frac{3}{2}}b^3x^{11}}{560\sqrt{1+\frac{bx^2}{a}}} + \frac{23\sqrt{a}b^4x^{13}}{56\sqrt{1+\frac{bx^2}{a}}} + \frac{9a^7\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2048b^{\frac{5}{2}}} + \frac{b^5x^{15}}{14\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} \end{aligned}$$

## 9 Test file number 20

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.2\_Quadratic/20\_1.1.2.3-a+b\_x^2-^p-c+d\_x^2-^q

### 9.1 Problem number 62

$$\int (a + bx^2)^{5/2} (c + dx^2)^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a(-5a^3d^3 + 36a^2bcd^2 - 120ab^2c^2d + 320b^3c^3)x(bx^2 + a)^{\frac{3}{2}}}{1536b^3} \\ & + \frac{(-5a^3d^3 + 36a^2bcd^2 - 120ab^2c^2d + 320b^3c^3)x(bx^2 + a)^{\frac{5}{2}}}{1920b^3} \\ & + \frac{d(15a^2d^2 - 68abcd + 152b^2c^2)x(bx^2 + a)^{\frac{7}{2}}}{960b^3} \\ & + \frac{d(-5ad + 16bc)x(bx^2 + a)^{\frac{7}{2}}(dx^2 + c)}{120b^2} + \frac{dx(bx^2 + a)^{\frac{7}{2}}(dx^2 + c)^2}{12b} \\ & + \frac{a^3(-5a^3d^3 + 36a^2bcd^2 - 120ab^2c^2d + 320b^3c^3)\operatorname{arctanh}\left(\frac{x\sqrt{b}}{\sqrt{bx^2 + a}}\right)}{1024b^{\frac{7}{2}}} \\ & + \frac{a^2(-5a^3d^3 + 36a^2bcd^2 - 120ab^2c^2d + 320b^3c^3)x\sqrt{bx^2 + a}}{1024b^3} \end{aligned}$$

command

```
integrate((b*x**2+a)**(5/2)*(d*x**2+c)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{5a^{\frac{11}{2}}d^3x}{1024b^3\sqrt{1+\frac{bx^2}{a}}} - \frac{9a^{\frac{9}{2}}cd^2x}{256b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{5a^{\frac{9}{2}}d^3x^3}{3072b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{15a^{\frac{7}{2}}c^2dx}{128b\sqrt{1+\frac{bx^2}{a}}} \\
& - \frac{3a^{\frac{7}{2}}cd^2x^3}{256b\sqrt{1+\frac{bx^2}{a}}} - \frac{a^{\frac{7}{2}}d^3x^5}{1536b\sqrt{1+\frac{bx^2}{a}}} + \frac{a^{\frac{5}{2}}c^3x\sqrt{1+\frac{bx^2}{a}}}{2} + \frac{3a^{\frac{5}{2}}c^3x}{16\sqrt{1+\frac{bx^2}{a}}} \\
& + \frac{133a^{\frac{5}{2}}c^2dx^3}{128\sqrt{1+\frac{bx^2}{a}}} + \frac{387a^{\frac{5}{2}}cd^2x^5}{640\sqrt{1+\frac{bx^2}{a}}} + \frac{55a^{\frac{5}{2}}d^3x^7}{384\sqrt{1+\frac{bx^2}{a}}} + \frac{35a^{\frac{3}{2}}bc^3x^3}{48\sqrt{1+\frac{bx^2}{a}}} \\
& + \frac{127a^{\frac{3}{2}}bc^2dx^5}{64\sqrt{1+\frac{bx^2}{a}}} + \frac{219a^{\frac{3}{2}}bcd^2x^7}{160\sqrt{1+\frac{bx^2}{a}}} + \frac{67a^{\frac{3}{2}}bd^3x^9}{192\sqrt{1+\frac{bx^2}{a}}} + \frac{17\sqrt{a}b^2c^3x^5}{24\sqrt{1+\frac{bx^2}{a}}} \\
& + \frac{23\sqrt{a}b^2c^2dx^7}{16\sqrt{1+\frac{bx^2}{a}}} + \frac{87\sqrt{a}b^2cd^2x^9}{80\sqrt{1+\frac{bx^2}{a}}} + \frac{7\sqrt{a}b^2d^3x^{11}}{24\sqrt{1+\frac{bx^2}{a}}} - \frac{5a^6d^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{1024b^{\frac{7}{2}}} \\
& + \frac{9a^5cd^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{\frac{5}{2}}} - \frac{15a^4c^2d\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}} + \frac{5a^3c^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{b}} \\
& + \frac{b^3c^3x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + \frac{3b^3c^2dx^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + \frac{3b^3cd^2x^{11}}{10\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + \frac{b^3d^3x^{13}}{12\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}
\end{aligned}$$

## 9.2 Problem number 63

$$\int (a + bx^2)^{5/2} (c + dx^2)^2 dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{a(3a^2d^2 - 20abcd + 80b^2c^2)x(bx^2 + a)^{\frac{3}{2}}}{384b^2} + \frac{(3a^2d^2 - 20abcd + 80b^2c^2)x(bx^2 + a)^{\frac{5}{2}}}{480b^2} \\
& + \frac{3d(-ad + 4bc)x(bx^2 + a)^{\frac{7}{2}}}{80b^2} + \frac{dx(bx^2 + a)^{\frac{7}{2}}(dx^2 + c)}{10b} \\
& + \frac{a^3(3a^2d^2 - 20abcd + 80b^2c^2) \operatorname{arctanh}\left(\frac{x\sqrt{b}}{\sqrt{bx^2 + a}}\right)}{256b^{\frac{5}{2}}} \\
& + \frac{a^2(3a^2d^2 - 20abcd + 80b^2c^2)x\sqrt{bx^2 + a}}{256b^2}
\end{aligned}$$

command

```
integrate((b*x**2+a)**(5/2)*(d*x**2+c)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& -\frac{3a^{\frac{9}{2}}d^2x}{256b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{5a^{\frac{7}{2}}cdx}{64b\sqrt{1+\frac{bx^2}{a}}} - \frac{a^{\frac{7}{2}}d^2x^3}{256b\sqrt{1+\frac{bx^2}{a}}} + \frac{a^{\frac{5}{2}}c^2x\sqrt{1+\frac{bx^2}{a}}}{2} \\
& + \frac{3a^{\frac{5}{2}}c^2x}{16\sqrt{1+\frac{bx^2}{a}}} + \frac{133a^{\frac{5}{2}}cdx^3}{192\sqrt{1+\frac{bx^2}{a}}} + \frac{129a^{\frac{5}{2}}d^2x^5}{640\sqrt{1+\frac{bx^2}{a}}} + \frac{35a^{\frac{3}{2}}bc^2x^3}{48\sqrt{1+\frac{bx^2}{a}}} \\
& + \frac{127a^{\frac{3}{2}}bcdx^5}{96\sqrt{1+\frac{bx^2}{a}}} + \frac{73a^{\frac{3}{2}}bd^2x^7}{160\sqrt{1+\frac{bx^2}{a}}} + \frac{17\sqrt{a}b^2c^2x^5}{24\sqrt{1+\frac{bx^2}{a}}} + \frac{23\sqrt{a}b^2cdx^7}{24\sqrt{1+\frac{bx^2}{a}}} \\
& + \frac{29\sqrt{a}b^2d^2x^9}{80\sqrt{1+\frac{bx^2}{a}}} + \frac{3a^5d^2 \operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{\frac{5}{2}}} - \frac{5a^4cd \operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}} \\
& + \frac{5a^3c^2 \operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{b}} + \frac{b^3c^2x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + \frac{b^3cdx^9}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + \frac{b^3d^2x^{11}}{10\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}
\end{aligned}$$

## 10 Test file number 21

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.2\_Quadratic/21\_1.1.2.4-  
e\_x<sup>-m</sup>+b\_x<sup>2</sup>-<sup>p</sup>+d\_x<sup>2</sup>-<sup>q</sup>

### 10.1 Problem number 46

$$\int \frac{(a + bx^2)^5 (A + Bx^2)}{x^{14}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{a^5 A}{13x^{13}} - \frac{a^4(5Ab + aB)}{11x^{11}} - \frac{5a^3b(2Ab + aB)}{9x^9} - \frac{10a^2b^2(Ab + aB)}{7x^7} \\ & - \frac{ab^3(Ab + 2aB)}{x^5} - \frac{b^4(Ab + 5aB)}{3x^3} - \frac{b^5 B}{x} \end{aligned}$$

command

```
integrate((b*x**2+a)**5*(B*x**2+A)/x**14,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-693Aa^5 - 9009Bb^5x^{12} + x^{10}(-3003Ab^5 - 15015Bab^4) + x^8(-9009Aab^4 - 18018Ba^2b^3) + x^6(-12870Aa^2b^3 - 1}{9009x^{13}}$$

### 10.2 Problem number 48

$$\int \frac{(a + bx^2)^5 (A + Bx^2)}{x^{16}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{a^5 A}{15x^{15}} - \frac{a^4(5Ab + aB)}{13x^{13}} - \frac{5a^3b(2Ab + aB)}{11x^{11}} - \frac{10a^2b^2(Ab + aB)}{9x^9} \\ & - \frac{5ab^3(Ab + 2aB)}{7x^7} - \frac{b^4(Ab + 5aB)}{5x^5} - \frac{b^5 B}{3x^3} \end{aligned}$$

command

```
integrate((b*x**2+a)**5*(B*x**2+A)/x**16,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-3003Aa^5 - 15015Bb^5x^{12} + x^{10}(-9009Ab^5 - 45045Bab^4) + x^8(-32175Aab^4 - 64350Ba^2b^3) + x^6(-50050Aa^2b^3)}{45045x^{15}}$$

### 10.3 Problem number 49

$$\int \frac{(a + bx^2)^5 (A + Bx^2)}{x^{17}} dx$$

Optimal antiderivative

$$-\frac{A(bx^2 + a)^6}{16ax^{16}} + \frac{(Ab - 4aB)(bx^2 + a)^6}{56a^2x^{14}} - \frac{b(Ab - 4aB)(bx^2 + a)^6}{336a^3x^{12}}$$

command

`integrate((b*x**2+a)**5*(B*x**2+A)/x**17, x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-21Aa^5 - 84Bb^5x^{12} + x^{10}(-56Ab^5 - 280Bab^4) + x^8(-210Aab^4 - 420Ba^2b^3) + x^6(-336Aa^2b^3 - 336Ba^3b^2) + x^4(-336Aa^2b^2 - 336Ba^3b)}{336x^{16}}$$

### 10.4 Problem number 50

$$\int \frac{(a + bx^2)^5 (A + Bx^2)}{x^{18}} dx$$

Optimal antiderivative

$$\begin{aligned} &-\frac{a^5 A}{17x^{17}} - \frac{a^4(5Ab + aB)}{15x^{15}} - \frac{5a^3b(2Ab + aB)}{13x^{13}} - \frac{10a^2b^2(Ab + aB)}{11x^{11}} \\ &-\frac{5ab^3(Ab + 2aB)}{9x^9} - \frac{b^4(Ab + 5aB)}{7x^7} - \frac{b^5 B}{5x^5} \end{aligned}$$

command

```
integrate((b*x**2+a)**5*(B*x**2+A)/x**18,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-45045Aa^5 - 153153Bb^5x^{12} + x^{10}(-109395Ab^5 - 546975Bab^4) + x^8(-425425Aab^4 - 850850Ba^2b^3) + x^6(-696150Aa^2b^3 - 1392300Aab^2) + x^4(-45045Aa^2b^2 - 135135Aab^2) + x^2(-45045Aa^2b^2 - 135135Aab^2)}{765765x^{17}}$$

## 10.5 Problem number 51

$$\int \frac{(a + bx^2)^5 (A + Bx^2)}{x^{19}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{a^5A}{18x^{18}} - \frac{a^4(5Ab + aB)}{16x^{16}} - \frac{5a^3b(2Ab + aB)}{14x^{14}} - \frac{5a^2b^2(Ab + aB)}{6x^{12}} \\ & - \frac{ab^3(Ab + 2aB)}{2x^{10}} - \frac{b^4(Ab + 5aB)}{8x^8} - \frac{b^5B}{6x^6} \end{aligned}$$

command

```
integrate((b*x**2+a)**5*(B*x**2+A)/x**19,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-56Aa^5 - 168Bb^5x^{12} + x^{10}(-126Ab^5 - 630Bab^4) + x^8(-504Aab^4 - 1008Ba^2b^3) + x^6(-840Aa^2b^3 - 840Ba^3b^2) + x^4(-45045Aa^2b^2 - 135135Aab^2) + x^2(-45045Aa^2b^2 - 135135Aab^2)}{1008x^{18}}$$

## 10.6 Problem number 53

$$\int \frac{(a + bx^2)^5 (A + Bx^2)}{x^{21}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{a^5A}{20x^{20}} - \frac{a^4(5Ab + aB)}{18x^{18}} - \frac{5a^3b(2Ab + aB)}{16x^{16}} - \frac{5a^2b^2(Ab + aB)}{7x^{14}} \\ & - \frac{5ab^3(Ab + 2aB)}{12x^{12}} - \frac{b^4(Ab + 5aB)}{10x^{10}} - \frac{b^5B}{8x^8} \end{aligned}$$

command

```
integrate((b*x**2+a)**5*(B*x**2+A)/x**21,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-252Aa^5 - 630Bb^5x^{12} + x^{10}(-504Ab^5 - 2520Bab^4) + x^8(-2100Aab^4 - 4200Ba^2b^3) + x^6(-3600Aa^2b^3 - 3600Bab^2)}{5040x^{20}}$$

## 10.7 Problem number 228

$$\int \frac{x^5}{(a + bx^2)(c + dx^2)} dx$$

Optimal antiderivative

$$\frac{x^2}{2bd} + \frac{a^2 \ln(bx^2 + a)}{2b^2(-ad + bc)} - \frac{c^2 \ln(dx^2 + c)}{2d^2(-ad + bc)}$$

command

```
integrate(x**5/(b*x**2+a)/(d*x**2+c),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{a^2 \log\left(x^2 + \frac{\frac{a^4 d^3}{b(ad-bc)} - \frac{2a^3 cd^2}{ad-bc} + \frac{a^2 bc^2 d}{ad-bc} + a^2 cd + abc^2}{a^2 d^2 + b^2 c^2}\right)}{2b^2(ad-bc)} + \frac{c^2 \log\left(x^2 + \frac{-\frac{a^2 bc^2 d}{ad-bc} + a^2 cd + \frac{2ab^2 c^3}{ad-bc} + abc^2 - \frac{b^3 c^4}{d(ad-bc)}}{a^2 d^2 + b^2 c^2}\right)}{2d^2(ad-bc)} + \frac{x^2}{2bd}$$

### 10.8 Problem number 237

$$\int \frac{1}{x^4 (a + bx^2) (c + dx^2)} dx$$

Optimal antiderivative

$$-\frac{1}{3acx^3} + \frac{ad+bc}{a^2c^2x} + \frac{b^{\frac{5}{2}} \arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}(-ad+bc)} - \frac{d^{\frac{5}{2}} \arctan\left(\frac{x\sqrt{d}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}(-ad+bc)}$$

command

```
integrate(1/x**4/(b*x**2+a)/(d*x**2+c), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

### 10.9 Problem number 335

$$\int \frac{x^m}{(a + bx^2)^2 (c + dx^2)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{bx^{1+m}}{2a(-ad+bc)(bx^2+a)} \\ & + \frac{b(bc(1-m) - ad(3-m))x^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{1}{2} + \frac{m}{2}\right], \left[\frac{3}{2} + \frac{m}{2}\right], -\frac{bx^2}{a}\right)}{2a^2(-ad+bc)^2(1+m)} \\ & + \frac{d^2x^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{1}{2} + \frac{m}{2}\right], \left[\frac{3}{2} + \frac{m}{2}\right], -\frac{dx^2}{c}\right)}{c(-ad+bc)^2(1+m)} \end{aligned}$$

command

```
integrate(x**m/(b*x**2+a)**2/(d*x**2+c), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display



### 10.10 Problem number 340

$$\int \frac{x^m}{(a + bx^2)^2 (c + dx^2)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{bx^{1+m}}{2a(-ad+bc)(bx^2+a)} \\ & + \frac{b(bc(1-m) - ad(3-m))x^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{1}{2} + \frac{m}{2}\right], \left[\frac{3}{2} + \frac{m}{2}\right], -\frac{bx^2}{a}\right)}{2a^2(-ad+bc)^2(1+m)} \\ & + \frac{d^2x^{1+m} \operatorname{hypergeom}\left(\left[1, \frac{1}{2} + \frac{m}{2}\right], \left[\frac{3}{2} + \frac{m}{2}\right], -\frac{dx^2}{c}\right)}{c(-ad+bc)^2(1+m)} \end{aligned}$$

command

```
integrate(x**m/(b*x**2+a)**2/(d*x**2+c), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

### 10.11 Problem number 539

$$\int x^4 (a + bx^2)^{5/2} (A + Bx^2) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a(12Ab - 5aB)x^5(bx^2+a)^{\frac{3}{2}}}{192b} + \frac{(12Ab - 5aB)x^5(bx^2+a)^{\frac{5}{2}}}{120b} + \frac{Bx^5(bx^2+a)^{\frac{7}{2}}}{12b} \\ & + \frac{a^5(12Ab - 5aB) \operatorname{arctanh}\left(\frac{x\sqrt{b}}{\sqrt{bx^2+a}}\right)}{1024b^{\frac{7}{2}}} - \frac{a^4(12Ab - 5aB)x\sqrt{bx^2+a}}{1024b^3} \\ & + \frac{a^3(12Ab - 5aB)x^3\sqrt{bx^2+a}}{1536b^2} + \frac{a^2(12Ab - 5aB)x^5\sqrt{bx^2+a}}{384b} \end{aligned}$$

command

```
integrate(x**4*(b*x**2+a)**(5/2)*(B*x**2+A), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
 & -\frac{3Aa^{\frac{9}{2}}x}{256b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{Aa^{\frac{7}{2}}x^3}{256b\sqrt{1+\frac{bx^2}{a}}} + \frac{129Aa^{\frac{5}{2}}x^5}{640\sqrt{1+\frac{bx^2}{a}}} + \frac{73Aa^{\frac{3}{2}}bx^7}{160\sqrt{1+\frac{bx^2}{a}}} \\
 & + \frac{29A\sqrt{a}b^2x^9}{80\sqrt{1+\frac{bx^2}{a}}} + \frac{3Aa^5 \operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{\frac{5}{2}}} + \frac{Ab^3x^{11}}{10\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} \\
 & + \frac{5Ba^{\frac{11}{2}}x}{1024b^3\sqrt{1+\frac{bx^2}{a}}} + \frac{5Ba^{\frac{9}{2}}x^3}{3072b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{Ba^{\frac{7}{2}}x^5}{1536b\sqrt{1+\frac{bx^2}{a}}} + \frac{55Ba^{\frac{5}{2}}x^7}{384\sqrt{1+\frac{bx^2}{a}}} \\
 & + \frac{67Ba^{\frac{3}{2}}bx^9}{192\sqrt{1+\frac{bx^2}{a}}} + \frac{7B\sqrt{a}b^2x^{11}}{24\sqrt{1+\frac{bx^2}{a}}} - \frac{5Ba^6 \operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{1024b^{\frac{7}{2}}} + \frac{Bb^3x^{13}}{12\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}
 \end{aligned}$$

## 10.12 Problem number 613

$$\int x^4(a+bx^2)^2(c+dx^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{(24a^2d^2 + bc(-24ad + 7bc))x^5(dx^2 + c)^{\frac{3}{2}}}{192d^2} - \frac{b(-24ad + 7bc)x^5(dx^2 + c)^{\frac{5}{2}}}{120d^2} \\
 & + \frac{b^2x^7(dx^2 + c)^{\frac{5}{2}}}{12d} + \frac{c^4(24a^2d^2 + bc(-24ad + 7bc)) \operatorname{arctanh}\left(\frac{x\sqrt{d}}{\sqrt{dx^2 + c}}\right)}{1024d^{\frac{9}{2}}} \\
 & - \frac{c^3(24a^2d^2 + bc(-24ad + 7bc))x\sqrt{dx^2 + c}}{1024d^4} \\
 & + \frac{c^2(24a^2d^2 + bc(-24ad + 7bc))x^3\sqrt{dx^2 + c}}{1536d^3} \\
 & + \frac{c(24a^2d^2 + bc(-24ad + 7bc))x^5\sqrt{dx^2 + c}}{384d^2}
 \end{aligned}$$

command

```
integrate(x**4*(b*x**2+a)**2*(d*x**2+c)**(3/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
 & -\frac{3a^2c^{\frac{7}{2}}x}{128d^2\sqrt{1+\frac{dx^2}{c}}} - \frac{a^2c^{\frac{5}{2}}x^3}{128d\sqrt{1+\frac{dx^2}{c}}} + \frac{13a^2c^{\frac{3}{2}}x^5}{64\sqrt{1+\frac{dx^2}{c}}} + \frac{5a^2\sqrt{c}dx^7}{16\sqrt{1+\frac{dx^2}{c}}} \\
 & + \frac{3a^2c^4\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{128d^{\frac{5}{2}}} + \frac{a^2d^2x^9}{8\sqrt{c}\sqrt{1+\frac{dx^2}{c}}} + \frac{3abc^{\frac{9}{2}}x}{128d^3\sqrt{1+\frac{dx^2}{c}}} \\
 & + \frac{abc^{\frac{7}{2}}x^3}{128d^2\sqrt{1+\frac{dx^2}{c}}} - \frac{abc^{\frac{5}{2}}x^5}{320d\sqrt{1+\frac{dx^2}{c}}} + \frac{23abc^{\frac{3}{2}}x^7}{80\sqrt{1+\frac{dx^2}{c}}} + \frac{19ab\sqrt{c}dx^9}{40\sqrt{1+\frac{dx^2}{c}}} \\
 & - \frac{3abc^5\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{128d^{\frac{7}{2}}} + \frac{abd^2x^{11}}{5\sqrt{c}\sqrt{1+\frac{dx^2}{c}}} - \frac{7b^2c^{\frac{11}{2}}x}{1024d^4\sqrt{1+\frac{dx^2}{c}}} \\
 & - \frac{7b^2c^{\frac{9}{2}}x^3}{3072d^3\sqrt{1+\frac{dx^2}{c}}} + \frac{7b^2c^{\frac{7}{2}}x^5}{7680d^2\sqrt{1+\frac{dx^2}{c}}} - \frac{b^2c^{\frac{5}{2}}x^7}{1920d\sqrt{1+\frac{dx^2}{c}}} \\
 & + \frac{107b^2c^{\frac{3}{2}}x^9}{960\sqrt{1+\frac{dx^2}{c}}} + \frac{23b^2\sqrt{c}dx^{11}}{120\sqrt{1+\frac{dx^2}{c}}} + \frac{7b^2c^6\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{1024d^{\frac{9}{2}}} + \frac{b^2d^2x^{13}}{12\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}
 \end{aligned}$$

### 10.13 Problem number 626

$$\int x^2(a+bx^2)^2(c+dx^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{c(40a^2d^2+bc(-24ad+5bc))x^3(dx^2+c)^{\frac{3}{2}}}{384d^2} + \frac{(40a^2d^2+bc(-24ad+5bc))x^3(dx^2+c)^{\frac{5}{2}}}{320d^2} \\
 & - \frac{b(-24ad+5bc)x^3(dx^2+c)^{\frac{7}{2}}}{120d^2} + \frac{b^2x^5(dx^2+c)^{\frac{7}{2}}}{12d} \\
 & - \frac{c^4(40a^2d^2+bc(-24ad+5bc))\operatorname{arctanh}\left(\frac{x\sqrt{d}}{\sqrt{dx^2+c}}\right)}{1024d^{\frac{7}{2}}} \\
 & + \frac{c^3(40a^2d^2+bc(-24ad+5bc))x\sqrt{dx^2+c}}{1024d^3} \\
 & + \frac{c^2(40a^2d^2+bc(-24ad+5bc))x^3\sqrt{dx^2+c}}{512d^2}
 \end{aligned}$$

command

`integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**(5/2), x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
 & \frac{5a^2c^{\frac{7}{2}}x}{128d\sqrt{1+\frac{dx^2}{c}}} + \frac{133a^2c^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{dx^2}{c}}} + \frac{127a^2c^{\frac{3}{2}}dx^5}{192\sqrt{1+\frac{dx^2}{c}}} + \frac{23a^2\sqrt{c}d^2x^7}{48\sqrt{1+\frac{dx^2}{c}}} \\
 & - \frac{5a^2c^4 \operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{128d^{\frac{3}{2}}} + \frac{a^2d^3x^9}{8\sqrt{c}\sqrt{1+\frac{dx^2}{c}}} - \frac{3abc^{\frac{9}{2}}x}{128d^2\sqrt{1+\frac{dx^2}{c}}} \\
 & - \frac{abc^{\frac{7}{2}}x^3}{128d\sqrt{1+\frac{dx^2}{c}}} + \frac{129abc^{\frac{5}{2}}x^5}{320\sqrt{1+\frac{dx^2}{c}}} + \frac{73abc^{\frac{3}{2}}dx^7}{80\sqrt{1+\frac{dx^2}{c}}} + \frac{29ab\sqrt{c}d^2x^9}{40\sqrt{1+\frac{dx^2}{c}}} \\
 & + \frac{3abc^5 \operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{128d^{\frac{5}{2}}} + \frac{abd^3x^{11}}{5\sqrt{c}\sqrt{1+\frac{dx^2}{c}}} + \frac{5b^2c^{\frac{11}{2}}x}{1024d^3\sqrt{1+\frac{dx^2}{c}}} \\
 & + \frac{5b^2c^{\frac{9}{2}}x^3}{3072d^2\sqrt{1+\frac{dx^2}{c}}} - \frac{b^2c^{\frac{7}{2}}x^5}{1536d\sqrt{1+\frac{dx^2}{c}}} + \frac{55b^2c^{\frac{5}{2}}x^7}{384\sqrt{1+\frac{dx^2}{c}}} + \frac{67b^2c^{\frac{3}{2}}dx^9}{192\sqrt{1+\frac{dx^2}{c}}} \\
 & + \frac{7b^2\sqrt{c}d^2x^{11}}{24\sqrt{1+\frac{dx^2}{c}}} - \frac{5b^2c^6 \operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{1024d^{\frac{7}{2}}} + \frac{b^2d^3x^{13}}{12\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}
 \end{aligned}$$

#### 10.14 Problem number 1084

$$\int \frac{x}{(-2+3x^2)(-1+3x^2)^{3/4}} dx$$

Optimal antiderivative

$$-\frac{\arctan\left((3x^2-1)^{\frac{1}{4}}\right)}{3} - \frac{\operatorname{arctanh}\left((3x^2-1)^{\frac{1}{4}}\right)}{3}$$

command

`integrate(x/(3*x**2-2)/(3*x**2-1)**(3/4),x)`

Sympy 1.10.1 under Python 3.10.4 output

$$\int \frac{x}{(3x^2 - 2)(3x^2 - 1)^{\frac{3}{4}}} dx$$

Sympy 1.8 under Python 3.8.8 output

$$\frac{\log\left(\sqrt[4]{3x^2 - 1} - 1\right)}{6} - \frac{\log\left(\sqrt[4]{3x^2 - 1} + 1\right)}{6} - \frac{\operatorname{atan}\left(\sqrt[4]{3x^2 - 1}\right)}{3}$$

## 11 Test file number 22

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.2\_Quadratic/22\_1.1.2.5-a+b\_x^2-^p-c+d\_x^2-^q-e+f\_x^2-^r

### 11.1 Problem number 15

$$\int \frac{(a + bx^2)(c + dx^2)^2}{(e + fx^2)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-af + be)x(dx^2 + c)^2}{6ef(fx^2 + e)^3} - \frac{(de(af + 5be) - cf(5af + be))x(dx^2 + c)}{24e^2f^2(fx^2 + e)^2} \\ & - \frac{(af(-15c^2f^2 + 4cdef + 3d^2e^2) + be(-3c^2f^2 - 4cdef + 15d^2e^2))x}{48e^3f^3(fx^2 + e)} \\ & + \frac{(be(c^2f^2 + 2cdef + 5d^2e^2) + af(5c^2f^2 + 2cdef + d^2e^2)) \arctan\left(\frac{x\sqrt{f}}{\sqrt{e}}\right)}{16e^{\frac{7}{2}}f^{\frac{7}{2}}} \end{aligned}$$

command

`integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**4,x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{\sqrt{-\frac{1}{e^7 f^7}} (5ac^2 f^3 + 2acde f^2 + ad^2 e^2 f + bc^2 e f^2 + 2bcde^2 f + 5bd^2 e^3) \log\left(-e^4 f^3 \sqrt{-\frac{1}{e^7 f^7}} + x\right)}{32}$$

$$+ \frac{\sqrt{-\frac{1}{e^7 f^7}} (5ac^2 f^3 + 2acde f^2 + ad^2 e^2 f + bc^2 e f^2 + 2bcde^2 f + 5bd^2 e^3) \log\left(e^4 f^3 \sqrt{-\frac{1}{e^7 f^7}} + x\right)}{32}$$

$$+ \frac{x^5 (15ac^2 f^5 + 6acde f^4 + 3ad^2 e^2 f^3 + 3bc^2 e f^4 + 6bcde^2 f^3 - 33bd^2 e^3 f^2) + x^3 (40ac^2 e f^4 + 16acde^2 f^3 - 8ad^2 e^3 f^2)}{48e^6 f^3 + 144e^5 f^4 x^2 + 144e^4 f^5 x^3}$$

## 12 Test file number 24

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.2\_Quadratic/24\_1.1.2.8\_P-x-c\_x^-m-a+b\_x^2-^p

### 12.1 Problem number 131

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^8 (a + bx^2)^2} dx$$

Optimal antiderivative

$$-\frac{c}{7a^2 x^7} + \frac{-ad + 2bc}{5a^3 x^5} + \frac{-a^2 e + 2abd - 3b^2 c}{3a^4 x^3} + \frac{-a^3 f + 2a^2 be - 3ab^2 d + 4b^3 c}{a^5 x}$$

$$+ \frac{b(-a^3 f + a^2 be - ab^2 d + b^3 c) x}{2a^5 (bx^2 + a)} + \frac{(-3a^3 f + 5a^2 be - 7ab^2 d + 9b^3 c) \arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right) \sqrt{b}}{2a^{\frac{11}{2}}}$$

command

`integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**2,x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{\sqrt{-\frac{b}{a^{11}}} (3a^3 f - 5a^2 be + 7ab^2 d - 9b^3 c) \log\left(-\frac{a^6 \sqrt{-\frac{b}{a^{11}}} (3a^3 f - 5a^2 be + 7ab^2 d - 9b^3 c)}{3a^3 bf - 5a^2 b^2 e + 7ab^3 d - 9b^4 c} + x\right)}{4}$$

$$- \frac{\sqrt{-\frac{b}{a^{11}}} (3a^3 f - 5a^2 be + 7ab^2 d - 9b^3 c) \log\left(\frac{a^6 \sqrt{-\frac{b}{a^{11}}} (3a^3 f - 5a^2 be + 7ab^2 d - 9b^3 c)}{3a^3 bf - 5a^2 b^2 e + 7ab^3 d - 9b^4 c} + x\right)}{4}$$

$$+ \frac{-30a^4 c + x^8 (-315a^3 bf + 525a^2 b^2 e - 735ab^3 d + 945b^4 c) + x^6 (-210a^4 f + 350a^3 be - 490a^2 b^2 d + 630ab^3 c) + x^4}{210a^6 x^7 + 210a^5 b x^9}$$

## 12.2 Problem number 139

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^4 (a + bx^2)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{c}{3a^3x^3} + \frac{-ad + 3bc}{a^4x} + \frac{\left(\frac{b^2c}{a^2} - \frac{bd}{a} + e - \frac{af}{b}\right)x}{4a(bx^2 + a)^2} + \frac{(a^3f + 3a^2be - 7ab^2d + 11b^3c)x}{8a^4b(bx^2 + a)} \\ & + \frac{(a^3f + 3a^2be - 15ab^2d + 35b^3c) \arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right)}{8a^{\frac{9}{2}}b^{\frac{3}{2}}} \end{aligned}$$

command

```
integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & -\frac{\sqrt{-\frac{1}{a^9b^3}} (a^3f + 3a^2be - 15ab^2d + 35b^3c) \log\left(-a^5b\sqrt{-\frac{1}{a^9b^3}} + x\right)}{16} \\ & + \frac{\sqrt{-\frac{1}{a^9b^3}} (a^3f + 3a^2be - 15ab^2d + 35b^3c) \log\left(a^5b\sqrt{-\frac{1}{a^9b^3}} + x\right)}{16} \\ & + \frac{-8a^3bc + x^6(3a^3bf + 9a^2b^2e - 45ab^3d + 105b^4c) + x^4(-3a^4f + 15a^3be - 75a^2b^2d + 175ab^3c) + x^2(-24a^3bd + 5}{24a^6bx^3 + 48a^5b^2x^5 + 24a^4b^3x^7} \end{aligned}$$

## 12.3 Problem number 140

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^6 (a + bx^2)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{c}{5a^3x^5} + \frac{-ad + 3bc}{3a^4x^3} + \frac{-a^2e + 3abd - 6b^2c}{a^5x} \\ & - \frac{(-a^3f + a^2be - ab^2d + b^3c)x}{4a^4(bx^2 + a)^2} - \frac{(-3a^3f + 7a^2be - 11ab^2d + 15b^3c)x}{8a^5(bx^2 + a)} \\ & - \frac{(-3a^3f + 15a^2be - 35ab^2d + 63b^3c) \arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right)}{8a^{\frac{11}{2}}\sqrt{b}} \end{aligned}$$

command

```
integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{\sqrt{-\frac{1}{a^{11}b}} (3a^3f - 15a^2be + 35ab^2d - 63b^3c) \log\left(-a^6\sqrt{-\frac{1}{a^{11}b}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{a^{11}b}} (3a^3f - 15a^2be + 35ab^2d - 63b^3c) \log\left(a^6\sqrt{-\frac{1}{a^{11}b}} + x\right)}{16} + \frac{-24a^4c + x^8(45a^3bf - 225a^2b^2e + 525ab^3d - 945b^4c) + x^6(75a^4f - 375a^3be + 875a^2b^2d - 1575ab^3c) + x^4(-120a^5b^2c + 120a^6b^3d - 120a^7b^4e + 120a^8b^5c)}{120a^7x^5 + 240a^6bx^7 + 120a^5b^2x^9}$$

## 12.4 Problem number 151

$$\int \frac{x^4(c + dx^2 + ex^4 + fx^6)}{\sqrt{a + bx^2}} dx$$

Optimal antiderivative

$$\frac{a^2(-63a^3f + 70a^2be - 80ab^2d + 96b^3c) \operatorname{arctanh}\left(\frac{x\sqrt{b}}{\sqrt{bx^2 + a}}\right)}{256b^{\frac{11}{2}}} - \frac{a(-63a^3f + 70a^2be - 80ab^2d + 96b^3c) x \sqrt{bx^2 + a}}{256b^5} + \frac{(-63a^3f + 70a^2be - 80ab^2d + 96b^3c) x^3 \sqrt{bx^2 + a}}{384b^4} + \frac{(63a^2f - 70abe + 80b^2d) x^5 \sqrt{bx^2 + a}}{480b^3} + \frac{(-9af + 10be) x^7 \sqrt{bx^2 + a}}{80b^2} + \frac{fx^9 \sqrt{bx^2 + a}}{10b}$$

command

```
integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output



$$\begin{aligned}
& \frac{63a^{\frac{9}{2}}fx}{256b^5\sqrt{1+\frac{bx^2}{a}}} - \frac{35a^{\frac{7}{2}}ex}{128b^4\sqrt{1+\frac{bx^2}{a}}} + \frac{21a^{\frac{7}{2}}fx^3}{256b^4\sqrt{1+\frac{bx^2}{a}}} + \frac{5a^{\frac{5}{2}}dx}{16b^3\sqrt{1+\frac{bx^2}{a}}} \\
& - \frac{35a^{\frac{5}{2}}ex^3}{384b^3\sqrt{1+\frac{bx^2}{a}}} - \frac{21a^{\frac{5}{2}}fx^5}{640b^3\sqrt{1+\frac{bx^2}{a}}} - \frac{3a^{\frac{3}{2}}cx}{8b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{5a^{\frac{3}{2}}dx^3}{48b^2\sqrt{1+\frac{bx^2}{a}}} \\
& + \frac{7a^{\frac{3}{2}}ex^5}{192b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{3a^{\frac{3}{2}}fx^7}{160b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{\sqrt{a}cx^3}{8b\sqrt{1+\frac{bx^2}{a}}} - \frac{\sqrt{a}dx^5}{24b\sqrt{1+\frac{bx^2}{a}}} \\
& - \frac{\sqrt{a}ex^7}{48b\sqrt{1+\frac{bx^2}{a}}} - \frac{\sqrt{a}fx^9}{80b\sqrt{1+\frac{bx^2}{a}}} - \frac{63a^5f \operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{\frac{11}{2}}} \\
& + \frac{35a^4e \operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{9}{2}}} - \frac{5a^3d \operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{7}{2}}} + \frac{3a^2c \operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}} \\
& + \frac{cx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + \frac{dx^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + \frac{ex^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + \frac{fx^{11}}{10\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}
\end{aligned}$$

### 13 Test file number 25

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.3\_General/25\_1.1.3.2-c\_x-  
 $\hat{m}-a+b_x\hat{n}-\hat{p}$

#### 13.1 Problem number 1309

$$\int \frac{x^{13/2}}{\sqrt{a+bx^5}} dx$$

Optimal antiderivative

$$-\frac{a \operatorname{arctanh}\left(\frac{x^{\frac{5}{2}}\sqrt{b}}{\sqrt{bx^5+a}}\right)}{5b^{\frac{3}{2}}} + \frac{x^{\frac{5}{2}}\sqrt{bx^5+a}}{5b}$$

command

```
integrate(x**(13/2)/(b*x**5+a)**(1/2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: SystemError

Sympy 1.8 under Python 3.8.8 output

$$\frac{\sqrt{a} x^{\frac{5}{2}} \sqrt{1 + \frac{bx^5}{a}}}{5b} - \frac{a \operatorname{asinh}\left(\frac{\sqrt{b} x^{\frac{5}{2}}}{\sqrt{a}}\right)}{5b^{\frac{3}{2}}}$$

### 13.2 Problem number 1314

$$\int \frac{x^{13/2}}{\sqrt{1+x^5}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)}{5} + \frac{x^{\frac{5}{2}} \sqrt{x^5 + 1}}{5}$$

command

```
integrate(x**(13/2)/(x**5+1)**(1/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: SystemError

Sympy 1.8 under Python 3.8.8 output

$$\frac{x^{\frac{5}{2}} \sqrt{x^5 + 1}}{5} - \frac{\operatorname{asinh}\left(x^{\frac{5}{2}}\right)}{5}$$

### 13.3 Problem number 1760

$$\int \left(a + \frac{b}{x}\right)^{3/2} x^{9/2} dx$$

Optimal antiderivative

$$-\frac{32b^3 \left(a + \frac{b}{x}\right)^{\frac{5}{2}} x^{\frac{5}{2}}}{1155a^4} + \frac{16b^2 \left(a + \frac{b}{x}\right)^{\frac{5}{2}} x^{\frac{7}{2}}}{231a^3} - \frac{4b \left(a + \frac{b}{x}\right)^{\frac{5}{2}} x^{\frac{9}{2}}}{33a^2} + \frac{2 \left(a + \frac{b}{x}\right)^{\frac{5}{2}} x^{\frac{11}{2}}}{11a}$$

command

```
integrate((a+b/x)**(3/2)*x**(9/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: SystemError

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
 & \frac{210a^8b^{\frac{19}{2}}x^8\sqrt{\frac{ax}{b}+1}}{1155a^7b^9x^3+3465a^6b^{10}x^2+3465a^5b^{11}x+1155a^4b^{12}} \\
 & + \frac{910a^7b^{\frac{21}{2}}x^7\sqrt{\frac{ax}{b}+1}}{1155a^7b^9x^3+3465a^6b^{10}x^2+3465a^5b^{11}x+1155a^4b^{12}} \\
 & + \frac{1480a^6b^{\frac{23}{2}}x^6\sqrt{\frac{ax}{b}+1}}{1155a^7b^9x^3+3465a^6b^{10}x^2+3465a^5b^{11}x+1155a^4b^{12}} \\
 & + \frac{1068a^5b^{\frac{25}{2}}x^5\sqrt{\frac{ax}{b}+1}}{1155a^7b^9x^3+3465a^6b^{10}x^2+3465a^5b^{11}x+1155a^4b^{12}} \\
 & + \frac{290a^4b^{\frac{27}{2}}x^4\sqrt{\frac{ax}{b}+1}}{1155a^7b^9x^3+3465a^6b^{10}x^2+3465a^5b^{11}x+1155a^4b^{12}} \\
 & - \frac{10a^3b^{\frac{29}{2}}x^3\sqrt{\frac{ax}{b}+1}}{1155a^7b^9x^3+3465a^6b^{10}x^2+3465a^5b^{11}x+1155a^4b^{12}} \\
 & - \frac{60a^2b^{\frac{31}{2}}x^2\sqrt{\frac{ax}{b}+1}}{1155a^7b^9x^3+3465a^6b^{10}x^2+3465a^5b^{11}x+1155a^4b^{12}} \\
 & - \frac{80ab^{\frac{33}{2}}x\sqrt{\frac{ax}{b}+1}}{1155a^7b^9x^3+3465a^6b^{10}x^2+3465a^5b^{11}x+1155a^4b^{12}} \\
 & - \frac{32b^{\frac{35}{2}}\sqrt{\frac{ax}{b}+1}}{1155a^7b^9x^3+3465a^6b^{10}x^2+3465a^5b^{11}x+1155a^4b^{12}}
 \end{aligned}$$

### 13.4 Problem number 2357

$$\int \frac{x^2}{a+b\sqrt[3]{x}} dx$$

Optimal antiderivative

$$-\frac{3a^7x^{\frac{1}{3}}}{b^8} + \frac{3a^6x^{\frac{2}{3}}}{2b^7} - \frac{a^5x}{b^6} + \frac{3a^4x^{\frac{4}{3}}}{4b^5} - \frac{3a^3x^{\frac{5}{3}}}{5b^4} + \frac{a^2x^2}{2b^3} - \frac{3ax^{\frac{7}{3}}}{7b^2} + \frac{3x^{\frac{8}{3}}}{8b} + \frac{3a^8 \ln(a+b x^{\frac{1}{3}})}{b^9}$$

command

`integrate(x**2/(a+b*x**(1/3)), x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{3a^8 \log\left(1 + \frac{b\sqrt[3]{x}}{a}\right)}{b^9} - \frac{3a^7 \sqrt[3]{x}}{b^8} + \frac{3a^6 x^{\frac{2}{3}}}{2b^7} - \frac{a^5 x}{b^6} + \frac{3a^4 x^{\frac{4}{3}}}{4b^5} - \frac{3a^3 x^{\frac{5}{3}}}{5b^4} + \frac{a^2 x^2}{2b^3} - \frac{3ax^{\frac{7}{3}}}{7b^2} + \frac{3x^{\frac{8}{3}}}{8b}$$

### 13.5 Problem number 2366

$$\int \frac{x^2}{(a + b\sqrt[3]{x})^2} dx$$

Optimal antiderivative

$$-\frac{3a^8}{b^9 (a + b x^{\frac{1}{3}})} + \frac{21a^6 x^{\frac{1}{3}}}{b^8} - \frac{9a^5 x^{\frac{2}{3}}}{b^7} + \frac{5a^4 x}{b^6} - \frac{3a^3 x^{\frac{4}{3}}}{b^5} + \frac{9a^2 x^{\frac{5}{3}}}{5b^4} - \frac{a x^2}{b^3} + \frac{3x^{\frac{7}{3}}}{7b^2} - \frac{24a^7 \ln(a + b x^{\frac{1}{3}})}{b^9}$$

command

`integrate(x**2/(a+b*x**(1/3))**2,x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & -\frac{840a^8 x^{\frac{176}{3}} \log\left(1 + \frac{b\sqrt[3]{x}}{a}\right)}{35ab^9 x^{\frac{176}{3}} + 35b^{10} x^{59}} - \frac{840a^7 b x^{59} \log\left(1 + \frac{b\sqrt[3]{x}}{a}\right)}{35ab^9 x^{\frac{176}{3}} + 35b^{10} x^{59}} \\ & + \frac{840a^7 b x^{59}}{35ab^9 x^{\frac{176}{3}} + 35b^{10} x^{59}} + \frac{420a^6 b^2 x^{\frac{178}{3}}}{35ab^9 x^{\frac{176}{3}} + 35b^{10} x^{59}} - \frac{140a^5 b^3 x^{\frac{179}{3}}}{35ab^9 x^{\frac{176}{3}} + 35b^{10} x^{59}} \\ & + \frac{70a^4 b^4 x^{60}}{35ab^9 x^{\frac{176}{3}} + 35b^{10} x^{59}} - \frac{42a^3 b^5 x^{\frac{181}{3}}}{35ab^9 x^{\frac{176}{3}} + 35b^{10} x^{59}} + \frac{28a^2 b^6 x^{\frac{182}{3}}}{35ab^9 x^{\frac{176}{3}} + 35b^{10} x^{59}} \\ & - \frac{20ab^7 x^{61}}{35ab^9 x^{\frac{176}{3}} + 35b^{10} x^{59}} + \frac{15b^8 x^{\frac{184}{3}}}{35ab^9 x^{\frac{176}{3}} + 35b^{10} x^{59}} \end{aligned}$$

## 14 Test file number 26

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.3\_General/26\_1.1.3.3-a+b\_x^n-  
^p-c+d\_x^n-q

### 14.1 Problem number 18

$$\int \frac{1}{(a + bx^3)(c + dx^3)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b^{\frac{2}{3}} \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{3a^{\frac{2}{3}}(-ad + bc)} - \frac{d^{\frac{2}{3}} \ln\left(c^{\frac{1}{3}} + d^{\frac{1}{3}}x\right)}{3c^{\frac{2}{3}}(-ad + bc)} - \frac{b^{\frac{2}{3}} \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}(-ad + bc)} \\ & + \frac{d^{\frac{2}{3}} \ln\left(c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2\right)}{6c^{\frac{2}{3}}(-ad + bc)} - \frac{b^{\frac{2}{3}} \arctan\left(\frac{(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}}{3a^{\frac{2}{3}}(-ad + bc)} \\ & + \frac{d^{\frac{2}{3}} \arctan\left(\frac{(c^{\frac{1}{3}} - 2d^{\frac{1}{3}}x)\sqrt{3}}{3c^{\frac{1}{3}}}\right)\sqrt{3}}{3c^{\frac{2}{3}}(-ad + bc)} \end{aligned}$$

command

`integrate(1/(b*x**3+a)/(d*x**3+c), x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \text{RootSum}\left(t^3(27a^5d^3 - 81a^4bcd^2 + 81a^3b^2c^2d - 27a^2b^3c^3) + b^2, \left(t \mapsto t \log\left(x + \frac{81t^4a^7c^2d^5 - 243t^4a^6bc^3d^4 + 162}{\dots}\right)\right)\right. \\ & \left. + \text{RootSum}\left(t^3(27a^3c^2d^3 - 81a^2bc^3d^2 + 81ab^2c^4d - 27b^3c^5) - d^2, \left(t \mapsto t \log\left(x + \frac{81t^4a^7c^2d^5 - 243t^4a^6bc^3d^4 + 162}{\dots}\right)\right)\right) \end{aligned}$$

## 14.2 Problem number 167

$$\int \frac{(c + dx^4)^4}{(a + bx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^2(3a^2d^2 - 8abcd + 6b^2c^2)x}{b^4} + \frac{2d^3(-ad + 2bc)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{(-ad + bc)^4x}{4ab^4(bx^4 + a)} \\ & + \frac{(-ad + bc)^3(13ad + 3bc) \arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{16a^{\frac{7}{4}}b^{\frac{17}{4}}} \\ & + \frac{(-ad + bc)^3(13ad + 3bc) \arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \sqrt{2}}{16a^{\frac{7}{4}}b^{\frac{17}{4}}} \\ & - \frac{(-ad + bc)^3(13ad + 3bc) \ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \sqrt{2}}{32a^{\frac{7}{4}}b^{\frac{17}{4}}} \\ & + \frac{(-ad + bc)^3(13ad + 3bc) \ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \sqrt{2}}{32a^{\frac{7}{4}}b^{\frac{17}{4}}} \end{aligned}$$

command

```
integrate((d*x**4+c)**4/(b*x**4+a)**2, x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & x^5 \left( -\frac{2ad^4}{5b^3} + \frac{4cd^3}{5b^2} \right) + x \left( \frac{3a^2d^4}{b^4} - \frac{8acd^3}{b^3} + \frac{6c^2d^2}{b^2} \right) \\ & + \frac{x(a^4d^4 - 4a^3bcd^3 + 6a^2b^2c^2d^2 - 4ab^3c^3d + b^4c^4)}{4a^2b^4 + 4ab^5x^4} \\ & + \text{RootSum} \left( 65536t^4a^7b^{17} + 28561a^{16}d^{16} - 316368a^{15}bcd^{15} + 1577784a^{14}b^2c^2d^{14} - 4651504a^{13}b^3c^3d^{13} + 8923164a^{12}b^4c^4d^{12} \right) \\ & + \frac{d^4x^9}{9b^2} \end{aligned}$$

### 14.3 Problem number 290

$$\int \frac{a + bx^n}{(c + dx^n)^3} dx$$

Optimal antiderivative

$$-\frac{(-ad + bc)x}{2cdn(c + dx^n)^2} + \frac{(bc - ad(1 - 2n))x \operatorname{hypergeom}\left(\left[2, \frac{1}{n}\right], \left[1 + \frac{1}{n}\right], -\frac{dx^n}{c}\right)}{2c^3dn}$$

command

```
integrate((a+b*x**n)/(c+d*x**n)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

### 14.4 Problem number 352

$$\int \frac{a + bx^2}{\sqrt{-1 + cx} \sqrt{1 + cx}} dx$$

Optimal antiderivative

$$\frac{(2ac^2 + b) \operatorname{arccosh}(cx)}{2c^3} + \frac{bx\sqrt{cx - 1} \sqrt{cx + 1}}{2c^2}$$

command

```
integrate((b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{aG_{6,6}^{6,2} \left( \begin{array}{c} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 \end{array} \middle| \frac{1}{c^2 x^2} \right) - iaG_{6,6}^{2,6} \left( \begin{array}{c} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 \\ -\frac{1}{4}, \frac{1}{4} \end{array} \middle| \frac{e^{2i\pi}}{c^2 x^2} \right)}{4\pi^{\frac{3}{2}} c} \\
& + \frac{bG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{3}{4}, -\frac{1}{4} \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{array} \middle| \frac{1}{c^2 x^2} \right)}{4\pi^{\frac{3}{2}} c^3} \\
& - \frac{ibG_{6,6}^{2,6} \left( \begin{array}{c} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \end{array} \middle| \frac{e^{2i\pi}}{c^2 x^2} \right)}{4\pi^{\frac{3}{2}} c^3}
\end{aligned}$$

### 14.5 Problem number 355

$$\int \frac{a + bx^2}{x^3 \sqrt{-1 + cx} \sqrt{1 + cx}} dx$$

Optimal antiderivative

$$\frac{(ac^2 + 2b) \arctan\left(\sqrt{cx - 1} \sqrt{cx + 1}\right)}{2} + \frac{a\sqrt{cx - 1} \sqrt{cx + 1}}{2x^2}$$

command

`integrate((b*x**2+a)/x**3/(c*x-1)**(1/2)/(c*x+1)**(1/2), x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{ac^2 G_{6,6}^{5,3} \left( \begin{array}{c} \frac{7}{4}, \frac{9}{4}, 1 \\ \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2} \end{array} \middle| \frac{1}{c^2 x^2} \right) - iac^2 G_{6,6}^{2,6} \left( \begin{array}{c} 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 1 \\ \frac{5}{4}, \frac{7}{4} \end{array} \middle| \frac{e^{2i\pi}}{c^2 x^2} \right)}{4\pi^{\frac{3}{2}}} \\
& + \frac{bG_{6,6}^{5,3} \left( \begin{array}{c} \frac{3}{4}, \frac{5}{4}, 1 \\ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2} \end{array} \middle| \frac{1}{c^2 x^2} \right) - ibG_{6,6}^{2,6} \left( \begin{array}{c} 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1 \\ \frac{1}{4}, \frac{3}{4} \end{array} \middle| \frac{e^{2i\pi}}{c^2 x^2} \right)}{4\pi^{\frac{3}{2}}}
\end{aligned}$$



## 14.6 Problem number 362

$$\int \frac{a + bx^2}{\sqrt{-c + dx} \sqrt{c + dx}} dx$$

Optimal antiderivative

$$\frac{(2a d^2 + b c^2) \operatorname{arctanh}\left(\frac{\sqrt{dx - c}}{\sqrt{dx + c}}\right)}{d^3} + \frac{bx\sqrt{dx - c} \sqrt{dx + c}}{2d^2}$$

command

```
integrate((b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{a G_{6,6}^{6,2} \left( \begin{matrix} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d} - \frac{ia G_{6,6}^{2,6} \left( \begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 \\ -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, 0, 0, 0 \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d}$$

$$+ \frac{bc^2 G_{6,6}^{6,2} \left( \begin{matrix} -\frac{3}{4}, -\frac{1}{4} & -\frac{1}{2}, -\frac{1}{2}, 0, 1 \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3}$$

$$- \frac{ibc^2 G_{6,6}^{2,6} \left( \begin{matrix} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} & -\frac{3}{2}, -1, -1, 0 \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3}$$

## 14.7 Problem number 365

$$\int \frac{a + bx^2}{x^3 \sqrt{-c + dx} \sqrt{c + dx}} dx$$

Optimal antiderivative

$$\frac{(a d^2 + 2b c^2) \operatorname{arctan}\left(\frac{\sqrt{dx - c} \sqrt{dx + c}}{c}\right)}{2c^3} + \frac{a\sqrt{dx - c} \sqrt{dx + c}}{2c^2 x^2}$$

command

```
integrate((b*x**2+a)/x**3/(d*x-c)**(1/2)/(d*x+c)**(1/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
 & -\frac{ad^2 G_{6,6}^{5,3} \left( \begin{matrix} \frac{7}{4}, \frac{9}{4}, 1 & 2, 2, \frac{5}{2} \\ \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2} & 0 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} c^3} + \frac{iad^2 G_{6,6}^{2,6} \left( \begin{matrix} 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 1 \\ \frac{5}{4}, \frac{7}{4} & 1, \frac{3}{2}, \frac{3}{2}, 0 \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} c^3} \\
 & -\frac{bG_{6,6}^{5,3} \left( \begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 & 1, 1, \frac{3}{2} \\ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2} & 0 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} c} + \frac{ibG_{6,6}^{2,6} \left( \begin{matrix} 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1 \\ \frac{1}{4}, \frac{3}{4} & 0, \frac{1}{2}, \frac{1}{2}, 0 \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} c}
 \end{aligned}$$

## 14.8 Problem number 369

$$\int \frac{x^3(a + bx^2)}{(-c + dx)^{3/2}(c + dx)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{(3ad^2 + 4bc^2)x^2}{3d^4\sqrt{dx-c}\sqrt{dx+c}} + \frac{bx^4}{3d^2\sqrt{dx-c}\sqrt{dx+c}} + \frac{2(3ad^2 + 4bc^2)\sqrt{dx-c}\sqrt{dx+c}}{3d^6}$$

command

```
integrate(x**3*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& a \left( \frac{{}_cG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{3}{4}, -\frac{1}{4} \\ -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{2}, 0 \end{array} \middle| \frac{c^2}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} d^4} \right. \\
& \quad \left. - \frac{{}_cG_{6,6}^{2,6} \left( \begin{array}{c} -2, -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \end{array} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} d^4} \right) \\
& + b \left( \frac{{}_c^3 G_{6,6}^{6,2} \left( \begin{array}{c} -\frac{7}{4}, -\frac{5}{4} \\ -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{1}{2}, 0 \end{array} \middle| \frac{c^2}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} d^6} \right. \\
& \quad \left. - \frac{{}_c^3 G_{6,6}^{2,6} \left( \begin{array}{c} -3, -\frac{5}{2}, -\frac{9}{4}, -2, -\frac{7}{4}, 1 \\ -\frac{9}{4}, -\frac{7}{4} \end{array} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} d^6} \right)
\end{aligned}$$

### 14.9 Problem number 371

$$\int \frac{x(a + bx^2)}{(-c + dx)^{3/2}(c + dx)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\left(\frac{a}{c^2} + \frac{b}{d^2}\right) x^2}{\sqrt{dx - c} \sqrt{dx + c}} + \frac{(a d^2 + 2b c^2) \sqrt{dx - c} \sqrt{dx + c}}{c^2 d^4}$$

command

```
integrate(x*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
 & a \left( \frac{G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{1}{4}, \frac{3}{4}, 1 & 0, 1, \frac{3}{2} \\ \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2} & 0 \end{array} \middle| \frac{c^2}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} c d^2} - \frac{iG_{6,6}^{2,6} \left( \begin{array}{c|c} -1, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, 1 & \\ -\frac{1}{4}, \frac{1}{4} & -1, -\frac{1}{2}, \frac{1}{2}, 0 \end{array} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} c d^2} \right) \\
 & + b \left( \frac{cG_{6,6}^{6,2} \left( \begin{array}{c|c} -\frac{3}{4}, -\frac{1}{4} & -1, 0, \frac{1}{2}, 1 \\ -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{2}, 0 & \end{array} \middle| \frac{c^2}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} d^4} \right. \\
 & \left. - \frac{i c G_{6,6}^{2,6} \left( \begin{array}{c|c} -2, -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, 1 & \\ -\frac{5}{4}, -\frac{3}{4} & -2, -\frac{3}{2}, -\frac{1}{2}, 0 \end{array} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} d^4} \right)
 \end{aligned}$$

#### 14.10 Problem number 372

$$\int \frac{a + bx^2}{(-c + dx)^{3/2}(c + dx)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2b \operatorname{arctanh}\left(\frac{\sqrt{dx-c}}{\sqrt{dx+c}}\right)}{d^3} - \frac{\left(\frac{a}{c^2} + \frac{b}{d^2}\right) x}{\sqrt{dx-c} \sqrt{dx+c}}$$

command

`integrate((b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2), x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& a \left( \frac{G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{3}{4}, \frac{5}{4}, 1 & \frac{1}{2}, \frac{3}{2}, 2 \\ \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 2 & 0 \end{array} \middle| \frac{c^2}{d^2 x^2} \right) + iG_{6,6}^{2,6} \left( \begin{array}{c|c} -\frac{1}{2}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1 & \\ \frac{1}{4}, \frac{3}{4} & -\frac{1}{2}, 0, 1, 0 \end{array} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} c^2 d} \right) \\
& + b \left( \frac{G_{6,6}^{6,2} \left( \begin{array}{c|c} -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, \frac{1}{2}, 1, 1 \\ -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1, 0 & \end{array} \middle| \frac{c^2}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} d^3} \right) \\
& + \left( \frac{iG_{6,6}^{2,6} \left( \begin{array}{c|c} -\frac{3}{2}, -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 1 & \\ -\frac{3}{4}, -\frac{1}{4} & -\frac{3}{2}, -1, 0, 0 \end{array} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} d^3} \right)
\end{aligned}$$

#### 14.11 Problem number 373

$$\int \frac{a + bx^2}{x(-c + dx)^{3/2}(c + dx)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{a \arctan\left(\frac{\sqrt{dx-c} \sqrt{dx+c}}{c}\right)}{c^3} + \frac{-\frac{a}{c^2} - \frac{b}{d^2}}{\sqrt{dx-c} \sqrt{dx+c}}$$

command

`integrate((b*x**2+a)/x/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& a \left( \frac{G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{5}{4}, \frac{7}{4}, 1 & 1, 2, \frac{5}{2} \\ \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, \frac{5}{2} & 0 \end{array} \middle| \frac{c^2}{d^2 x^2} \right) - iG_{6,6}^{2,6} \left( \begin{array}{c|c} 0, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, 1 & \\ \frac{3}{4}, \frac{5}{4} & 0, \frac{1}{2}, \frac{3}{2}, 0 \end{array} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} c^3} \right) \\
& + b \left( \frac{G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{1}{4}, \frac{3}{4}, 1 & 0, 1, \frac{3}{2} \\ \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2} & 0 \end{array} \middle| \frac{c^2}{d^2 x^2} \right) - iG_{6,6}^{2,6} \left( \begin{array}{c|c} -1, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, 1 & \\ -\frac{1}{4}, \frac{1}{4} & -1, -\frac{1}{2}, \frac{1}{2}, 0 \end{array} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} cd^2} \right)
\end{aligned}$$

### 14.12 Problem number 374

$$\int \frac{a + bx^2}{x^2(-c + dx)^{3/2}(c + dx)^{3/2}} dx$$

Optimal antiderivative

$$\frac{a}{c^2 x \sqrt{dx - c} \sqrt{dx + c}} - \frac{(2a d^2 + b c^2) x}{c^4 \sqrt{dx - c} \sqrt{dx + c}}$$

command

`integrate((b*x**2+a)/x**2/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& a \left( \frac{dG_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{7}{4}, \frac{9}{4}, 1 & \frac{3}{2}, \frac{5}{2}, 3 \\ \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2}, 3 & 0 \end{array} \middle| \frac{c^2}{d^2 x^2} \right) + idG_{6,6}^{2,6} \left( \begin{array}{c|c} \frac{1}{2}, 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 1 & \\ \frac{5}{4}, \frac{7}{4} & \frac{1}{2}, 1, 2, 0 \end{array} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} c^4} \right) \\
& + b \left( \frac{G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{3}{4}, \frac{5}{4}, 1 & \frac{1}{2}, \frac{3}{2}, 2 \\ \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 2 & 0 \end{array} \middle| \frac{c^2}{d^2 x^2} \right) + iG_{6,6}^{2,6} \left( \begin{array}{c|c} -\frac{1}{2}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1 & \\ \frac{1}{4}, \frac{3}{4} & -\frac{1}{2}, 0, 1, 0 \end{array} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} c^2 d} \right)
\end{aligned}$$

## 15 Test file number 27

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.3\_General/27\_1.1.3.4-e\_x-  
 $\hat{m}-a+b_x\hat{n}-\hat{p}-c+d_x\hat{n}-\hat{q}$

### 15.1 Problem number 46

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{14}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{a^5 A}{13x^{13}} - \frac{a^4(5Ab + aB)}{10x^{10}} - \frac{5a^3b(2Ab + aB)}{7x^7} - \frac{5a^2b^2(Ab + aB)}{2x^4} \\ & - \frac{5a b^3(Ab + 2aB)}{x} + \frac{b^4(Ab + 5aB)x^2}{2} + \frac{b^5 B x^5}{5} \end{aligned}$$

command

```
integrate((b*x**3+a)**5*(B*x**3+A)/x**14,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{Bb^5x^5}{5} + x^2 \left( \frac{Ab^5}{2} + \frac{5Bab^4}{2} \right) \\ & + \frac{-70Aa^5 + x^{12}(-4550Aab^4 - 9100Ba^2b^3) + x^9(-2275Aa^2b^3 - 2275Ba^3b^2) + x^6(-1300Aa^3b^2 - 650Ba^4b) + x^3}{910x^{13}} \end{aligned}$$

### 15.2 Problem number 47

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{15}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{a^5 A}{14x^{14}} - \frac{a^4(5Ab + aB)}{11x^{11}} - \frac{5a^3b(2Ab + aB)}{8x^8} - \frac{2a^2b^2(Ab + aB)}{x^5} \\ & - \frac{5a b^3(Ab + 2aB)}{2x^2} + b^4(Ab + 5aB)x + \frac{b^5 B x^4}{4} \end{aligned}$$

command

```
integrate((b*x**3+a)**5*(B*x**3+A)/x**15,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{Bb^5x^4}{4} + x(Ab^5 + 5Bab^4) + \frac{-44Aa^5 + x^{12}(-1540Aab^4 - 3080Ba^2b^3) + x^9(-1232Aa^2b^3 - 1232Ba^3b^2) + x^6(-770Aa^3b^2 - 385Ba^4b) + x^3(-770Aa^4b - 385Ba^5)}{616x^{14}}$$

### 15.3 Problem number 48

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{16}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{a^5 A}{15x^{15}} - \frac{a^4(5Ab + aB)}{12x^{12}} - \frac{5a^3b(2Ab + aB)}{9x^9} - \frac{5a^2b^2(Ab + aB)}{3x^6} \\ & - \frac{5ab^3(Ab + 2aB)}{3x^3} + \frac{b^5 B x^3}{3} + b^4(Ab + 5aB) \ln(x) \end{aligned}$$

command

```
integrate((b*x**3+a)**5*(B*x**3+A)/x**16,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{Bb^5x^3}{3} + b^4(Ab + 5Ba) \log(x) + \frac{-12Aa^5 + x^{12}(-300Aab^4 - 600Ba^2b^3) + x^9(-300Aa^2b^3 - 300Ba^3b^2) + x^6(-200Aa^3b^2 - 100Ba^4b) + x^3(-75Aa^4b - 375Ba^5)}{180x^{15}}$$



## 15.4 Problem number 49

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{17}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{a^5 A}{16x^{16}} - \frac{a^4(5Ab + aB)}{13x^{13}} - \frac{a^3b(2Ab + aB)}{2x^{10}} - \frac{10a^2b^2(Ab + aB)}{7x^7} \\ & - \frac{5ab^3(Ab + 2aB)}{4x^4} - \frac{b^4(Ab + 5aB)}{x} + \frac{b^5 B x^2}{2} \end{aligned}$$

command

```
integrate((b*x**3+a)**5*(B*x**3+A)/x**17,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{Bb^5x^2}{2} \\ & + \frac{-91Aa^5 + x^{15}(-1456Ab^5 - 7280Bab^4) + x^{12}(-1820Aab^4 - 3640Ba^2b^3) + x^9(-2080Aa^2b^3 - 2080Ba^3b^2) + x^6}{1456x^{16}} \end{aligned}$$

## 15.5 Problem number 111

$$\int \frac{x^4}{(a + bx^3)(c + dx^3)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{\frac{2}{3}} \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{3b^{\frac{2}{3}}(-ad + bc)} - \frac{c^{\frac{2}{3}} \ln\left(c^{\frac{1}{3}} + d^{\frac{1}{3}}x\right)}{3d^{\frac{2}{3}}(-ad + bc)} - \frac{a^{\frac{2}{3}} \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{6b^{\frac{2}{3}}(-ad + bc)} \\ & + \frac{c^{\frac{2}{3}} \ln\left(c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2\right)}{6d^{\frac{2}{3}}(-ad + bc)} + \frac{a^{\frac{2}{3}} \arctan\left(\frac{(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}}{3b^{\frac{2}{3}}(-ad + bc)} \\ & - \frac{c^{\frac{2}{3}} \arctan\left(\frac{(c^{\frac{1}{3}} - 2d^{\frac{1}{3}}x)\sqrt{3}}{3c^{\frac{1}{3}}}\right)\sqrt{3}}{3d^{\frac{2}{3}}(-ad + bc)} \end{aligned}$$

command

`integrate(x**4/(b*x**3+a)/(d*x**3+c), x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \text{RootSum} \left( t^3(27a^3d^5 - 81a^2bcd^4 + 81ab^2c^2d^3 - 27b^3c^3d^2) - c^2, \left( t \mapsto t \log \left( x + \frac{243t^5a^6b^2d^8 - 1458t^5a^5b^3cd^7 + 3}{t^3(27a^3d^5 - 81a^2bcd^4 + 81ab^2c^2d^3 - 27b^3c^3d^2) - c^2} \right) \right) \right. \\ & \left. + \text{RootSum} \left( t^3(27a^3b^2d^3 - 81a^2b^3cd^2 + 81ab^4c^2d - 27b^5c^3) + a^2, \left( t \mapsto t \log \left( x + \frac{243t^5a^6b^2d^8 - 1458t^5a^5b^3cd^7 + 3}{t^3(27a^3b^2d^3 - 81a^2b^3cd^2 + 81ab^4c^2d - 27b^5c^3) + a^2} \right) \right) \right) \end{aligned}$$

## 15.6 Problem number 115

$$\int \frac{1}{(a + bx^3)(c + dx^3)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b^{\frac{2}{3}} \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{3a^{\frac{2}{3}}(-ad + bc)} - \frac{d^{\frac{2}{3}} \ln\left(c^{\frac{1}{3}} + d^{\frac{1}{3}}x\right)}{3c^{\frac{2}{3}}(-ad + bc)} - \frac{b^{\frac{2}{3}} \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{6a^{\frac{2}{3}}(-ad + bc)} \\ & + \frac{d^{\frac{2}{3}} \ln\left(c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2\right)}{6c^{\frac{2}{3}}(-ad + bc)} - \frac{b^{\frac{2}{3}} \arctan\left(\frac{(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}}{3a^{\frac{2}{3}}(-ad + bc)} \\ & + \frac{d^{\frac{2}{3}} \arctan\left(\frac{(c^{\frac{1}{3}} - 2d^{\frac{1}{3}}x)\sqrt{3}}{3c^{\frac{1}{3}}}\right)\sqrt{3}}{3c^{\frac{2}{3}}(-ad + bc)} \end{aligned}$$

command

`integrate(1/(b*x**3+a)/(d*x**3+c), x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \text{RootSum} \left( t^3(27a^5d^3 - 81a^4bcd^2 + 81a^3b^2c^2d - 27a^2b^3c^3) + b^2, \left( t \mapsto t \log \left( x + \frac{81t^4a^7c^2d^5 - 243t^4a^6bc^3d^4 + 162}{t^3(27a^5d^3 - 81a^4bcd^2 + 81a^3b^2c^2d - 27a^2b^3c^3) + b^2} \right) \right) \right. \\ & \left. + \text{RootSum} \left( t^3(27a^3c^2d^3 - 81a^2bc^3d^2 + 81ab^2c^4d - 27b^3c^5) - d^2, \left( t \mapsto t \log \left( x + \frac{81t^4a^7c^2d^5 - 243t^4a^6bc^3d^4 + 162}{t^3(27a^3c^2d^3 - 81a^2bc^3d^2 + 81ab^2c^4d - 27b^3c^5) - d^2} \right) \right) \right) \end{aligned}$$

### 15.7 Problem number 769

$$\int \frac{x^7}{(a + bx^4)(c + dx^4)} dx$$

Optimal antiderivative

$$-\frac{a \ln(bx^4 + a)}{4b(-ad + bc)} + \frac{c \ln(dx^4 + c)}{4d(-ad + bc)}$$

command

```
integrate(x**7/(b*x**4+a)/(d*x**4+c), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{a \log \left( x^4 + \frac{\frac{a^3 d^2}{b(ad-bc)} - \frac{2a^2 cd}{ad-bc} + \frac{abc^2}{ad-bc} + 2ac}{ad+bc} \right)}{4b(ad-bc)} - \frac{c \log \left( x^4 + \frac{-\frac{a^2 cd}{ad-bc} + \frac{2abc^2}{ad-bc} + 2ac - \frac{b^2 c^3}{d(ad-bc)}}{ad+bc} \right)}{4d(ad-bc)}$$

### 15.8 Problem number 774

$$\int \frac{x^9}{(a + bx^4)(c + dx^4)} dx$$

Optimal antiderivative

$$\frac{x^2}{2bd} + \frac{a^{\frac{3}{2}} \arctan\left(\frac{x^2 \sqrt{b}}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}(-ad + bc)} - \frac{c^{\frac{3}{2}} \arctan\left(\frac{x^2 \sqrt{d}}{\sqrt{c}}\right)}{2d^{\frac{3}{2}}(-ad + bc)}$$

command

```
integrate(x**9/(b*x**4+a)/(d*x**4+c), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{\sqrt{-\frac{a^3}{b^3}} \log \left( x^2 + \frac{\frac{a^4 d^4 \sqrt{-\frac{a^3}{b^3}}}{ad-bc} - \frac{a^3 b^3 d^6 \left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3} + \frac{a^2 b^4 c d^5 \left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3} + \frac{a b^5 c^2 d^4 \left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3} - \frac{b^6 c^3 d^3 \left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3} - \frac{b^4 c^4 \sqrt{-\frac{a^3}{b^3}}}{ad-bc}}{a^3 c d^2 + a^2 b c^2 d + a b^2 c^3}}{4(ad-bc)} \\
& + \frac{\sqrt{-\frac{a^3}{b^3}} \log \left( x^2 + \frac{\frac{a^4 d^4 \sqrt{-\frac{a^3}{b^3}}}{ad-bc} + \frac{a^3 b^3 d^6 \left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3} - \frac{a^2 b^4 c d^5 \left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3} - \frac{a b^5 c^2 d^4 \left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3} + \frac{b^6 c^3 d^3 \left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3} + \frac{b^4 c^4 \sqrt{-\frac{a^3}{b^3}}}{ad-bc}}{a^3 c d^2 + a^2 b c^2 d + a b^2 c^3}}{4(ad-bc)} \\
& + \frac{\sqrt{-\frac{c^3}{d^3}} \log \left( x^2 + \frac{\frac{a^4 d^4 \sqrt{-\frac{c^3}{d^3}}}{ad-bc} - \frac{a^3 b^3 d^6 \left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3} + \frac{a^2 b^4 c d^5 \left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3} + \frac{a b^5 c^2 d^4 \left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3} - \frac{b^6 c^3 d^3 \left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3} - \frac{b^4 c^4 \sqrt{-\frac{c^3}{d^3}}}{ad-bc}}{a^3 c d^2 + a^2 b c^2 d + a b^2 c^3}}{4(ad-bc)} \\
& + \frac{\sqrt{-\frac{c^3}{d^3}} \log \left( x^2 + \frac{\frac{a^4 d^4 \sqrt{-\frac{c^3}{d^3}}}{ad-bc} + \frac{a^3 b^3 d^6 \left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3} - \frac{a^2 b^4 c d^5 \left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3} - \frac{a b^5 c^2 d^4 \left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3} + \frac{b^6 c^3 d^3 \left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3} + \frac{b^4 c^4 \sqrt{-\frac{c^3}{d^3}}}{ad-bc}}{a^3 c d^2 + a^2 b c^2 d + a b^2 c^3}}{4(ad-bc)} \\
& + \frac{x^2}{2bd}
\end{aligned}$$

### 15.9 Problem number 777

$$\int \frac{1}{x^3 (a + b x^4) (c + d x^4)} dx$$

Optimal antiderivative

$$-\frac{1}{2acx^2} - \frac{b^{\frac{3}{2}} \arctan\left(\frac{x^2 \sqrt{b}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}} (-ad + bc)} + \frac{d^{\frac{3}{2}} \arctan\left(\frac{x^2 \sqrt{d}}{\sqrt{c}}\right)}{2c^{\frac{3}{2}} (-ad + bc)}$$

command

`integrate(1/x**3/(b*x**4+a)/(d*x**4+c), x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

## 15.10 Problem number 1013

$$\int \frac{\sqrt{x}}{\sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}} dx$$

Optimal antiderivative

$$\operatorname{arccosh}(\sqrt{x}) + \sqrt{x} \sqrt{-1 + \sqrt{x}} \sqrt{1 + \sqrt{x}}$$

command

```
integrate(x**(1/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{G_{6,6}^{6,2} \left( \begin{matrix} -\frac{3}{4}, -\frac{1}{4} & -\frac{1}{2}, -\frac{1}{2}, 0, 1 \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{matrix} \middle| \frac{1}{x} \right)}{2\pi^{\frac{3}{2}}} - \frac{iG_{6,6}^{2,6} \left( \begin{matrix} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} & -\frac{3}{2}, -1, -1, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{x} \right)}{2\pi^{\frac{3}{2}}}$$

## 16 Test file number 29

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.3\_General/29\_1.1.3.8\_P-  
x-c\_x^-m-a+b\_x^n-p

### 16.1 Problem number 72

$$\int \frac{(a + bx)^4}{c + dx^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4ab^3x}{d} + \frac{b^4x^2}{2d} + \frac{\left(bc^{\frac{1}{3}}(-4a^3d + b^3c) - d^{\frac{1}{3}}(-a^4d + 4ab^3c)\right) \ln\left(c^{\frac{1}{3}} + d^{\frac{1}{3}}x\right)}{3c^{\frac{2}{3}}d^{\frac{5}{3}}} \\ & - \frac{\left(bc^{\frac{1}{3}}(-4a^3d + b^3c) - d^{\frac{1}{3}}(-a^4d + 4ab^3c)\right) \ln\left(c^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2\right)}{6c^{\frac{2}{3}}d^{\frac{5}{3}}} + \frac{2a^2b^2 \ln(dx^3 + c)}{d} \\ & + \frac{\left(b^4c^{\frac{4}{3}} + 4ab^3cd^{\frac{1}{3}} - 4a^3bc^{\frac{1}{3}}d - a^4d^{\frac{4}{3}}\right) \arctan\left(\frac{(c^{\frac{1}{3}} - 2d^{\frac{1}{3}}x)\sqrt{3}}{3c^{\frac{1}{3}}}\right) \sqrt{3}}{3c^{\frac{2}{3}}d^{\frac{5}{3}}} \end{aligned}$$

command

```
integrate((b*x+a)**4/(d*x**3+c), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{4ab^3x}{d} + \frac{b^4x^2}{2d} \\ & + \text{RootSum}\left(27t^3c^2d^5 - 162t^2a^2b^2c^2d^4 + t(36a^7bcd^4 + 171a^4b^4c^2d^3 + 36ab^7c^3d^2) - a^{12}d^4 + 4a^9b^3cd^3 - 6a^6b^6c^2d^2 - \dots\right) \end{aligned}$$

## 16.2 Problem number 129

$$\int \frac{c + dx + ex^2}{(a - bx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(ex^2 + dx + c)}{8a(-bx^4 + a)^2} + \frac{x(5ex^2 + 6dx + 7c)}{32a^2(-bx^4 + a)} + \frac{3d \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{b}} \\ & + \frac{\operatorname{arctan}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right) (-5e\sqrt{a} + 21c\sqrt{b})}{64a^{\frac{11}{4}}b^{\frac{3}{4}}} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right) (5e\sqrt{a} + 21c\sqrt{b})}{64a^{\frac{11}{4}}b^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((e*x**2+d*x+c)/(-b*x**4+a)**3, x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$- \text{RootSum} \left( 268435456t^4a^{11}b^3 + t^2(-6881280a^6b^2ce - 4718592a^6b^2d^2) + t(-153600a^4bde^2 - 2709504a^3b^2c^2d) - \frac{-11acx - 10adx^2 - 9aex^3 + 7bcx^5 + 6bdx^6 + 5bex^7}{32a^4 - 64a^3bx^4 + 32a^2b^2x^8} \right)$$

**16.3 Problem number 130**

$$\int \frac{c + dx + ex^2}{(a + bx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(ex^2 + dx + c)}{8a(bx^4 + a)^2} + \frac{x(5ex^2 + 6dx + 7c)}{32a^2(bx^4 + a)} + \frac{3d \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{b}} \\ & - \frac{\ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \left(-5e\sqrt{a} + 21c\sqrt{b}\right) \sqrt{2}}{256a^{\frac{11}{4}}b^{\frac{3}{4}}} \\ & + \frac{\ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) \left(-5e\sqrt{a} + 21c\sqrt{b}\right) \sqrt{2}}{256a^{\frac{11}{4}}b^{\frac{3}{4}}} \\ & + \frac{\arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \left(5e\sqrt{a} + 21c\sqrt{b}\right) \sqrt{2}}{128a^{\frac{11}{4}}b^{\frac{3}{4}}} \\ & + \frac{\arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) \left(5e\sqrt{a} + 21c\sqrt{b}\right) \sqrt{2}}{128a^{\frac{11}{4}}b^{\frac{3}{4}}} \end{aligned}$$

command`integrate((e*x**2+d*x+c)/(b*x**4+a)**3,x)`Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\text{RootSum} \left( 268435456t^4a^{11}b^3 + t^2(6881280a^6b^2ce + 4718592a^6b^2d^2) + t(153600a^4bde^2 - 2709504a^3b^2c^2d) + 625a^5 - \frac{11acx + 10adx^2 + 9aex^3 + 7bcx^5 + 6bdx^6 + 5bex^7}{32a^4 + 64a^3bx^4 + 32a^2b^2x^8} \right)$$

### 16.4 Problem number 131

$$\int \frac{c + dx + ex^2}{(a - bx^4)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(ex^2 + dx + c)}{12a(-bx^4 + a)^3} + \frac{x(9ex^2 + 10dx + 11c)}{96a^2(-bx^4 + a)^2} + \frac{x(45ex^2 + 60dx + 77c)}{384a^3(-bx^4 + a)} + \frac{5d \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}\sqrt{b}} \\ & + \frac{\arctan\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right) (-15e\sqrt{a} + 77c\sqrt{b})}{256a^{\frac{15}{4}}b^{\frac{3}{4}}} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right) (15e\sqrt{a} + 77c\sqrt{b})}{256a^{\frac{15}{4}}b^{\frac{3}{4}}} \end{aligned}$$

command

`integrate((e*x**2+d*x+c)/(-b*x**4+a)**4,x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \operatorname{RootSum}\left(68719476736t^4a^{15}b^3 + t^2(-1211105280a^8b^2ce - 838860800a^8b^2d^2) + t(18432000a^5bde^2 + 485703680a^4e^2)\right. \\ & \left. + \frac{-153a^2cx - 132a^2dx^2 - 113a^2ex^3 + 198abcx^5 + 160abdx^6 + 126abex^7 - 77b^2cx^9 - 60b^2dx^{10} - 45b^2ex^{11}}{-384a^6 + 1152a^5bx^4 - 1152a^4b^2x^8 + 384a^3b^3x^{12}}\right) \end{aligned}$$

### 16.5 Problem number 132

$$\int \frac{c + dx + ex^2}{(a + bx^4)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(ex^2 + dx + c)}{12a(bx^4 + a)^3} + \frac{x(9ex^2 + 10dx + 11c)}{96a^2(bx^4 + a)^2} + \frac{x(45ex^2 + 60dx + 77c)}{384a^3(bx^4 + a)} \\ & + \frac{5d \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{32a^{\frac{7}{2}}\sqrt{b}} - \frac{\ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) (-15e\sqrt{a} + 77c\sqrt{b}) \sqrt{2}}{1024a^{\frac{15}{4}}b^{\frac{3}{4}}} \\ & + \frac{\ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right) (-15e\sqrt{a} + 77c\sqrt{b}) \sqrt{2}}{1024a^{\frac{15}{4}}b^{\frac{3}{4}}} \\ & + \frac{\arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) (15e\sqrt{a} + 77c\sqrt{b}) \sqrt{2}}{512a^{\frac{15}{4}}b^{\frac{3}{4}}} \\ & + \frac{\arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right) (15e\sqrt{a} + 77c\sqrt{b}) \sqrt{2}}{512a^{\frac{15}{4}}b^{\frac{3}{4}}} \end{aligned}$$



command

`integrate((e*x**2+d*x+c)/(b*x**4+a)**4,x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\text{RootSum} \left( 68719476736t^4 a^{15} b^3 + t^2 (1211105280a^8 b^2 ce + 838860800a^8 b^2 d^2) + t(18432000a^5 bde^2 - 485703680a^4 b^2 d^2) - 4a^2 f - 11abcx - 10abdx^2 + 113a^2 ex^3 + 198abcx^5 + 160abdx^6 + 126abex^7 + 77b^2 cx^9 + 60b^2 dx^{10} + 45b^2 ex^{11} \right) \\ + \frac{153a^2 cx + 132a^2 dx^2 + 113a^2 ex^3 + 198abcx^5 + 160abdx^6 + 126abex^7 + 77b^2 cx^9 + 60b^2 dx^{10} + 45b^2 ex^{11}}{384a^6 + 1152a^5 bx^4 + 1152a^4 b^2 x^8 + 384a^3 b^3 x^{12}}$$

## 16.6 Problem number 150

$$\int \frac{c + dx + ex^2 + fx^3}{(a - bx^4)^3} dx$$

Optimal antiderivative

$$\frac{x(5ex^2 + 6dx + 7c)}{32a^2(-bx^4 + a)} + \frac{af + bx(ex^2 + dx + c)}{8ab(-bx^4 + a)^2} + \frac{3d \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{b}} \\ + \frac{\operatorname{arctan}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right) (-5e\sqrt{a} + 21c\sqrt{b})}{64a^{\frac{11}{4}}b^{\frac{3}{4}}} + \frac{\operatorname{arctanh}\left(\frac{b^{\frac{1}{4}}x}{a^{\frac{1}{4}}}\right) (5e\sqrt{a} + 21c\sqrt{b})}{64a^{\frac{11}{4}}b^{\frac{3}{4}}}$$

command

`integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$-\text{RootSum} \left( 268435456t^4 a^{11} b^3 + t^2 (-6881280a^6 b^2 ce - 4718592a^6 b^2 d^2) + t(-153600a^4 bde^2 - 2709504a^3 b^2 c^2 d) - 4a^2 f - 11abcx - 10abdx^2 - 9abex^3 + 7b^2 cx^5 + 6b^2 dx^6 + 5b^2 ex^7 \right) \\ - \frac{-4a^2 f - 11abcx - 10abdx^2 - 9abex^3 + 7b^2 cx^5 + 6b^2 dx^6 + 5b^2 ex^7}{32a^4 b - 64a^3 b^2 x^4 + 32a^2 b^3 x^8}$$

### 16.7 Problem number 229

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^7(a + bx^3)} dx$$

Optimal antiderivative

$$-\frac{c}{6ax^6} + \frac{-ad + bc}{3a^2x^3} + \frac{(a^2e - abd + b^2c) \ln(x)}{a^3} - \frac{(-a^3f + a^2be - ab^2d + b^3c) \ln(bx^3 + a)}{3a^3b}$$

command

```
integrate((f*x**9+e*x**6+d*x**3+c)/x**7/(b*x**3+a),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-ac + x^3(-2ad + 2bc)}{6a^2x^6} + \frac{(a^2e - abd + b^2c) \log(x)}{a^3} + \frac{(a^3f - a^2be + ab^2d - b^3c) \log\left(\frac{a}{b} + x^3\right)}{3a^3b}$$

### 16.8 Problem number 245

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^9(a + bx^3)} dx$$

Optimal antiderivative

$$\begin{aligned} &-\frac{c}{8ax^8} + \frac{-ad + bc}{5a^2x^5} + \frac{-a^2e + abd - b^2c}{2a^3x^2} - \frac{(-a^3f + a^2be - ab^2d + b^3c) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{3a^{\frac{11}{3}}b^{\frac{1}{3}}} \\ &+ \frac{(-a^3f + a^2be - ab^2d + b^3c) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{6a^{\frac{11}{3}}b^{\frac{1}{3}}} \\ &+ \frac{(-a^3f + a^2be - ab^2d + b^3c) \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{11}{3}}b^{\frac{1}{3}}} \sqrt{3} \end{aligned}$$

command

```
integrate((f*x**9+e*x**6+d*x**3+c)/x**9/(b*x**3+a),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} &\text{RootSum}\left(27t^3a^{11}b - a^9f^3 + 3a^8bef^2 - 3a^7b^2df^2 - 3a^7b^2e^2f + 3a^6b^3cf^2 + 6a^6b^3def + a^6b^3e^3 - 6a^5b^4cef - 3a^5b^4cf^2\right. \\ &+ \left.\frac{-5a^2c + x^6(-20a^2e + 20abd - 20b^2c) + x^3(-8a^2d + 8abc)}{40a^3x^8}\right) \end{aligned}$$

### 16.9 Problem number 255

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x(a + bx^3)^2} dx$$

Optimal antiderivative

$$\frac{fx^3}{3b^2} + \frac{-a^3f + a^2be - ab^2d + b^3c}{3ab^3(bx^3 + a)} + \frac{c \ln(x)}{a^2} - \frac{(2a^3f - a^2be + b^3c) \ln(bx^3 + a)}{3a^2b^3}$$

command

```
integrate((f*x**9+e*x**6+d*x**3+c)/x/(b*x**3+a)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-a^3f + a^2be - ab^2d + b^3c}{3a^2b^3 + 3ab^4x^3} + \frac{fx^3}{3b^2} + \frac{c \log(x)}{a^2} - \frac{(2a^3f - a^2be + b^3c) \log\left(\frac{a}{b} + x^3\right)}{3a^2b^3}$$

### 16.10 Problem number 261

$$\int \frac{x^7(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-4a^3f + 3a^2be - 2ab^2d + b^3c)x^2}{2b^5} + \frac{(3a^2f - 2abe + b^2d)x^5}{5b^4} + \frac{(-2af + be)x^8}{8b^3} + \frac{fx^{11}}{11b^2} \\ & + \frac{a(-a^3f + a^2be - ab^2d + b^3c)x^2}{3b^5(bx^3 + a)} + \frac{a^{\frac{2}{3}}(-14a^3f + 11a^2be - 8ab^2d + 5b^3c) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{9b^{\frac{17}{3}}} \\ & - \frac{a^{\frac{2}{3}}(-14a^3f + 11a^2be - 8ab^2d + 5b^3c) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{18b^{\frac{17}{3}}} \\ & + \frac{a^{\frac{2}{3}}(-14a^3f + 11a^2be - 8ab^2d + 5b^3c) \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)}{9b^{\frac{17}{3}}} \sqrt{3} \end{aligned}$$

command

```
integrate(x**7*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
 & x^8 \left( -\frac{af}{4b^3} + \frac{e}{8b^2} \right) + x^5 \left( \frac{3a^2f}{5b^4} - \frac{2ae}{5b^3} + \frac{d}{5b^2} \right) \\
 & + x^2 \left( -\frac{2a^3f}{b^5} + \frac{3a^2e}{2b^4} - \frac{ad}{b^3} + \frac{c}{2b^2} \right) + \frac{x^2(-a^4f + a^3be - a^2b^2d + ab^3c)}{3ab^5 + 3b^6x^3} \\
 & + \text{RootSum} \left( 729t^3b^{17} + 2744a^{11}f^3 - 6468a^{10}bef^2 + 4704a^9b^2df^2 + 5082a^9b^2e^2f - 2940a^8b^3cf^2 - 7392a^8b^3def - 1 \right) \\
 & + \frac{fx^{11}}{11b^2}
 \end{aligned}$$

### 16.11 Problem number 263

$$\int \frac{x^4(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{(3a^2f - 2abe + b^2d)x^2}{2b^4} + \frac{(-2af + be)x^5}{5b^3} + \frac{fx^8}{8b^2} - \frac{(-a^3f + a^2be - ab^2d + b^3c)x^2}{3b^4(bx^3 + a)} \\
 & - \frac{(-11a^3f + 8a^2be - 5ab^2d + 2b^3c) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{9a^{\frac{1}{3}}b^{\frac{14}{3}}} \\
 & + \frac{(-11a^3f + 8a^2be - 5ab^2d + 2b^3c) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{18a^{\frac{1}{3}}b^{\frac{14}{3}}} \\
 & - \frac{(-11a^3f + 8a^2be - 5ab^2d + 2b^3c) \arctan\left(\frac{(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x)\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{9a^{\frac{1}{3}}b^{\frac{14}{3}}}
 \end{aligned}$$

command

```
integrate(x**4*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
 & x^5 \left( -\frac{2af}{5b^3} + \frac{e}{5b^2} \right) + x^2 \left( \frac{3a^2f}{2b^4} - \frac{ae}{b^3} + \frac{d}{2b^2} \right) + \frac{x^2(a^3f - a^2be + ab^2d - b^3c)}{3ab^4 + 3b^5x^3} \\
 & + \text{RootSum} \left( 729t^3ab^{14} - 1331a^9f^3 + 2904a^8bef^2 - 1815a^7b^2df^2 - 2112a^7b^2e^2f + 726a^6b^3cf^2 + 2640a^6b^3def + 51 \right) \\
 & + \frac{fx^8}{8b^2}
 \end{aligned}$$

## 16.12 Problem number 265

$$\int \frac{x(c + dx^3 + ex^6 + fx^9)}{(a + bx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-2af + be)x^2}{2b^3} + \frac{fx^5}{5b^2} + \frac{(-a^3f + a^2be - ab^2d + b^3c)x^2}{3ab^3(bx^3 + a)} \\ & - \frac{(8a^3f - 5a^2be + 2ab^2d + b^3c) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{9a^{\frac{4}{3}}b^{\frac{11}{3}}} \\ & + \frac{(8a^3f - 5a^2be + 2ab^2d + b^3c) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{18a^{\frac{4}{3}}b^{\frac{11}{3}}} \\ & - \frac{(8a^3f - 5a^2be + 2ab^2d + b^3c) \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{9a^{\frac{4}{3}}b^{\frac{11}{3}}} \end{aligned}$$

command

```
integrate(x*(f*x**9+e*x**6+d*x**3+c)/(b*x**3+a)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & x^2 \left( -\frac{af}{b^3} + \frac{e}{2b^2} \right) + \frac{x^2(-a^3f + a^2be - ab^2d + b^3c)}{3a^2b^3 + 3ab^4x^3} \\ & + \text{RootSum} \left( 729t^3a^4b^{11} + 512a^9f^3 - 960a^8bef^2 + 384a^7b^2df^2 + 600a^7b^2e^2f + 192a^6b^3cf^2 - 480a^6b^3def - 125a^6b^3 \right) \\ & + \frac{fx^5}{5b^2} \end{aligned}$$

## 16.13 Problem number 268

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^3(a + bx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{c}{2a^2x^2} + \frac{fx}{b^2} - \frac{(-a^3f + a^2be - ab^2d + b^3c)x}{3a^2b^2(bx^3 + a)} \\
 & - \frac{(4a^3f - a^2be - 2ab^2d + 5b^3c) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{9a^{\frac{8}{3}}b^{\frac{7}{3}}} \\
 & + \frac{(4a^3f - a^2be - 2ab^2d + 5b^3c) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{18a^{\frac{8}{3}}b^{\frac{7}{3}}} \\
 & + \frac{(4a^3f - a^2be - 2ab^2d + 5b^3c) \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{9a^{\frac{8}{3}}b^{\frac{7}{3}}}
 \end{aligned}$$

command

```
integrate((f*x**9+e*x**6+d*x**3+c)/x**3/(b*x**3+a)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
 & \frac{-3ab^2c + x^3(2a^3f - 2a^2be + 2ab^2d - 5b^3c)}{6a^3b^2x^2 + 6a^2b^3x^5} \\
 & + \text{RootSum}\left(729t^3a^8b^7 + 64a^9f^3 - 48a^8bef^2 - 96a^7b^2df^2 + 12a^7b^2e^2f + 240a^6b^3cf^2 + 48a^6b^3def - a^6b^3e^3 - 120a^5b^3ef^2 - 120a^5b^3e^2f - 120a^5b^3ef^2 - 120a^5b^3e^2f\right) \\
 & + \frac{fx}{b^2}
 \end{aligned}$$

## 16.14 Problem number 269

$$\int \frac{c + dx^3 + ex^6 + fx^9}{x^5(a + bx^3)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{c}{4a^2x^4} + \frac{-ad + 2bc}{a^3x} + \frac{(-a^3f + a^2be - ab^2d + b^3c)x^2}{3a^3b(bx^3 + a)} \\
 & - \frac{(2a^3f + a^2be - 4ab^2d + 7b^3c) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{9a^{\frac{10}{3}}b^{\frac{5}{3}}} \\
 & + \frac{(2a^3f + a^2be - 4ab^2d + 7b^3c) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{18a^{\frac{10}{3}}b^{\frac{5}{3}}} \\
 & - \frac{(2a^3f + a^2be - 4ab^2d + 7b^3c) \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{9a^{\frac{10}{3}}b^{\frac{5}{3}}}
 \end{aligned}$$

command

```
integrate((f*x**9+e*x**6+d*x**3+c)/x**5/(b*x**3+a)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\text{RootSum}\left(729t^3a^{10}b^5 + 8a^9f^3 + 12a^8bef^2 - 48a^7b^2df^2 + 6a^7b^2e^2f + 84a^6b^3cf^2 - 48a^6b^3def + a^6b^3e^3 + 84a^5b^4ce\right. \\ \left. - 3a^2bc + x^6(-4a^3f + 4a^2be - 16ab^2d + 28b^3c) + x^3(-12a^2bd + 21ab^2c)\right) \\ + \frac{-3a^2bc + x^6(-4a^3f + 4a^2be - 16ab^2d + 28b^3c) + x^3(-12a^2bd + 21ab^2c)}{12a^4bx^4 + 12a^3b^2x^7}$$

### 16.15 Problem number 403

$$\int \frac{x^4(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{a + bx^3} dx$$

Optimal antiderivative

$$-\frac{a(-ah + be)x}{b^3} + \frac{(-af + bc)x^2}{2b^2} + \frac{(-ag + bd)x^3}{3b^2} + \frac{(-ah + be)x^4}{4b^2} \\ + \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b} + \frac{a^{\frac{2}{3}}\left(b^{\frac{2}{3}}(-af + bc) + a^{\frac{2}{3}}(-ah + be)\right)\ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{3b^{\frac{10}{3}}} \\ - \frac{a^{\frac{2}{3}}\left(b^{\frac{2}{3}}(-af + bc) + a^{\frac{2}{3}}(-ah + be)\right)\ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{6b^{\frac{10}{3}}} - \frac{a(-ag + bd)\ln(bx^3 + a)}{3b^3} \\ + \frac{a^{\frac{2}{3}}\left(b^{\frac{5}{3}}c - a^{\frac{2}{3}}be - ab^{\frac{2}{3}}f + a^{\frac{5}{3}}h\right)\arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}}{3b^{\frac{10}{3}}}$$

command

```
integrate(x**4*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$x^4\left(-\frac{ah}{4b^2} + \frac{e}{4b}\right) + x^3\left(-\frac{ag}{3b^2} + \frac{d}{3b}\right) + x^2\left(-\frac{af}{2b^2} + \frac{c}{2b}\right) + x\left(\frac{a^2h}{b^3} - \frac{ae}{b^2}\right) \\ + \text{RootSum}\left(27t^3b^{10} + t^2(-27a^2b^7g + 27ab^8d) + t(-9a^4b^4fh + 9a^4b^4g^2 + 9a^3b^5ch - 18a^3b^5dg + 9a^3b^5ef - 9a^2b^6c)\right) \\ + \frac{fx^5}{5b} + \frac{gx^6}{6b} + \frac{hx^7}{7b}$$

### 16.16 Problem number 404

$$\int \frac{x^3(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{a + bx^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-af + bc)x}{b^2} + \frac{(-ag + bd)x^2}{2b^2} + \frac{(-ah + be)x^3}{3b^2} + \frac{fx^4}{4b} + \frac{gx^5}{5b} \\ & + \frac{hx^6}{6b} - \frac{a^{\frac{1}{3}} \left( b^{\frac{1}{3}}(-af + bc) - a^{\frac{1}{3}}(-ag + bd) \right) \ln \left( a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{3b^{\frac{8}{3}}} \\ & + \frac{a^{\frac{1}{3}} \left( b^{\frac{1}{3}}(-af + bc) - a^{\frac{1}{3}}(-ag + bd) \right) \ln \left( a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2 \right)}{6b^{\frac{8}{3}}} - \frac{a(-ah + be) \ln(bx^3 + a)}{3b^3} \\ & + \frac{a^{\frac{1}{3}} \left( b^{\frac{4}{3}}c + a^{\frac{1}{3}}bd - ab^{\frac{1}{3}}f - a^{\frac{4}{3}}g \right) \arctan \left( \frac{\left( a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x \right) \sqrt{3}}{3a^{\frac{1}{3}}} \right) \sqrt{3}}{3b^{\frac{8}{3}}} \end{aligned}$$

command

```
integrate(x**3*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & x^3 \left( -\frac{ah}{3b^2} + \frac{e}{3b} \right) + x^2 \left( -\frac{ag}{2b^2} + \frac{d}{2b} \right) + x \left( -\frac{af}{b^2} + \frac{c}{b} \right) \\ & + \text{RootSum} \left( 27t^3b^9 + t^2(-27a^2b^6h + 27ab^7e) + t(9a^4b^3h^2 - 18a^3b^4eh + 9a^3b^4fg - 9a^2b^5cg - 9a^2b^5df + 9a^2b^5e^2 + \right. \\ & \left. + \frac{fx^4}{4b} + \frac{gx^5}{5b} + \frac{hx^6}{6b} \right) \end{aligned}$$

### 16.17 Problem number 405

$$\int \frac{x^2(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{a + bx^3} dx$$



Optimal antiderivative

$$\begin{aligned} & \frac{(-ag + bd)x}{b^2} + \frac{(-ah + be)x^2}{2b^2} + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} \\ & - \frac{a^{\frac{1}{3}} \left( b^{\frac{1}{3}}(-ag + bd) - a^{\frac{1}{3}}(-ah + be) \right) \ln \left( a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{3b^{\frac{8}{3}}} \\ & + \frac{a^{\frac{1}{3}} \left( b^{\frac{1}{3}}(-ag + bd) - a^{\frac{1}{3}}(-ah + be) \right) \ln \left( a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2 \right)}{6b^{\frac{8}{3}}} + \frac{(-af + bc) \ln(bx^3 + a)}{3b^2} \\ & + \frac{a^{\frac{1}{3}} \left( b^{\frac{4}{3}}d + a^{\frac{1}{3}}be - ab^{\frac{1}{3}}g - a^{\frac{4}{3}}h \right) \arctan \left( \frac{\left( a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x \right) \sqrt{3}}{3a^{\frac{1}{3}}} \right) \sqrt{3}}{3b^{\frac{8}{3}}} \end{aligned}$$

command

```
integrate(x**2*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & x^2 \left( -\frac{ah}{2b^2} + \frac{e}{2b} \right) + x \left( -\frac{ag}{b^2} + \frac{d}{b} \right) \\ & + \text{RootSum} \left( 27t^3b^8 + t^2(27ab^6f - 27b^7c) + t(9a^3b^3gh - 9a^2b^4dh - 9a^2b^4eg + 9a^2b^4f^2 - 18ab^5cf + 9ab^5de + 9b^6c^2) \right) \\ & + \frac{fx^3}{3b} + \frac{gx^4}{4b} + \frac{hx^5}{5b} \end{aligned}$$

## 16.18 Problem number 406

$$\int \frac{x(c + dx + ex^2 + fx^3 + gx^4 + hx^5)}{a + bx^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-ah + be)x}{b^2} + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b} - \frac{\left( b^{\frac{2}{3}}(-af + bc) + a^{\frac{2}{3}}(-ah + be) \right) \ln \left( a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{3a^{\frac{1}{3}}b^{\frac{7}{3}}} \\ & + \frac{\left( b^{\frac{2}{3}}(-af + bc) + a^{\frac{2}{3}}(-ah + be) \right) \ln \left( a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2 \right)}{6a^{\frac{1}{3}}b^{\frac{7}{3}}} + \frac{(-ag + bd) \ln(bx^3 + a)}{3b^2} \\ & - \frac{\left( b^{\frac{5}{3}}c - a^{\frac{2}{3}}be - ab^{\frac{2}{3}}f + a^{\frac{5}{3}}h \right) \arctan \left( \frac{\left( a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x \right) \sqrt{3}}{3a^{\frac{1}{3}}} \right) \sqrt{3}}{3a^{\frac{1}{3}}b^{\frac{7}{3}}} \end{aligned}$$

command

```
integrate(x*(h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$x \left( -\frac{ah}{b^2} + \frac{e}{b} \right) + \text{RootSum} \left( 27t^3ab^7 + t^2(27a^2b^5g - 27ab^6d) + t(-9a^3b^3fh + 9a^3b^3g^2 + 9a^2b^4ch - 18a^2b^4dg + 9a^2b^4ef - 9ab^5ce - \dots) \right) + \frac{fx^2}{2b} + \frac{gx^3}{3b} + \frac{hx^4}{4b}$$

### 16.19 Problem number 407

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4 + hx^5}{a + bx^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{fx}{b} + \frac{gx^2}{2b} + \frac{hx^3}{3b} + \frac{\left( b^{\frac{1}{3}}(-af + bc) - a^{\frac{1}{3}}(-ag + bd) \right) \ln\left( a^{\frac{1}{3}} + b^{\frac{1}{3}}x \right)}{3a^{\frac{2}{3}}b^{\frac{5}{3}}} \\ & - \frac{\left( b^{\frac{1}{3}}(-af + bc) - a^{\frac{1}{3}}(-ag + bd) \right) \ln\left( a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2 \right)}{6a^{\frac{2}{3}}b^{\frac{5}{3}}} + \frac{(-ah + be) \ln(bx^3 + a)}{3b^2} \\ & - \frac{\left( b^{\frac{4}{3}}c + a^{\frac{1}{3}}bd - ab^{\frac{1}{3}}f - a^{\frac{4}{3}}g \right) \arctan\left( \frac{\left( a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x \right) \sqrt{3}}{3a^{\frac{1}{3}}} \right) \sqrt{3}}{3a^{\frac{2}{3}}b^{\frac{5}{3}}} \end{aligned}$$

command

```
integrate((h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\text{RootSum} \left( 27t^3a^2b^6 + t^2(27a^3b^4h - 27a^2b^5e) + t(9a^4b^2h^2 - 18a^3b^3eh + 9a^3b^3fg - 9a^2b^4cg - 9a^2b^4df + 9a^2b^4e^2 + \dots) \right) + \frac{fx}{b} + \frac{gx^2}{2b} + \frac{hx^3}{3b}$$

## 16.20 Problem number 491

$$\int \frac{c + dx + ex^2 + fx^3}{(a + bx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(5ex^2 + 6dx + 7c)}{32a^2(bx^4 + a)} + \frac{-af + bx(ex^2 + dx + c)}{8ab(bx^4 + a)^2} + \frac{3d \arctan\left(\frac{x^2\sqrt{b}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}\sqrt{b}} \\ & - \frac{\ln\left(-a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right)\left(-5e\sqrt{a} + 21c\sqrt{b}\right)\sqrt{2}}{256a^{\frac{11}{4}}b^{\frac{3}{4}}} \\ & + \frac{\ln\left(a^{\frac{1}{4}}b^{\frac{1}{4}}x\sqrt{2} + \sqrt{a} + x^2\sqrt{b}\right)\left(-5e\sqrt{a} + 21c\sqrt{b}\right)\sqrt{2}}{256a^{\frac{11}{4}}b^{\frac{3}{4}}} \\ & + \frac{\arctan\left(-1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)\left(5e\sqrt{a} + 21c\sqrt{b}\right)\sqrt{2}}{128a^{\frac{11}{4}}b^{\frac{3}{4}}} \\ & + \frac{\arctan\left(1 + \frac{b^{\frac{1}{4}}x\sqrt{2}}{a^{\frac{1}{4}}}\right)\left(5e\sqrt{a} + 21c\sqrt{b}\right)\sqrt{2}}{128a^{\frac{11}{4}}b^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \text{RootSum}\left(268435456t^4a^{11}b^3 + t^2(6881280a^6b^2ce + 4718592a^6b^2d^2) + t(153600a^4bde^2 - 2709504a^3b^2c^2d) + 625a^7\right) \\ & + \frac{-4a^2f + 11abcx + 10abdx^2 + 9abex^3 + 7b^2cx^5 + 6b^2dx^6 + 5b^2ex^7}{32a^4b + 64a^3b^2x^4 + 32a^2b^3x^8} \end{aligned}$$

## 17 Test file number 33

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.1\_Quadratic/33\_1.2.1.2-d+e\_x-^m-a+b\_x+c\_x^2-^p

### 17.1 Problem number 255

$$\int \frac{(bx + cx^2)^3}{(d + ex)^7} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{d^3(-be + cd)^3}{6e^7 (ex + d)^6} + \frac{3d^2(-be + cd)^2(-be + 2cd)}{5e^7 (ex + d)^5} \\ & -\frac{3d(-be + cd)(b^2e^2 - 5bcde + 5c^2d^2)}{4e^7 (ex + d)^4} + \frac{(-be + 2cd)(b^2e^2 - 10bcde + 10c^2d^2)}{3e^7 (ex + d)^3} \\ & -\frac{3c(b^2e^2 - 5bcde + 5c^2d^2)}{2e^7 (ex + d)^2} + \frac{3c^2(-be + 2cd)}{e^7 (ex + d)} + \frac{c^3 \ln(ex + d)}{e^7} \end{aligned}$$

command

```
integrate((c*x**2+b*x)**3/(e*x+d)**7,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{c^3 \log(d + ex)}{e^7} \\ & + \frac{-b^3d^3e^3 - 6b^2cd^4e^2 - 30bc^2d^5e + 147c^3d^6 + x^5(-180bc^2e^6 + 360c^3de^5) + x^4(-90b^2ce^6 - 450bc^2de^5 + 1350c^3d^2e^4)}{60d^6e^7 + 360d^5e^6} \end{aligned}$$

### 17.2 Problem number 256

$$\int \frac{(bx + cx^2)^3}{(d + ex)^8} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{d^3(-be + cd)^3}{7e^7 (ex + d)^7} + \frac{d^2(-be + cd)^2(-be + 2cd)}{2e^7 (ex + d)^6} \\ & -\frac{3d(-be + cd)(b^2e^2 - 5bcde + 5c^2d^2)}{5e^7 (ex + d)^5} + \frac{(-be + 2cd)(b^2e^2 - 10bcde + 10c^2d^2)}{4e^7 (ex + d)^4} \\ & -\frac{c(b^2e^2 - 5bcde + 5c^2d^2)}{e^7 (ex + d)^3} + \frac{3c^2(-be + 2cd)}{2e^7 (ex + d)^2} - \frac{c^3}{e^7 (ex + d)} \end{aligned}$$

command

```
integrate((c*x**2+b*x)**3/(e*x+d)**8,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-b^3 d^3 e^3 - 4b^2 c d^4 e^2 - 10bc^2 d^5 e - 20c^3 d^6 - 140c^3 e^6 x^6 + x^5(-210bc^2 e^6 - 420c^3 d e^5) + x^4(-140b^2 c e^6 - 350bc^2 d e^5 - 140d^7 e^7 + 980d^6 e^8 x + \dots}{140d^7 e^7 + 980d^6 e^8 x + \dots}$$

### 17.3 Problem number 275

$$\int \frac{(d+ex)^7}{(bx+cx^2)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{d^7}{2b^3 x^2} + \frac{d^6(-7be+3cd)}{b^4 x} + \frac{e^6(-3be+7cd)x}{c^4} + \frac{e^7 x^2}{2c^3} + \frac{(-be+cd)^7}{2b^3 c^5 (cx+b)^2} \\ & + \frac{(-be+cd)^6(4be+3cd)}{b^4 c^5 (cx+b)} + \frac{3d^5(7b^2 e^2 - 7bcde + 2c^2 d^2) \ln(x)}{b^5} \\ & - \frac{3(-be+cd)^5(2b^2 e^2 + 3bcde + 2c^2 d^2) \ln(cx+b)}{b^5 c^5} \end{aligned}$$

command

```
integrate((e*x+d)**7/(c*x**2+b*x)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & x \left( -\frac{3be^7}{c^4} + \frac{7de^6}{c^3} \right) \\ & + \frac{-b^3 c^5 d^7 + x^3(8b^7 ce^7 - 42b^6 c^2 de^6 + 84b^5 c^3 d^2 e^5 - 70b^4 c^4 d^3 e^4 + 42b^2 c^6 d^5 e^2 - 42bc^7 d^6 e + 12c^8 d^7) + x^2(7b^8 e^7 - 35b^7 c e^6 + 35b^6 c^2 d e^5 - 35b^5 c^3 d^2 e^4 + 35b^4 c^4 d^3 e^3 - 35b^3 c^5 d^4 e^2 + 35b^2 c^6 d^5 e - 35b c^7 d^6 + 12c^8 d^7)}{2b^6 c^5 x^2 + 4b^5 c^4 x + 4b^4 c^3} \\ & + \frac{e^7 x^2}{2c^3} + \frac{3d^5(7b^2 e^2 - 7bcde + 2c^2 d^2) \log \left( x + \frac{-21b^3 c^4 d^5 e^2 + 21b^2 c^5 d^6 e - 6bc^6 d^7 + 3bc^4 d^5(7b^2 e^2 - 7bcde + 2c^2 d^2)}{6b^7 e^7 - 21b^6 c d e^6 + 21b^5 c^2 d^2 e^5 - 42b^2 c^5 d^5 e^2 + 42bc^6 d^6 e - 12c^7 d^7} \right)}{b^5} \\ & + \frac{3(be-cd)^5(2b^2 e^2 + 3bcde + 2c^2 d^2) \log \left( x + \frac{-21b^3 c^4 d^5 e^2 + 21b^2 c^5 d^6 e - 6bc^6 d^7 + 3b(be-cd)^5(2b^2 e^2 + 3bcde + 2c^2 d^2)}{6b^7 e^7 - 21b^6 c d e^6 + 21b^5 c^2 d^2 e^5 - 42b^2 c^5 d^5 e^2 + 42bc^6 d^6 e - 12c^7 d^7} \right)}{b^5 c^5} \end{aligned}$$

### 17.4 Problem number 486

$$\int \frac{(a + cx^2)^3}{(d + ex)^9} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(ae^2 + cd^2)^3}{8e^7(ex + d)^8} + \frac{6cd(ae^2 + cd^2)^2}{7e^7(ex + d)^7} - \frac{c(ae^2 + cd^2)(ae^2 + 5cd^2)}{2e^7(ex + d)^6} \\ & + \frac{4c^2d(3ae^2 + 5cd^2)}{5e^7(ex + d)^5} - \frac{3c^2(ae^2 + 5cd^2)}{4e^7(ex + d)^4} + \frac{2c^3d}{e^7(ex + d)^3} - \frac{c^3}{2e^7(ex + d)^2} \end{aligned}$$

command

```
integrate((c*x**2+a)**3/(e*x+d)**9, x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-35a^3e^6 - 5a^2cd^2e^4 - 3ac^2d^4e^2 - 5c^3d^6 - 280c^3de^5x^5 - 140c^3e^6x^6 + x^4(-210ac^2e^6 - 350c^3d^2e^4) + x^3(-168ac^2d^2e^4 - 168ac^3d^2e^4) + x^2(-168ac^2d^2e^4 - 168ac^3d^2e^4) + x(-168ac^2d^2e^4 - 168ac^3d^2e^4) - 168ac^2d^2e^4 - 168ac^3d^2e^4}{280d^8e^7 + 2240d^7e^8x + 7840d^6e^9x^2 + 15680d^5e^{10}x^3 + 19600d^4e^{11}x^4 + 15680d^3e^{12}x^5 + 11200d^2e^{13}x^6 + 5600de^{14}x^7 + 1120e^{15}x^8}$$

### 17.5 Problem number 487

$$\int \frac{(a + cx^2)^3}{(d + ex)^{10}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(ae^2 + cd^2)^3}{9e^7(ex + d)^9} + \frac{3cd(ae^2 + cd^2)^2}{4e^7(ex + d)^8} - \frac{3c(ae^2 + cd^2)(ae^2 + 5cd^2)}{7e^7(ex + d)^7} \\ & + \frac{2c^2d(3ae^2 + 5cd^2)}{3e^7(ex + d)^6} - \frac{3c^2(ae^2 + 5cd^2)}{5e^7(ex + d)^5} + \frac{3c^3d}{2e^7(ex + d)^4} - \frac{c^3}{3e^7(ex + d)^3} \end{aligned}$$

command

```
integrate((c*x**2+a)**3/(e*x+d)**10, x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-140a^3e^6 - 15a^2cd^2e^4 - 6ac^2d^4e^2 - 5c^3d^6 - 630c^3de^5x^5 - 420c^3e^6x^6 + x^4(-756ac^2e^6 - 630c^3d^2e^4) + x^3(-504ac^2d^2e^4 - 504ac^3d^2e^4) + x^2(-504ac^2d^2e^4 - 504ac^3d^2e^4) + x(-504ac^2d^2e^4 - 504ac^3d^2e^4) - 504ac^2d^2e^4 - 504ac^3d^2e^4}{1260d^9e^7 + 11340d^8e^8x + 45360d^7e^9x^2 + 105840d^6e^{10}x^3 + 158760d^5e^{11}x^4 + 158760d^4e^{12}x^5 + 11340d^3e^{13}x^6 + 45360d^2e^{14}x^7 + 105840de^{15}x^8 + 158760e^{16}x^9}$$

## 17.6 Problem number 1288

$$\int \frac{(bd + 2cdx)^{5/2}}{a + bx + cx^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4d(2cdx + bd)^{\frac{3}{2}}}{3} + 2(-4ac + b^2)^{\frac{3}{4}} d^{\frac{5}{2}} \arctan\left(\frac{\sqrt{d}(2cx + b)}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}\right) \\ & - 2(-4ac + b^2)^{\frac{3}{4}} d^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d}(2cx + b)}{(-4ac + b^2)^{\frac{1}{4}} \sqrt{d}}\right) \end{aligned}$$

command

```
integrate((2*c*d*x+b*d)**(5/2)/(c*x**2+b*x+a),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & 32abcd^4 \operatorname{RootSum}\left(t^4(16384a^3c^3d^6 - 12288a^2b^2c^2d^6 + 3072ab^4cd^6 - 256b^6d^6) + 1, \left(t \mapsto t \log\left(16tacd^2 - 4tb^2d^2 + \sqrt{bd + 2cdx}\right)\right)\right) \\ & - 32abcd^4 \operatorname{RootSum}\left(t^4(16384a^3c^3d^6 - 12288a^2b^2c^2d^6 + 3072ab^4cd^6 - 256b^6d^6) + 1, \left(t \mapsto t \log\left(16tacd^2 - 4tb^2d^2 + \sqrt{bd + 2cdx}\right)\right)\right) \\ & - 16acd^3 \operatorname{RootSum}\left(t^4(1024acd^2 - 256b^2d^2) + 1, \left(t \mapsto t \log\left(256t^3acd^2 - 64t^3b^2d^2 + \sqrt{bd + 2cdx}\right)\right)\right) \\ & - 8b^3d^4 \operatorname{RootSum}\left(t^4(16384a^3c^3d^6 - 12288a^2b^2c^2d^6 + 3072ab^4cd^6 - 256b^6d^6) + 1, \left(t \mapsto t \log\left(16tacd^2 - 4tb^2d^2 + \sqrt{bd + 2cdx}\right)\right)\right) \\ & + 8b^3d^4 \operatorname{RootSum}\left(t^4(16384a^3c^3d^6 - 12288a^2b^2c^2d^6 + 3072ab^4cd^6 - 256b^6d^6) + 1, \left(t \mapsto t \log\left(16tacd^2 - 4tb^2d^2 + \sqrt{bd + 2cdx}\right)\right)\right) \\ & + 8b^2d^3 \operatorname{RootSum}\left(t^4(1024acd^2 - 256b^2d^2) + 1, \left(t \mapsto t \log\left(256t^3acd^2 - 64t^3b^2d^2 + \sqrt{bd + 2cdx}\right)\right)\right) \\ & - 8b^2d^3 \operatorname{RootSum}\left(t^4(1024acd^2 - 256b^2d^2) + 1, \left(t \mapsto t \log\left(256t^3acd^2 - 64t^3b^2d^2 + \sqrt{bd + 2cdx}\right)\right)\right) \\ & + 4b^2d^3 \operatorname{RootSum}\left(t^4(1024acd^2 - 256b^2d^2) + 1, \left(t \mapsto t \log\left(256t^3acd^2 - 64t^3b^2d^2 + \sqrt{bd + 2cdx}\right)\right)\right) \\ & + \frac{4d(bd + 2cdx)^{\frac{3}{2}}}{3} \end{aligned}$$





### 17.9 Problem number 1480

$$\int \frac{(a^2 + 2abx + b^2x^2)^2}{(d + ex)^{11}} dx$$

Optimal antiderivative

$$-\frac{(-ae + bd)^4}{10e^5 (ex + d)^{10}} + \frac{4b(-ae + bd)^3}{9e^5 (ex + d)^9} - \frac{3b^2(-ae + bd)^2}{4e^5 (ex + d)^8} + \frac{4b^3(-ae + bd)}{7e^5 (ex + d)^7} - \frac{b^4}{6e^5 (ex + d)^6}$$

command

```
integrate((b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**11,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-126a^4e^4 - 56a^3bde^3 - 21a^2b^2d^2e^2 - 6ab^3d^3e - b^4d^4 - 210b^4e^4x^4 + x^3(-720ab^3e^4 - 120b^4de^3) + x^2(-945a^2b^2e^4}{1260d^{10}e^5 + 12600d^9e^6x + 56700d^8e^7x^2 + 151200d^7e^8x^3 + 264600d^6e^9x^4 + 317520d^5e^{10}x^5 + 264600d^4e^{11}}$$

### 17.10 Problem number 1495

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^6} dx$$

Optimal antiderivative

$$\frac{b^6x}{e^6} - \frac{(-ae + bd)^6}{5e^7 (ex + d)^5} + \frac{3b(-ae + bd)^5}{2e^7 (ex + d)^4} - \frac{5b^2(-ae + bd)^4}{e^7 (ex + d)^3} + \frac{10b^3(-ae + bd)^3}{e^7 (ex + d)^2} - \frac{15b^4(-ae + bd)^2}{e^7 (ex + d)} - \frac{6b^5(-ae + bd) \ln(ex + d)}{e^7}$$

command

```
integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**6,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{b^6x}{e^6} + \frac{6b^5(ae - bd) \log(d + ex)}{e^7} - \frac{2a^6e^6 - 3a^5bde^5 - 5a^4b^2d^2e^4 - 10a^3b^3d^3e^3 - 30a^2b^4d^4e^2 + 137ab^5d^5e - 87b^6d^6 + x^4(-150a^2b^4e^6 + 300ab^5de^5}{e^7}$$

### 17.11 Problem number 1496

$$\int \frac{(a^2 + 2abx + b^2x^2)^3}{(d + ex)^7} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-ae + bd)^6}{6e^7 (ex + d)^6} + \frac{6b(-ae + bd)^5}{5e^7 (ex + d)^5} - \frac{15b^2(-ae + bd)^4}{4e^7 (ex + d)^4} + \frac{20b^3(-ae + bd)^3}{3e^7 (ex + d)^3} \\ & - \frac{15b^4(-ae + bd)^2}{2e^7 (ex + d)^2} + \frac{6b^5(-ae + bd)}{e^7 (ex + d)} + \frac{b^6 \ln(ex + d)}{e^7} \end{aligned}$$

command

```
integrate((b**2*x**2+2*a*b*x+a**2)**3/(e*x+d)**7,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{b^6 \log(d + ex)}{e^7} \\ & + \frac{-10a^6e^6 - 12a^5bde^5 - 15a^4b^2d^2e^4 - 20a^3b^3d^3e^3 - 30a^2b^4d^4e^2 - 60ab^5d^5e + 147b^6d^6 + x^5(-360ab^5e^6 + 360b^6de^5)}{e^7} \end{aligned}$$

### 17.12 Problem number 1526

$$\int \frac{(d + ex)^7}{(a^2 + 2abx + b^2x^2)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{e^6(-6ae + 7bd)x}{b^7} + \frac{e^7x^2}{2b^6} - \frac{(-ae + bd)^7}{5b^8 (bx + a)^5} - \frac{7e(-ae + bd)^6}{4b^8 (bx + a)^4} - \frac{7e^2(-ae + bd)^5}{b^8 (bx + a)^3} \\ & - \frac{35e^3(-ae + bd)^4}{2b^8 (bx + a)^2} - \frac{35e^4(-ae + bd)^3}{b^8 (bx + a)} + \frac{21e^5(-ae + bd)^2 \ln(bx + a)}{b^8} \end{aligned}$$

command

```
integrate((e*x+d)**7/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& x \left( -\frac{6ae^7}{b^7} + \frac{7de^6}{b^6} \right) \\
& + \frac{459a^7e^7 - 1218a^6bde^6 + 959a^5b^2d^2e^5 - 140a^4b^3d^3e^4 - 35a^3b^4d^4e^3 - 14a^2b^5d^5e^2 - 7ab^6d^6e - 4b^7d^7 + x^4(700a^3b^4}{2b^6} \\
& + \frac{e^7x^2}{2b^6} + \frac{21e^5(ae - bd)^2 \log(a + bx)}{b^8}
\end{aligned}$$

**17.13 Problem number 1527**

$$\int \frac{(d + ex)^6}{(a^2 + 2abx + b^2x^2)^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{e^6x}{b^6} - \frac{(-ae + bd)^6}{5b^7(bx + a)^5} - \frac{3e(-ae + bd)^5}{2b^7(bx + a)^4} - \frac{5e^2(-ae + bd)^4}{b^7(bx + a)^3} \\
& - \frac{10e^3(-ae + bd)^3}{b^7(bx + a)^2} - \frac{15e^4(-ae + bd)^2}{b^7(bx + a)} + \frac{6e^5(-ae + bd) \ln(bx + a)}{b^7}
\end{aligned}$$

command

```
integrate((e*x+d)**6/(b**2*x**2+2*a*b*x+a**2)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{-87a^6e^6 + 137a^5bde^5 - 30a^4b^2d^2e^4 - 10a^3b^3d^3e^3 - 5a^2b^4d^4e^2 - 3ab^5d^5e - 2b^6d^6 + x^4(-150a^2b^4e^6 + 300ab^5de^5 -}{b^6} \\
& + \frac{e^6x}{b^6} - \frac{6e^5(ae - bd) \log(a + bx)}{b^7}
\end{aligned}$$

### 17.14 Problem number 1798

$$\int \frac{(ac + (bc + ad)x + bdx^2)^3}{(a + bx)^{12}} dx$$

Optimal antiderivative

$$-\frac{(-ad + bc)^3}{8b^4 (bx + a)^8} - \frac{3d(-ad + bc)^2}{7b^4 (bx + a)^7} - \frac{d^2(-ad + bc)}{2b^4 (bx + a)^6} - \frac{d^3}{5b^4 (bx + a)^5}$$

command

```
integrate((a*c+(a*d+b*c)*x+b*d*x**2)**3/(b*x+a)**12,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-a^3 d^3 - 5a^2 b c d^2 - 15a b^2 c^2 d - 35b^3 c^3 - 56b^3 d^3 x^3 + x^2(-28a b^2 d^3 - 140b^3 c d^2) + x(-8a^2 b d^3 - 40a b^2 c d^2 - 120b^3 c^2 d) + 120a^2 b^2 c^2 d^2 + 120a^2 b^3 c^3}{280a^8 b^4 + 2240a^7 b^5 x + 7840a^6 b^6 x^2 + 15680a^5 b^7 x^3 + 19600a^4 b^8 x^4 + 15680a^3 b^9 x^5 + 7840a^2 b^{10} x^6 + 2240a b^{11} x^7 + 280b^{12}}$$

### 17.15 Problem number 1863

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^{10}} dx$$

Optimal antiderivative

$$\frac{(-a e^2 + c d^2)^3}{6e^4 (ex + d)^6} - \frac{3cd(-a e^2 + c d^2)^2}{5e^4 (ex + d)^5} + \frac{3c^2 d^2 (-a e^2 + c d^2)}{4e^4 (ex + d)^4} - \frac{c^3 d^3}{3e^4 (ex + d)^3}$$

command

```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**10,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-10a^3 e^6 - 6a^2 c d^2 e^4 - 3a c^2 d^4 e^2 - c^3 d^6 - 20c^3 d^3 e^3 x^3 + x^2(-45a c^2 d^2 e^4 - 15c^3 d^4 e^2) + x(-36a^2 c d e^5 - 18a c^2 d^3 e^3 - 12a^2 c^2 d^2 e) + 12a^2 c^2 d^2 e^2 + 12a^2 c^3 d^3 e}{60d^6 e^4 + 360d^5 e^5 x + 900d^4 e^6 x^2 + 1200d^3 e^7 x^3 + 900d^2 e^8 x^4 + 360d e^9 x^5 + 60e^{10} x^6}$$

### 17.16 Problem number 1864

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^3}{(d + ex)^{11}} dx$$

Optimal antiderivative

$$\frac{(-ae^2 + cd^2)^3}{7e^4(ex + d)^7} - \frac{cd(-ae^2 + cd^2)^2}{2e^4(ex + d)^6} + \frac{3c^2d^2(-ae^2 + cd^2)}{5e^4(ex + d)^5} - \frac{c^3d^3}{4e^4(ex + d)^4}$$

command

```
integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**3/(e*x+d)**11,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-20a^3e^6 - 10a^2cd^2e^4 - 4ac^2d^4e^2 - c^3d^6 - 35c^3d^3e^3x^3 + x^2(-84ac^2d^2e^4 - 21c^3d^4e^2) + x(-70a^2cde^5 - 28ac^2d^3e^3)}{140d^7e^4 + 980d^6e^5x + 2940d^5e^6x^2 + 4900d^4e^7x^3 + 4900d^3e^8x^4 + 2940d^2e^9x^5 + 980de^{10}x^6 + 140e^{11}x^7}$$

### 17.17 Problem number 1896

$$\int \frac{(d + ex)^{10}}{(ade + (cd^2 + ae^2)x + cdex^2)^4} dx$$

Optimal antiderivative

$$\frac{e^4(10a^2e^4 - 24acd^2e^2 + 15c^2d^4)x}{c^6d^6} + \frac{e^5(-2ae^2 + 3cd^2)x^2}{c^5d^5} + \frac{e^6x^3}{3c^4d^4} - \frac{(-ae^2 + cd^2)^6}{3c^7d^7(cdx + ae)^3} - \frac{3e(-ae^2 + cd^2)^5}{c^7d^7(cdx + ae)^2} - \frac{15e^2(-ae^2 + cd^2)^4}{c^7d^7(cdx + ae)} + \frac{20e^3(-ae^2 + cd^2)^3 \ln(cdx + ae)}{c^7d^7}$$

command

```
integrate((e*x+d)**10/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$x^2 \left( -\frac{2ae^7}{c^5d^5} + \frac{3e^5}{c^4d^3} \right) + x \left( \frac{10a^2e^8}{c^6d^6} - \frac{24ae^6}{c^5d^4} + \frac{15e^4}{c^4d^2} \right) + \frac{-37a^6e^{12} + 141a^5cd^2e^{10} - 195a^4c^2d^4e^8 + 110a^3c^3d^6e^6 - 15a^2c^4d^8e^4 - 3ac^5d^{10}e^2 - c^6d^{12} + x^2(-45a^4c^2d^2e^{10} + 1}{3a^3c^7d^7e^3} - \frac{e^6x^3}{3c^4d^4} - \frac{20e^3(ae^2 - cd^2)^3 \log(ae + cdx)}{c^7d^7}$$

## 17.18 Problem number 1897

$$\int \frac{(d+ex)^9}{(ade+(cd^2+ae^2)x+cdex^2)^4} dx$$

Optimal antiderivative

$$\frac{e^4(-4ae^2+5cd^2)x}{c^5d^5} + \frac{e^5x^2}{2c^4d^4} - \frac{(-ae^2+cd^2)^5}{3c^6d^6(cd^2+ae^2)^3} - \frac{5e(-ae^2+cd^2)^4}{2c^6d^6(cd^2+ae^2)^2} - \frac{10e^2(-ae^2+cd^2)^3}{c^6d^6(cd^2+ae^2)} + \frac{10e^3(-ae^2+cd^2)^2 \ln(cd^2+ae^2)}{c^6d^6}$$

command

```
integrate((e*x+d)**9/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**4,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$x \left( -\frac{4ae^6}{c^5d^5} + \frac{5e^4}{c^4d^3} \right) + \frac{47a^5e^{10} - 130a^4cd^2e^8 + 110a^3c^2d^4e^6 - 20a^2c^3d^6e^4 - 5ac^4d^8e^2 - 2c^5d^{10} + x^2(60a^3c^2d^2e^8 - 180a^2c^3d^4e^6 + 180ac^4d^6e^4 - 60a^3c^6d^6e^3 + 18a^2c^7d^7e^2x + 18ac^8d^8e^2x^2)}{6a^3c^6d^6e^3 + 18a^2c^7d^7e^2x + 18ac^8d^8e^2x^2} + \frac{e^5x^2}{2c^4d^4} + \frac{10e^3(ae^2 - cd^2)^2 \log(ae + cd^2x)}{c^6d^6}$$

## 17.19 Problem number 2014

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^2} dx$$

Optimal antiderivative

$$\frac{e \operatorname{arctanh} \left( \frac{\sqrt{c} \sqrt{d} \sqrt{ex+d}}{\sqrt{-ae^2+cd^2}} \right)}{(-ae^2+cd^2)^{3/2} \sqrt{c} \sqrt{d}} - \frac{\sqrt{ex+d}}{(-ae^2+cd^2)(cd^2+ae^2)}$$

command

```
integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{e \sqrt{-\frac{1}{cd(ae^2 - cd^2)^3}} \log\left(-a^2 e^4 \sqrt{-\frac{1}{cd(ae^2 - cd^2)^3}} + 2acd^2 e^2 \sqrt{-\frac{1}{cd(ae^2 - cd^2)^3}} - c^2 d^4 \sqrt{-\frac{1}{cd(ae^2 - cd^2)^3}}\right)}{e \sqrt{-\frac{1}{cd(ae^2 - cd^2)^3}} \log\left(a^2 e^4 \sqrt{-\frac{1}{cd(ae^2 - cd^2)^3}} - 2acd^2 e^2 \sqrt{-\frac{1}{cd(ae^2 - cd^2)^3}} + c^2 d^4 \sqrt{-\frac{1}{cd(ae^2 - cd^2)^3}}\right) + \frac{2e\sqrt{d+ex}}{2a^2e^4 - 2acd^2e^2 + 2acde^3x - 2c^2d^3ex}}$$

**17.20 Problem number 2129**

$$\int \frac{(a + bx + cx^2)^2}{(d + ex)^7} dx$$

Optimal antiderivative

$$-\frac{(ae^2 - bde + cd^2)^2}{6e^5(ex + d)^6} + \frac{2(-be + 2cd)(ae^2 - bde + cd^2)}{5e^5(ex + d)^5} + \frac{-6c^2d^2 - b^2e^2 + 2ce(-ae + 3bd)}{4e^5(ex + d)^4} + \frac{2c(-be + 2cd)}{3e^5(ex + d)^3} - \frac{c^2}{2e^5(ex + d)^2}$$

command`integrate((c*x**2+b*x+a)**2/(e*x+d)**7, x)`Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-10a^2e^4 - 4abde^3 - 2acd^2e^2 - b^2d^2e^2 - 2bcd^3e - 2c^2d^4 - 30c^2e^4x^4 + x^3(-40bce^4 - 40c^2de^3) + x^2(-30ace^4 - 150a^2ce^3 - 30bde^3 - 30c^2d^2e^2 - 30cd^3e - 30d^4) + x(-30ace^4 - 150a^2ce^3 - 30bde^3 - 30c^2d^2e^2 - 30cd^3e - 30d^4) - 30d^4}{60d^6e^5 + 360d^5e^6x + 900d^4e^7x^2 + 1200d^3e^8x^3 + 900d^2e^9x^4}$$

## 17.21 Problem number 2130

$$\int \frac{(a + bx + cx^2)^2}{(d + ex)^8} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(ae^2 - bde + cd^2)^2}{7e^5(ex + d)^7} + \frac{(-be + 2cd)(ae^2 - bde + cd^2)}{3e^5(ex + d)^6} \\ & + \frac{-6c^2d^2 - b^2e^2 + 2ce(-ae + 3bd)}{5e^5(ex + d)^5} + \frac{c(-be + 2cd)}{2e^5(ex + d)^4} - \frac{c^2}{3e^5(ex + d)^3} \end{aligned}$$

command

```
integrate((c*x**2+b*x+a)**2/(e*x+d)**8,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-30a^2e^4 - 10abde^3 - 4acd^2e^2 - 2b^2d^2e^2 - 3bcd^3e - 2c^2d^4 - 70c^2e^4x^4 + x^3(-105bce^4 - 70c^2de^3) + x^2(-84ace^4 - 210d^7e^5 + 1470d^6e^6x + 4410d^5e^7x^2 + 7350d^4e^8x^3 + 7350d^3e^9x^4 + \dots}{210d^7e^5 + 1470d^6e^6x + 4410d^5e^7x^2 + 7350d^4e^8x^3 + 7350d^3e^9x^4 + \dots}$$

## 17.22 Problem number 2140

$$\int \frac{(a + bx + cx^2)^3}{(d + ex)^5} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{c^2(-3be + 5cd)x}{e^6} + \frac{c^3x^2}{2e^5} - \frac{(ae^2 - bde + cd^2)^3}{4e^7(ex + d)^4} + \frac{(-be + 2cd)(ae^2 - bde + cd^2)^2}{e^7(ex + d)^3} \\ & - \frac{3(ae^2 - bde + cd^2)(5c^2d^2 + b^2e^2 - ce(-ae + 5bd))}{2e^7(ex + d)^2} \\ & + \frac{(-be + 2cd)(10c^2d^2 + b^2e^2 - 2ce(-3ae + 5bd))}{e^7(ex + d)} \\ & + \frac{3c(5c^2d^2 + b^2e^2 - ce(-ae + 5bd)) \ln(ex + d)}{e^7} \end{aligned}$$

command

```
integrate((c*x**2+b*x+a)**3/(e*x+d)**5,x)
```



Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{c^3 x^2}{2e^5} + \frac{3c(ace^2 + b^2e^2 - 5bcde + 5c^2d^2) \log(d + ex)}{e^7} + x \left( \frac{3bc^2}{e^5} - \frac{5c^3d}{e^6} \right) + \frac{-a^3e^6 - a^2bde^5 - a^2cd^2e^4 - ab^2d^2e^4 - 6abcd^3e^3 + 25ac^2d^4e^2 - b^3d^3e^3 + 25b^2cd^4e^2 - 77bc^2d^5e + 57c^3d^6 + x^3(-2$$

### 17.23 Problem number 2154

$$\int \frac{(a + bx + cx^2)^4}{(d + ex)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(35c^4d^4 + b^4e^4 - 4b^2ce^3(-3ae + 4bd) - 40c^3d^2e(-ae + 2bd) + 6c^2e^2(a^2e^2 - 8abde + 10b^2d^2)) x}{e^8} \\ & - \frac{2c(5c^3d^3 - b^3e^3 - 2c^2de(-2ae + 5bd) + 3bc e^2(-ae + 2bd)) x^2}{e^7} \\ & + \frac{2c^2(5c^2d^2 + 3b^2e^2 - 2ce(-ae + 4bd)) x^3}{3e^6} - \frac{c^3(-be + cd) x^4}{e^5} + \frac{c^4 x^5}{5e^4} - \frac{(a e^2 - bde + c d^2)^4}{3e^9 (ex + d)^3} \\ & + \frac{2(-be + 2cd) (a e^2 - bde + c d^2)^3}{e^9 (ex + d)^2} - \frac{2(a e^2 - bde + c d^2)^2 (14c^2d^2 + 3b^2e^2 - 2ce(-ae + 7bd))}{e^9 (ex + d)} \\ & - \frac{4(-be + 2cd) (a e^2 - bde + c d^2) (7c^2d^2 + b^2e^2 - ce(-3ae + 7bd)) \ln(ex + d)}{e^9} \end{aligned}$$

command

`integrate((c*x**2+b*x+a)**4/(e*x+d)**4, x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{c^4 x^5}{5e^4} + x^4 \left( \frac{bc^3}{e^4} - \frac{c^4 d}{e^5} \right) + x^3 \left( \frac{4ac^3}{3e^4} + \frac{2b^2c^2}{e^4} - \frac{16bc^3d}{3e^5} + \frac{10c^4d^2}{3e^6} \right) \\ & + x^2 \left( \frac{6abc^2}{e^4} - \frac{8ac^3d}{e^5} + \frac{2b^3c}{e^4} - \frac{12b^2c^2d}{e^5} + \frac{20bc^3d^2}{e^6} - \frac{10c^4d^3}{e^7} \right) \\ & + x \left( \frac{6a^2c^2}{e^4} + \frac{12ab^2c}{e^4} - \frac{48abc^2d}{e^5} + \frac{40ac^3d^2}{e^6} + \frac{b^4}{e^4} - \frac{16b^3cd}{e^5} + \frac{60b^2c^2d^2}{e^6} - \frac{80bc^3d^3}{e^7} + \frac{35c^4d^4}{e^8} \right) \\ & + \frac{-a^4e^8 - 2a^3bde^7 - 4a^3cd^2e^6 - 6a^2b^2d^2e^6 + 66a^2bcd^3e^5 - 78a^2c^2d^4e^4 + 22ab^3d^3e^5 - 156ab^2cd^4e^4 + 282abc^2d^5e^3 - 156a^2b^2cd^4e^4 + 282abc^2d^5e^3 - 156a^2b^2cd^4e^4 + 282abc^2d^5e^3 - 156a^2b^2cd^4e^4 + 282abc^2d^5e^3}{e^9} \\ & + \frac{4(be - 2cd) (ae^2 - bde + cd^2) (3ace^2 + b^2e^2 - 7bcde + 7c^2d^2) \log(d + ex)}{e^9} \end{aligned}$$

## 17.24 Problem number 2201

$$\int \frac{(d+ex)^5}{(a+bx+cx^2)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{e^2(-be+2cd)(3c^2d^2-b^2e^2-ce(-7ae+3bd))x}{c^2(-4ac+b^2)^2} - \frac{(ex+d)^4(bd-2ae+(-be+2cd)x)}{2(-4ac+b^2)(cx^2+bx+a)^2} \\ & - \frac{(ex+d)^2(8ace(2ae^2+cd^2)-6bcd(3ae^2+cd^2)+b^2(-ae^3+7cd^2e)-(-be+2cd)(6c^2d^2-b^2e^2-2ce(-5ae^2+2cd^2)))}{2c(-4ac+b^2)^2(cx^2+bx+a)} \\ & - \frac{(12c^5d^5-b^5e^5+10ab^3ce^5-30a^2bc^2e^5-10c^4d^3e(-4ae+3bd)+20c^3de^2(3a^2e^2-3abde+b^2d^2)) \operatorname{arctanh}\left(\frac{2cx+b}{\sqrt{-4ac+b^2}}\right)}{c^3(-4ac+b^2)^{\frac{5}{2}}} \\ & + \frac{e^5 \ln(cx^2+bx+a)}{2c^3} \end{aligned}$$

command

`integrate((e*x+d)**5/(c*x**2+b*x+a)**3,x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

## 17.25 Problem number 2214

$$\int \frac{(d+ex)^4}{(a+bx+cx^2)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(2cx+b)(ex+d)^4}{3(-4ac+b^2)(cx^2+bx+a)^3} + \frac{(ex+d)^3(5bcd-2b^2e-2ace+5c(-be+2cd)x)}{3(-4ac+b^2)^2(cx^2+bx+a)^2} \\ & - \frac{2(5c^2d^2+b^2e^2-ce(-ae+5bd))(ex+d)(bd-2ae+(-be+2cd)x)}{(-4ac+b^2)^3(cx^2+bx+a)} \\ & + \frac{8(ae^2-bde+cd^2)(5c^2d^2+b^2e^2-ce(-ae+5bd)) \operatorname{arctanh}\left(\frac{2cx+b}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{7}{2}}} \end{aligned}$$

command

```
integrate((e*x+d)**4/(c*x**2+b*x+a)**4,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

## 17.26 Problem number 2291

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{ex+d}}{\sqrt{2cd-e(b-\sqrt{-4ac+b^2})}}\right)\sqrt{2}\sqrt{2cd-e(b-\sqrt{-4ac+b^2})}}{\sqrt{c}\sqrt{-4ac+b^2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{ex+d}}{\sqrt{2cd-e(b+\sqrt{-4ac+b^2})}}\right)\sqrt{2}\sqrt{2cd-e(b+\sqrt{-4ac+b^2})}}{\sqrt{c}\sqrt{-4ac+b^2}}$$

command

```
integrate((e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$2e \operatorname{RootSum}\left(t^4(256a^2c^3e^4 - 128ab^2c^2e^4 + 16b^4ce^4) + t^2(-16abce^3 + 32ac^2de^2 + 4b^3e^3 - 8b^2cde^2) + ae^2 - bde + c\right)$$

## 18 Test file number 34

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.1\_Quadratic/34\_1.2.1.3-d+e\_x-^m-f+g\_x-a+b\_x+c\_x^2-^p

### 18.1 Problem number 877

$$\int \frac{(A + Bx)(a + bx + cx^2)^3}{x^8} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{a^3 A}{7x^7} - \frac{a^2(3Ab + aB)}{6x^6} - \frac{3a(abB + A(ac + b^2))}{5x^5} + \frac{-3aB(ac + b^2) - A(6abc + b^3)}{4x^4} \\ & + \frac{-3aAc^2 - 3Ab^2c - 6abBc - b^3B}{3x^3} - \frac{3c(Abc + aBc + b^2B)}{2x^2} - \frac{c^2(Ac + 3bB)}{x} + Bc^3 \ln(x) \end{aligned}$$

command

```
integrate((B*x+A)*(c*x**2+b*x+a)**3/x**8,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & Bc^3 \log(x) \\ & + \frac{-60Aa^3 + x^6(-420Ac^3 - 1260Bbc^2) + x^5(-630Abc^2 - 630Bac^2 - 630Bb^2c) + x^4(-420Aac^2 - 420Ab^2c - 840Bc^2)}{\dots} \end{aligned}$$

### 18.2 Problem number 878

$$\int \frac{(A + Bx)(a + bx + cx^2)^3}{x^9} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{a^3 A}{8x^8} - \frac{a^2(3Ab + aB)}{7x^7} - \frac{a(abB + A(ac + b^2))}{2x^6} + \frac{-3aB(ac + b^2) - A(6abc + b^3)}{5x^5} \\ & + \frac{-3aAc^2 - 3Ab^2c - 6abBc - b^3B}{4x^4} - \frac{c(Abc + aBc + b^2B)}{x^3} - \frac{c^2(Ac + 3bB)}{2x^2} - \frac{Bc^3}{x} \end{aligned}$$

command

`integrate((B*x+A)*(c*x**2+b*x+a)**3/x**9,x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$-35Aa^3 - 280Bc^3x^7 + x^6(-140Ac^3 - 420Bbc^2) + x^5(-280Abc^2 - 280Bac^2 - 280Bb^2c) + x^4(-210Aac^2 - 210A$$

### 18.3 Problem number 1123

$$\int \frac{(A + Bx)(bx + cx^2)^2}{(d + ex)^5} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{Bc^2x}{e^5} + \frac{d^2(-Ae + Bd)(-be + cd)^2}{4e^6(ex + d)^4} - \frac{d(-be + cd)(Bd(-3be + 5cd) - 2Ae(-be + 2cd))}{3e^6(ex + d)^3} \\ & + \frac{-Ae(b^2e^2 - 6bcde + 6c^2d^2) + Bd(3b^2e^2 - 12bcde + 10c^2d^2)}{2e^6(ex + d)^2} \\ & + \frac{2Ace(-be + 2cd) - B(b^2e^2 - 8bcde + 10c^2d^2)}{e^6(ex + d)} - \frac{c(-Ace - 2bBe + 5Bcd) \ln(ex + d)}{e^6} \end{aligned}$$

command

`integrate((B*x+A)*(c*x**2+b*x)**2/(e*x+d)**5,x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{Bc^2x}{e^5} + \frac{c(Ace + 2Bbe - 5Bcd) \log(d + ex)}{e^6} \\ & + \frac{-Ab^2d^2e^3 - 6Abcd^3e^2 + 25Ac^2d^4e - 3Bb^2d^3e^2 + 50Bbcd^4e - 77Bc^2d^5 + x^3(-24Abce^5 + 48Ac^2de^4 - 12Bb^2e^5 + \dots}{\dots} \end{aligned}$$

### 18.4 Problem number 1124

$$\int \frac{(A + Bx)(bx + cx^2)^2}{(d + ex)^6} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^2(-Ae + Bd)(-be + cd)^2}{5e^6(ex + d)^5} - \frac{d(-be + cd)(Bd(-3be + 5cd) - 2Ae(-be + 2cd))}{4e^6(ex + d)^4} \\ & + \frac{-Ae(b^2e^2 - 6bcde + 6c^2d^2) + Bd(3b^2e^2 - 12bcde + 10c^2d^2)}{3e^6(ex + d)^3} \\ & + \frac{2Ace(-be + 2cd) - B(b^2e^2 - 8bcde + 10c^2d^2)}{2e^6(ex + d)^2} + \frac{c(-Ace - 2bBe + 5Bcd)}{e^6(ex + d)} + \frac{Bc^2 \ln(ex + d)}{e^6} \end{aligned}$$

command

```
integrate((B*x+A)*(c*x**2+b*x)**2/(e*x+d)**6,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{Bc^2 \log(d + ex)}{e^6} \\ & + \frac{-2Ab^2d^2e^3 - 6Abcd^3e^2 - 12Ac^2d^4e - 3Bb^2d^3e^2 - 24Bbcd^4e + 137Bc^2d^5 + x^4(-60Ac^2e^5 - 120Bbce^5 + 300Bc^2d^5)}{e^6} \end{aligned}$$

### 18.5 Problem number 1155

$$\int \frac{(A + Bx)(d + ex)^4}{(bx + cx^2)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{Ad^4}{2b^3x^2} - \frac{d^3(4Abe - 3Acd + Bbd)}{b^4x} - \frac{(-Ac + bB)(-be + cd)^4}{2b^3c^3(cx + b)^2} \\ & - \frac{(-be + cd)^3(-Abce - 3Ac^2d + 2b^2Be + 2Bbcd)}{b^4c^3(cx + b)} \\ & + \frac{d^2(6Ac^2d^2 + 2b^2e(3Ae + 2Bd) - 3bcd(4Ae + Bd)) \ln(x)}{b^5} \\ & + \frac{(-be + cd)^2(-6Ac^3d^2 + b^3Be^2 + 2Bb^2cde + 3Bbc^2d^2) \ln(cx + b)}{b^5c^3} \end{aligned}$$

command

```
integrate((B*x+A)*(e*x+d)**4/(c*x**2+b*x)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{-Ab^3c^3d^4 + x^3(-2Ab^4c^2e^4 + 12Ab^2c^4d^2e^2 - 24Abc^5d^3e + 12Ac^6d^4 + 4Bb^5ce^4 - 8Bb^4c^2de^3 + 8Bb^2c^4d^3e - 6Bbc^5d^4)}{b^5} \\ & + \frac{d^2(6Ab^2e^2 - 12Abcde + 6Ac^2d^2 + 4Bb^2de - 3Bbcd^2) \log\left(x + \frac{-6Ab^3c^2d^2e^2 + 12Ab^2c^3d^3e - 6Abc^4d^4 - 4Bb^3c^2d^3e + 3Bb^2c^3d^4 + 6Bbc^4d^5}{-12Ab^2c^3d^2e^2 + 24Abc^4d^3e - 12Ac^5d^4 + 12Ac^6d^5}\right)}{b^5} \\ & + \frac{(be - cd)^2(-6Ac^3d^2 + Bb^3e^2 + 2Bb^2cde + 3Bbc^2d^2) \log\left(x + \frac{-6Ab^3c^2d^2e^2 + 12Ab^2c^3d^3e - 6Abc^4d^4 - 4Bb^3c^2d^3e + 3Bb^2c^3d^4 + 6Bbc^4d^5}{-12Ab^2c^3d^2e^2 + 24Abc^4d^3e - 12Ac^5d^4 + 12Ac^6d^5}\right)}{b^5c^3} \end{aligned}$$

## 18.6 Problem number 1308

$$\int \frac{(A + Bx)(a + cx^2)^2}{(d + ex)^6} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-Ae + Bd)(ae^2 + cd^2)^2}{5e^6(ex + d)^5} - \frac{(ae^2 + cd^2)(-4Acde + aBe^2 + 5Bcd^2)}{4e^6(ex + d)^4} \\ & + \frac{2c(-aAe^3 - 3Acde + 3aBde^2 + 5Bcd^3)}{3e^6(ex + d)^3} \\ & - \frac{c(-2Acde + aBe^2 + 5Bcd^2)}{e^6(ex + d)^2} + \frac{c^2(-Ae + 5Bd)}{e^6(ex + d)} + \frac{Bc^2 \ln(ex + d)}{e^6} \end{aligned}$$

command

```
integrate((B*x+A)*(c*x**2+a)**2/(e*x+d)**6,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{Bc^2 \log(d + ex)}{e^6} \\ & + \frac{-12Aa^2e^5 - 4Aacd^2e^3 - 12Ac^2d^4e - 3Ba^2de^4 - 6Bacd^3e^2 + 137Bc^2d^5 + x^4(-60Ac^2e^5 + 300Bc^2de^4) + x^3(-12Aa^2e^5 - 4Aacd^2e^3 - 12Ac^2d^4e - 3Ba^2de^4 - 6Bacd^3e^2 + 137Bc^2d^5)}{60d^5e^6} \end{aligned}$$

### 18.7 Problem number 1309

$$\int \frac{(A + Bx)(a + cx^2)^2}{(d + ex)^7} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-Ae + Bd)(ae^2 + cd^2)^2}{6e^6(ex + d)^6} - \frac{(ae^2 + cd^2)(-4Acde + aBe^2 + 5Bcd^2)}{5e^6(ex + d)^5} \\ & + \frac{c(-aAe^3 - 3Ac d^2e + 3aBde^2 + 5Bcd^3)}{2e^6(ex + d)^4} \\ & - \frac{2c(-2Acde + aBe^2 + 5Bcd^2)}{3e^6(ex + d)^3} + \frac{c^2(-Ae + 5Bd)}{2e^6(ex + d)^2} - \frac{Bc^2}{e^6(ex + d)} \end{aligned}$$

command

```
integrate((B*x+A)*(c*x**2+a)**2/(e*x+d)**7,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-5Aa^2e^5 - Aacd^2e^3 - Ac^2d^4e - Ba^2de^4 - Bacd^3e^2 - 5Bc^2d^5 - 30Bc^2e^5x^5 + x^4(-15Ac^2e^5 - 75Bc^2de^4) + x^3(-20Ac^2e^5 - 60Bc^2de^4) + x^2(-15Ac^2e^5 - 75Bc^2de^4) + x(-10Ac^2e^5 - 50Bc^2de^4) - 5Ac^2e^5}{30d^6e^6 + 180d^5e^7x}$$

### 18.8 Problem number 1323

$$\int \frac{(A + Bx)(a + cx^2)^3}{(d + ex)^5} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{c^2(5Acde - 3B(ae^2 + 5cd^2))x}{e^7} - \frac{c^3(-Ae + 5Bd)x^2}{2e^6} + \frac{Bc^3x^3}{3e^5} \\ & + \frac{(-Ae + Bd)(ae^2 + cd^2)^3}{4e^8(ex + d)^4} - \frac{(ae^2 + cd^2)^2(-6Acde + aBe^2 + 7Bcd^2)}{3e^8(ex + d)^3} \\ & + \frac{3c(ae^2 + cd^2)(-aAe^3 - 5Ac d^2e + 3aBde^2 + 7Bcd^3)}{2e^8(ex + d)^2} \\ & + \frac{c(4Acde(3ae^2 + 5cd^2) - B(3a^2e^4 + 30acd^2e^2 + 35c^2d^4))}{e^8(ex + d)} \\ & - \frac{c^2(-3aAe^3 - 15Ac d^2e + 15aBde^2 + 35Bcd^3) \ln(ex + d)}{e^8} \end{aligned}$$



command

```
integrate((B*x+A)*(c*x**2+a)**3/(e*x+d)**5,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{Bc^3x^3}{3e^5} - \frac{c^2(-3Aae^3 - 15Acd^2e + 15Bade^2 + 35Bcd^3) \log(d + ex)}{e^8} + x^2 \left( \frac{Ac^3}{2e^5} - \frac{5Bc^3d}{2e^6} \right) + x \left( -\frac{5Ac^3d}{e^6} + \frac{3Bac^2}{e^5} + \frac{15Bc^3d^2}{e^7} \right) + \frac{-3Aa^3e^7 - 3Aa^2cd^2e^5 + 75Aac^2d^4e^3 + 171Ac^3d^6e - Ba^3de^6 - 9Ba^2cd^3e^4 - 231Bac^2d^5e^2 - 319Bc^3d^7 + x^3(144$$

## 18.9 Problem number 1346

$$\int \frac{(A + Bx)(d + ex)^5}{(a + cx^2)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{e^2(5aBe(-3ae^2 + cd^2) + Acd(7ae^2 + 3cd^2))x}{8a^2c^3} - \frac{(ex + d)^4(a(Ae + Bd) - (Acd - Bae)x)}{4ac(cx^2 + a)^2} \\ & - \frac{(ex + d)^2(2ae(2Aae^2 + Acd^2 + 5aBde) - (5aBe(-ae^2 + cd^2) + Acd(5ae^2 + 3cd^2))x)}{8a^2c^2(cx^2 + a)} \\ & + \frac{(5aBe(-3a^2e^4 + 6acd^2e^2 + c^2d^4) + Acd(15a^2e^4 + 10acd^2e^2 + 3c^2d^4)) \arctan\left(\frac{x\sqrt{c}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}c^{\frac{7}{2}}} \\ & + \frac{e^4(Ae + 5Bd) \ln(cx^2 + a)}{2c^3} \end{aligned}$$

command

```
integrate((B*x+A)*(e*x+d)**5/(c*x**2+a)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{Be^5x}{c^3} + \left( \frac{e^4(Ae + 5Bd)}{2c^3} \right. \\ & \left. - \frac{\sqrt{-a^5c^7} (-15Aa^2cde^4 - 10Aac^2d^3e^2 - 3Ac^3d^5 + 15Ba^3e^5 - 30Ba^2cd^2e^3 - 5Bac^2d^4e)}{16a^5c^7} \right) \log \left( x + \frac{8Aa^3e^5 + 40}{\dots} \right) \\ & + \left( \frac{e^4(Ae + 5Bd)}{2c^3} \right. \\ & \left. + \frac{\sqrt{-a^5c^7} (-15Aa^2cde^4 - 10Aac^2d^3e^2 - 3Ac^3d^5 + 15Ba^3e^5 - 30Ba^2cd^2e^3 - 5Bac^2d^4e)}{16a^5c^7} \right) \log \left( x + \frac{8Aa^3e^5 + 40}{\dots} \right) \\ & + \frac{6Aa^4e^5 - 20Aa^3cd^2e^3 - 10Aa^2c^2d^4e + 30Ba^4de^4 - 20Ba^3cd^3e^2 - 2Ba^2c^2d^5 + x^3(-25Aa^2c^2de^4 + 10Aac^3d^3e^2 + \dots)}{\dots} \end{aligned}$$

### 18.10 Problem number 1347

$$\int \frac{(A + Bx)(d + ex)^4}{(a + cx^2)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(ex + d)^3 (a(Ae + Bd) - (Acd - Bae) x)}{4ac (cx^2 + a)^2} \\ & - \frac{(ex + d) (ae(8aBde + 3A(ae^2 + cd^2)) + (4a^2Be^3 - cd(3Acd^2 + ae(3Ae + 4Bd))) x)}{8a^2c^2 (cx^2 + a)} \\ & + \frac{\left( 3A(ae^2 + cd^2)^2 + 4aBde(3ae^2 + cd^2) \right) \arctan\left(\frac{x\sqrt{c}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}c^{\frac{5}{2}}} + \frac{Be^4 \ln(cx^2 + a)}{2c^3} \end{aligned}$$

command

```
integrate((B*x+A)*(e*x+d)**4/(c*x**2+a)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \left( \frac{Be^4}{2c^3} \right. \\
& \left. - \frac{\sqrt{-a^5c^7} (3Aa^2e^4 + 6Aacd^2e^2 + 3Ac^2d^4 + 12Ba^2de^3 + 4Bacd^3e)}{16a^5c^6} \right) \log \left( x + \frac{-8Ba^3e^4 + 16a^3c^3 \left( \frac{Be^4}{2c^3} - \frac{\sqrt{-a^5c^7}}{16a^5c^6} \right)}{3Aa^2ce^4 + 6Aac^2d^2e^2 + \dots} \right) \\
& + \left( \frac{Be^4}{2c^3} \right. \\
& \left. + \frac{\sqrt{-a^5c^7} (3Aa^2e^4 + 6Aacd^2e^2 + 3Ac^2d^4 + 12Ba^2de^3 + 4Bacd^3e)}{16a^5c^6} \right) \log \left( x + \frac{-8Ba^3e^4 + 16a^3c^3 \left( \frac{Be^4}{2c^3} + \frac{\sqrt{-a^5c^7}}{16a^5c^6} \right)}{3Aa^2ce^4 + 6Aac^2d^2e^2 + \dots} \right) \\
& + \frac{-8Aa^3cde^3 - 8Aa^2c^2d^3e + 6Ba^4e^4 - 12Ba^3cd^2e^2 - 2Ba^2c^2d^4 + x^3(-5Aa^2c^2e^4 + 6Aac^3d^2e^2 + 3Ac^4d^4 - 20Ba^2c^2d^2e^2 + 8a^5c^5)}{8a^5c^5}
\end{aligned}$$

### 18.11 Problem number 1513

$$\int \frac{(b + 2cx)(a + bx + cx^2)^2}{(d + ex)^5} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2c^3x}{e^5} + \frac{(-be + 2cd)(ae^2 - bde + cd^2)^2}{4e^6(ex + d)^4} \\
& - \frac{2(ae^2 - bde + cd^2)(5c^2d^2 + b^2e^2 - ce(-ae + 5bd))}{3e^6(ex + d)^3} \\
& + \frac{(-be + 2cd)(10c^2d^2 + b^2e^2 - 2ce(-3ae + 5bd))}{2e^6(ex + d)^2} \\
& - \frac{4c(5c^2d^2 + b^2e^2 - ce(-ae + 5bd))}{e^6(ex + d)} - \frac{5c^2(-be + 2cd) \ln(ex + d)}{e^6}
\end{aligned}$$

command

```
integrate((2*c*x+b)*(c*x**2+b*x+a)**2/(e*x+d)**5,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{2c^3x}{e^5} + \frac{5c^2(be - 2cd) \log(d + ex)}{e^6} \\
& + \frac{-3a^2be^5 - 2a^2cde^4 - 2ab^2de^4 - 6abcd^2e^3 - 12ac^2d^3e^2 - b^3d^2e^3 - 12b^2cd^3e^2 + 125bc^2d^4e - 154c^3d^5 + x^3(-48ac^2d^2e^2 + 8a^5c^5)}{8a^5c^5}
\end{aligned}$$

## 18.12 Problem number 1522

$$\int \frac{(b + 2cx)(a + bx + cx^2)^3}{(d + ex)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{c(40c^3d^3 - 5b^3e^3 - 2c^2de(-12ae + 35bd) + 3bce^2(-5ae + 12bd))x}{e^7} \\ & + \frac{c^2(6ace^2 + 9b^2e^2 - 28bcde + 20c^2d^2)x^2}{2e^6} - \frac{c^3(-7be + 8cd)x^3}{3e^5} + \frac{c^4x^4}{2e^4} \\ & + \frac{(-be + 2cd)(ae^2 - bde + cd^2)^3}{3e^8(ex + d)^3} - \frac{(ae^2 - bde + cd^2)^2(14c^2d^2 + 3b^2e^2 - 2ce(-ae + 7bd))}{2e^8(ex + d)^2} \\ & + \frac{3(-be + 2cd)(ae^2 - bde + cd^2)(7c^2d^2 + b^2e^2 - ce(-3ae + 7bd))}{e^8(ex + d)} \\ & + \frac{(70c^4d^4 + b^4e^4 - 4b^2ce^3(-3ae + 5bd) - 20c^3d^2e(-3ae + 7bd) + 6c^2e^2(a^2e^2 - 10abde + 15b^2d^2)) \ln(ex + d)}{e^8} \end{aligned}$$

command

```
integrate((2*c*x+b)*(c*x**2+b*x+a)**3/(e*x+d)**4,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{c^4x^4}{2e^4} + x^3 \left( \frac{7bc^3}{3e^4} - \frac{8c^4d}{3e^5} \right) + x^2 \left( \frac{3ac^3}{e^4} + \frac{9b^2c^2}{2e^4} - \frac{14bc^3d}{e^5} + \frac{10c^4d^2}{e^6} \right) \\ & + x \left( \frac{15abc^2}{e^4} - \frac{24ac^3d}{e^5} + \frac{5b^3c}{e^4} - \frac{36b^2c^2d}{e^5} + \frac{70bc^3d^2}{e^6} - \frac{40c^4d^3}{e^7} \right) \\ & + \frac{-2a^3be^7 - 2a^3cde^6 - 3a^2b^2de^6 - 18a^2bcd^2e^5 + 66a^2c^2d^3e^4 - 6ab^3d^2e^5 + 132ab^2cd^3e^4 - 390abc^2d^4e^3 + 282ac^3d^5e^2}{e^8} \\ & + \frac{(6a^2c^2e^4 + 12ab^2ce^4 - 60abc^2de^3 + 60ac^3d^2e^2 + b^4e^4 - 20b^3cde^3 + 90b^2c^2d^2e^2 - 140bc^3d^3e + 70c^4d^4) \log(d + ex)}{e^8} \end{aligned}$$

## 18.13 Problem number 1524

$$\int \frac{(b + 2cx)(d + ex)^4}{a + bx + cx^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{e(8c^3d^3 - b^3e^3 + bc e^2(3ae + 4bd) - 2c^2de(4ae + 3bd)) x}{c^3} \\ & + \frac{e^2(12c^2d^2 + b^2e^2 - 2ce(ae + 2bd)) x^2}{2c^2} + \frac{e^3(-be + 8cd) x^3}{3c} + \frac{e^4x^4}{2} \\ & + \frac{(2c^4d^4 + b^4e^4 - 4b^2ce^3(ae + bd) - 4c^3d^2e(3ae + bd) + 2c^2e^2(a^2e^2 + 6abde + 3b^2d^2)) \ln(cx^2 + bx + a)}{2c^4} \\ & - \frac{e(-be + 2cd)(2c^2d^2 + b^2e^2 - 2ce(ae + bd)) \operatorname{arctanh}\left(\frac{2cx+b}{\sqrt{-4ac+b^2}}\right) \sqrt{-4ac+b^2}}{c^4} \end{aligned}$$

command

```
integrate((2*c*x+b)*(e*x+d)**4/(c*x**2+b*x+a), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{e^4x^4}{2} + x^3\left(-\frac{be^4}{3c} + \frac{8de^3}{3}\right) + x^2\left(-\frac{ae^4}{c} + \frac{b^2e^4}{2c^2} - \frac{2bde^3}{c} + 6d^2e^2\right) \\ & + x\left(\frac{3abe^4}{c^2} - \frac{8ade^3}{c} - \frac{b^3e^4}{c^3} + \frac{4b^2de^3}{c^2} - \frac{6bd^2e^2}{c} + 8d^3e\right) \\ & + \left(-\frac{e\sqrt{-4ac+b^2}(be-2cd)(2ace^2-b^2e^2+2bcde-2c^2d^2)}{2c^4}\right. \\ & + \left.\frac{2a^2c^2e^4-4ab^2ce^4+12abc^2de^3-12ac^3d^2e^2+b^4e^4-4b^3cde^3+6b^2c^2d^2e^2-4bc^3d^3e+2c^4d^4}{2c^4}\right) \log\left(x + \frac{a^2ce^4 -}{\dots}\right) \\ & + \left(\frac{e\sqrt{-4ac+b^2}(be-2cd)(2ace^2-b^2e^2+2bcde-2c^2d^2)}{2c^4}\right. \\ & + \left.\frac{2a^2c^2e^4-4ab^2ce^4+12abc^2de^3-12ac^3d^2e^2+b^4e^4-4b^3cde^3+6b^2c^2d^2e^2-4bc^3d^3e+2c^4d^4}{2c^4}\right) \log\left(x + \frac{a^2ce^4 -}{\dots}\right) \end{aligned}$$

### 18.14 Problem number 1540

$$\int \frac{(b + 2cx)(d + ex)^4}{(a + bx + cx^2)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e^3(-be + 2cd)x}{c(-4ac + b^2)} - \frac{(ex + d)^4}{2(cx^2 + bx + a)^2} - \frac{2e(ex + d)^2 (bd - 2ae + (-be + 2cd)x)}{(-4ac + b^2)(cx^2 + bx + a)} \\ & + \frac{2e(-be + 2cd)(2c^2d^2 - b^2e^2 - 2ce(-3ae + bd)) \operatorname{arctanh}\left(\frac{2cx + b}{\sqrt{-4ac + b^2}}\right)}{c^2(-4ac + b^2)^{\frac{3}{2}}} \\ & + \frac{e^4 \ln(cx^2 + bx + a)}{c^2} \end{aligned}$$

command

```
integrate((2*c*x+b)*(e*x+d)**4/(c*x**2+b*x+a)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

### 18.15 Problem number 1672

$$\int \frac{(A + Bx)(a^2 + 2abx + b^2x^2)}{(d + ex)^8} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-ae + bd)^2 (-Ae + Bd)}{7e^4 (ex + d)^7} - \frac{(-ae + bd)(-2Abe - Bae + 3Bbd)}{6e^4 (ex + d)^6} \\ & + \frac{b(-Abe - 2Bae + 3Bbd)}{5e^4 (ex + d)^5} - \frac{b^2B}{4e^4 (ex + d)^4} \end{aligned}$$

command

```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)/(e*x+d)**8,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-60Aa^2e^3 - 20Aabde^2 - 4Ab^2d^2e - 10Ba^2de^2 - 8Babd^2e - 3Bb^2d^3 - 105Bb^2e^3x^3 + x^2(-84Ab^2e^3 - 168Babe^3 - 420d^7e^4 + 2940d^6e^5x + 8820d^5e^6x^2 + 14700d^4e^7x^3 + 14700d^3e^8x^4 + 8820d^2e^9x^5 + 8820de^{10}x^6 + 8820e^{11}x^7)}{420d^7e^4 + 2940d^6e^5x + 8820d^5e^6x^2 + 14700d^4e^7x^3 + 14700d^3e^8x^4 + 8820d^2e^9x^5 + 8820de^{10}x^6 + 8820e^{11}x^7}$$

## 18.16 Problem number 1685

$$\int \frac{(A + Bx)(a^2 + 2abx + b^2x^2)^2}{(d + ex)^5} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b^4 Bx}{e^5} + \frac{(-ae + bd)^4 (-Ae + Bd)}{4e^6 (ex + d)^4} - \frac{(-ae + bd)^3 (-4Abe - Bae + 5Bbd)}{3e^6 (ex + d)^3} \\ & + \frac{b(-ae + bd)^2 (-3Abe - 2Bae + 5Bbd)}{e^6 (ex + d)^2} - \frac{2b^2(-ae + bd)(-2Abe - 3Bae + 5Bbd)}{e^6 (ex + d)} \\ & - \frac{b^3(-Abe - 4Bae + 5Bbd) \ln(ex + d)}{e^6} \end{aligned}$$

command

```
integrate((B*x+A)*(b**2*x**2+2*a*b*x+a**2)**2/(e*x+d)**5,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{Bb^4x}{e^5} + \frac{b^3(Abe + 4Bae - 5Bbd) \log(d + ex)}{e^6} \\ & + \frac{-3Aa^4e^5 - 4Aa^3bde^4 - 6Aa^2b^2d^2e^3 - 12Aab^3d^3e^2 + 25Ab^4d^4e - Ba^4de^4 - 4Ba^3bd^2e^3 - 18Ba^2b^2d^3e^2 + 100Bab}{e^6} \end{aligned}$$

## 19 Test file number 35

Test folder name:

```
test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.1_Quadratic/35_1.2.1.4-  
d+e_x-^m-f+g_x-^n-a+b_x+c_x^2-^p
```

### 19.1 Problem number 65

$$\int x^5(d + ex)^3 (d^2 - e^2x^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{35d^{10}x(-e^2x^2 + d^2)^{\frac{3}{2}}}{3072e^5} + \frac{7d^8x(-e^2x^2 + d^2)^{\frac{5}{2}}}{768e^5} - \frac{124d^5x^2(-e^2x^2 + d^2)^{\frac{7}{2}}}{1287e^4} \\ & - \frac{7d^4x^3(-e^2x^2 + d^2)^{\frac{7}{2}}}{48e^3} - \frac{31d^3x^4(-e^2x^2 + d^2)^{\frac{7}{2}}}{143e^2} - \frac{7d^2x^5(-e^2x^2 + d^2)^{\frac{7}{2}}}{24e} \\ & - \frac{3dx^6(-e^2x^2 + d^2)^{\frac{7}{2}}}{13} - \frac{ex^7(-e^2x^2 + d^2)^{\frac{7}{2}}}{14} - \frac{d^6(63063ex + 31744d)(-e^2x^2 + d^2)^{\frac{7}{2}}}{1153152e^6} \\ & + \frac{35d^{14} \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right)}{2048e^6} + \frac{35d^{12}x\sqrt{-e^2x^2 + d^2}}{2048e^5} \end{aligned}$$

command

```
integrate(x**5*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

## 19.2 Problem number 66

$$\int x^4(d + ex)^3 (d^2 - e^2x^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{9d^9x(-e^2x^2 + d^2)^{\frac{3}{2}}}{512e^4} + \frac{9d^7x(-e^2x^2 + d^2)^{\frac{5}{2}}}{640e^4} - \frac{20d^4x^2(-e^2x^2 + d^2)^{\frac{7}{2}}}{143e^3} \\ & - \frac{9d^3x^3(-e^2x^2 + d^2)^{\frac{7}{2}}}{40e^2} - \frac{45d^2x^4(-e^2x^2 + d^2)^{\frac{7}{2}}}{143e} - \frac{dx^5(-e^2x^2 + d^2)^{\frac{7}{2}}}{4} \\ & - \frac{ex^6(-e^2x^2 + d^2)^{\frac{7}{2}}}{13} - \frac{d^5(27027ex + 12800d)(-e^2x^2 + d^2)^{\frac{7}{2}}}{320320e^5} \\ & + \frac{27d^{13} \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right)}{1024e^5} + \frac{27d^{11}x\sqrt{-e^2x^2 + d^2}}{1024e^4} \end{aligned}$$

command

```
integrate(x**4*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2), x)
```

Sympy 1.10.1 under Python 3.10.4 output



Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

**19.3 Problem number 67**

$$\int x^3(d+ex)^3(d^2-e^2x^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{41d^8x(-e^2x^2+d^2)^{\frac{3}{2}}}{1536e^3} + \frac{41d^6x(-e^2x^2+d^2)^{\frac{5}{2}}}{1920e^3} - \frac{23d^3x^2(-e^2x^2+d^2)^{\frac{7}{2}}}{99e^2} \\ & - \frac{41d^2x^3(-e^2x^2+d^2)^{\frac{7}{2}}}{120e} - \frac{3dx^4(-e^2x^2+d^2)^{\frac{7}{2}}}{11} \\ & - \frac{ex^5(-e^2x^2+d^2)^{\frac{7}{2}}}{12} - \frac{d^4(28413ex+14720d)(-e^2x^2+d^2)^{\frac{7}{2}}}{221760e^4} \\ & + \frac{41d^{12} \arctan\left(\frac{ex}{\sqrt{-e^2x^2+d^2}}\right)}{1024e^4} + \frac{41d^{10}x\sqrt{-e^2x^2+d^2}}{1024e^3} \end{aligned}$$

command

```
integrate(x**3*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

**19.4 Problem number 267**

$$\int \frac{x^4(d^2-e^2x^2)^p}{d+ex} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^4(-e^2x^2+d^2)^p}{2e^5p} - \frac{d^2(-e^2x^2+d^2)^{1+p}}{e^5(1+p)} + \frac{(-e^2x^2+d^2)^{2+p}}{2e^5(2+p)} \\ & + \frac{x^5(-e^2x^2+d^2)^p \operatorname{hypergeom}\left(\left[\frac{5}{2}, 1-p\right], \left[\frac{7}{2}\right], \frac{e^2x^2}{d^2}\right) \left(1 - \frac{e^2x^2}{d^2}\right)^{-p}}{5d} \end{aligned}$$

command

```
integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

## 19.5 Problem number 529

$$\int \frac{\sqrt{d+ex}}{a+bx+cx^2} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{ex+d}}{\sqrt{2cd-e(b-\sqrt{-4ac+b^2})}}\right)\sqrt{2}\sqrt{2cd-e(b-\sqrt{-4ac+b^2})}}{\sqrt{c}\sqrt{-4ac+b^2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{ex+d}}{\sqrt{2cd-e(b+\sqrt{-4ac+b^2})}}\right)\sqrt{2}\sqrt{2cd-e(b+\sqrt{-4ac+b^2})}}{\sqrt{c}\sqrt{-4ac+b^2}}$$

command

```
integrate((e*x+d)**(1/2)/(c*x**2+b*x+a),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$2e \operatorname{RootSum}\left(t^4(256a^2c^3e^4 - 128ab^2c^2e^4 + 16b^4ce^4) + t^2(-16abce^3 + 32ac^2de^2 + 4b^3e^3 - 8b^2cde^2) + ae^2 - bde + c\right)$$

## 19.6 Problem number 604

$$\int \frac{-1 + 2x^2}{\sqrt{-1+x} \sqrt{1+x}} dx$$

Optimal antiderivative

$$x\sqrt{-1+x} \sqrt{1+x}$$

command

```
integrate((2*x**2-1)/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$- \begin{cases} 2 \operatorname{acosh} \left( \frac{\sqrt{2} \sqrt{x+1}}{2} \right) & \text{for } \frac{|x+1|}{2} > 1 \\ -2i \operatorname{asin} \left( \frac{\sqrt{2} \sqrt{x+1}}{2} \right) & \text{otherwise} \end{cases} + \frac{G_{6,6}^{6,2} \left( \begin{matrix} -\frac{3}{4}, -\frac{1}{4} \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{matrix} \middle| \begin{matrix} -\frac{1}{2}, -\frac{1}{2}, 0, 1 \\ \frac{1}{x^2} \end{matrix} \right)}{2\pi^{\frac{3}{2}}}$$

$$- \frac{iG_{6,6}^{2,6} \left( \begin{matrix} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \end{matrix} \middle| \begin{matrix} -\frac{3}{2}, -1, -1, 0 \\ \frac{e^{2i\pi}}{x^2} \end{matrix} \right)}{2\pi^{\frac{3}{2}}}$$

## 19.7 Problem number 794

$$\int \frac{a + bx + cx^2}{\sqrt{1-dx} \sqrt{1+dx}} dx$$

Optimal antiderivative

$$\frac{(2ad^2 + c) \arcsin(dx)}{2d^3} - \frac{b\sqrt{-d^2x^2 + 1}}{d^2} - \frac{cx\sqrt{-d^2x^2 + 1}}{2d^2}$$

command

```
integrate((c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
 & \frac{iaG_{6,6}^{6,2} \left( \begin{array}{c} \frac{1}{4}, \frac{3}{4} \quad \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d} + \frac{aG_{6,6}^{2,6} \left( \begin{array}{c} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 \\ -\frac{1}{4}, \frac{1}{4} \quad -\frac{1}{2}, 0, 0, 0 \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d} \\
 & - \frac{ibG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{1}{4}, \frac{1}{4} \quad 0, 0, \frac{1}{2}, 1 \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^2} \\
 & - \frac{bG_{6,6}^{2,6} \left( \begin{array}{c} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 \\ -\frac{3}{4}, -\frac{1}{4} \quad -1, -\frac{1}{2}, -\frac{1}{2}, 0 \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^2} \\
 & - \frac{icG_{6,6}^{6,2} \left( \begin{array}{c} -\frac{3}{4}, -\frac{1}{4} \quad -\frac{1}{2}, -\frac{1}{2}, 0, 1 \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{array} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3} \\
 & + \frac{cG_{6,6}^{2,6} \left( \begin{array}{c} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \quad -\frac{3}{2}, -1, -1, 0 \end{array} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3}
 \end{aligned}$$

## 20 Test file number 37

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.1\_Quadratic/37\_1.2.1.6-g+h\_x-^m-a+b\_x+c\_x^2-^p-d+e\_x+f\_x^2-^q

## 20.1 Problem number 3

$$\int \frac{(A + Bx)(a + bx + cx^2)^3}{d + fx^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(b^3 Bdf + 3A b^2 f(-af + cd) - 3bB(-af + cd)^2 - Ac(3a^2 f^2 - 3acdf + c^2 d^2)) x}{f^3} \\ & - \frac{(Abf(-6acf - b^2 f + 3c^2 d) - B(c^3 d^2 - 3a c^2 df + 3a b^2 f^2 - 3cf(-a^2 f + b^2 d))) x^2}{2f^3} \\ & + \frac{(b^3 Bf + 3A b^2 cf - A c^2(-3af + cd) - 3bBc(-2af + cd)) x^3}{3f^2} \\ & + \frac{c(3Abcf - B(-3acf - 3b^2 f + c^2 d)) x^4}{4f^2} + \frac{c^2(Ac + 3bB) x^5}{5f} + \frac{B c^3 x^6}{6f} \\ & + \frac{(Abf(3c^2 d^2 - 6acdf - f(-3a^2 f + b^2 d)) - B(-af + cd)(c^2 d^2 - 2acdf - f(-a^2 f + 3b^2 d))) \ln(f x^2 + d)}{2f^4} \\ & + \frac{(b^3 B d^2 f + 3A b^2 df(-af + cd) - 3bBd(-af + cd)^2 - A(-af + cd)^3) \arctan\left(\frac{x\sqrt{f}}{\sqrt{d}}\right)}{f^{\frac{7}{2}} \sqrt{d}} \end{aligned}$$

command

```
integrate((B*x+A)*(c*x**2+b*x+a)**3/(f*x**2+d),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

## 21 Test file number 38

Test folder name:

```
test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.1_Quadratic/38_1.2.1.9_P-
x-d+e_x-^m-a+b_x+c_x^2-^p
```

### 21.1 Problem number 58

$$\int \frac{(d + ex)^2 (A + Bx + Cx^2)}{(a + cx^2)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(aB - (Ac - aC)x)(ex + d)^2}{4ac(cx^2 + a)^2} - \frac{(ex + d)(a(Ac + 3aC)e - c(3Acd + 2Bae + aCd)x)}{8a^2c^2(cx^2 + a)} \\ & + \frac{(a(Ac + 3aC)e^2 + cd(3Acd + 2Bae + aCd)) \arctan\left(\frac{x\sqrt{c}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}c^{\frac{5}{2}}} \end{aligned}$$

command

```
integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & -\frac{\sqrt{-\frac{1}{a^5c^5}}(Aace^2 + 3Ac^2d^2 + 2Bacde + 3Ca^2e^2 + Cacd^2) \log\left(-a^3c^2\sqrt{-\frac{1}{a^5c^5}} + x\right)}{16} \\ & + \frac{\sqrt{-\frac{1}{a^5c^5}}(Aace^2 + 3Ac^2d^2 + 2Bacde + 3Ca^2e^2 + Cacd^2) \log\left(a^3c^2\sqrt{-\frac{1}{a^5c^5}} + x\right)}{16} \\ & + \frac{-4Aa^2cde - 2Ba^3e^2 - 2Ba^2cd^2 - 4Ca^3de + x^3(Aac^2e^2 + 3Ac^3d^2 + 2Bac^2de - 5Ca^2ce^2 + Cac^2d^2) + x^2(-4Ba^2c^2e + 2Aac^2d^2 + 2Bac^2d^2)}{8a^4c^2 + 16a^3c^3x^2 + 8a^2c^4x^4} \end{aligned}$$

### 21.2 Problem number 67

$$\int \frac{(d + ex)(A + Bx + Cx^2)}{(a + cx^2)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(aB - (Ac - aC)x)(ex + d)}{6ac(cx^2 + a)^3} + \frac{-2a(2Ac + aC)e + c(5Acd + Bae + aCd)x}{24a^2c^2(cx^2 + a)^2} \\ & + \frac{(5Acd + Bae + aCd)x}{16a^3c(cx^2 + a)} + \frac{(5Acd + Bae + aCd) \arctan\left(\frac{x\sqrt{c}}{\sqrt{a}}\right)}{16a^{\frac{7}{2}}c^{\frac{3}{2}}} \end{aligned}$$

command

```
integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{\sqrt{-\frac{1}{a^7c^3}} (5Acd + Bae + Cad) \log\left(-a^4c\sqrt{-\frac{1}{a^7c^3}} + x\right)}{32} + \frac{\sqrt{-\frac{1}{a^7c^3}} (5Acd + Bae + Cad) \log\left(a^4c\sqrt{-\frac{1}{a^7c^3}} + x\right)}{32} + \frac{-8Aa^3ce - 8Ba^3cd - 4Ca^4e - 12Ca^3cex^2 + x^5(15Ac^4d + 3Bac^3e + 3Cac^3d) + x^3(40Aac^3d + 8Ba^2c^2e + 8Ca^2c^2d)}{48a^6c^2 + 144a^5c^3x^2 + 144a^4c^4x^4 + 48a^3c^5x^6}$$

## 22 Test file number 39

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.2\_Quartic/39\_1.2.2.2-d\_x-  
^m-a+b\_x^2+c\_x^4-^p

### 22.1 Problem number 853

$$\int \frac{1}{x^3(a + bx^2 + cx^4)} dx$$

Optimal antiderivative

$$-\frac{1}{2ax^2} - \frac{b \ln(x)}{a^2} + \frac{b \ln(cx^4 + bx^2 + a)}{4a^2} - \frac{(-2ac + b^2) \operatorname{arctanh}\left(\frac{2cx^2 + b}{\sqrt{-4ac + b^2}}\right)}{2a^2 \sqrt{-4ac + b^2}}$$

command

```
integrate(1/x**3/(c*x**4+b*x**2+a),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \left( \frac{b}{4a^2} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2 (4ac - b^2)} \right) \log \left( x^2 + \frac{-8a^3c \left( \frac{b}{4a^2} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2 (4ac - b^2)} \right) + 2a^2b^2 \left( \frac{b}{4a^2} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2 (4ac - b^2)} \right)}{2ac^2 - b^2c} \right) \\
& + \left( \frac{b}{4a^2} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2 (4ac - b^2)} \right) \log \left( x^2 + \frac{-8a^3c \left( \frac{b}{4a^2} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2 (4ac - b^2)} \right) + 2a^2b^2 \left( \frac{b}{4a^2} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2 (4ac - b^2)} \right)}{2ac^2 - b^2c} \right) \\
& - \frac{1}{2ax^2} - \frac{b \log(x)}{a^2}
\end{aligned}$$

## 22.2 Problem number 861

$$\int \frac{x^7}{(a + bx^2 + cx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{bx^2}{2c(-4ac + b^2)} + \frac{x^4(bx^2 + 2a)}{2(-4ac + b^2)(cx^4 + bx^2 + a)} \\
& + \frac{b(-6ac + b^2) \operatorname{arctanh}\left(\frac{2cx^2 + b}{\sqrt{-4ac + b^2}}\right)}{2c^2(-4ac + b^2)^{\frac{3}{2}}} + \frac{\ln(cx^4 + bx^2 + a)}{4c^2}
\end{aligned}$$

command

```
integrate(x**7/(c*x**4+b*x**2+a)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output



$$\begin{aligned}
& \left( -\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} \right. \\
& \left. + \frac{1}{4c^2} \right) \log \left( x^2 + \frac{-32a^2c^3 \left( -\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} + \frac{1}{4c^2} \right) + 8a^2c + 16ab^2c^2 \left( -\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} \right)}{6abc-b^3} \right) \\
& + \left( \frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} \right. \\
& \left. + \frac{1}{4c^2} \right) \log \left( x^2 + \frac{-32a^2c^3 \left( \frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} + \frac{1}{4c^2} \right) + 8a^2c + 16ab^2c^2 \left( \frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} \right)}{6abc-b^3} \right) \\
& + \frac{2a^2c-ab^2+x^2(3abc-b^3)}{8a^2c^3-2ab^2c^2+x^4(8ac^4-2b^2c^3)+x^2(8abc^3-2b^3c^2)}
\end{aligned}$$

### 22.3 Problem number 868

$$\int \frac{x^6}{(a+bx^2+cx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{bx}{2c(-4ac+b^2)} + \frac{x^3(bx^2+2a)}{2(-4ac+b^2)(cx^4+bx^2+a)} \\
& + \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right) \left(b^2-6ac-\frac{b(-8ac+b^2)}{\sqrt{-4ac+b^2}}\right) \sqrt{2}}{4c^{\frac{3}{2}}(-4ac+b^2)\sqrt{b-\sqrt{-4ac+b^2}}} \\
& + \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right) \left(b^2-6ac+\frac{b(-8ac+b^2)}{\sqrt{-4ac+b^2}}\right) \sqrt{2}}{4c^{\frac{3}{2}}(-4ac+b^2)\sqrt{b+\sqrt{-4ac+b^2}}}
\end{aligned}$$

command

`integrate(x**6/(c*x**4+b*x**2+a)**2,x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{abx + x^3(-2ac + b^2)}{8a^2c^2 - 2ab^2c + x^4(8ac^3 - 2b^2c^2) + x^2(8abc^2 - 2b^3c)} + \text{RootSum} \left( t^4(1048576a^6c^9 - 1572864a^5b^2c^8 + 983040a^4b^4c^7 - 327680a^3b^6c^6 + 61440a^2b^8c^5 - 6144ab^{10}c^4 + 256b^{12}c^3) \right)$$

**22.4 Problem number 870**

$$\int \frac{x^2}{(a + bx^2 + cx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{x(2cx^2 + b)}{2(-4ac + b^2)(cx^4 + bx^2 + a)} \\ & + \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \sqrt{c} (2b - \sqrt{-4ac + b^2}) \sqrt{2}}{2(-4ac + b^2)^{\frac{3}{2}} \sqrt{b - \sqrt{-4ac + b^2}}} \\ & - \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \sqrt{c} (2b + \sqrt{-4ac + b^2}) \sqrt{2}}{2(-4ac + b^2)^{\frac{3}{2}} \sqrt{b + \sqrt{-4ac + b^2}}} \end{aligned}$$

command`integrate(x**2/(c*x**4+b*x**2+a)**2,x)`Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{bx + 2cx^3}{8a^2c - 2ab^2 + x^4(8ac^2 - 2b^2c) + x^2(8abc - 2b^3)} + \text{RootSum} \left( t^4(1048576a^7c^6 - 1572864a^6b^2c^5 + 983040a^5b^4c^4 - 327680a^4b^6c^3 + 61440a^3b^8c^2 - 6144a^2b^{10}c + 256ab^{12}) \right)$$

## 22.5 Problem number 883

$$\int \frac{x^6}{(a + bx^2 + cx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x^3(bx^2 + 2a)}{4(-4ac + b^2)(cx^4 + bx^2 + a)^2} + \frac{3x(4ab + (4ac + b^2)x^2)}{8(-4ac + b^2)^2(cx^4 + bx^2 + a)} \\ & + \frac{3 \arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \left(b^2 + 4ac - \frac{b(12ac + b^2)}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{16(-4ac + b^2)^2 \sqrt{c} \sqrt{b - \sqrt{-4ac + b^2}}} \\ & + \frac{3 \arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \left(b^2 + 4ac + \frac{b(12ac + b^2)}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{16(-4ac + b^2)^2 \sqrt{c} \sqrt{b + \sqrt{-4ac + b^2}}} \end{aligned}$$

command

`integrate(x**6/(c*x**4+b*x**2+a)**3,x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{12a^2bx + x^7(12ac^2 + 3b^2c) + x^5(16abc + 5b^3) + x^3(-4a^2c + 19ab^2)}{128a^4c^2 - 64a^3b^2c + 8a^2b^4 + x^8(128a^2c^4 - 64ab^2c^3 + 8b^4c^2) + x^6(256a^2bc^3 - 128ab^3c^2 + 16b^5c) + x^4(256a^3c^3 - 4a^2b^3c^2 + 16ab^5c)} \\ & + \text{RootSum}\left(t^4(68719476736a^{10}c^{11} - 171798691840a^9b^2c^{10} + 193273528320a^8b^4c^9 - 128849018880a^7b^6c^8 + 56371200a^6b^8c^7 - 128849018880a^5b^{10}c^6 + 128849018880a^4b^{12}c^5 - 128849018880a^3b^{14}c^4 + 128849018880a^2b^{16}c^3 - 128849018880ab^{18}c^2 + 128849018880a^{20}c) \right) \end{aligned}$$

## 22.6 Problem number 892

$$\int \frac{1}{x^3(a - bx^2 + cx^4)} dx$$

Optimal antiderivative

$$-\frac{1}{2ax^2} + \frac{b \ln(x)}{a^2} - \frac{b \ln(cx^4 - bx^2 + a)}{4a^2} + \frac{(-2ac + b^2) \operatorname{arctanh}\left(\frac{-2cx^2 + b}{\sqrt{-4ac + b^2}}\right)}{2a^2 \sqrt{-4ac + b^2}}$$

command

```
integrate(1/x**3/(c*x**4-b*x**2+a), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \left( -\frac{b}{4a^2} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2 (4ac - b^2)} \right) \log \left( x^2 + \frac{-8a^3c \left( -\frac{b}{4a^2} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2 (4ac - b^2)} \right) + 2a^2b^2 \left( -\frac{b}{4a^2} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2 (4ac - b^2)} \right)}{2ac^2 - b^2c} \right) \\ & + \left( -\frac{b}{4a^2} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2 (4ac - b^2)} \right) \log \left( x^2 + \frac{-8a^3c \left( -\frac{b}{4a^2} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2 (4ac - b^2)} \right) + 2a^2b^2 \left( -\frac{b}{4a^2} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2 (4ac - b^2)} \right)}{2ac^2 - b^2c} \right) \\ & - \frac{1}{2ax^2} + \frac{b \log(x)}{a^2} \end{aligned}$$

## 23 Test file number 40

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.2\_Quartic/40\_1.2.2.3-d+e\_x^2-  
^m-a+b\_x^2+c\_x^4-^p

### 23.1 Problem number 259

$$\int \frac{(a + bx^2 + cx^4)^2}{(d + ex^2)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{c^2x}{e^4} + \frac{(ae^2 - bde + cd^2)^2x}{6de^4(e^2x^2 + d)^3} - \frac{(-5ae^2 - 7bde + 19cd^2)(ae^2 - bde + cd^2)x}{24d^2e^4(e^2x^2 + d)^2} \\ & + \frac{(29c^2d^4 - 2cd^2e(-ae + 11bd) + e^2(5a^2e^2 + 2abde + b^2d^2))x}{16d^3e^4(e^2x^2 + d)} \\ & - \frac{(35c^2d^4 - 2cd^2e(ae + 5bd) - e^2(5a^2e^2 + 2abde + b^2d^2)) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{16d^{\frac{7}{2}}e^{\frac{9}{2}}} \end{aligned}$$

command

```
integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**4,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{c^2 x}{e^4} - \frac{\sqrt{-\frac{1}{d^7 e^9}} (5a^2 e^4 + 2abde^3 + 2acd^2 e^2 + b^2 d^2 e^2 + 10bcd^3 e - 35c^2 d^4) \log\left(-d^4 e^4 \sqrt{-\frac{1}{d^7 e^9}} + x\right)}{32} + \frac{\sqrt{-\frac{1}{d^7 e^9}} (5a^2 e^4 + 2abde^3 + 2acd^2 e^2 + b^2 d^2 e^2 + 10bcd^3 e - 35c^2 d^4) \log\left(d^4 e^4 \sqrt{-\frac{1}{d^7 e^9}} + x\right)}{32} + \frac{x^5 (15a^2 e^6 + 6abde^5 + 6acd^2 e^4 + 3b^2 d^2 e^4 - 66bcd^3 e^3 + 87c^2 d^4 e^2) + x^3 (40a^2 de^5 + 16abd^2 e^4 - 16acd^3 e^3 - 8b^2 d^3 e^3)}{48d^6 e^4 + 144d^5 e^5 x^2 + 144d^4 e^6 x^4}$$

### 23.2 Problem number 266

$$\int \frac{d + ex^2}{a + bx^2 + cx^4} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \left(e + \frac{-be+2cd}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2\sqrt{c} \sqrt{b - \sqrt{-4ac + b^2}}} + \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \left(e + \frac{be-2cd}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2\sqrt{c} \sqrt{b + \sqrt{-4ac + b^2}}}$$

command

```
integrate((e*x**2+d)/(c*x**4+b*x**2+a),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\text{RootSum}\left(t^4(256a^3c^3 - 128a^2b^2c^2 + 16ab^4c) + t^2(-16a^2bce^2 + 64a^2c^2de + 4ab^3e^2 - 16ab^2cde - 16abc^2d^2 + 4b^3c^2d^2) + t(-16a^2bce^2 + 64a^2c^2de + 4ab^3e^2 - 16ab^2cde - 16abc^2d^2 + 4b^3c^2d^2) + 16a^2bce^2 - 64a^2c^2de - 4ab^3e^2 + 16ab^2cde + 16abc^2d^2 - 4b^3c^2d^2\right)$$

### 23.3 Problem number 273

$$\int \frac{1}{(a + bx^2 + cx^4)^2} dx$$

Optimal antiderivative

$$\frac{x(bc x^2 - 2ac + b^2)}{2a(-4ac + b^2)(cx^4 + bx^2 + a)}$$

$$+ \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \sqrt{c} (b^2 - 12ac + b\sqrt{-4ac + b^2}) \sqrt{2}}{4a(-4ac + b^2)^{\frac{3}{2}} \sqrt{b - \sqrt{-4ac + b^2}}}$$

$$- \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \sqrt{c} (b^2 - 12ac - b\sqrt{-4ac + b^2}) \sqrt{2}}{4a(-4ac + b^2)^{\frac{3}{2}} \sqrt{b + \sqrt{-4ac + b^2}}}$$

command

```
integrate(1/(c*x**4+b*x**2+a)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{-bcx^3 + x(2ac - b^2)}{8a^3c - 2a^2b^2 + x^4(8a^2c^2 - 2ab^2c) + x^2(8a^2bc - 2ab^3)}$$

$$+ \text{RootSum}\left(t^4(1048576a^9c^6 - 1572864a^8b^2c^5 + 983040a^7b^4c^4 - 327680a^6b^6c^3 + 61440a^5b^8c^2 - 6144a^4b^{10}c + 256a^3b^{12})\right)$$

## 24 Test file number 41

Test folder name:

```
test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.2_Quartic/41_1.2.2.4-f_x-
~m-d+e_x^2-~q-a+b_x^2+c_x^4-~p
```

## 24.1 Problem number 102

$$\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-Ac+bB)x^2}{2c^2} + \frac{Bx^4}{4c} + \frac{(-Abc-aBc+b^2B)\ln(cx^4+bx^2+a)}{4c^3} \\ & + \frac{(2aAc^2-Ab^2c-3abBc+b^3B)\operatorname{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right)}{2c^3\sqrt{-4ac+b^2}} \end{aligned}$$

command

```
integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{Bx^4}{4c} + x^2\left(\frac{A}{2c} - \frac{Bb}{2c^2}\right) + \left(-\frac{\sqrt{-4ac+b^2}(-2Aac^2+Ab^2c+3Babc-Bb^3)}{4c^3(4ac-b^2)}\right. \\ & \left.-\frac{Abc+Bac-Bb^2}{4c^3}\right) \log\left(x^2 + \frac{Aabc+2Ba^2c-Bab^2+8ac^3\left(-\frac{\sqrt{-4ac+b^2}(-2Aac^2+Ab^2c+3Babc-Bb^3)}{4c^3(4ac-b^2)} - \frac{Abc+Bac}{4c^3}\right)}{-2Aac^2+Ab^2c+3Babc-Bb^3}\right) \\ & + \left(\frac{\sqrt{-4ac+b^2}(-2Aac^2+Ab^2c+3Babc-Bb^3)}{4c^3(4ac-b^2)}\right. \\ & \left.-\frac{Abc+Bac-Bb^2}{4c^3}\right) \log\left(x^2 + \frac{Aabc+2Ba^2c-Bab^2+8ac^3\left(\frac{\sqrt{-4ac+b^2}(-2Aac^2+Ab^2c+3Babc-Bb^3)}{4c^3(4ac-b^2)} - \frac{Abc+Bac}{4c^3}\right)}{-2Aac^2+Ab^2c+3Babc-Bb^3}\right) \end{aligned}$$

## 24.2 Problem number 109

$$\int \frac{A + Bx^2}{a + bx^2 + cx^4} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \left(B + \frac{2Ac - bB}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2\sqrt{c} \sqrt{b - \sqrt{-4ac + b^2}}} + \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \left(B + \frac{-2Ac + bB}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2\sqrt{c} \sqrt{b + \sqrt{-4ac + b^2}}}$$

command

```
integrate((B*x**2+A)/(c*x**4+b*x**2+a),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\text{RootSum}\left(t^4(256a^3c^3 - 128a^2b^2c^2 + 16ab^4c) + t^2(-16A^2abc^2 + 4A^2b^3c + 64ABa^2c^2 - 16ABab^2c - 16B^2a^2bc + \dots)\right)$$

## 24.3 Problem number 126

$$\int \frac{x^7(A + Bx^2)}{(a + bx^2 + cx^4)^3} dx$$

Optimal antiderivative

$$-\frac{x^6(Ab - 2aB - (-2Ac + bB)x^2)}{4(-4ac + b^2)(cx^4 + bx^2 + a)^2} + \frac{3(Ab - 2aB)x^2(bx^2 + 2a)}{4(-4ac + b^2)^2(cx^4 + bx^2 + a)} + \frac{3a(Ab - 2aB) \operatorname{arctanh}\left(\frac{2cx^2 + b}{\sqrt{-4ac + b^2}}\right)}{(-4ac + b^2)^{\frac{5}{2}}}$$

command



`integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
 & 3a \sqrt{-\frac{1}{(4ac-b^2)^5}} (-Ab+2Ba) \log \left( x^2 + \frac{-3Aab^2+6Ba^2b-192a^4c^3 \sqrt{-\frac{1}{(4ac-b^2)^5}} (-Ab+2Ba)+144a^3b^2c^2 \sqrt{-\frac{1}{(4ac-b^2)^5}}}{\dots} \right) \\
 & - \frac{3a \sqrt{-\frac{1}{(4ac-b^2)^5}} (-Ab+2Ba) \log \left( x^2 + \frac{-3Aab^2+6Ba^2b+192a^4c^3 \sqrt{-\frac{1}{(4ac-b^2)^5}} (-Ab+2Ba)-144a^3b^2c^2 \sqrt{-\frac{1}{(4ac-b^2)^5}}}{\dots} \right)}{2} \\
 & + \frac{-8Aa^3c^2 - Aa^2b^2c + 10Ba^3bc - Ba^2b^3 + x^6(-6Aabc^3 - 20Ba^2c^3 + 16Bab^2c^2 - 2Bb^4c) + x^4(-16Aa^2c^3 - Aab^2c^2)}{64a^4c^4 - 32a^3b^2c^3 + 4a^2b^4c^2 + x^8(64a^2c^6 - 32ab^2c^5 + 4b^4c^4) + x^6(128a^2bc^5 - 64ab^3c^4 + \dots)}
 \end{aligned}$$

#### 24.4 Problem number 127

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{x^4(Ab-2aB-(-2Ac+bB)x^2)}{4(-4ac+b^2)(cx^4+bx^2+a)^2} \\
 & + \frac{-a(-6Abc+8aBc+b^2B)-(4aAc^2-4Ab^2c+2abBc+b^3B)x^2}{4c(-4ac+b^2)^2(cx^4+bx^2+a)} \\
 & + \frac{(3abB-A(2ac+b^2)) \operatorname{arctanh}\left(\frac{2cx^2+b}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{5}{2}}}
 \end{aligned}$$

command

`integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\sqrt{-\frac{1}{(4ac-b^2)^5}} (-2Aac - Ab^2 + 3Bab) \log \left( x^2 + \frac{-2Aabc - Ab^3 + 3Bab^2 - 64a^3c^3 \sqrt{-\frac{1}{(4ac-b^2)^5}} (-2Aac - Ab^2 + 3Bab) + 48a^4c^3}{(4ac-b^2)^5} \right)$$

$$\sqrt{-\frac{1}{(4ac-b^2)^5}} (-2Aac - Ab^2 + 3Bab) \log \left( x^2 + \frac{-2Aabc - Ab^3 + 3Bab^2 + 64a^3c^3 \sqrt{-\frac{1}{(4ac-b^2)^5}} (-2Aac - Ab^2 + 3Bab) - 48a^4c^3}{(4ac-b^2)^5} \right)$$

$$+ \frac{6Aa^2bc - 8Ba^3c - Ba^2b^2 + x^6(4Aac^3 + 2Ab^2c^2 - 6Babc^2) + x^4(6Aabc^2 + 3Ab^3c - 16Ba^2c^2 - Bab^2c - Bb^4)}{64a^4c^3 - 32a^3b^2c^2 + 4a^2b^4c + x^8(64a^2c^5 - 32ab^2c^4 + 4b^4c^3) + x^6(128a^2bc^4 - 64ab^3c^3 + 8b^5c^2) + x^4(128a^3c^4 - 2}$$

## 25 Test file number 42

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.2\_Quartic/42\_1.2.2.5\_P-x-a+b\_x^2+c\_x^4-^p

### 25.1 Problem number 27

$$\int \frac{d + ex + fx^2}{(4 - 5x^2 + x^4)^2} dx$$

Optimal antiderivative

$$\frac{e(-2x^2 + 5)}{18x^4 - 90x^2 + 72} + \frac{x(17d + 20f - (5d + 8f)x^2)}{72x^4 - 360x^2 + 288} + \frac{(19d + 52f) \operatorname{arctanh}\left(\frac{x}{2}\right)}{432} - \frac{(d + 7f) \operatorname{arctanh}(x)}{54} + \frac{e \ln(-x^2 + 1)}{27} - \frac{e \ln(-x^2 + 4)}{27}$$

command

```
integrate((f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

## 25.2 Problem number 43

$$\int \frac{d + ex + fx^2}{(4 - 5x^2 + x^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{e(-2x^2 + 5)}{36(x^4 - 5x^2 + 4)^2} + \frac{x(17d + 20f - (5d + 8f)x^2)}{144(x^4 - 5x^2 + 4)^2} - \frac{e(-2x^2 + 5)}{54(x^4 - 5x^2 + 4)} \\ & - \frac{x(59d + 380f - 35(d + 4f)x^2)}{3456(x^4 - 5x^2 + 4)} - \frac{(313d + 820f) \operatorname{arctanh}\left(\frac{x}{2}\right)}{20736} \\ & + \frac{(13d + 25f) \operatorname{arctanh}(x)}{648} - \frac{e \ln(-x^2 + 1)}{81} + \frac{e \ln(-x^2 + 4)}{81} \end{aligned}$$

command

```
integrate((f*x**2+e*x+d)/(x**4-5*x**2+4)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

## 26 Test file number 43

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.2\_Quartic/43\_1.2.2.6\_P-x-d\_x-^m-a+b\_x^2+c\_x^4-^p

### 26.1 Problem number 64

$$\int \frac{x(d + ex^2 + fx^4)}{(a + bx^2 + cx^4)^2} dx$$

Optimal antiderivative

$$\frac{2ace - b(af + cd) - (-2acf + b^2f - bce + 2c^2d)x^2}{2c(-4ac + b^2)(cx^4 + bx^2 + a)} + \frac{(2af - be + 2cd) \operatorname{arctanh}\left(\frac{2cx^2 + b}{\sqrt{-4ac + b^2}}\right)}{(-4ac + b^2)^{\frac{3}{2}}}$$

command

```
integrate(x*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}(2af-be+2cd) \log\left(x^2 + \frac{-16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}(2af-be+2cd)+8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}(2af-be+2cd)}{4acf-2bce+4c^2d}\right)}{\sqrt{-\frac{1}{(4ac-b^2)^3}}(2af-be+2cd) \log\left(x^2 + \frac{16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}(2af-be+2cd)-8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}(2af-be+2cd)}{4acf-2bce+4c^2d}\right)} + \frac{abf-2ace+bcd+x^2(-2acf+b^2f-bce+2c^2d)}{8a^2c^2-2ab^2c+x^4(8ac^3-2b^2c^2)+x^2(8abc^2-2b^3c)}$$

## 26.2 Problem number 136

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}} dx$$

Optimal antiderivative

$$\frac{(ae^2+2bd^2) \operatorname{arctanh}\left(\frac{\sqrt{-ex+d}\sqrt{ex+d}}{d}\right)}{2d^3} - \frac{c\sqrt{-ex+d}\sqrt{ex+d}}{e^2} - \frac{a\sqrt{-ex+d}\sqrt{ex+d}}{2d^2x^2}$$

command

```
integrate((c*x**4+b*x**2+a)/x**3/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{iae^2 G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{7}{4}, \frac{9}{4}, 1 & 2, 2, \frac{5}{2} \\ \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2} & 0 \end{array} \middle| \frac{d^2}{e^2 x^2} \right) - ae^2 G_{6,6}^{2,6} \left( \begin{array}{c|c} 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 1 & \\ \frac{5}{4}, \frac{7}{4} & 1, \frac{3}{2}, \frac{3}{2}, 0 \end{array} \middle| \frac{d^2 e^{-2i\pi}}{e^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3} \\
& + \frac{ibG_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{3}{4}, \frac{5}{4}, 1 & 1, 1, \frac{3}{2} \\ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2} & 0 \end{array} \middle| \frac{d^2}{e^2 x^2} \right) - bG_{6,6}^{2,6} \left( \begin{array}{c|c} 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1 & \\ \frac{1}{4}, \frac{3}{4} & 0, \frac{1}{2}, \frac{1}{2}, 0 \end{array} \middle| \frac{d^2 e^{-2i\pi}}{e^2 x^2} \right)}{4\pi^{\frac{3}{2}} d} \\
& - \frac{icdG_{6,6}^{6,2} \left( \begin{array}{c|c} -\frac{1}{4}, \frac{1}{4} & 0, 0, \frac{1}{2}, 1 \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 \end{array} \middle| \frac{d^2}{e^2 x^2} \right)}{4\pi^{\frac{3}{2}} e^2} \\
& - \frac{cdG_{6,6}^{2,6} \left( \begin{array}{c|c} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 & \\ -\frac{3}{4}, -\frac{1}{4} & -1, -\frac{1}{2}, -\frac{1}{2}, 0 \end{array} \middle| \frac{d^2 e^{-2i\pi}}{e^2 x^2} \right)}{4\pi^{\frac{3}{2}} e^2}
\end{aligned}$$

### 26.3 Problem number 141

$$\int \frac{a + bx^2 + cx^4}{x^2 \sqrt{d - ex} \sqrt{d + ex}} dx$$

Optimal antiderivative

$$-\frac{(2be^2 + cd^2) \arctan\left(\frac{\sqrt{-ex + d}}{\sqrt{ex + d}}\right)}{e^3} + \frac{cx(ex - d) \sqrt{ex + d}}{2e^2 \sqrt{-ex + d}} - \frac{a\sqrt{-ex + d} \sqrt{ex + d}}{d^2 x}$$

command

```
integrate((c*x**4+b*x**2+a)/x**2/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{iaeG_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{5}{4}, \frac{7}{4}, 1 & \frac{3}{2}, \frac{3}{2}, 2 \\ \hline 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2 & 0 \end{array} \middle| \frac{d^2}{e^2 x^2} \right) + aeG_{6,6}^{2,6} \left( \begin{array}{c|c} \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 1 & \\ \hline \frac{3}{4}, \frac{5}{4} & \frac{1}{2}, 1, 1, 0 \end{array} \middle| \frac{d^2 e^{-2i\pi}}{e^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^2} \\
& - \frac{ibG_{6,6}^{6,2} \left( \begin{array}{c|c} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ \hline 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 & \end{array} \middle| \frac{d^2}{e^2 x^2} \right) + bG_{6,6}^{2,6} \left( \begin{array}{c|c} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 & \\ \hline -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, 0, 0, 0 \end{array} \middle| \frac{d^2 e^{-2i\pi}}{e^2 x^2} \right)}{4\pi^{\frac{3}{2}} e} \\
& - \frac{icd^2 G_{6,6}^{6,2} \left( \begin{array}{c|c} -\frac{3}{4}, -\frac{1}{4} & -\frac{1}{2}, -\frac{1}{2}, 0, 1 \\ \hline -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 & \end{array} \middle| \frac{d^2}{e^2 x^2} \right)}{4\pi^{\frac{3}{2}} e^3} \\
& + \frac{cd^2 G_{6,6}^{2,6} \left( \begin{array}{c|c} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 & \\ \hline -\frac{5}{4}, -\frac{3}{4} & -\frac{3}{2}, -1, -1, 0 \end{array} \middle| \frac{d^2 e^{-2i\pi}}{e^2 x^2} \right)}{4\pi^{\frac{3}{2}} e^3}
\end{aligned}$$

## 27 Test file number 46

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.3\_General/46\_1.2.3.2-d\_x-  
 $\hat{m}-a+b_x^{\hat{n}}+c_x^{-2}_n^{-\hat{p}}$

### 27.1 Problem number 142

$$\int \frac{1}{x^4 (a + bx^3 + cx^6)} dx$$

Optimal antiderivative

$$-\frac{1}{3ax^3} - \frac{b \ln(x)}{a^2} + \frac{b \ln(cx^6 + bx^3 + a)}{6a^2} - \frac{(-2ac + b^2) \operatorname{arctanh}\left(\frac{2cx^3 + b}{\sqrt{-4ac + b^2}}\right)}{3a^2 \sqrt{-4ac + b^2}}$$

command

`integrate(1/x**4/(c*x**6+b*x**3+a), x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \left( \frac{b}{6a^2} \right. \\
& \left. - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{6a^2 (4ac - b^2)} \right) \log \left( x^3 + \frac{-12a^3c \left( \frac{b}{6a^2} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{6a^2 (4ac - b^2)} \right) + 3a^2b^2 \left( \frac{b}{6a^2} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{6a^2 (4ac - b^2)} \right)}{2ac^2 - b^2c} \right) \\
& + \left( \frac{b}{6a^2} \right. \\
& \left. + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{6a^2 (4ac - b^2)} \right) \log \left( x^3 + \frac{-12a^3c \left( \frac{b}{6a^2} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{6a^2 (4ac - b^2)} \right) + 3a^2b^2 \left( \frac{b}{6a^2} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{6a^2 (4ac - b^2)} \right)}{2ac^2 - b^2c} \right) \\
& - \frac{1}{3ax^3} - \frac{b \log(x)}{a^2}
\end{aligned}$$

## 27.2 Problem number 150

$$\int \frac{1}{x^3 (a + bx^3 + cx^6)} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{1}{2ax^2} - \frac{c^{\frac{2}{3}} \ln\left(2^{\frac{1}{3}}c^{\frac{1}{3}}x + (b - \sqrt{-4ac + b^2})^{\frac{1}{3}}\right) \left(1 + \frac{b}{\sqrt{-4ac + b^2}}\right) 2^{\frac{2}{3}}}{6a \left(b - \sqrt{-4ac + b^2}\right)^{\frac{2}{3}}} \\
& + \frac{c^{\frac{2}{3}} \ln\left(2^{\frac{2}{3}}c^{\frac{2}{3}}x^2 - 2^{\frac{1}{3}}c^{\frac{1}{3}}x \left(b - \sqrt{-4ac + b^2}\right)^{\frac{1}{3}} + \left(b - \sqrt{-4ac + b^2}\right)^{\frac{2}{3}}\right) \left(1 + \frac{b}{\sqrt{-4ac + b^2}}\right) 2^{\frac{2}{3}}}{12a \left(b - \sqrt{-4ac + b^2}\right)^{\frac{2}{3}}} \\
& + \frac{c^{\frac{2}{3}} \arctan\left(\frac{\left(1 - \frac{2 \cdot 2^{\frac{1}{3}}c^{\frac{1}{3}}x}{\left(b - \sqrt{-4ac + b^2}\right)^{\frac{1}{3}}}\right) \sqrt{3}}{3}\right) \left(1 + \frac{b}{\sqrt{-4ac + b^2}}\right) 2^{\frac{2}{3}} \sqrt{3}}{6a \left(b - \sqrt{-4ac + b^2}\right)^{\frac{2}{3}}} \\
& - \frac{c^{\frac{2}{3}} \ln\left(2^{\frac{1}{3}}c^{\frac{1}{3}}x + (b + \sqrt{-4ac + b^2})^{\frac{1}{3}}\right) \left(1 - \frac{b}{\sqrt{-4ac + b^2}}\right) 2^{\frac{2}{3}}}{6a \left(b + \sqrt{-4ac + b^2}\right)^{\frac{2}{3}}} \\
& + \frac{c^{\frac{2}{3}} \ln\left(2^{\frac{2}{3}}c^{\frac{2}{3}}x^2 - 2^{\frac{1}{3}}c^{\frac{1}{3}}x \left(b + \sqrt{-4ac + b^2}\right)^{\frac{1}{3}} + \left(b + \sqrt{-4ac + b^2}\right)^{\frac{2}{3}}\right) \left(1 - \frac{b}{\sqrt{-4ac + b^2}}\right) 2^{\frac{2}{3}}}{12a \left(b + \sqrt{-4ac + b^2}\right)^{\frac{2}{3}}} \\
& + \frac{c^{\frac{2}{3}} \arctan\left(\frac{\left(1 - \frac{2 \cdot 2^{\frac{1}{3}}c^{\frac{1}{3}}x}{\left(b + \sqrt{-4ac + b^2}\right)^{\frac{1}{3}}}\right) \sqrt{3}}{3}\right) \left(1 - \frac{b}{\sqrt{-4ac + b^2}}\right) 2^{\frac{2}{3}} \sqrt{3}}{6a \left(b + \sqrt{-4ac + b^2}\right)^{\frac{2}{3}}}
\end{aligned}$$

command

```
integrate(1/x**3/(c*x**6+b*x**3+a), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output



$$\text{RootSum} \left( t^6 (46656a^8c^3 - 34992a^7b^2c^2 + 8748a^6b^4c - 729a^5b^6) + t^3 (-432a^4c^4 + 1512a^3b^2c^3 - 1107a^2b^4c^2 + 297a^2b^6) - \frac{1}{2ax^2} \right)$$

### 27.3 Problem number 321

$$\int \frac{x^6}{a + bx^4 + cx^8} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\arctan \left( \frac{2^{\frac{1}{4}} c^{\frac{1}{4}} x}{(-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}} \right) (-b - \sqrt{-4ac + b^2})^{\frac{3}{4}} 2^{\frac{1}{4}}}{4c^{\frac{3}{4}} \sqrt{-4ac + b^2}} \\ & + \frac{\operatorname{arctanh} \left( \frac{2^{\frac{1}{4}} c^{\frac{1}{4}} x}{(-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}} \right) (-b - \sqrt{-4ac + b^2})^{\frac{3}{4}} 2^{\frac{1}{4}}}{4c^{\frac{3}{4}} \sqrt{-4ac + b^2}} \\ & + \frac{\arctan \left( \frac{2^{\frac{1}{4}} c^{\frac{1}{4}} x}{(-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}} \right) (-b + \sqrt{-4ac + b^2})^{\frac{3}{4}} 2^{\frac{1}{4}}}{4c^{\frac{3}{4}} \sqrt{-4ac + b^2}} \\ & - \frac{\operatorname{arctanh} \left( \frac{2^{\frac{1}{4}} c^{\frac{1}{4}} x}{(-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}} \right) (-b + \sqrt{-4ac + b^2})^{\frac{3}{4}} 2^{\frac{1}{4}}}{4c^{\frac{3}{4}} \sqrt{-4ac + b^2}} \end{aligned}$$

command

`integrate(x**6/(c*x**8+b*x**4+a), x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\text{RootSum} \left( t^8 (16777216a^4c^7 - 16777216a^3b^2c^6 + 6291456a^2b^4c^5 - 1048576ab^6c^4 + 65536b^8c^3) + t^4 (-12288a^3bc^3 - 12288a^2b^3c^2 + 12288a^2b^5c - 12288ab^7c) - \frac{1}{2ax^2} \right)$$

## 27.4 Problem number 324

$$\int \frac{1}{a + bx^4 + cx^8} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{c^{\frac{3}{4}} \arctan\left(\frac{2^{\frac{1}{4}} c^{\frac{1}{4}} x}{(-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}}\right) 2^{\frac{3}{4}}}{2(-b - \sqrt{-4ac + b^2})^{\frac{3}{4}} \sqrt{-4ac + b^2}} + \frac{c^{\frac{3}{4}} \operatorname{arctanh}\left(\frac{2^{\frac{1}{4}} c^{\frac{1}{4}} x}{(-b - \sqrt{-4ac + b^2})^{\frac{1}{4}}}\right) 2^{\frac{3}{4}}}{2(-b - \sqrt{-4ac + b^2})^{\frac{3}{4}} \sqrt{-4ac + b^2}} \\ & - \frac{c^{\frac{3}{4}} \arctan\left(\frac{2^{\frac{1}{4}} c^{\frac{1}{4}} x}{(-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}}\right) 2^{\frac{3}{4}}}{2\sqrt{-4ac + b^2} (-b + \sqrt{-4ac + b^2})^{\frac{3}{4}}} - \frac{c^{\frac{3}{4}} \operatorname{arctanh}\left(\frac{2^{\frac{1}{4}} c^{\frac{1}{4}} x}{(-b + \sqrt{-4ac + b^2})^{\frac{1}{4}}}\right) 2^{\frac{3}{4}}}{2\sqrt{-4ac + b^2} (-b + \sqrt{-4ac + b^2})^{\frac{3}{4}}} \end{aligned}$$

command

```
integrate(1/(c*x**8+b*x**4+a), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\operatorname{RootSum}\left(t^8(16777216a^7c^4 - 16777216a^6b^2c^3 + 6291456a^5b^4c^2 - 1048576a^4b^6c + 65536a^3b^8) + t^4(-12288a^3bc^3 - \dots)\right)$$

## 27.5 Problem number 502

$$\int \frac{x^{-1+\frac{n}{2}}}{bx^n + cx^{2n}} dx$$

Optimal antiderivative

$$-\frac{2x^{-\frac{n}{2}}}{bn} + \frac{2 \arctan\left(\frac{\sqrt{b} x^{-\frac{n}{2}}}{\sqrt{c}}\right) \sqrt{c}}{b^{\frac{3}{2}} n}$$

command

```
integrate(x**(-1+1/2*n)/(b*x**n+c*x**(2*n)), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$-\frac{2 \operatorname{atan}\left(\frac{x^{\frac{n}{2}}}{\sqrt{\frac{b}{c}}}\right)}{bn\sqrt{\frac{b}{c}}} - \frac{2x^{-\frac{n}{2}}}{bn}$$

## 27.6 Problem number 503

$$\int \frac{x^{-1-\frac{n}{2}}}{bx^n + cx^{2n}} dx$$

Optimal antiderivative

$$-\frac{2x^{-\frac{3n}{2}}}{3bn} + \frac{2cx^{-\frac{n}{2}}}{b^2n} - \frac{2c^{\frac{3}{2}} \arctan\left(\frac{\sqrt{b}x^{-\frac{n}{2}}}{\sqrt{c}}\right)}{b^{\frac{5}{2}}n}$$

command

```
integrate(x**(-1-1/2*n)/(b*x**n+c*x**(2*n)), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$-\frac{2x^{-\frac{3n}{2}}}{3bn} + \frac{2cx^{-\frac{n}{2}}}{b^2n} - \frac{2c^2 \operatorname{atan}\left(\frac{x^{-\frac{n}{2}}}{\sqrt{\frac{c}{b}}}\right)}{b^3n\sqrt{\frac{c}{b}}}$$

## 27.7 Problem number 619

$$\int \frac{1}{(d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{1}{2ae(ex+d)^2} - \frac{b \ln(ex+d)}{a^2e} + \frac{b \ln(a+b(ex+d)^2+c(ex+d)^4)}{4a^2e} \\ & - \frac{(-2ac+b^2) \operatorname{arctanh}\left(\frac{b+2c(ex+d)^2}{\sqrt{-4ac+b^2}}\right)}{2a^2e\sqrt{-4ac+b^2}} \end{aligned}$$

command

```
integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \left( \frac{b}{4a^2e} \right. \\ & \left. - \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2e(4ac-b^2)} \right) \log \left( \frac{2dx}{e} + x^2 + \frac{-8a^3ce \left( \frac{b}{4a^2e} - \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2e(4ac-b^2)} \right) + 2a^2b^2e \left( \frac{b}{4a^2e} - \frac{\sqrt{-4ac+b^2}}{4a^2e(4ac-b^2)} \right)}{2ac^2e^2 - b^2ce^2} \right) \\ & + \left( \frac{b}{4a^2e} \right. \\ & \left. + \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2e(4ac-b^2)} \right) \log \left( \frac{2dx}{e} + x^2 + \frac{-8a^3ce \left( \frac{b}{4a^2e} + \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2e(4ac-b^2)} \right) + 2a^2b^2e \left( \frac{b}{4a^2e} + \frac{\sqrt{-4ac+b^2}}{4a^2e(4ac-b^2)} \right)}{2ac^2e^2 - b^2ce^2} \right) \\ & - \frac{1}{2ad^2e + 4ade^2x + 2ae^3x^2} - \frac{b \log\left(\frac{d}{e} + x\right)}{a^2e} \end{aligned}$$

**27.8 Problem number 623**

$$\int \frac{(d+ex)^2}{(a+b(d+ex)^2+c(d+ex)^4)^2} dx$$

Optimal antiderivative

$$\frac{(ex+d)(b+2c(ex+d)^2)}{2(-4ac+b^2)e(a+b(ex+d)^2+c(ex+d)^4)}$$

$$+ \frac{\arctan\left(\frac{(ex+d)\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\sqrt{c}(2b-\sqrt{-4ac+b^2})\sqrt{2}}{2(-4ac+b^2)^{\frac{3}{2}}e\sqrt{b-\sqrt{-4ac+b^2}}}$$

$$- \frac{\arctan\left(\frac{(ex+d)\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\sqrt{c}(2b+\sqrt{-4ac+b^2})\sqrt{2}}{2(-4ac+b^2)^{\frac{3}{2}}e\sqrt{b+\sqrt{-4ac+b^2}}}$$

command

```
integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{bd+2cd^3+6cde^2x^2+2ce^3x^3+x(bd+2cd^3+6cde^2x^2+2ce^3x^3+x^4(8ac^2e^5-2b^2ce^5)+x^3(32ac^2de^4-8b^2cde^4)+x^2(8abd^2e^4-8b^2cd^2e^4)+x(8abd^2e^4-8b^2cd^2e^4)+8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4(8ac^2e^5-2b^2ce^5)+x^3(32ac^2de^4-8b^2cde^4)+x^2(8abd^2e^4-8b^2cd^2e^4)+x(8abd^2e^4-8b^2cd^2e^4)+8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e}{(8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4(8ac^2e^5-2b^2ce^5)+x^3(32ac^2de^4-8b^2cde^4)+x^2(8abd^2e^4-8b^2cd^2e^4)+x(8abd^2e^4-8b^2cd^2e^4)+8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e)^2}$$

$$+ \text{RootSum}\left(t^4(1048576a^7c^6e^4-1572864a^6b^2c^5e^4+983040a^5b^4c^4e^4-327680a^4b^6c^3e^4+61440a^3b^8c^2e^4-6144a^2b^{10}c^1e^4)\right)$$

**27.9 Problem number 644**

$$\int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)} dx$$

Optimal antiderivative

$$\frac{1}{2ae f^3 (ex+d)^2} - \frac{b \ln(ex+d)}{a^2e f^3} + \frac{b \ln(a+b(ex+d)^2+c(ex+d)^4)}{4a^2e f^3}$$

$$- \frac{(-2ac+b^2) \operatorname{arctanh}\left(\frac{b+2c(ex+d)^2}{\sqrt{-4ac+b^2}}\right)}{2a^2e f^3 \sqrt{-4ac+b^2}}$$

command

```
integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \left( \frac{b}{4a^2ef^3} \right. \\ & \left. - \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2ef^3(4ac-b^2)} \right) \log \left( \frac{2dx}{e} + x^2 + \frac{-8a^3cef^3 \left( \frac{b}{4a^2ef^3} - \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2ef^3(4ac-b^2)} \right) + 2a^2b^2ef^3 \left( \frac{b}{4a^2ef^3} - \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2ef^3(4ac-b^2)} \right)}{2ac^2e^2 - b^2ce^2} \right) \\ & + \left( \frac{b}{4a^2ef^3} \right. \\ & \left. + \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2ef^3(4ac-b^2)} \right) \log \left( \frac{2dx}{e} + x^2 + \frac{-8a^3cef^3 \left( \frac{b}{4a^2ef^3} + \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2ef^3(4ac-b^2)} \right) + 2a^2b^2ef^3 \left( \frac{b}{4a^2ef^3} + \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2ef^3(4ac-b^2)} \right)}{2ac^2e^2 - b^2ce^2} \right) \\ & - \frac{1}{2ad^2ef^3 + 4ade^2f^3x + 2ae^3f^3x^2} - \frac{b \log \left( \frac{d}{e} + x \right)}{a^2ef^3} \end{aligned}$$

## 27.10 Problem number 648

$$\int \frac{(df + efx)^2}{(a + b(d + ex)^2 + c(d + ex)^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{f^2(ex+d)(b+2c(ex+d)^2)}{2(-4ac+b^2)e(a+b(ex+d)^2+c(ex+d)^4)} \\ & + \frac{f^2 \arctan \left( \frac{(ex+d)\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}} \right) \sqrt{c} (2b - \sqrt{-4ac+b^2}) \sqrt{2}}{2(-4ac+b^2)^{\frac{3}{2}} e \sqrt{b-\sqrt{-4ac+b^2}}} \\ & - \frac{f^2 \arctan \left( \frac{(ex+d)\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}} \right) \sqrt{c} (2b + \sqrt{-4ac+b^2}) \sqrt{2}}{2(-4ac+b^2)^{\frac{3}{2}} e \sqrt{b+\sqrt{-4ac+b^2}}} \end{aligned}$$

command

```
integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{bdf^2 + 2cd^3f^2 + 6cde^2f^2x^2 + 2ce^3f^2x^3 + x^4(8a^2ce - 2ab^2e + 8abcd^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2cd^4e + x^4(8ac^2e^5 - 2b^2ce^5) + x^3(32ac^2de^4 - 8b^2cde^4) + x^2(8a^2ce - 2ab^2e + 8abcd^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2cd^4e) + x(8ac^2e^5 - 2b^2ce^5) + e^5)}{8a^2ce - 2ab^2e + 8abcd^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2cd^4e + x^4(8ac^2e^5 - 2b^2ce^5) + x^3(32ac^2de^4 - 8b^2cde^4) + x^2(8a^2ce - 2ab^2e + 8abcd^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2cd^4e) + x(8ac^2e^5 - 2b^2ce^5) + e^5}$$

$$+\text{RootSum}\left(t^4(1048576a^7c^6e^4 - 1572864a^6b^2c^5e^4 + 983040a^5b^4c^4e^4 - 327680a^4b^6c^3e^4 + 61440a^3b^8c^2e^4 - 6144a^2b^{10}c^2e^4 + 1536a^2b^{12}ce^4 - 1536ab^{14}e^4 + 256b^{16}e^4)\right)$$

## 28 Test file number 48

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.3\_General/48\_1.2.3.4-f\_x-  
^m-d+e\_x^n-q-a+b\_x^n+c\_x^-2\_n-p

### 28.1 Problem number 9

$$\int \frac{x^8(d + ex^3)}{a + bx^3 + cx^6} dx$$

Optimal antiderivative

$$\frac{(-be + cd)x^3}{3c^2} + \frac{ex^6}{6c} - \frac{(ace - b^2e + bcd) \ln(cx^6 + bx^3 + a)}{6c^3}$$

$$- \frac{(3abce - 2ac^2d - b^3e + b^2cd) \operatorname{arctanh}\left(\frac{2cx^3 + b}{\sqrt{-4ac + b^2}}\right)}{3c^3 \sqrt{-4ac + b^2}}$$

command

```
integrate(x**8*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& x^3 \left( -\frac{be}{3c^2} + \frac{d}{3c} \right) + \left( -\frac{\sqrt{-4ac+b^2} (3abce - 2ac^2d - b^3e + b^2cd)}{6c^3 (4ac - b^2)} \right. \\
& \left. - \frac{ace - b^2e + bcd}{6c^3} \right) \log \left( x^3 + \frac{2a^2ce - ab^2e + abcd + 12ac^3 \left( -\frac{\sqrt{-4ac+b^2} (3abce - 2ac^2d - b^3e + b^2cd)}{6c^3 (4ac - b^2)} - \frac{ace - b^2e + bcd}{6c^3} \right)}{3abce - 2ac^2d - b^3e + b^2cd} \right) \\
& + \left( \frac{\sqrt{-4ac+b^2} (3abce - 2ac^2d - b^3e + b^2cd)}{6c^3 (4ac - b^2)} \right. \\
& \left. - \frac{ace - b^2e + bcd}{6c^3} \right) \log \left( x^3 + \frac{2a^2ce - ab^2e + abcd + 12ac^3 \left( \frac{\sqrt{-4ac+b^2} (3abce - 2ac^2d - b^3e + b^2cd)}{6c^3 (4ac - b^2)} - \frac{ace - b^2e + bcd}{6c^3} \right)}{3abce - 2ac^2d - b^3e + b^2cd} \right) \\
& + \frac{ex^6}{6c}
\end{aligned}$$

## 29 Test file number 50

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.4\_Improper/50\_1.2.4.2-d\_x~m-a\_x~q+b\_x~n+c\_x~-2\_n-q~p

### 29.1 Problem number 88

$$\int \frac{1}{x^2(ax + bx^3 + cx^5)} dx$$

Optimal antiderivative

$$-\frac{1}{2ax^2} - \frac{b \ln(x)}{a^2} + \frac{b \ln(cx^4 + bx^2 + a)}{4a^2} - \frac{(-2ac + b^2) \operatorname{arctanh}\left(\frac{2cx^2 + b}{\sqrt{-4ac + b^2}}\right)}{2a^2 \sqrt{-4ac + b^2}}$$

command

```
integrate(1/x**2/(c*x**5+b*x**3+a*x), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output



$$\begin{aligned}
& \left( \frac{b}{4a^2} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2 (4ac - b^2)} \right) \log \left( x^2 + \frac{-8a^3c \left( \frac{b}{4a^2} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2 (4ac - b^2)} \right) + 2a^2b^2 \left( \frac{b}{4a^2} - \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2 (4ac - b^2)} \right)}{2ac^2 - b^2c} \right) \\
& + \left( \frac{b}{4a^2} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2 (4ac - b^2)} \right) \log \left( x^2 + \frac{-8a^3c \left( \frac{b}{4a^2} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2 (4ac - b^2)} \right) + 2a^2b^2 \left( \frac{b}{4a^2} + \frac{\sqrt{-4ac + b^2} (2ac - b^2)}{4a^2 (4ac - b^2)} \right)}{2ac^2 - b^2c} \right) \\
& - \frac{1}{2ax^2} - \frac{b \log(x)}{a^2}
\end{aligned}$$

## 29.2 Problem number 89

$$\int \frac{x^{11}}{(ax + bx^3 + cx^5)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{(-3ac + b^2) x^2}{c^2 (-4ac + b^2)} - \frac{bx^4}{2c(-4ac + b^2)} + \frac{x^6(bx^2 + 2a)}{2(-4ac + b^2)(cx^4 + bx^2 + a)} \\
& - \frac{(6a^2c^2 - 6ab^2c + b^4) \operatorname{arctanh}\left(\frac{2cx^2 + b}{\sqrt{-4ac + b^2}}\right)}{c^3 (-4ac + b^2)^{\frac{3}{2}}} - \frac{b \ln(cx^4 + bx^2 + a)}{2c^3}
\end{aligned}$$

command

```
integrate(x**11/(c*x**5+b*x**3+a*x)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \left( -\frac{b}{2c^3} \right. \\
& \left. - \frac{\sqrt{-(4ac-b^2)^3} (6a^2c^2 - 6ab^2c + b^4)}{2c^3 (64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)} \right) \log \left( x^2 + \frac{-5a^2bc - 16a^2c^4 \left( -\frac{b}{2c^3} - \frac{\sqrt{-(4ac-b^2)^3} (6a^2c^2 - 6ab^2c + b^4)}{2c^3 (64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)} \right)}{\dots} \right) \\
& + \left( -\frac{b}{2c^3} \right. \\
& \left. + \frac{\sqrt{-(4ac-b^2)^3} (6a^2c^2 - 6ab^2c + b^4)}{2c^3 (64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)} \right) \log \left( x^2 + \frac{-5a^2bc - 16a^2c^4 \left( -\frac{b}{2c^3} + \frac{\sqrt{-(4ac-b^2)^3} (6a^2c^2 - 6ab^2c + b^4)}{2c^3 (64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)} \right)}{\dots} \right) \\
& + \frac{-3a^2bc + ab^3 + x^2(2a^2c^2 - 4ab^2c + b^4)}{8a^2c^4 - 2ab^2c^3 + x^4(8ac^5 - 2b^2c^4) + x^2(8abc^4 - 2b^3c^3)} + \frac{x^2}{2c^2}
\end{aligned}$$

### 29.3 Problem number 91

$$\int \frac{x^9}{(ax + bx^3 + cx^5)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{bx^2}{2c(-4ac + b^2)} + \frac{x^4(bx^2 + 2a)}{2(-4ac + b^2)(cx^4 + bx^2 + a)} \\
& + \frac{b(-6ac + b^2) \operatorname{arctanh}\left(\frac{2cx^2 + b}{\sqrt{-4ac + b^2}}\right)}{2c^2(-4ac + b^2)^{\frac{3}{2}}} + \frac{\ln(cx^4 + bx^2 + a)}{4c^2}
\end{aligned}$$

command

```
integrate(x**9/(c*x**5+b*x**3+a*x)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \left( -\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} \right. \\
& + \frac{1}{4c^2} \left. \right) \log \left( x^2 + \frac{-32a^2c^3 \left( -\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} + \frac{1}{4c^2} \right) + 8a^2c + 16ab^2c^2 \left( -\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} \right)}{6abc-b^3} \right) \\
& + \left( \frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} \right. \\
& + \frac{1}{4c^2} \left. \right) \log \left( x^2 + \frac{-32a^2c^3 \left( \frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} + \frac{1}{4c^2} \right) + 8a^2c + 16ab^2c^2 \left( \frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} \right)}{6abc-b^3} \right) \\
& + \frac{2a^2c-ab^2+x^2(3abc-b^3)}{8a^2c^3-2ab^2c^2+x^4(8ac^4-2b^2c^3)+x^2(8abc^3-2b^3c^2)}
\end{aligned}$$

## 29.4 Problem number 92

$$\int \frac{x^8}{(ax+bx^3+cx^5)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{bx}{2c(-4ac+b^2)} + \frac{x^3(bx^2+2a)}{2(-4ac+b^2)(cx^4+bx^2+a)} \\
& + \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right) \left(b^2-6ac-\frac{b(-8ac+b^2)}{\sqrt{-4ac+b^2}}\right) \sqrt{2}}{4c^{\frac{3}{2}}(-4ac+b^2)\sqrt{b-\sqrt{-4ac+b^2}}} \\
& + \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right) \left(b^2-6ac+\frac{b(-8ac+b^2)}{\sqrt{-4ac+b^2}}\right) \sqrt{2}}{4c^{\frac{3}{2}}(-4ac+b^2)\sqrt{b+\sqrt{-4ac+b^2}}}
\end{aligned}$$

command

`integrate(x**8/(c*x**5+b*x**3+a*x)**2,x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{abx + x^3(-2ac + b^2)}{8a^2c^2 - 2ab^2c + x^4(8ac^3 - 2b^2c^2) + x^2(8abc^2 - 2b^3c)} + \text{RootSum} \left( t^4(1048576a^6c^9 - 1572864a^5b^2c^8 + 983040a^4b^4c^7 - 327680a^3b^6c^6 + 61440a^2b^8c^5 - 6144ab^{10}c^4 + 256b^{12}c^3) \right)$$

**29.5 Problem number 96**

$$\int \frac{x^4}{(ax + bx^3 + cx^5)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{x(2cx^2 + b)}{2(-4ac + b^2)(cx^4 + bx^2 + a)} \\ & + \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \sqrt{c} (2b - \sqrt{-4ac + b^2}) \sqrt{2}}{2(-4ac + b^2)^{\frac{3}{2}} \sqrt{b - \sqrt{-4ac + b^2}}} \\ & - \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \sqrt{c} (2b + \sqrt{-4ac + b^2}) \sqrt{2}}{2(-4ac + b^2)^{\frac{3}{2}} \sqrt{b + \sqrt{-4ac + b^2}}} \end{aligned}$$

command`integrate(x**4/(c*x**5+b*x**3+a*x)**2,x)`Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{bx + 2cx^3}{8a^2c - 2ab^2 + x^4(8ac^2 - 2b^2c) + x^2(8abc - 2b^3)} + \text{RootSum} \left( t^4(1048576a^7c^6 - 1572864a^6b^2c^5 + 983040a^5b^4c^4 - 327680a^4b^6c^3 + 61440a^3b^8c^2 - 6144a^2b^{10}c + 256ab^{12}) \right)$$

## 29.6 Problem number 98

$$\int \frac{x^2}{(ax + bx^3 + cx^5)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(bc x^2 - 2ac + b^2)}{2a(-4ac + b^2)(cx^4 + bx^2 + a)} \\ & + \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \sqrt{c} (b^2 - 12ac + b\sqrt{-4ac + b^2}) \sqrt{2}}{4a(-4ac + b^2)^{\frac{3}{2}} \sqrt{b - \sqrt{-4ac + b^2}}} \\ & - \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \sqrt{c} (b^2 - 12ac - b\sqrt{-4ac + b^2}) \sqrt{2}}{4a(-4ac + b^2)^{\frac{3}{2}} \sqrt{b + \sqrt{-4ac + b^2}}} \end{aligned}$$

command

```
integrate(x**2/(c*x**5+b*x**3+a*x)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{-bcx^3 + x(2ac - b^2)}{8a^3c - 2a^2b^2 + x^4(8a^2c^2 - 2ab^2c) + x^2(8a^2bc - 2ab^3)} \\ & + \text{RootSum}\left(t^4(1048576a^9c^6 - 1572864a^8b^2c^5 + 983040a^7b^4c^4 - 327680a^6b^6c^3 + 61440a^5b^8c^2 - 6144a^4b^{10}c + 256a^3b^{12})\right) \end{aligned}$$

## 30 Test file number 51

Test folder name:

test\_cases/1\_Algebraic\_functions/1.3\_Miscellaneous/51\_1.3.1\_Rational\_functions

## 30.1 Problem number 38

$$\int \frac{1}{(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(\frac{c}{d} + x\right) \left(c^3 - 4ad^2 - cd^2\left(\frac{c}{d} + x\right)^2\right)}{16ac(4ad^2 + c^3)(d^2x^4 + 4cdx^3 + 4c^2x^2 + 4ac)} \\ & - \frac{d \operatorname{arctanh}\left(\frac{c\sqrt{2} + dx\sqrt{2} + c^{\frac{1}{4}}\sqrt{c^{\frac{3}{2}} + \sqrt{4ad^2 + c^3}}}{c^{\frac{1}{4}}\sqrt{c^{\frac{3}{2}} - \sqrt{4ad^2 + c^3}}}\right) \left(c^3 + 12ad^2 + c^{\frac{3}{2}}\sqrt{4ad^2 + c^3}\right) \sqrt{2}}{64ac^{\frac{7}{4}}(4ad^2 + c^3)^{\frac{3}{2}}\sqrt{c^{\frac{3}{2}} - \sqrt{4ad^2 + c^3}}} \\ & + \frac{d \operatorname{arctanh}\left(\frac{-(dx+c)\sqrt{2} + c^{\frac{1}{4}}\sqrt{c^{\frac{3}{2}} + \sqrt{4ad^2 + c^3}}}{c^{\frac{1}{4}}\sqrt{c^{\frac{3}{2}} - \sqrt{4ad^2 + c^3}}}\right) \left(c^3 + 12ad^2 + c^{\frac{3}{2}}\sqrt{4ad^2 + c^3}\right) \sqrt{2}}{64ac^{\frac{7}{4}}(4ad^2 + c^3)^{\frac{3}{2}}\sqrt{c^{\frac{3}{2}} - \sqrt{4ad^2 + c^3}}} \\ & - \frac{d \ln\left(d^2\left(\frac{c}{d} + x\right)^2 + \sqrt{c}\sqrt{4ad^2 + c^3} - c^{\frac{1}{4}}d\left(\frac{c}{d} + x\right)\sqrt{2}\sqrt{c^{\frac{3}{2}} + \sqrt{4ad^2 + c^3}}\right) \left(c^3 + 12ad^2 - c^{\frac{3}{2}}\sqrt{4ad^2 + c^3}\right)}{128ac^{\frac{7}{4}}(4ad^2 + c^3)^{\frac{3}{2}}\sqrt{c^{\frac{3}{2}} + \sqrt{4ad^2 + c^3}}} \\ & + \frac{d \ln\left(d^2\left(\frac{c}{d} + x\right)^2 + \sqrt{c}\sqrt{4ad^2 + c^3} + c^{\frac{1}{4}}d\left(\frac{c}{d} + x\right)\sqrt{2}\sqrt{c^{\frac{3}{2}} + \sqrt{4ad^2 + c^3}}\right) \left(c^3 + 12ad^2 - c^{\frac{3}{2}}\sqrt{4ad^2 + c^3}\right)}{128ac^{\frac{7}{4}}(4ad^2 + c^3)^{\frac{3}{2}}\sqrt{c^{\frac{3}{2}} + \sqrt{4ad^2 + c^3}}} \end{aligned}$$

command

```
integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{4acd + 3c^2dx^2 + cd^2x^3 + x(4ad^2 + 2c^3)}{256a^3c^2d^2 + 64a^2c^5 + x^4(64a^2cd^4 + 16ac^4d^2) + x^3(256a^2c^2d^3 + 64ac^5d) + x^2(256a^2c^3d^2 + 64ac^6)} \\ & + \operatorname{RootSum}\left(t^4(1073741824a^9c^7d^6 + 805306368a^8c^{10}d^4 + 201326592a^7c^{13}d^2 + 16777216a^6c^{16}) + t^2(491520a^5c^5d^4\right. \end{aligned}$$

## 31 Test file number 52

Test folder name:

test\_cases/1\_Algebraic\_functions/1.3\_Miscellaneous/52\_1.3.2\_Algebraic\_functions

### 31.1 Problem number 289

$$\int x \sqrt{\frac{1-x^2}{1+x^2}} dx$$

Optimal antiderivative

$$-\arctan\left(\sqrt{\frac{-x^2+1}{x^2+1}}\right) + \frac{(x^2+1)\sqrt{\frac{-x^2+1}{x^2+1}}}{2}$$

command

```
integrate(x*((-x**2+1)/(x**2+1))**(1/2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

$$\int x \sqrt{-\frac{(x-1)(x+1)}{x^2+1}} dx$$

Sympy 1.8 under Python 3.8.8 output

$$\left\{ \frac{\sqrt{1-x^2}\sqrt{x^2+1}}{2} - \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{1-x^2}}{2}\right) \right. \text{ for } x > -1 \wedge x < 1$$

### 31.2 Problem number 290

$$\int x \sqrt{\frac{5-7x^2}{7+5x^2}} dx$$

Optimal antiderivative

$$-\frac{37 \arctan\left(\frac{\sqrt{35}\sqrt{\frac{-7x^2+5}{5x^2+7}}}{7}\right)\sqrt{35}}{175} + \frac{(5x^2+7)\sqrt{\frac{-7x^2+5}{5x^2+7}}}{10}$$

command

`integrate(x*((-7*x**2+5)/(5*x**2+7))**(1/2),x)`

Sympy 1.10.1 under Python 3.10.4 output

$$\int x \sqrt{-\frac{7x^2 - 5}{5x^2 + 7}} dx$$

Sympy 1.8 under Python 3.8.8 output

$$\left\{ \frac{5\sqrt{35} \left( \frac{\sqrt{25 - 35x^2} \sqrt{35x^2 + 49}}{125} - \frac{{}^{74} \operatorname{asin} \left( \frac{\sqrt{74} \sqrt{25 - 35x^2}}{74} \right)}{125} \right)}{14} \right. \text{ for } x > -\frac{\sqrt{35}}{7} \wedge x < \frac{\sqrt{35}}{7}$$

### 31.3 Problem number 573

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}} + \frac{1}{\sqrt[4]{x}}} dx$$

Optimal antiderivative

$$\begin{aligned} & 12x^{\frac{1}{12}} - 6x^{\frac{1}{6}} + 4x^{\frac{1}{4}} - 3x^{\frac{1}{3}} + \frac{12x^{\frac{5}{12}}}{5} + \frac{12x^{\frac{7}{12}}}{7} - \frac{3x^{\frac{2}{3}}}{2} + \frac{4x^{\frac{3}{4}}}{3} - \frac{6x^{\frac{5}{6}}}{5} \\ & + \frac{12x^{\frac{11}{12}}}{11} - x + \frac{12x^{\frac{13}{12}}}{13} - \frac{6x^{\frac{7}{6}}}{7} + \frac{4x^{\frac{5}{4}}}{5} - 12 \ln\left(1 + x^{\frac{1}{12}}\right) - 2\sqrt{x} \end{aligned}$$

command

`integrate(1/(1/x**(1/3)+1/x**(1/4)),x)`

Sympy 1.10.1 under Python 3.10.4 output

$$\int \frac{x^{\frac{7}{12}}}{\sqrt[4]{x} + \sqrt[3]{x}} dx$$

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & \frac{12x^{\frac{13}{12}}}{13} + \frac{12x^{\frac{11}{12}}}{11} + \frac{12x^{\frac{7}{12}}}{7} + \frac{12x^{\frac{5}{12}}}{5} + 12 \sqrt[12]{x} - \frac{6x^{\frac{7}{6}}}{7} - \frac{6x^{\frac{5}{6}}}{5} - 6\sqrt[6]{x} \\ & + \frac{4x^{\frac{5}{4}}}{5} + \frac{4x^{\frac{3}{4}}}{3} + 4\sqrt[4]{x} - \frac{3x^{\frac{2}{3}}}{2} - 3\sqrt[3]{x} - 2\sqrt{x} - x - 12 \log\left(\sqrt[12]{x} + 1\right) \end{aligned}$$



### 31.4 Problem number 577

$$\int \frac{\sqrt{x}}{\sqrt[3]{x} + x} dx$$

Optimal antiderivative

$$\frac{3 \arctan\left(-1 + x^{\frac{1}{6}} \sqrt{2}\right) \sqrt{2}}{2} - \frac{3 \arctan\left(1 + x^{\frac{1}{6}} \sqrt{2}\right) \sqrt{2}}{2} - \frac{3 \ln\left(1 + x^{\frac{1}{3}} - x^{\frac{1}{6}} \sqrt{2}\right) \sqrt{2}}{4} + \frac{3 \ln\left(1 + x^{\frac{1}{3}} + x^{\frac{1}{6}} \sqrt{2}\right) \sqrt{2}}{4} + 2\sqrt{x}$$

command

`integrate(x**(1/2)/(x**(1/3)+x), x)`

Sympy 1.10.1 under Python 3.10.4 output

$$\int \frac{\sqrt{x}}{\sqrt[3]{x} + x} dx$$

Sympy 1.8 under Python 3.8.8 output

$$2\sqrt{x} - \frac{3\sqrt{2} \log\left(-4\sqrt{2} \sqrt[6]{x} + 4\sqrt[3]{x} + 4\right)}{4} + \frac{3\sqrt{2} \log\left(4\sqrt{2} \sqrt[6]{x} + 4\sqrt[3]{x} + 4\right)}{4} - \frac{3\sqrt{2} \operatorname{atan}\left(\sqrt{2} \sqrt[6]{x} - 1\right)}{2} - \frac{3\sqrt{2} \operatorname{atan}\left(\sqrt{2} \sqrt[6]{x} + 1\right)}{2}$$

### 31.5 Problem number 580

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}} + \sqrt{x}} dx$$

Optimal antiderivative

$$6x^{\frac{1}{6}} + x + \frac{6 \ln\left(1 - x^{\frac{1}{6}}\right)}{5} - \frac{3 \ln\left(2 + x^{\frac{1}{6}} + 2x^{\frac{1}{3}} - x^{\frac{1}{6}} \sqrt{5}\right) \left(-\sqrt{5} + 1\right)}{10} - \frac{3 \ln\left(2 + x^{\frac{1}{6}} + 2x^{\frac{1}{3}} + x^{\frac{1}{6}} \sqrt{5}\right) \left(\sqrt{5} + 1\right)}{10} - \frac{3 \arctan\left(\frac{\left(1+4x^{\frac{1}{6}}+\sqrt{5}\right)\sqrt{50+10\sqrt{5}}}{20}\right) \sqrt{10-2\sqrt{5}}}{5} - \frac{3 \arctan\left(\frac{1+4x^{\frac{1}{6}}-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right) \sqrt{10+2\sqrt{5}}}{5}$$

command

```
integrate(x**(1/2)/(-1/x**(1/3)+x**(1/2)),x)
```

Sympy 1.10.1 under Python 3.10.4 output

$$\int \frac{x^{\frac{5}{6}}}{(\sqrt[6]{x} - 1) (\sqrt[6]{x} + x^{\frac{2}{3}} + \sqrt[3]{x} + \sqrt{x} + 1)} dx$$

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & 6\sqrt[6]{x} + x + \frac{6 \log(\sqrt[6]{x} - 1)}{5} - \frac{3\sqrt{5} \log(8\sqrt[6]{x} + 8\sqrt{5}\sqrt[6]{x} + 16\sqrt[3]{x} + 16)}{10} \\ & - \frac{3 \log(8\sqrt[6]{x} + 8\sqrt{5}\sqrt[6]{x} + 16\sqrt[3]{x} + 16)}{10} - \frac{3 \log(-8\sqrt{5}\sqrt[6]{x} + 8\sqrt[6]{x} + 16\sqrt[3]{x} + 16)}{10} \\ & + \frac{3\sqrt{5} \log(-8\sqrt{5}\sqrt[6]{x} + 8\sqrt[6]{x} + 16\sqrt[3]{x} + 16)}{10} \\ & - \frac{3\sqrt{2} \sqrt{5 - \sqrt{5}} \operatorname{atan}\left(\frac{2\sqrt{2}\sqrt[6]{x}}{\sqrt{5 - \sqrt{5}}} + \frac{\sqrt{2}}{2\sqrt{5 - \sqrt{5}}} + \frac{\sqrt{10}}{2\sqrt{5 - \sqrt{5}}}\right)}{5} \\ & - \frac{3\sqrt{2} \sqrt{\sqrt{5} + 5} \operatorname{atan}\left(\frac{2\sqrt{2}\sqrt[6]{x}}{\sqrt{\sqrt{5} + 5}} - \frac{\sqrt{10}}{2\sqrt{\sqrt{5} + 5}} + \frac{\sqrt{2}}{2\sqrt{\sqrt{5} + 5}}\right)}{5} \end{aligned}$$

### 31.6 Problem number 915

$$\int \frac{-4 + x}{(1 + \sqrt[3]{x}) \sqrt{x}} dx$$

Optimal antiderivative

$$-30x^{\frac{1}{6}} - \frac{6x^{\frac{5}{6}}}{5} + \frac{6x^{\frac{7}{6}}}{7} + 30 \arctan\left(x^{\frac{1}{6}}\right) + 2\sqrt{x}$$

command

```
integrate((-4+x)/(1+x**(1/3))/x**(1/2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

$$\int \frac{x - 4}{\sqrt{x} (\sqrt[3]{x} + 1)} dx$$

Sympy 1.8 under Python 3.8.8 output

$$\frac{6x^{\frac{7}{6}}}{7} - \frac{6x^{\frac{5}{6}}}{5} - 30\sqrt[6]{x} + 2\sqrt{x} + 30 \operatorname{atan}(\sqrt[6]{x})$$

## 32 Test file number 57

Test folder name:

test\_cases/3\_Logarithms/57\_3.1.4-f\_x-^m-d+e\_x^r-^q-a+b\_log-c\_x^n-^p

### 32.1 Problem number 120

$$\int \frac{x \log^2(x)}{(d + ex)^4} dx$$

Optimal antiderivative

$$-\frac{x}{3d^2e(ex+d)} + \frac{x \ln(x)}{3de(ex+d)^2} + \frac{x^2(ex+3d) \ln(x)^2}{6d^2(ex+d)^3} - \frac{\ln(x) \ln\left(1 + \frac{ex}{d}\right)}{3d^2e^2} - \frac{\text{polylog}\left(2, -\frac{ex}{d}\right)}{3d^2e^2}$$

command

```
integrate(x*ln(x)**2/(e*x+d)**4,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{(-d - 3ex) \log(x)^2}{6d^3e^2 + 18d^2e^3x + 18de^4x^2 + 6e^5x^3} + \frac{\left( \begin{cases} \frac{x}{d^3} & \text{for } e = 0 \\ -\frac{1}{2e(d+ex)^2} & \text{otherwise} \end{cases} \right) \log(x)}{e} \\
& - \frac{\begin{cases} \frac{x}{d^3} & \text{for } e = 0 \\ -\frac{1}{2d^2e+2de^2x} - \frac{\log(x)}{2d^2e} + \frac{\log\left(\frac{d}{e}+x\right)}{2d^2e} & \text{otherwise} \end{cases}}{e} \\
& + \frac{\begin{cases} -\frac{1}{e^3x} & \text{for } d = 0 \\ -\frac{1}{2de^2+2e^3x} - \frac{\log(d+ex)}{2de^2} & \text{otherwise} \end{cases}}{3d} - \frac{\left( \begin{cases} \frac{1}{e^3x} & \text{for } d = 0 \\ -\frac{1}{2d\left(\frac{d}{x}+e\right)^2} & \text{otherwise} \end{cases} \right) \log(x)}{3d} \\
& - \frac{2 \left( \begin{cases} -\frac{1}{e^2x} & \text{for } d = 0 \\ -\frac{\log(d^2+dex)}{de} & \text{otherwise} \end{cases} \right)}{3de} + \frac{2 \left( \begin{cases} \frac{1}{e^2x} & \text{for } d = 0 \\ -\frac{1}{\frac{d^2}{x}+de} & \text{otherwise} \end{cases} \right) \log(x)}{3de} \\
& + \frac{\begin{cases} -\frac{1}{ex} & \text{for } d = 0 \\ \log(e) \log(x) + \text{Li}_2\left(\frac{de^{i\pi}}{ex}\right) & \text{for } |x| < 1 \\ -\log(e) \log\left(\frac{1}{x}\right) + \text{Li}_2\left(\frac{de^{i\pi}}{ex}\right) & \text{for } \frac{1}{|x|} < 1 \\ -G_{2,2}^{2,0}\left(0,0 \left| \begin{matrix} 1,1 \\ x \end{matrix} \right. \right) \log(e) + G_{2,2}^{0,2}\left(1,1 \left| \begin{matrix} 1,1 \\ 0,0 \end{matrix} \right. \right) \log(e) + \text{Li}_2\left(\frac{de^{i\pi}}{ex}\right) & \text{otherwise} \end{cases}}{d} \text{ otherwise} \\
& - \frac{\left( \begin{cases} \frac{1}{ex} & \text{for } d = 0 \\ \frac{\log\left(\frac{d}{x}+e\right)}{d} & \text{otherwise} \end{cases} \right) \log(x)}{3de^2}
\end{aligned}$$

### 32.2 Problem number 343

$$\int \frac{a + b \log(cx)}{(d + \frac{e}{x}) x^3} dx$$

Optimal antiderivative

$$-\frac{b}{ex} + \frac{-a - b \ln(cx)}{ex} - \frac{d(a + b \ln(cx))^2}{2be^2} + \frac{d(a + b \ln(cx)) \ln\left(1 + \frac{dx}{e}\right)}{e^2} + \frac{bd \text{polylog}\left(2, -\frac{dx}{e}\right)}{e^2}$$

command

`integrate((a+b*ln(c*x))/(d+e/x)/x**3,x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& \frac{ad^2 \left( \begin{cases} \frac{x}{e} & \text{for } d = 0 \\ \frac{\log(dx+e)}{d} & \text{otherwise} \end{cases} \right)}{e^2} - \frac{ad \log(x)}{e^2} - \frac{a}{ex} \\
& + \frac{bd^2 \left( \begin{cases} \frac{x}{e} & \text{for } d = 0 \\ \begin{cases} \log(e) \log(x) - \text{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } |x| < 1 \\ -\log(e) \log\left(\frac{1}{x}\right) - \text{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } \frac{1}{|x|} < 1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} 1, 1 \\ 0, 0 \end{matrix} \middle| x \right) \log(e) + G_{2,2}^{0,2}\left(\begin{matrix} 1, 1 \\ 0, 0 \end{matrix} \middle| x \right) \log(e) - \text{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{otherwise} \end{cases} & \text{otherwise} \end{cases} \right)}{e^2} \\
& + \frac{bd^2 \left( \begin{cases} \frac{x}{e} & \text{for } d = 0 \\ \frac{\log(dx+e)}{d} & \text{otherwise} \end{cases} \right) \log(cx)}{e^2} + \frac{bd \log(x)^2}{2e^2} - \frac{bd \log(x) \log(cx)}{e^2} - \frac{b \log(cx)}{ex} - \frac{b}{ex}
\end{aligned}$$

**32.3 Problem number 344**

$$\int \frac{a + b \log(cx)}{\left(d + \frac{e}{x}\right) x^4} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{b}{4e x^2} + \frac{bd}{e^2 x} + \frac{-a - b \ln(cx)}{2e x^2} + \frac{d(a + b \ln(cx))}{e^2 x} + \frac{d^2(a + b \ln(cx))^2}{2b e^3} \\
& - \frac{d^2(a + b \ln(cx)) \ln\left(1 + \frac{dx}{e}\right)}{e^3} - \frac{b d^2 \text{polylog}\left(2, -\frac{dx}{e}\right)}{e^3}
\end{aligned}$$

command

```
integrate((a+b*ln(c*x))/(d+e/x)/x**4,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned}
& - \frac{ad^3 \left( \begin{cases} \frac{x}{e} & \text{for } d = 0 \\ \frac{\log(dx+e)}{d} & \text{otherwise} \end{cases} \right)}{e^3} + \frac{ad^2 \log(x)}{e^3} + \frac{ad}{e^2 x} - \frac{a}{2ex^2} \\
& + \frac{bd^3 \left( \begin{cases} \frac{x}{e} & \text{for } d = 0 \\ \begin{cases} \log(e) \log(x) - \text{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } |x| < 1 \\ -\log(e) \log\left(\frac{1}{x}\right) - \text{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{for } \frac{1}{|x|} < 1 \\ -G_{2,2}^{2,0}\left(0, 0 \left| \begin{matrix} 1, 1 \\ x \end{matrix} \right. \right) \log(e) + G_{2,2}^{0,2}\left(1, 1 \left| \begin{matrix} 1, 1 \\ 0, 0 \end{matrix} \right. \right) \log(e) - \text{Li}_2\left(\frac{dxe^{i\pi}}{e}\right) & \text{otherwise} \end{cases} & \text{otherwise} \end{cases} \right)}{e^3} \\
& - \frac{bd^3 \left( \begin{cases} \frac{x}{e} & \text{for } d = 0 \\ \frac{\log(dx+e)}{d} & \text{otherwise} \end{cases} \right) \log(cx)}{e^3} - \frac{bd^2 \log(x)^2}{2e^3} \\
& + \frac{bd^2 \log(x) \log(cx)}{e^3} + \frac{bd \log(cx)}{e^2 x} + \frac{bd}{e^2 x} - \frac{b \log(cx)}{2ex^2} - \frac{b}{4ex^2}
\end{aligned}$$

### 33 Test file number 59

Test folder name:

test\_cases/3\_Logarithms/59\_3.2.1-f+g\_x~m-A+B\_log-e-a+b\_x-over-c+d\_x~n~p

#### 33.1 Problem number 3

$$\int (ag + bgx)^2 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{B(-ad + bc)^2 g^2 n x}{3d^2} - \frac{B(-ad + bc) g^2 n (bx + a)^2}{6bd} \\
& + \frac{g^2 (bx + a)^3 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{3b} - \frac{B(-ad + bc)^3 g^2 n \ln(dx + c)}{3b d^3}
\end{aligned}$$

command

```
integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\left\{ \begin{array}{l} a^2 g^2 x (A + B \log(e(\frac{a}{c})^n)) \\ Aa^2 g^2 x + Aabg^2 x^2 + \frac{Ab^2 g^2 x^3}{3} + \frac{Ba^3 g^2 n \log(\frac{a}{c} + \frac{bx}{c})}{3b} + Ba^2 g^2 nx \log(\frac{a}{c} + \frac{bx}{c}) - \frac{Ba^2 g^2 nx}{3} + Ba^2 g^2 x \log(e) + Babg^2 nx \\ a^2 g^2 \left( Ax - \frac{Bcn \log(c+dx)}{d} + Bnx \log(a) - Bnx \log(c+dx) + Bnx + Bx \log(e) \right) \\ Aa^2 g^2 x + Aabg^2 x^2 + \frac{Ab^2 g^2 x^3}{3} + \frac{Ba^3 g^2 n \log(\frac{a}{c+dx} + \frac{bx}{c+dx})}{3b} + \frac{Ba^3 g^2 n \log(\frac{c}{d} + x)}{3b} - \frac{Ba^2 cg^2 n \log(\frac{c}{d} + x)}{d} + Ba^2 g^2 nx \log\left(\frac{a}{c+dx}\right) \end{array} \right.$$

### 33.2 Problem number 31

$$\int (cg + dgx)^2 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{B(-ad+bc)^2 g^2 nx}{3b^2} - \frac{B(-ad+bc) g^2 n(dx+c)^2}{6bd} \\ & - \frac{B(-ad+bc)^3 g^2 n \ln(bx+a)}{3b^3 d} + \frac{g^2(dx+c)^3 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{3d} \end{aligned}$$

command

`integrate((d*g*x+c*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\left\{ \begin{array}{l} c^2 g^2 x (A + B \log(e(\frac{a}{c})^n)) \\ Ac^2 g^2 x + Acdg^2 x^2 + \frac{Ad^2 g^2 x^3}{3} - \frac{Bc^3 g^2 n \log(c+dx)}{3d} + Bc^2 g^2 nx \log(a) - Bc^2 g^2 nx \log(c+dx) + \frac{Bc^2 g^2 nx}{3} + Bc^2 g^2 x \log(e) \\ c^2 g^2 \left( Ax + \frac{Ban \log(\frac{a}{c} + \frac{bx}{c})}{b} + Bnx \log(\frac{a}{c} + \frac{bx}{c}) - Bnx + Bx \log(e) \right) \\ Ac^2 g^2 x + Acdg^2 x^2 + \frac{Ad^2 g^2 x^3}{3} + \frac{Ba^3 d^2 g^2 n \log(\frac{a}{c+dx} + \frac{bx}{c+dx})}{3b^3} + \frac{Ba^3 d^2 g^2 n \log(\frac{c}{d} + x)}{3b^3} - \frac{Ba^2 cdg^2 n \log(\frac{a}{c+dx} + \frac{bx}{c+dx})}{b^2} - \frac{Ba^2 cdg^2 n}{b} \end{array} \right.$$

### 33.3 Problem number 59

$$\int (f + gx)^2 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{B(-ad + bc)g(-adg - bcg + 3bdf)nx}{3b^2d^2} - \frac{B(-ad + bc)g^2nx^2}{6bd} - \frac{B(-ag + bf)^3n \ln(bx + a)}{3b^3g} \\ & + \frac{(gx + f)^3 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{3g} + \frac{B(-cg + df)^3n \ln(dx + c)}{3d^3g} \end{aligned}$$

command

```
integrate((g*x+f)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

## 34 Test file number 60

Test folder name:

test\_cases/3\_Logarithms/60\_3.2.2-f+g\_x-^m-h+i\_x-^q-A+B\_log-e-a+b\_x-over-c+d\_x-^n-^p

### 34.1 Problem number 45

$$\int \frac{A + B \log \left( \frac{e(a+bx)}{c+dx} \right)}{(ag + bgx)^3 (ci + dix)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{B d^3 (bx + a)}{(-ad + bc)^4 g^3 i^2 (dx + c)} + \frac{3b^2 B d (dx + c)}{(-ad + bc)^4 g^3 i^2 (bx + a)} \\ & - \frac{b^3 B (dx + c)^2}{4(-ad + bc)^4 g^3 i^2 (bx + a)^2} - \frac{3b B d^2 \ln \left( \frac{bx+a}{dx+c} \right)^2}{2(-ad + bc)^4 g^3 i^2} \\ & - \frac{d^3 (bx + a) \left( A + B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{(-ad + bc)^4 g^3 i^2 (dx + c)} + \frac{3b^2 d (dx + c) \left( A + B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{(-ad + bc)^4 g^3 i^2 (bx + a)} \\ & - \frac{b^3 (dx + c)^2 \left( A + B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{2(-ad + bc)^4 g^3 i^2 (bx + a)^2} + \frac{3b d^2 \ln \left( \frac{bx+a}{dx+c} \right) \left( A + B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{(-ad + bc)^4 g^3 i^2} \end{aligned}$$



command

```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3/(d*i*x+c*i)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

### 34.2 Problem number 52

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^2(ci + dix)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{B d^3 (bx + a)^2}{4 (-ad + bc)^4 g^2 i^3 (dx + c)^2} - \frac{3bB d^2 (bx + a)}{(-ad + bc)^4 g^2 i^3 (dx + c)} \\ & - \frac{b^3 B (dx + c)}{(-ad + bc)^4 g^2 i^3 (bx + a)} + \frac{3b^2 B d \ln\left(\frac{bx+a}{dx+c}\right)^2}{2 (-ad + bc)^4 g^2 i^3} \\ & - \frac{d^3 (bx + a)^2 \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{2 (-ad + bc)^4 g^2 i^3 (dx + c)^2} + \frac{3b d^2 (bx + a) \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{(-ad + bc)^4 g^2 i^3 (dx + c)} \\ & - \frac{b^3 (dx + c) \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{(-ad + bc)^4 g^2 i^3 (bx + a)} - \frac{3b^2 d \ln\left(\frac{bx+a}{dx+c}\right) \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{(-ad + bc)^4 g^2 i^3} \end{aligned}$$

command

```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2/(d*i*x+c*i)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

## 34.3 Problem number 106

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^3 (ci + dix)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{B^2 d^4 (bx+a)^2}{4(-ad+bc)^5 g^3 i^3 (dx+c)^2} + \frac{8AbB d^3 (bx+a)}{(-ad+bc)^5 g^3 i^3 (dx+c)} - \frac{8b B^2 d^3 (bx+a)}{(-ad+bc)^5 g^3 i^3 (dx+c)} \\ & + \frac{8b^3 B^2 d(dx+c)}{(-ad+bc)^5 g^3 i^3 (bx+a)} - \frac{b^4 B^2 (dx+c)^2}{4(-ad+bc)^5 g^3 i^3 (bx+a)^2} + \frac{8b B^2 d^3 (bx+a) \ln\left(\frac{e(bx+a)}{dx+c}\right)}{(-ad+bc)^5 g^3 i^3 (dx+c)} \\ & - \frac{B d^4 (bx+a)^2 \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{2(-ad+bc)^5 g^3 i^3 (dx+c)^2} + \frac{8b^3 B d(dx+c) \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{(-ad+bc)^5 g^3 i^3 (bx+a)} \\ & - \frac{b^4 B (dx+c)^2 \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{2(-ad+bc)^5 g^3 i^3 (bx+a)^2} + \frac{d^4 (bx+a)^2 \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{2(-ad+bc)^5 g^3 i^3 (dx+c)^2} \\ & - \frac{4b d^3 (bx+a) \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(-ad+bc)^5 g^3 i^3 (dx+c)} + \frac{4b^3 d(dx+c) \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(-ad+bc)^5 g^3 i^3 (bx+a)} \\ & - \frac{b^4 (dx+c)^2 \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{2(-ad+bc)^5 g^3 i^3 (bx+a)^2} + \frac{2b^2 d^2 \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^3}{B(-ad+bc)^5 g^3 i^3} \end{aligned}$$

command

```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3/(d*i*x+c*i)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

### 34.4 Problem number 110

$$\int (ag + bgx)(ci + dix) \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{B(-ad + bc)^2 ginx}{6bd} + \frac{gi(bx + a)^2 (dx + c) \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{3b} \\ & + \frac{(-ad + bc) gi(bx + a)^2 \left( A - Bn + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{6b^2} + \frac{B(-ad + bc)^3 gin \ln(dx + c)}{6b^2 d^2} \end{aligned}$$

command

```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\left\{ \begin{array}{l} acgix(A + B \log(e(\frac{a}{c})^n)) \\ ag \left( Acix + \frac{Adix^2}{2} - \frac{Bc^2 in \log(c+dx)}{2d} + Bcinx \log(a) - Bcinx \log(c + dx) + \frac{Bcinx}{2} + Bcix \log(e) + \frac{Bdinx^2 \log(a)}{2} - \dots \right) \\ ci \left( Aagx + \frac{Abgx^2}{2} + \frac{Ba^2 gn \log(\frac{a}{c} + \frac{bx}{c})}{2b} + Bagnx \log(\frac{a}{c} + \frac{bx}{c}) - \frac{Bagnx}{2} + Bagx \log(e) + \frac{Bbgnx^2 \log(\frac{a}{c} + \frac{bx}{c})}{2} - \frac{Bbgnx^2}{4} \right) \\ Aacgix + \frac{Aadgix^2}{2} + \frac{Abcgix^2}{2} + \frac{Abdgiix^3}{3} - \frac{Ba^3 dgin \log(\frac{a}{c+dx} + \frac{bx}{c+dx})}{6b^2} - \frac{Ba^3 dgin \log(\frac{c}{d} + x)}{6b^2} + \frac{Ba^2 cgin \log(\frac{a}{c+dx} + \frac{bx}{c+dx})}{2b} + \frac{Ba^2}{4} \end{array} \right.$$

### 34.5 Problem number 120

$$\int (ci + dix)^2 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{B(-ad + bc)^2 i^2 nx}{3b^2} - \frac{B(-ad + bc) i^2 n(dx + c)^2}{6bd} \\ & - \frac{B(-ad + bc)^3 i^2 n \ln(bx + a)}{3b^3 d} + \frac{i^2 (dx + c)^3 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{3d} \end{aligned}$$

command

`integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\left\{ \begin{array}{l} c^2 i^2 x (A + B \log(e(\frac{a}{c})^n)) \\ Ac^2 i^2 x + Ac d i^2 x^2 + \frac{Ad^2 i^2 x^3}{3} - \frac{Bc^3 i^2 n \log(c+dx)}{3d} + Bc^2 i^2 n x \log(a) - Bc^2 i^2 n x \log(c+dx) + \frac{Bc^2 i^2 n x}{3} + Bc^2 i^2 x \log(e) \\ c^2 i^2 \left( Ax + \frac{Ban \log(\frac{a}{c} + \frac{bx}{c})}{b} + Bnx \log(\frac{a}{c} + \frac{bx}{c}) - Bnx + Bx \log(e) \right) \\ Ac^2 i^2 x + Ac d i^2 x^2 + \frac{Ad^2 i^2 x^3}{3} + \frac{Ba^3 d^2 i^2 n \log(\frac{a}{c+dx} + \frac{bx}{c+dx})}{3b^3} + \frac{Ba^3 d^2 i^2 n \log(\frac{c}{d} + x)}{3b^3} - \frac{Ba^2 c d i^2 n \log(\frac{a}{c+dx} + \frac{bx}{c+dx})}{b^2} - \frac{Ba^2 c d i^2 n \log(e)}{b^2} \end{array} \right.$$

### 34.6 Problem number 245

$$\int \frac{1}{(a+bx)(c+dx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

Optimal antiderivative

$$\frac{\ln\left(\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(-ad+bc)n}$$

command

`integrate(1/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))**n),x)`

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\left\{ \begin{array}{ll} -\frac{1}{(bc+bdx) \log(e)} & \text{for } a = \frac{bc}{d} \wedge n = 0 \\ -\frac{1}{bcn \log\left(\frac{bc}{cd+d^2x} + \frac{bx}{c+dx}\right) + bc \log(e) + bdnx \log\left(\frac{bc}{cd+d^2x} + \frac{bx}{c+dx}\right) + bdx \log(e)} & \text{for } a = \frac{bc}{d} \\ -\frac{\log\left(\frac{a}{b} + x\right) + \log\left(\frac{c}{d} + x\right)}{\log(e) \frac{ad-bc}{ad-bc}} & \text{for } n = 0 \\ -\frac{\log\left(n \log\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right) + \log(e)\right)}{adn-bcn} & \text{otherwise} \end{array} \right.$$

## 35 Test file number 61

Test folder name:

test\_cases/3\_Logarithms/61\_3.2.3\_u\_log-e-f-a+b\_x-^p-c+d\_x-^q-r-^s

### 35.1 Problem number 74

$$\int \left( \frac{1}{(c+dx)(-a+c+(-b+d)x) \log\left(\frac{a+bx}{c+dx}\right)} + \frac{\log\left(1 - \frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx) \log^2\left(\frac{a+bx}{c+dx}\right)} \right) dx$$

Optimal antiderivative

$$-\frac{\ln\left(1 + \frac{-bx-a}{dx+c}\right)}{(-ad+bc) \ln\left(\frac{bx+a}{dx+c}\right)}$$

command

```
integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{\log\left(\frac{-a-bx}{c+dx} + 1\right)}{ad \log\left(\frac{a+bx}{c+dx}\right) - bc \log\left(\frac{a+bx}{c+dx}\right)}$$

### 35.2 Problem number 75

$$\int \left( -\frac{1}{(a+bx)(a-c+(b-d)x) \log\left(\frac{a+bx}{c+dx}\right)} + \frac{\log\left(1 - \frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx) \log^2\left(\frac{a+bx}{c+dx}\right)} \right) dx$$

Optimal antiderivative

$$-\frac{\ln\left(1 + \frac{-dx-c}{bx+a}\right)}{(-ad+bc) \ln\left(\frac{bx+a}{dx+c}\right)}$$

command

```
integrate(-1/(b*x+a)/(a-c+(b-d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{\log\left(1 + \frac{-c-dx}{a+bx}\right)}{ad \log\left(\frac{a+bx}{c+dx}\right) - bc \log\left(\frac{a+bx}{c+dx}\right)}$$

## 36 Test file number 64

Test folder name:

test\_cases/3\_Logarithms/64\_3.5\_Logarithm\_functions

### 36.1 Problem number 26

$$\int \frac{ax + 2bn \log(cx^n)}{ax^2 + bx \log^2(cx^n)} dx$$

Optimal antiderivative

$$\ln(ax + b \ln(cx^n)^2)$$

command

`integrate((a*x+2*b*n*ln(c*x**n))/(a*x**2+b*x*ln(c*x**n)**2), x)`

Sympy 1.10.1 under Python 3.10.4 output

$$\int \frac{ax + 2bn \log(cx^n)}{x(ax + b \log(cx^n)^2)} dx$$

Sympy 1.8 under Python 3.8.8 output

$$\begin{cases} \log\left(x + \frac{bn^2 \log(x)^2}{a} + \frac{2bn \log(c) \log(x)}{a} + \frac{b \log(c)^2}{a}\right) & \text{for } a \neq 0 \\ 2 \log(n \log(x) + \log(c)) & \text{otherwise} \end{cases}$$

## 37 Test file number 106

Test folder name:

test\_cases/4\_Trig\_functions/4.3\_Tangent/106\_4.3.7-d\_trig-<sup>m</sup>-a+b-c\_tan-<sup>n</sup>-<sup>p</sup>

### 37.1 Problem number 216

$$\int \frac{\cot^5(e + fx)}{a + b \tan^2(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(a + b) (\cot^2(fx + e))}{2a^2 f} - \frac{\cot^4(fx + e)}{4af} + \frac{\ln(\cos(fx + e))}{(a - b) f} \\ & + \frac{(a^2 + ab + b^2) \ln(\tan(fx + e))}{a^3 f} + \frac{b^3 \ln(a + b(\tan^2(fx + e)))}{2a^3 (a - b) f} \end{aligned}$$

command

```
integrate(cot(f*x+e)**5/(a+b*tan(f*x+e)**2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

output too large to display

## 38 Test file number 113

Test folder name:

test\_cases/4\_Trig\_functions/4.4\_Cotangent/113\_4.4.7-d\_trig-<sup>m</sup>-a+b-c\_cot-<sup>n</sup>-<sup>p</sup>

### 38.1 Problem number 13

$$\int \frac{1}{\sqrt{-1 - \cot^2(x)}} dx$$

Optimal antiderivative

$$-\frac{\cot(x)}{\sqrt{-(\csc^2(x))}}$$

command

```
integrate(1/(-1-cot(x)**2)**(1/2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

$$\int \frac{1}{\sqrt{-\cot^2(x) - 1}} dx$$

Sympy 1.8 under Python 3.8.8 output

$$-\frac{\cot(x)}{\sqrt{-\cot^2(x) - 1}}$$

## 39 Test file number 141

Test folder name:

test\_cases/4\_Trig\_functions/4.7\_Miscellaneous/141\_4.7.7\_Trig\_functions

### 39.1 Problem number 591

$$\int \frac{x^2}{(ax \cos(ax) - \sin(ax))^2} dx$$

Optimal antiderivative

$$-\frac{\cot(ax)}{a^3} + \frac{x \csc(ax)}{a^2 (ax \cos(ax) - \sin(ax))}$$

command

```
integrate(x**2/(a*x*cos(a*x)-sin(a*x))**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

$$\int \frac{x^2}{(ax \cos(ax) - \sin(ax))^2} dx$$

Sympy 1.8 under Python 3.8.8 output

$$-\frac{2ax \tan\left(\frac{ax}{2}\right)}{a^4x \tan^2\left(\frac{ax}{2}\right) - a^4x + 2a^3 \tan\left(\frac{ax}{2}\right)} + \frac{\tan^2\left(\frac{ax}{2}\right)}{a^4x \tan^2\left(\frac{ax}{2}\right) - a^4x + 2a^3 \tan\left(\frac{ax}{2}\right)} - \frac{1}{a^4x \tan^2\left(\frac{ax}{2}\right) - a^4x + 2a^3 \tan\left(\frac{ax}{2}\right)}$$



## 40 Test file number 144

Test folder name:

test\_cases/5\_Inverse\_trig\_functions/5.1\_Inverse\_sine/144\_5.1.5\_Inverse\_sine\_functions

### 40.1 Problem number 474

$$\int \frac{\sqrt{1-x^2} + x \operatorname{ArcSin}(x)}{\operatorname{ArcSin}(x) - x^2 \operatorname{ArcSin}(x)} dx$$

Optimal antiderivative

$$-\frac{\ln(-x^2 + 1)}{2} + \ln(\arcsin(x))$$

command

```
integrate((x*asin(x)+(-x**2+1)**(1/2))/(asin(x)-x**2*asin(x)),x)
```

Sympy 1.10.1 under Python 3.10.4 output

$$-\int \frac{\sqrt{1-x^2}}{x^2 \operatorname{asin}(x) - \operatorname{asin}(x)} dx - \int \frac{x \operatorname{asin}(x)}{x^2 \operatorname{asin}(x) - \operatorname{asin}(x)} dx$$

Sympy 1.8 under Python 3.8.8 output

$$-\frac{\log(2x^2 - 2)}{2} + \log(\operatorname{asin}(x))$$

## 41 Test file number 173

Test folder name:

test\_cases/6\_Hyperbolic\_functions/6.3\_Hyperbolic\_tangent/173\_6.3.7-d\_hyper-<sup>m</sup>-a+b-c\_tanh-<sup>n</sup>-<sup>p</sup>

### 41.1 Problem number 206

$$\int \frac{1}{\sqrt{1 - \tanh^2(x)}} dx$$

Optimal antiderivative

$$\frac{\tanh(x)}{\sqrt{\operatorname{sech}(x)^2}}$$

command

```
integrate(1/(1-tanh(x)**2)**(1/2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

$$\int \frac{1}{\sqrt{1 - \tanh^2(x)}} dx$$

Sympy 1.8 under Python 3.8.8 output

$$\frac{\tanh(x)}{\sqrt{1 - \tanh^2(x)}}$$

## 42 Test file number 196

Test folder name:

test\_cases/7\_Inverse\_hyperbolic\_functions/7.3\_Inverse\_hyperbolic\_tangent/196\_7.3.6\_Exponential

### 42.1 Problem number 201

$$\int \frac{e^{-\tanh^{-1}(ax)}}{c - acx} dx$$

Optimal antiderivative

$$\frac{\arcsin(ax)}{ac}$$

command

```
integrate(1/(a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*c*x+c),x)
```

Sympy 1.10.1 under Python 3.10.4 output

$$\frac{\int \frac{\sqrt{-a^2x^2+1}}{a^2x^2-1} dx}{c}$$

Sympy 1.8 under Python 3.8.8 output

$$\frac{\begin{cases} \sqrt{\frac{1}{a^2}} \operatorname{asin}\left(x\sqrt{a^2}\right) & \text{for } a^2 > 0 \\ \sqrt{-\frac{1}{a^2}} \operatorname{asinh}\left(x\sqrt{-a^2}\right) & \text{for } a^2 < 0 \end{cases}}{c}$$

### 43 Test file number 199

Test folder name:

test\_cases/7\_Inverse\_hyperbolic\_functions/7.4\_Inverse\_hyperbolic\_cotangent/199\_7.4.2\_Exponent

#### 43.1 Problem number 256

$$\int e^{-\coth^{-1}(ax)} \sqrt{c - acx} \, dx$$

Optimal antiderivative

$$\frac{8cx \sqrt{1 - \frac{1}{a^2 x^2}}}{3\sqrt{-acx + c}} + \frac{2x \sqrt{1 - \frac{1}{a^2 x^2}} \sqrt{-acx + c}}{3}$$

command

```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

$$\int \sqrt{\frac{ax - 1}{ax + 1}} \sqrt{-c(ax - 1)} \, dx$$

Sympy 1.8 under Python 3.8.8 output

$$\frac{4icx \sqrt{\frac{1}{acx + c}}}{3} + \frac{4ic \sqrt{\frac{1}{acx + c}}}{a} - \frac{2i(-acx + c)^2 \sqrt{\frac{1}{acx + c}}}{3ac}$$

#### 43.2 Problem number 326

$$\int e^{\coth^{-1}(x)} \sqrt{1 - x} \, dx$$

Optimal antiderivative

$$\frac{2(1+x) \sqrt{1-x}}{3 \sqrt{\frac{-1+x}{1+x}}}$$

command

```
integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**(1/2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

$$\int \frac{\sqrt{1-x}}{\sqrt{\frac{x-1}{x+1}}} dx$$

Sympy 1.8 under Python 3.8.8 output

$$-\frac{2ix}{3\sqrt{\frac{1}{x+1}}} - \frac{2i}{3\sqrt{\frac{1}{x+1}}}$$

### 43.3 Problem number 338

$$\int e^{-\coth^{-1}(ax)} \sqrt{c-acx} dx$$

Optimal antiderivative

$$\frac{8cx\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{-acx+c}} + \frac{2x\sqrt{1-\frac{1}{a^2x^2}}\sqrt{-acx+c}}{3}$$

command

```
integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

$$\int \sqrt{\frac{ax-1}{ax+1}} \sqrt{-c(ax-1)} dx$$

Sympy 1.8 under Python 3.8.8 output

$$\frac{4icx\sqrt{\frac{1}{acx+c}}}{3} + \frac{4ic\sqrt{\frac{1}{acx+c}}}{a} - \frac{2i(-acx+c)^2\sqrt{\frac{1}{acx+c}}}{3ac}$$

## 44 Test file number 210

Test folder name:

test\_cases/210\_Hebisch

#### 44.1 Problem number 86

$$\int \frac{e^{\frac{18}{2x^2+x^2\log(x)}} (-90 + 4x^2 + (-36 + 4x^2) \log(x) + x^2 \log^2(x))}{4x^2 + 4x^2 \log(x) + x^2 \log^2(x)} dx$$

Optimal antiderivative

$$x e^{\frac{9}{\ln(x) \left( \frac{x}{2} + \frac{x}{\ln(x)} \right) x}}$$

command

```
integrate((x**2*ln(x)**2+(4*x**2-36)*ln(x)+4*x**2-90)*exp(18/(x**2*ln(x)+2*x**2))/(x**2*ln(x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$x e^{\frac{18}{x^2 \log(x) + 2x^2}}$$

#### 44.2 Problem number 89

$$\int \frac{e^{14x/5} (-360 - 165x - 15x^2) + (45x + 15x^2 + e^{14x/5} (888x + 447x^2 + 42x^3)) \log(x) + (-360 - 165x - 15x^2 + (360x + 285x^2 + 70x^3 + 5x^4) \log^2(x))}{(360x + 285x^2 + 70x^3 + 5x^4) \log^2(x)}$$

Optimal antiderivative

$$\frac{3 e^{2x} e^{\frac{4x}{5}} + 3 \ln(8 + x)}{(3 + x) \ln(x)}$$

command

```
integrate(((((-15*x**2-120*x)*ln(x)-15*x**2-165*x-360)*ln(x+8)+((42*x**3+447*x**2+888*x)*exp(2
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{3e^{\frac{14x}{5}}}{x \log(x) + 3 \log(x)} + \frac{3 \log(x + 8)}{x \log(x) + 3 \log(x)}$$

### 44.3 Problem number 152

$$\int \frac{e^{\frac{4x^3}{11x-2e^2x-2x^2+2\log(x)}} (-8x^2 + 88x^3 - 16e^2x^3 - 8x^4 + 24x^2 \log(x))}{121x^2 + 4e^4x^2 - 44x^3 + 4x^4 + e^2(-44x^2 + 8x^3) + (44x - 8e^2x - 8x^2) \log(x) + 4\log^2(x)} dx$$

Optimal antiderivative

$$e^{\frac{\frac{2x^2}{\ln(x)} + \frac{11}{2} - x - e^2}{x}} - 4e^{-2}$$

command

```
integrate((24*x**2*ln(x)-16*x**3*exp(2)-8*x**4+88*x**3-8*x**2)*exp(2*x**3/(2*ln(x)-2*exp(2)*x-2*x**2+11*x))**2/(4*ln(x)**2+(-8*exp(2)*x-8*x**2+44*x)*ln(x)+4*x**2*exp(2)**2+(8*x+44*x**2)*exp(2)+4*x**4-44*x**3+121*x**2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$e^{\frac{4x^3}{-2x^2-2xe^2+11x+2\log(x)}}$$

### 44.4 Problem number 222

$$\int \frac{-2x + 2\log(x) + (x - \log(x)) \log(2x^2) + (1 - x) \log(2x^2) \log\left(\frac{x}{\log(2x^2)}\right)}{(x^3 - 2x^2 \log(x) + x \log^2(x)) \log(2x^2)} dx$$

Optimal antiderivative

$$1 + \frac{\ln\left(\frac{x}{\ln(2x^2)}\right)}{x - \ln(x)}$$

command

```
integrate(((1-x)*ln(2*x**2)*ln(x/ln(2*x**2)))+(x-ln(x))*ln(2*x**2)+2*ln(x)-2*x)/(x*ln(x)**2-2*x**2*ln(x)+x**3)/ln(2*x**2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{\log\left(\frac{x}{2\log(x)+\log(2)}\right)}{x - \log(x)}$$

#### 44.5 Problem number 255

$$\int \frac{3750x + 300x^2 + 6x^3 + (4000x^2 + 300x^3 + 6x^4) \log(x) + ((4000x^2 + 310x^3 + 6x^4) \log(x) + (3750x + 300x^2 + (2000x + 155x^2 + 3x^3) \log(x) + (1875 +$$

Optimal antiderivative

$$x^2 \ln \left( \left( \frac{x}{\frac{3x}{5} + 15} + x + \ln(\ln(x)) \right)^2 \right)$$

command

```
integrate((((6*x**3+300*x**2+3750*x)*ln(x)*ln(ln(x)))+(6*x**4+310*x**3+4000*x**2)*ln(x))*ln(((2000*x+155*x**2+3*x**3)*ln(x)+(1875+3*x**3)))
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\left( x^2 - \frac{625}{6} \right) \log \left( \frac{9x^4 + 480x^3 + 6400x^2 + (9x^2 + 450x + 5625) \log(\log(x))^2 + (18x^3 + 930x^2 + 12000x) \log(\log(x))}{9x^2 + 450x + 5625} \right) + \frac{625 \log \left( \log(\log(x)) + \frac{3x^2 + 80x}{3x + 75} \right)}{3}$$

#### 44.6 Problem number 284

$$\int \frac{e^{\frac{x}{4+2x+2\log(x^2)}} (-16 + 8x + 20x^2 + 6x^3 + (-16 + 17x + 11x^2) \log(x^2) + (-4 + 6x) \log^2(x^2))}{8x^3 - 8x^4 - 6x^5 + 4x^6 + 2x^7 + (8x^3 - 12x^4 + 4x^6) \log(x^2) + (2x^3 - 4x^4 + 2x^5) \log^2(x^2)} dx$$

Optimal antiderivative

$$\frac{e^{\frac{x}{2\ln(x^2)+2x+4}}}{(-x^2 + x)x}$$

command

```
integrate((((6*x-4)*ln(x**2)**2+(11*x**2+17*x-16)*ln(x**2)+6*x**3+20*x**2+8*x-16)*exp(x/(2*ln(4*x**4+2*x**3))*ln(x**2)**2+(4*x**6-12*x**4+8*x**3)*ln(x**2)+2*x**7+4*x**6-6*x**5-8*x**4+8*x**3)))
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$-\frac{e^{\frac{x}{2x+2\log(x^2)+4}}}{x^3 - x^2}$$

#### 44.7 Problem number 307

$$\int \frac{e^3(-3 + 5x - x^2) + e^6(-12 + 20x - 4x^2) \log(x) + (e^6(-12x + 4x^2) + e^6(12 - 4x) \log(x))}{30x^2 - 10x^3 + (-30x + 10x^2) \log(3x - x^2) + \log(x) (e^3(240x^2 - 80x^3) + e^3(-240x + 80x^2) \log(3x - x^2)) + 10}$$

Optimal antiderivative

$$\frac{\ln(\ln((2-x)x+x)-x)}{10e^{-3} + 40\ln(x)}$$

command

```
integrate(((((-4*x+12)*exp(3)**2*ln(-x**2+3*x)+(4*x**2-12*x)*exp(3)**2)*ln(ln(-x**2+3*x)-x)+(-4*x**2+20*x-12)*exp(3)**2*ln(x)+(-x**2+5*x-3)*exp(3))/((160*x**2-480*x)*exp(3)**2*ln(-x**2+3*x)+(-160*x**3+480*x**2)*exp(3)**2)*ln(x)**2+((80*x**2-240*x)*exp(3)*ln(-x**2+3*x)+(-80*x**3+240*x**2)*exp(3))*ln(x)+(10*x**2-30*x)*ln(-x**2+3*x)-10*x**3+30*x**2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{e^3 \log(-x + \log(-x^2 + 3x))}{40e^3 \log(x) + 10}$$

#### 44.8 Problem number 330

$$\int \frac{e^{-4x^2+x^2 \log(\log(x))} (-5 + 40 \log(x) - 10 \log(x) \log(\log(x)))}{16x^3 \log(x) - 8x^3 \log(x) \log(\log(x)) + x^3 \log(x) \log^2(\log(x))} dx$$

Optimal antiderivative

$$e^{\frac{5}{x^2(\ln(\ln(x))-4)}}$$

command

```
integrate((-10*ln(x)*ln(ln(x))+40*ln(x)-5)*exp(5/(x**2*ln(ln(x))-4*x**2))/(x**3*ln(x)*ln(ln(x))-8*x**3*ln(x)*ln(ln(x))+16*x**3*ln(x)),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$e^{\frac{5}{x^2 \log(\log(x)) - 4x^2}}$$



#### 44.9 Problem number 473

$$\int \frac{4x^3 \log(x) \log^2(\log(x)) + e^{\frac{-8x-5e^3x+x^2-10\log(\log(x))}{\log(\log(x))}} (8 + 5e^3 - x + (-8 - 5e^3 + 2x) \log(x) \log(\log(x))) + e^{\frac{-8x-5e^3x}{2\log(\log(x))}}}{\log(x) \log^2(\log(x))}$$

Optimal antiderivative

$$\left( x^2 + e^{\frac{x\left(\frac{x}{2} - \frac{5e^3}{2} - 4\right)}{\ln(\ln(x))} - 5} \right)^2 + 3$$

command

```
integrate(((((-5*exp(3)+2*x-8)*ln(x)*ln(ln(x))+5*exp(3)+8-x)*exp(1/2*(-10*ln(ln(x))-5*x*exp(3)+x**2-8*x)/ln(ln(x)))**2+(4*x*ln(x)*ln(ln(x))**2+(-5*x**2*exp(3)+2*x**3-8*x**2)*ln(x)*ln(ln(x))+5*x**2*exp(3)-x**3+8*x**2)*exp(1/2*(-10*ln(ln(x))-5*x*exp(3)+x**2-8*x)/ln(ln(x)))+4*x**3*ln(x)*ln(ln(x))**2)/ln(x)/ln(ln(x))**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$x^4 + 2x^2 e^{\frac{\frac{x^2}{2} - \frac{5xe^3}{2} - 4x - 5\log(\log(x))}{\log(\log(x))}} + e^{\frac{2\left(\frac{x^2}{2} - \frac{5xe^3}{2} - 4x - 5\log(\log(x))\right)}{\log(\log(x))}}$$

#### 44.10 Problem number 598

$$\int \frac{(12 + 6x) \log^2\left(\frac{x}{2+x}\right) + e^{\frac{5x}{\log\left(\frac{x}{2+x}\right)}} \left(-20 + (20 + 10x) \log\left(\frac{x}{2+x}\right)\right)}{(2+x) \log^2\left(\frac{x}{2+x}\right)} dx$$

Optimal antiderivative

$$2e^{\frac{5x}{\ln\left(\frac{x}{2+x}\right)}} + 6x$$

command

```
integrate((((10*x+20)*ln(x/(2+x))-20)*exp(5*x/ln(x/(2+x)))+(6*x+12)*ln(x/(2+x))**2)/(2+x)/ln(x/(2+x))
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$6x + 2e^{\frac{5x}{\log\left(\frac{x}{x+2}\right)}}$$

### 44.11 Problem number 675

$$\int \frac{(4 - 4x^2) \log(5) + (-1 + x^2) \log^2(5) \log(-1 + x) + ((-4 + 4x^2 + e^7(-4 - 4x + 8x^2)) \log(5) + e^7(-x - x^2) \log(5))}{(-16x^2 - 16x^3 + 16x^4 + 16x^5 + (8x^2 - 4x + 1) \log(5)) \log(-1 + x)}$$

Optimal antiderivative

$$\frac{e^7 + \ln\left(\frac{\ln(x)}{x}\right)}{(x^2 + x) \left(\ln(-1 + x) - \frac{4}{\ln(5)}\right)}$$

command

```
integrate((((-2*x**2+x+1)*ln(5)**2*ln(-1+x)+(-x**2-x)*ln(5)**2+(8*x**2-4*x-4)*ln(5))*ln(x)*ln(2*x**2+x+1)*exp(7)-x**2+1)*ln(5)**2*ln(-1+x)+(-x**2-x)*exp(7)*ln(5)**2+((8*x**2-4*x-4)*exp(7)+4*x**2-4)*ln(5))*ln(x)+(x**2-1)*ln(5)**2*ln(-1+x)+(-4*x**2+4)*ln(5))/((x**5+x**4-x**3-x**2)*ln(5)**2*ln(-1+x)**2+(-8*x**5-8*x**4+8*x**3+8*x**2)*ln(5)*ln(-1+x)+16*x**5+16*x**4-16*x**3-16*x**2)/ln(x), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{\log(5) \log\left(\frac{\log(x)}{x}\right)}{x^2 \log(5) \log(x-1) - 4x^2 + x \log(5) \log(x-1) - 4x} + \frac{e^7 \log(5)}{-4x^2 - 4x + (x^2 \log(5) + x \log(5)) \log(x-1)}$$

### 44.12 Problem number 700

$$\int \frac{(-8x^2 + 2x^3 - 2x^4) \log(5) \log\left(\frac{1}{4}(-4x + x^2 - x^3)\right) + (16 - 8x + 12x^2 + (-4x^2 + 2x^3 - 3x^4) \log(5)) \log(-4 + x)}{(-16x + 4x^2 - 4x^3 + (4x^3 - x^4 + x^5) \log(5)) \log\left(\frac{1}{4}(-4x + x^2 - x^3)\right)}$$

Optimal antiderivative

$$\frac{1 - \ln\left(\ln\left(\frac{x(-x^2+x)}{4} - x\right)\right)}{\ln(x^2 \ln(5) - 4)}$$

command

```
integrate(((2*x**4-2*x**3+8*x**2)*ln(5)*ln(-1/4*x**3+1/4*x**2-x)*ln(ln(-1/4*x**3+1/4*x**2-x)))+((-3*x**4+2*x**3-4*x**2)*ln(5)+12*x**2-8*x+16)*ln(x**2*ln(5)-4)+(-2*x**4+2*x**3-8*x**2)*ln(5)*ln(-1/4*x**3+1/4*x**2-x))/((x**5-x**4+4*x**3)*ln(5)-4*x**3+4*x**2-16*x)/ln(-1/4*x**3+1/4*x**2-x)/ln(x**2*ln(5)-4)**2, x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$-\frac{\log\left(\log\left(-\frac{x^3}{4} + \frac{x^2}{4} - x\right)\right)}{\log(x^2 \log(5) - 4)} + \frac{1}{\log(x^2 \log(5) - 4)}$$

#### 44.13 Problem number 778

$$\int \frac{-10 + 4x + (-2 + 2x - x^2) \log(x) + (2 - x) \log\left(-\frac{x}{-2+x}\right)}{-2x + x^2 + (-4x + 2x^2) \log(x) + (-2x + x^2) \log^2(x)} dx$$

Optimal antiderivative

$$\frac{-4 - x + \ln\left(\frac{x}{2-x}\right)}{1 + \ln(x)}$$

command

```
integrate(((x**2+2*x-2)*ln(x)+(2-x)*ln(-x/(-2+x))+4*x-10)/((x**2-2*x)*ln(x)**2+(2*x**2-4*x)*ln(x)+x**2-2*x), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{-x - 4}{\log(x) + 1} + \frac{\log\left(-\frac{x}{x-2}\right)}{\log(x) + 1}$$

#### 44.14 Problem number 835

$$\int 4^{\frac{-16-8x-x^2}{-2-2x+2\log(\log(x^2))}} \frac{((32 + 48x + 18x^2 + 2x^3) \log(4) + (2 + 8x + 10x^2 + 4x^3 + (-8x - 6x^2 + 3x^3 + x^4) \log(4)) \log(4))}{(2 + 4x + 2x^2) \log(x^2) + (-4 - 4x) \log(x)}$$

Optimal antiderivative

$$(x^2 + x) e^{\frac{(4+x)^2 \ln(2)}{x+1-\ln(\ln(x^2))}}$$

command

```
integrate(((4*x+2)*ln(x**2)*ln(ln(x**2))**2+(2*(-2*x**3-10*x**2-8*x)*ln(2)-8*x**2-
12*x-4)*ln(x**2)*ln(ln(x**2)))+(2*(x**4+3*x**3-6*x**2-8*x)*ln(2)+4*x**3+10*x**2+8*x+2)*ln(x**2
x**2-8*x-16)*ln(2)/(2*ln(ln(x**2))-2*x-2))/(2*ln(x**2)*ln(ln(x**2))**2+(-4*x-4)*ln(x**2)*ln(1
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$(x^2 + x) e^{\frac{(-2x^2 - 16x - 32) \log(2)}{-2x + 2 \log(\log(x^2)) - 2}}$$

#### 44.15 Problem number 917

$$\int \frac{-8 - 2x + (7 + 9x - 15x^2 - 2x^3) \log(x) + (-x - 2x^2) \log^2(x) + (1 + 2x) \log(x) \log(\log(x))}{(64x + 32x^2 + 4x^3) \log(x) + (16x + 4x^2) \log^2(x) + x \log^3(x)} dx$$

Optimal antiderivative

$$\frac{x - \ln(\ln(x)) + \ln(x) - x^2}{8 + 2x + \ln(x)}$$

command

```
integrate(((1+2*x)*ln(x)*ln(ln(x))+(-2*x**2-x)*ln(x)**2+(-2*x**3-15*x**2+9*x+7)*ln(x)-
2*x-8)/(x*ln(x)**3+(4*x**2+16*x)*ln(x)**2+(4*x**3+32*x**2+64*x)*ln(x)),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{-x^2 - x - 8}{2x + \log(x) + 8} - \frac{\log(\log(x))}{2x + \log(x) + 8}$$

#### 44.16 Problem number 1003

$$\int \frac{e^4(-1 + 17x - 4x^2 - 12x^3 + 3x^4) + e^4(4x - 3x^3) \log(x) + (e^4(20 - 5x - 12x^2 + 3x^3) + e^4(5 - 3x^2) \log(x)) \log(x)}{4x - x^2 + x \log(x) + \dots}$$

Optimal antiderivative

$$((5 - x^2 - \ln(\ln(-\ln(x) + x - 4) + x)) x + 9) e^4$$

command

```
integrate((((-exp(4)*ln(x)+(x-4)*exp(4))*ln(-ln(x)+x-4)-x*exp(4)*ln(x)+(x**2-4*x)*exp(4))*ln(ln(x)+x-4)+x)+((-3*x**2+5)*exp(4)*ln(x)+(3*x**3-12*x**2-5*x+20)*exp(4))*ln(-ln(x)+x-4)+(-3*x**3+4*x)*exp(4)*ln(x)+(3*x**4-12*x**3-4*x**2+17*x-1)*exp(4))/((ln(x)-x+4)*ln(-ln(x)+x-4)+x*ln(x)-x**2+4*x), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$-x^3 e^4 - x e^4 \log(x + \log(x - \log(x) - 4)) + 5x e^4$$

#### 44.17 Problem number 1043

$$\int e^{x+e^{90+10e^2}x+2e^{45+5e^2}x \log(e^{-x}x)+x \log^2(e^{-x}x)} \left(1 + e^{90+10e^2} + e^{45+5e^2}(2 - 2x) + (2 + 2e^{45+5e^2} - 2x) \log(e^{-x}x) + \log^2(e^{-x}x)\right) dx$$

Optimal antiderivative

$$e^{x(\ln(xe^{-x})+e^5e^2+45)^2+x}$$

command

```
integrate((ln(x/exp(x))**2+(2*exp(5*exp(1)**2+45)-2*x+2)*ln(x/exp(x))+exp(5*exp(1)**2+45)**2+2*x)*exp(5*exp(1)**2+45)+1)*exp(x*ln(x/exp(x))**2+2*x*exp(5*exp(1)**2+45)*ln(x/exp(x))+x*exp(5*exp(1)**2+45)), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: SystemError

Sympy 1.8 under Python 3.8.8 output

$$e^{x \log(xe^{-x})^2+2xe^{5e^2+45} \log(xe^{-x})+x+xe^{10e^2+90}}$$

#### 44.18 Problem number 1049

$$\int \frac{\left( \frac{x+4 \log\left(\frac{-4+4x \log(4)}{5x}\right)}{\log\left(\frac{-4+4x \log(4)}{5x}\right)} \right)^{\frac{1}{\log(x^2)}} \left( -x \log(x^2) + (-x+x^2) \log(x^2) \log\left(\frac{-4+4x \log(4)}{5x}\right) + (2x-2x^2) \log\left(\frac{-4+4x \log(4)}{5x}\right) \right)}{(-x^2+x^3) \log^2(x^2) \log\left(\frac{-4+4x \log(4)}{5x}\right) + (-4x+4x^2) \log^2(x^2)}$$

Optimal antiderivative

$$e^{\frac{\ln\left(4 + \frac{x}{\ln\left(\frac{8(-1+x)\ln(2)}{5x}\right)}\right)}{\ln(x^2)}}$$

command

```
integrate((((-8*x+8)*ln(2/5*(-4+4*x)*ln(2)/x)**2+(-2*x**2+2*x)*ln(2/5*(-4+4*x)*ln(2)/x))*ln((4+4*x)*ln(2)/x+x)/ln(2/5*(-4+4*x)*ln(2)/x)+(x**2-x)*ln(x**2)*ln(2/5*(-4+4*x)*ln(2)/x)-x*ln(x**2))*exp(ln((4*ln(2/5*(-4+4*x)*ln(2)/x)+x)/ln(2/5*(-4+4*x)*ln(2)/x)/ln(x**2))/((4*x**4*x)*ln(x**2)**2*ln(2/5*(-4+4*x)*ln(2)/x)**2+(x**3-x**2)*ln(x**2)**2*ln(2/5*(-4+4*x)*ln(2)/x))
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$e^{\frac{\log\left(\frac{x+4 \log\left(\frac{\left(\frac{8x}{5}-\frac{8}{5}\right) \log(2)}{x}\right)}{\log\left(\frac{\left(\frac{8x}{5}-\frac{8}{5}\right) \log(2)}{x}\right)}\right)}{\log(x^2)}}$$

#### 44.19 Problem number 1259

$$\int \frac{e^{-\frac{x}{\log\left(\frac{48x}{4+3x}\right)}} \left( 1036 + (-1036 - 777x) \log\left(\frac{48x}{4+3x}\right) \right)}{(4+3x) \log^2\left(\frac{48x}{4+3x}\right)} dx$$

Optimal antiderivative

$$259 e^{-\frac{x}{\ln\left(\frac{x}{16} + \frac{1}{12}\right)}}$$

command

```
integrate((( -777*x-1036)*ln(48*x/(4+3*x))+1036)/(4+3*x)/ln(48*x/(4+3*x))**2/exp(x/ln(48*x/(4+3*x))))
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$259e^{-\frac{x}{\log\left(\frac{48x}{3x+4}\right)}}$$

#### 44.20 Problem number 1276

$$\int e^{\frac{3-x+3\log\left(\frac{-20-5x-2x^2}{6x}\right)}{\log\left(\frac{-20-5x-2x^2}{6x}\right)}} \frac{\left(60-20x-6x^2+2x^3+(-20x-5x^2-2x^3)\log\left(\frac{-20-5x-2x^2}{6x}\right)\right)}{(20x+5x^2+2x^3)\log^2\left(\frac{-20-5x-2x^2}{6x}\right)} dx$$

Optimal antiderivative

$$e^{\frac{3-x}{\ln\left(-\frac{5}{6}-\frac{10}{3x}-\frac{x}{3}\right)}}+3$$

command

```
integrate((( -2*x**3-5*x**2-20*x)*ln(1/6*(-2*x**2-5*x-20)/x)+2*x**3-6*x**2-20*x+60)*exp((3*ln(2*x**2-5*x-20)/x)+3-x)/ln(1/6*(-2*x**2-5*x-20)/x))/(2*x**3+5*x**2+20*x)/ln(1/6*(-2*x**2-5*x-20)/x)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$e^{\frac{-x+3\log\left(\frac{-\frac{x^2}{3}-\frac{5x}{6}-\frac{10}{3}}{x}\right)+3}{\log\left(\frac{-\frac{x^2}{3}-\frac{5x}{6}-\frac{10}{3}}{x}\right)}}$$

#### 44.21 Problem number 1331

$$\int \frac{100+40x+4x^2+(-50-40x-6x^2)\log(x)+(10x+2x^2)\log^2(x)-4x\log^3(x)+(-4+2x)\log^4(x)+2\log^5(x)}{\dots} dx$$

Optimal antiderivative

$$\left(\ln(x) + \frac{x + \frac{5x-5x\ln(\ln(x)-2)^2}{x}}{\ln(x)}\right)^2$$

command

```
integrate(((−50∗ln(x)+100)∗ln(ln(x)−2)**4+100∗ln(x)∗ln(ln(x)−2)**3+(−10∗x∗ln(x)**2+(40∗x+100)
40∗x−200)∗ln(ln(x)−2)**2+(−20∗ln(x)**3+(−20∗x−100)∗ln(x))∗ln(ln(x)−2)+2∗ln(x)**5+(2∗x−
4)∗ln(x)**4−4∗x∗ln(x)**3+(2∗x**2+10∗x)∗ln(x)**2+(−6∗x**2−40∗x−50)∗ln(x)+4∗x**2+40∗x+100)/(x∗l
2∗x∗ln(x)**3),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$2x + \frac{(-10x - 10 \log(x)^2 - 50) \log(\log(x) - 2)^2}{\log(x)^2} + \frac{x^2 + 10x + 25}{\log(x)^2} + \log(x)^2 + \frac{25 \log(\log(x) - 2)^4}{\log(x)^2}$$

#### 44.22 Problem number 1387

$$\int \frac{(64 - 16x) \log(x) + (-64x + 16x^2) \log(x) \log(2x) + (64 - 16x + (64 - 48x) \log(x)) \log(2x) \log\left(\frac{e^x}{\log(2x)}\right)}{(-64x^2 + 48x^3 - 12x^4 + x^5) \log^2(x) \log(2x)} dx$$

Optimal antiderivative

$$\frac{16 \ln\left(\frac{e^x}{\ln(2x)}\right)}{x(4-x)^2 \ln(x)}$$

command

```
integrate((((−48∗x+64)∗ln(x)−16∗x+64)∗ln(2∗x)∗ln(exp(x)/ln(2∗x)))+(16∗x**2−64∗x)∗ln(x)∗ln(2∗x)
16∗x+64)∗ln(x))/(x**5−12∗x**4+48∗x**3−64∗x**2)/ln(x)**2/ln(2∗x),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{16 \log\left(\frac{e^x}{\log(x) + \log(2)}\right)}{x^3 \log(x) - 8x^2 \log(x) + 16x \log(x)}$$



#### 44.23 Problem number 1455

$$\int \frac{-240x - 48x^2 + (240x + 48x^2) \log(x) + ((240x + 96x^2) \log(x) + (45 + 18x) \log^2(x)) \log\left(\frac{16x+3\log(x)}{\log(x)}\right) \log\left(2 \log\left(\frac{16x+3\log(x)}{\log(x)}\right)\right)}{(16x \log(x) + 3 \log^2(x)) \log\left(\frac{16x+3\log(x)}{\log(x)}\right)}$$

Optimal antiderivative

$$3 \ln\left(2 \ln\left(3 + \frac{16x}{\ln(x)}\right)\right) (5+x)x + \ln(5)$$

command

```
integrate((((18*x+45)*ln(x)**2+(96*x**2+240*x)*ln(x))*ln((3*ln(x)+16*x)/ln(x))*ln(2*ln((3*ln(x)+16*x)/ln(x)))/(3*ln(x)**2+16*x*ln(x))/ln((3*ln(x)+16*x)/ln(x)),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$(3x^2 + 15x) \log\left(2 \log\left(\frac{16x + 3 \log(x)}{\log(x)}\right)\right)$$

#### 44.24 Problem number 1587

$$\int \frac{x - 21x^2 + 20x^3 + e^5(-20x + 20x^2) + x \log(4x) + (-2x + 2x^2 + e^5(-2 + 2x) + (-e^5 - x) \log(4x)) \log(e^5 - 20x^3 - 40x^4 + 20x^5 + e^5(20x^2 - 40x^3 + 20x^4)) + (40x^3 - 40x^4 + e^5(40x^2 - 40x^3)) \log(4x) + (20e^5x^2 + 20x^3) \log(4x)}{20x^3 - 40x^4 + 20x^5 + e^5(20x^2 - 40x^3 + 20x^4) + (40x^3 - 40x^4 + e^5(40x^2 - 40x^3)) \log(4x) + (20e^5x^2 + 20x^3) \log(4x)}$$

Optimal antiderivative

$$\frac{1 + \frac{\ln(e^5+x)}{20x}}{1 + \ln(4x) - x}$$

command

```
integrate(((((-exp(5)-x)*ln(4*x)+(-2+2*x)*exp(5)+2*x**2-2*x)*ln(exp(5)+x)+x*ln(4*x)+(20*x**2-20*x)*exp(5)+20*x**3-21*x**2+x)/((20*x**2*exp(5)+20*x**3)*ln(4*x)**2+((-40*x**3+40*x**2)*exp(5)+20*x**4+40*x**3)*ln(4*x)+(20*x**4-40*x**3+20*x**2)*exp(5)+20*x**5-40*x**4+20*x**3),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$-\frac{\log(x + e^5)}{20x^2 - 20x \log(4x) - 20x} + \frac{1}{-x + \log(4x) + 1}$$

#### 44.25 Problem number 1657

$$\int \frac{(5x + 4e^3x) \log(x) + (-19 + e^3(8 - 4x) - 5x) \log\left(\frac{1}{5}(38 + 10x + e^3(-16 + 8x))\right)}{(19x + 5x^2 + e^3(-8x + 4x^2)) \log^2(x)} dx$$

Optimal antiderivative

$$\frac{\ln\left(\frac{38}{5} + 2x - \frac{8(2-x)e^3}{5}\right)}{\ln(x)}$$

command

```
integrate(((((-4*x+8)*exp(3)-5*x-19)*ln(1/5*(8*x-16)*exp(3)+2*x+38/5)+(4*x*exp(3)+5*x)*ln(x))/
8*x)*exp(3)+5*x**2+19*x)/ln(x)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{\log\left(2x + \left(\frac{8x}{5} - \frac{16}{5}\right)e^3 + \frac{38}{5}\right)}{\log(x)}$$

#### 44.26 Problem number 1661

$$\int \frac{(-8x + 4x^2) \log(-2 + x) \log\left(\frac{1}{2}(1 + 2x)\right) + (-2x - 4x^2 + (2 + 3x - 2x^2) \log(-2 + x)) \log^2\left(\frac{1}{2}(1 + 2x)\right)}{(-2x^2 - 3x^3 + 2x^4) \log^3(-2 + x)} dx$$

Optimal antiderivative

$$\frac{\ln\left(\frac{1}{2} + x\right)^2}{x \ln(-2 + x)^2}$$

command

```
integrate(((((-2*x**2+3*x+2)*ln(-2+x)-4*x**2-2*x)*ln(1/2+x)**2+(4*x**2-8*x)*ln(-2+x)*ln(1/2+x)
3*x**3-2*x**2)/ln(-2+x)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{\log\left(x + \frac{1}{2}\right)^2}{x \log(x - 2)^2}$$

#### 44.27 Problem number 1705

$$\int \frac{4x^5 + x^3 \log(x) + (-32x^3 \log(\frac{4}{x}) + 64x^3 \log(\frac{4}{x}) \log(x)) \log(\log(\frac{4}{x})) \log(\log(\log(\frac{4}{x}))) \log(\log(\log(\log(\frac{4}{x}))))}{(64x^6 \log(\frac{4}{x}) + 48x^4 \log(\frac{4}{x}) \log(x) + 12x^2 \log(\frac{4}{x}) \log^2(x) + \dots)}$$

Optimal antiderivative

$$\frac{16 - \ln(\ln(\ln(\ln(\ln(\frac{4}{x}))))))}{\left(4 + \frac{\ln(x)}{x^2}\right)^2}$$

command

```
integrate((( -4*x**3*ln(4/x)*ln(x)+2*x**3*ln(4/x))*ln(ln(4/x))*ln(ln(ln(4/x)))*ln(ln(ln(ln(4/x)/
32*x**3*ln(4/x))*ln(ln(4/x))*ln(ln(ln(4/x)))*ln(ln(ln(ln(4/x)))))+x**3*ln(x)+4*x**5)/(ln(4/x)*
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$-\frac{x^4 \log(\log(\log(\log(-\log(x) + \log(4))))))}{16x^4 + 8x^2 \log(x) + \log(x)^2} + \frac{16x^4}{16x^4 + 8x^2 \log(x) + \log(x)^2}$$

#### 44.28 Problem number 1820

$$\int \frac{\log(x) \log^2(\log(x)) + e^{\frac{2x^2}{\log(\log(x))}} (-2x + 4x \log(x) \log(\log(x)))}{\log(x) \log^2(\log(x))} dx$$

Optimal antiderivative

$$e^5 + x - 41 + e^{\frac{2x^2}{\ln(\ln(x))}}$$

command

```
integrate(((4*x*ln(x)*ln(ln(x))-2*x)*exp(x**2/ln(ln(x)))*2+ln(x)*ln(ln(x))*2)/ln(x)/ln(ln(x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$x + e^{\frac{2x^2}{\log(\log(x))}}$$

## 44.29 Problem number 2019

$$\int \frac{e^{\frac{-2-x}{(-x+2x^3)\log(x)}} (-2-x+4x^2+2x^3+(-2+12x^2+4x^3)\log(x))}{(x^2-4x^4+4x^6)\log^2(x)} dx$$

Optimal antiderivative

$$e^{\frac{2+x}{\ln(x)(-2x^3+x)}}$$

command

```
integrate(((4*x**3+12*x**2-2)*ln(x)+2*x**3+4*x**2-x-2)*exp((-2-x)/(2*x**3-x)/ln(x))/(4*x**6-4*x**4+x**2)/ln(x)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$e^{\frac{-x-2}{(2x^3-x)\log(x)}}$$

## 44.30 Problem number 2117

$$\int \frac{4x^3 + 4x^2 \log(5) + (x - 8x^4 + (1 - 8x^2 - 8x^3) \log(5)) \log(x) \log(\log(x)) + (4x^2 + 4x \log(5) + (-12x^3 + (-8x - 12x^2 + 4x^3) \log(5)) \log(x) \log(\log(x)))}{(x^2 + 2x \log(5) + \log(5)^2) \log(\log(x))} dx$$

Optimal antiderivative

$$\frac{x}{2} - \left( \ln \left( \frac{x^2}{(\ln(5) + x)^2 \ln(\ln(x))} \right) + x \right)^2 x^2$$

command

```
integrate((((-4*x*ln(5)-4*x**2)*ln(x)*ln(ln(x))*ln(x**2/(ln(5)**2+2*x*ln(5)+x**2)/ln(ln(x)))*12*x**2-8*x)*ln(5)-12*x**3)*ln(x)*ln(ln(x))+4*x*ln(5)+4*x**2)*ln(x**2/(ln(5)**2+2*x*ln(5)+x**2)+8*x**3-8*x**2+1)*ln(5)-8*x**4+x)*ln(x)*ln(ln(x))+4*x**2*ln(5)+4*x**3)/(2*ln(5)+2*x)/ln(x)/ln(x))
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$-x^4 - 2x^3 \log \left( \frac{x^2}{(x^2 + 2x \log(5) + \log(5)^2) \log(\log(x))} \right) - x^2 \log \left( \frac{x^2}{(x^2 + 2x \log(5) + \log(5)^2) \log(\log(x))} \right)^2 + \frac{x}{2}$$

### 44.31 Problem number 2240

$$\int \frac{e^{\frac{135x}{-5x^2+\log(x)}} (-270x + 1350x^3 + 50x^4 + (270x - 20x^2) \log(x) + 2 \log^2(x))}{375x^4 - 150x^2 \log(x) + 15 \log^2(x)} dx$$

Optimal antiderivative

$$\frac{2x e^{\frac{27}{\frac{\ln(x)}{5x}-x}}}{15}$$

command

```
integrate((2*ln(x)**2+(-20*x**2+270*x)*ln(x)+50*x**4+1350*x**3-270*x)*exp(135*x/(ln(x)-5*x**2))/(15*ln(x)**2-150*x**2*ln(x)+375*x**4),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{2xe^{\frac{135x}{-5x^2+\log(x)}}}{15}$$

### 44.32 Problem number 2277

$$\int \frac{8x - 16x^3 - 16x^4 + e^8(-48x^3 - 48x^4) + e^{12}(16x^3 + 16x^4) + e^4(-8x + 48x^3 + 48x^4) + (-4 + 24x^2 + 16x^3 - 8x^4)}{-16x^4 + 48e^4x^4 - 48e^8x^4} dx$$

Optimal antiderivative

$$x + \ln(x) - \frac{\ln(\ln(x) + 2)}{\left(xe^4 - x + \frac{\ln(x)}{2}\right)^2}$$

command

```
integrate((((16*x*exp(4)-16*x+8)*ln(x)+32*x*exp(4)-32*x+16)*ln(ln(x)+2)+(1+x)*ln(x)**4+((6*x**6*x**2-4*x+2)*ln(x)**3+((12*x**3+12*x**2)*exp(4)**2+(-24*x**3-12*x**2+12*x)*exp(4)+12*x**3-12*x)*ln(x)**2+((8*x**4+8*x**3)*exp(4)**3+(-24*x**4+24*x**2)*exp(4)**2+(24*x**4-24*x**3-48*x**2)*exp(4)-8*x**4+16*x**3+24*x**2-4)*ln(x)+(16*x**4+16*x**3)*exp(4)**3+(-48*x**4-48*x**3)*exp(4)**2+(48*x**4+48*x**3-8*x)*exp(4)-16*x**4-16*x**3+8*x)/(x*ln(x)**4+(6*x**2*exp(6*x**2+2*x)*ln(x)**3+(12*x**3*exp(4)**2+(-24*x**3+12*x**2)*exp(4)+12*x**3-12*x**2)*ln(x)**2+(24*x**4+24*x**3)*exp(4)**2+(24*x**4-48*x**3)*exp(4)-8*x**4+24*x**3)*ln(x)+16*x**4*exp(4)**3-48*x**4*exp(4)**2+48*x**4*exp(4)-16*x**4),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$x + \log(x) - \frac{4 \log(\log(x) + 2)}{-8x^2e^4 + 4x^2 + 4x^2e^8 - 4x \log(x) + 4xe^4 \log(x) + \log(x)^2}$$

## 44.33 Problem number 2318

$$\int \frac{e^{e^{-\frac{2e^{-x}}{\log(x)} - x - \frac{2e^{-x}}{\log(x)}}} \left( 10 - 2e^x + 2x + (10x - 2e^x x + 2x^2) \log(x) + e^{\frac{2e^{-x}}{\log(x)}} (e^x x - e^{2x} x) \log^2(x) \right)}{x \log^2(x)} dx$$

Optimal antiderivative

$$e^e e^{-\frac{2e^{-x}}{\ln(x)}} (x - e^x + 5)$$

command

```
integrate((( -x*exp(x)**2+exp(x)*x)*ln(x)**2*exp(1/exp(x)/ln(x))**2+(-2*exp(x)*x+2*x**2+10*x)*2*exp(x)+2*x+10)*exp(1/exp(1/exp(x)/ln(x))**2)/x/exp(x)/ln(x)**2/exp(1/exp(x)/ln(x))**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$(xe^{-x} - 1 + 5e^{-x}) e^x e^e e^{-\frac{2e^{-x}}{\log(x)}}$$

## 44.34 Problem number 2344

$$\int \frac{x^2 + 2x \log(x) + \log^2(x) + e^{\frac{2(25+ex^2+ex \log(x))}{x^2+x \log(x)}} (-25 - 50x + x^3 + (-25 + 2x^2) \log(x) + x \log^2(x))}{x^2 + 2x \log(x) + \log^2(x)} dx$$

Optimal antiderivative

$$x + \frac{x^2 e^{\frac{50}{x(x+\ln(x))} + 2e}}{2} + 4$$

command

```
integrate(((x*ln(x)**2+(2*x**2-25)*ln(x)+x**3-50*x-25)*exp((x*exp(1)*ln(x)+x**2*exp(1)+25)/(x
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{x^2 e^{\frac{2(ex^2+ex \log(x)+25)}{x^2+x \log(x)}}}{2} + x$$

## 44.35 Problem number 2398

$$\int \frac{-4x \log(x) + 4e^{2x+4e^{2x}x^2} x^2 (8+8x) \log(x) + \left(2x + e^{4e^{2x}x^2} (-2 - 2\log(x)) + 2x \log(x)\right) \log\left(e^{8e^{2x}x^2} - 2e^{4e^{2x}x^2} x\right)}{e^{4e^{2x}x^2} x^2 \log^2(x) - x^3 \log^2(x)}$$

Optimal antiderivative

$$\frac{2 \ln\left(\left(e^{4e^{2x}x^2} - x\right)^2\right)}{x \ln(x)}$$

command

```
integrate((((-2*ln(x)-2)*exp(exp(ln(2*x)+x)**2)+2*x*ln(x)+2*x)*ln(exp(exp(ln(2*x)+x)**2)**2-2*x*exp(exp(ln(2*x)+x)**2)+x**2)+(8*x+8)*ln(x)*exp(ln(2*x)+x)**2*exp(exp(ln(2*x)+x)**2)-4*x*ln(x))/(x**2*ln(x)**2*exp(exp(ln(2*x)+x)**2)-x**3*ln(x)**2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{2 \log\left(x^2 - 2xe^{4x^2e^{2x}} + e^{8x^2e^{2x}}\right)}{x \log(x)}$$

## 44.36 Problem number 2405

$$\int e^{\frac{x^2+x \log(x)}{-x+\log\left(\frac{e^6(9x^2-6x^3+x^4)+e^3(-6x+2x^2) \log(x)+\log^2(x)}{e^6x^2}\right)}} \frac{\left(-2x + e^3(3x^2 - x^4) + (-2 + x - x^2 - 2e^3x^2) \log(x) + 2 \log^2(x)\right)}{e^3(-3x^3 + x^4) + x^2 \log(x) + (e^3(6x^2 - 2x^3) - 2x \log(x)) \log\left(\frac{e^6(9x^2-6x^3+x^4)+e^3(-6x+\log^2(x))}{e^6x^2}\right)}$$

Optimal antiderivative

$$\frac{\frac{(x+\ln(x))x}{\ln\left(\left(-3+x+\frac{\ln(x)e^{-3}}{x}\right)^2\right)}-x}{e^{-x}}$$

command

```
integrate(((ln(x)**2+((x**2-3*x)*exp(3)+2*x+1)*ln(x)+(2*x**3-5*x**2-3*x)*exp(3))*ln((ln(x)**2+6*x)*exp(3)*ln(x)+(x**4-6*x**3+9*x**2)*exp(3)**2)/x**2/exp(3)**2)+2*ln(x)**2+(-2*x**2*exp(3)-x**2+x-2)*ln(x)+(-x**4+3*x**2)*exp(3)-2*x)*exp((x*ln(x)+x**2)/(ln((ln(x)**2+(2*x**2-6*x)*exp(3)*ln(x)+(x**4-6*x**3+9*x**2)*exp(3)**2)/x**2/exp(3)-x)))/((ln(x)+(x**2-3*x)*exp(3))*ln((ln(x)**2+(2*x**2-6*x)*exp(3)*ln(x)+(x**4-6*x**3+9*x**2)*exp(3)**2)/x**2/exp(3)+2*x*ln(x)+(-2*x**3+6*x**2)*exp(3))*ln((ln(x)**2+(2*x**2-6*x)*exp(3)*ln(x)+(x**4-6*x**3+9*x**2-3*x**3)*exp(3))), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$e^{-x + \log\left(\frac{x^2 + x \log(x)}{(2x^2 - 6x)e^3 \log(x) + \frac{(x^4 - 6x^3 + 9x^2)e^6 + \log(x)^2}{x^2 e^6}}\right)}$$

#### 44.37 Problem number 2423

$$\int \frac{(16 - 12x^2 + 4x^3 + e^3(-16x^2 - 16x^3 + 28x^4 - 8x^5) + e^3(-16x - 16x^2 + 28x^3 - 8x^4) \log(x)) \log\left(\log^2\left(e^{-e^3(2x^2 + x \log(x))}\right)\right)}{(x^2 + x \log(x)) \log^2\left(e^{-e^3(2x^2 + x \log(x))}\right)}$$

Optimal antiderivative

$$(-2 + x)^2 \ln\left(\ln\left((x + \ln(x)) e^{-(x^2 + x + 2)e^3}\right)^2\right)^2$$

command

```
integrate((((2*x**2-4*x)*ln(x)+2*x**3-4*x**2)*ln((x+ln(x))/exp((x**2+x+2)*exp(3)))*ln(ln((x+ln(x))/exp((x**2+x+2)*exp(3))))*ln(ln((x+ln(x))/exp((x**2+x+2)*exp(3))))*ln(x)+(-8*x**5+28*x**4-16*x**3-16*x**2)*exp(3)+4*x**3-12*x**2+16)*ln(ln((x+ln(x))/exp((x**2+x+2)*exp(3))))**2)/(x*ln(x)+x**2)/ln((x+ln(x))/exp((x**2+x+2)*exp(3))))
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$(x^2 - 4x + 4) \log\left(\log\left((x + \log(x)) e^{-(x^2 + x + 2)e^3}\right)^2\right)^2$$

#### 44.38 Problem number 2508

$$\int \frac{240x^2 + 100x^3 + e^x(240x + 100x^2) + (-120x + 50x^2 + e^x(-20x - 50x^2)) \log(x) + (e^x(-120 - 50x) - 120x - 120x^2) \log^2(x)}{(12x^3 + 5x^4 + e^x(12x^2 + 5x^3)) \log^2(x)}$$

Optimal antiderivative

$$\frac{10 \ln\left(\frac{(12+5x)^2}{x+e^x}\right) - 20x}{x \ln(x)}$$



command

```
integrate(((((-50*x-120)*exp(x)-50*x**2-120*x)*ln(x)+(-50*x-120)*exp(x)-50*x**2-120*x)*ln((25
50*x**2-20*x)*exp(x)+50*x**2-120*x)*ln(x)+(100*x**2+240*x)*exp(x)+100*x**3+240*x**2)/((5*x**3
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$-\frac{20}{\log(x)} + \frac{10 \log\left(\frac{25x^2+120x+144}{x+e^x}\right)}{x \log(x)}$$

#### 44.39 Problem number 2569

$$\int \frac{16 \log^2(x^2) + e^{2e^{\frac{x}{\log(x^2)}} + 4x} \left( (1+4x) \log^2(x^2) + e^{\frac{x}{\log(x^2)} + \frac{x}{\log(x^2)}} (-4x + 2x \log(x^2)) \right) + e^{e^{\frac{x}{\log(x^2)} + 2x}} (8 + 1)}{\log^2(x^2)}$$

Optimal antiderivative

$$x \left( e^{e^{\frac{x}{\ln(x^2)} + 2x} + 4} \right)^2 - 4$$

command

```
integrate((((2*x*ln(x**2)-4*x)*exp(x/ln(x**2))*exp(exp(x/ln(x**2))))+(1+4*x)*ln(x**2)**2)*exp(
16*x)*exp(x/ln(x**2))*exp(exp(x/ln(x**2))))+(16*x+8)*ln(x**2)**2)*exp(exp(exp(x/ln(x**2))))+2*x
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$8xe^{2x+e^{\frac{x}{\log(x^2)}}} + xe^{4x+2e^{\frac{x}{\log(x^2)}}} + 16x$$

#### 44.40 Problem number 2570

$$\int e^{\frac{-15+5x}{(39x-8x^2-4x^3+x^4)\log(x)}} \frac{(-585+315x+20x^2-35x^3+5x^4+(-585+240x+140x^2-100x^3+15x^4)\log(x))}{(1521x^2-624x^3-248x^4+142x^5-8x^7+x^8)\log^2(x)} dx$$

Optimal antiderivative

$$\ln(2) - e^{\frac{5}{(3+x)\left(-\frac{1}{3+x}+x-4\right)x\ln(x)}}$$

command

```
integrate(((15*x**4-100*x**3+140*x**2+240*x-585)*ln(x)+5*x**4-35*x**3+20*x**2+315*x-585)*exp((5*x-15)/(x**4-4*x**3-8*x**2+39*x)/ln(x))/(x**8-8*x**7+142*x**5-248*x**4-624*x**3+1521*x**2)/ln(x)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$-e^{\frac{5x-15}{(x^4-4x^3-8x^2+39x)\log(x)}}$$

#### 44.41 Problem number 2671

$$\int \frac{e^{-4+2e^x+2x+4x^2}(3-3x)+2x-x^2-2e^xx^2-9x^3+\left(-3e^{-4+2e^x+2x+4x^2}-x^2\right)\log\left(e^{4-2e^x-2x-4x^2}\left(3e^{-4+2e^x+2x}\right)\right)}{3e^{-4+2e^x+3x+4x^2}+e^xx^2}$$

Optimal antiderivative

$$\left(\ln\left(x^2e^{-2x\left(2x-\frac{2}{x}\right)-2x-2e^x}+3\right)+x\right)e^{-x}$$

command

```
integrate(((3*exp(exp(x)+2*x**2+x-2)**2-x**2)*ln((3*exp(exp(x)+2*x**2+x-2)**2+x**2)/exp(exp(x)+2*x**2))+(-3*x+3)*exp(exp(x)+2*x**2+x-2)**2-2*exp(x)*x**2-9*x**3-x**2+2*x)/(3*exp(x)*exp(exp(x)+2*x**2+exp(x)*x**2)),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$xe^{-x} + e^{-x} \log\left(\left(x^2 + 3e^{4x^2+2x+2e^x-4}\right)e^{-4x^2-2x-2e^x+4}\right)$$

#### 44.42 Problem number 2849

$$\int \frac{-6 - 24x + (-1 - 6x) \log(2x) + (24x + 4 \log(x) + (6x + \log(x)) \log(2x) + (8 + 2 \log(2x)) \log(4 + \log(2x))) \log}{24x + 4 \log(x) + (6x + \log(x)) \log(2x) + (8 + 2 \log(2x)) \log(4 + \log(2x))} dx$$

Optimal antiderivative

$$x \ln \left( \frac{4}{\frac{\ln(x)}{2} + 3x + \ln(\ln(2x) + 4)} \right)$$

command

```
integrate((((2*ln(2*x)+8)*ln(ln(2*x)+4)+(ln(x)+6*x)*ln(2*x)+4*ln(x)+24*x)*ln(8/(2*ln(ln(2*x)+6*x-1)*ln(2*x)-24*x-6))/((2*ln(2*x)+8)*ln(ln(2*x)+4)+(ln(x)+6*x)*ln(2*x)+4*ln(x)+24*x)),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$x \log \left( \frac{8}{6x + \log(x) + 2 \log(\log(x) + \log(2) + 4)} \right)$$

#### 44.43 Problem number 2893

$$\int \frac{20x + 129x^2 + 50x^3 + 5x^4 + (60 + 730x + 171x^2 - 20x^3 - 5x^4) \log(x) + (-60 - 387x - 150x^2 - 15x^3) \log \left( \frac{4x}{(20x + 129x^2 + 50x^3 + 5x^4) \log^2(x)} \right)}{(20x + 129x^2 + 50x^3 + 5x^4) \log^2(x)} dx$$

Optimal antiderivative

$$\frac{3 \ln \left( \frac{\left( x + \frac{4}{25+5x} \right) x}{3} \right) - x - \ln(x)}{\ln(x)}$$

command

```
integrate(((((-15*x**3-150*x**2-387*x-60)*ln(((5*x**3+25*x**2+4*x)/(15*x+75)))+(-5*x**4-20*x**3+171*x**2+730*x+60)*ln(x)+5*x**4+50*x**3+129*x**2+20*x)/(5*x**4+50*x**3+129*x**2+20*x)),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$-\frac{x}{\log(x)} + \frac{3 \log \left( \frac{5x^3 + 25x^2 + 4x}{15x + 75} \right)}{\log(x)}$$

## 44.44 Problem number 2897

$$\int \frac{e^{-\frac{e^5+5x}{x}} \left( 4e^{5+4e^{-\frac{e^5+5x}{x}}} + e^{\frac{e^5+5x}{x}} (2x^2 - 2x^3 - 6x^4 + 4x^5) \right) + e^{2e^{-\frac{e^5+5x}{x}}} \left( e^5 (4 + 4x - 4x^2) + e^{\frac{e^5+5x}{x}} (2x^2 - 4x^3) \right)}{x^2}$$

Optimal antiderivative

$$\left( 1 - x^2 + e^{2e^{-5-\frac{e^5}{x}}} + x \right)^2 - 2 + e^4$$

command

```
integrate((4*exp(5)*exp(2/exp((exp(5)+5*x)/x))**2+((-4*x**3+2*x**2)*exp((exp(5)+5*x)/x)+(-4*x**2+4*x+4)*exp(5))*exp(2/exp((exp(5)+5*x)/x))+(4*x**5-6*x**4-2*x**3+2*x**2)*exp((exp(5)+5*x)/x))
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: SystemError

Sympy 1.8 under Python 3.8.8 output

$$x^4 - 2x^3 - x^2 + 2x + (-2x^2 + 2x + 2) e^{2e^{-\frac{5x+e^5}{x}}} + e^{4e^{-\frac{5x+e^5}{x}}}$$

## 44.45 Problem number 2907

$$\int \frac{(-40x^2 + 40x^3 - 10x^4) \log^2(x) + (2 - 3x + x^2) \log(x - x^2) + \log(x) (-2 + 5x - 2x^2) + (2 - 2x + x^2) \log(x - x^2)}{(120x^2 - 120x^3 + 30x^4) \log^2(x)}$$

Optimal antiderivative

$$\frac{\ln((1-x)x)}{30 \left( \frac{x}{-1+x} - x \right) \ln(x)} - \frac{x}{3}$$

command

```
integrate((( -10*x**4+40*x**3-40*x**2)*ln(x)**2+((x**2-2*x+2)*ln(-x**2+x)-2*x**2+5*x-2)*ln(x)+(x**2-3*x+2)*ln(-x**2+x))/(30*x**4-120*x**3+120*x**2)/ln(x)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$-\frac{x}{3} + \frac{(1-x) \log(-x^2+x)}{30x^2 \log(x) - 60x \log(x)}$$

#### 44.46 Problem number 3063

$$\int \frac{-2x - x^2 + (-2 - x - 2x^2 - 2x^3 - x^4) \log(x) + (-2x - 4x^2 - 2x^3) \log^2(x) + (-2x - x^2) \log^3(x) + (2 + 3x + (2x^3 + x^4) \log(x) + (4x^2 + 2x^3) \log^2(x) + (2x + x^2) \log^3(x))}{(2x^3 + x^4) \log(x) + (4x^2 + 2x^3) \log^2(x) + (2x + x^2) \log^3(x)}$$

Optimal antiderivative

$$13 - x - \frac{\ln(2 \ln(x) (2 + x)^2)}{x + \ln(x)}$$

command

```
integrate(((x**2+3*x+2)*ln(x)*ln((2*x**2+8*x+8)*ln(x))+(-x**2-2*x)*ln(x)**3+(-2*x**3-4*x**2-2*x)*ln(x)**2+(-x**4-2*x**3-2*x**2-x-2)*ln(x)-x**2-2*x)/((x**2+2*x)*ln(x)**3+(2*x**3+4*x**2+2*x)*ln(x)**2+(2*x**2+2*x)*ln(x)), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$-x - \frac{\log((2x^2 + 8x + 8) \log(x))}{x + \log(x)}$$

#### 44.47 Problem number 3087

$$\int \frac{108x^3 + 30x^4 + 2x^5 + (-108x^3 - 21x^4 - x^5) \log(x) + (27x^3 + 3x^4) \log^2(x) + ((432x^3 + 105x^4 + 7x^5) \log(x) + (36 + 12x + x^2) \log(x) + (-36 - 6x) \log^2(x) + 9)}{(36 + 12x + x^2) \log(x) + (-36 - 6x) \log^2(x) + 9}$$

Optimal antiderivative

$$\frac{x^4(x+9) \ln(\ln(x))}{3 - \frac{x}{\ln(x)-2}}$$

command

```
integrate((((15*x**4+108*x**3)*ln(x)**3+(-4*x**5-87*x**4-432*x**3)*ln(x)**2+(7*x**5+105*x**4+4*x**5-21*x**4-108*x**3)*ln(x)+2*x**5+30*x**4+108*x**3)/(9*ln(x)**3+(-6*x-36)*ln(x)**2+(x**2+12*x+36)*ln(x)**2+(x**2+12*x+36)*ln(x)), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{(-x^5 \log(x) + 2x^5 - 9x^4 \log(x) + 18x^4) \log(\log(x))}{x - 3 \log(x) + 6}$$

#### 44.48 Problem number 3089

$$\int \frac{16x^2 - 112x^3 - 128x^2 \log(x) + (-64x^2 + 32x^3 + (-64x + 32x^2) \log(x)) \log(x + \log(x)) + (-8x + 12x^2 + (-8$$

Optimal antiderivative

$$\left(-2 - \ln\left(\frac{x}{\frac{4}{\ln(x+\ln(x))} + \frac{1}{x}} - x\right)\right)^2 x$$

command

```
integrate(((((-1+x)*ln(x)+x**2-x)*ln(x+ln(x))**2+((4*x**2-8*x)*ln(x)+4*x**3-8*x**2)*ln(x+ln(x))-16*x**2*ln(x)-16*x**3)*ln((x**2-x)*ln(x+ln(x))-4*x**2)/(ln(x+ln(x))+4*x))**2+(((8*x-6)*ln(x)+8*x**2-6*x)*ln(x+ln(x))**2+((24*x**2-48*x)*ln(x)+24*x**3-48*x**2)*ln(x+ln(x))-96*x**2*ln(x)-88*x**3+8*x**2)*ln(((x**2-x)*ln(x+ln(x))-4*x**2)/(ln(x+ln(x))+4*x))+((12*x-8)*ln(x)+12*x**2-8*x)*ln(x+ln(x))**2+((32*x**2-64*x)*ln(x)+32*x**3-64*x**2)*ln(x+ln(x))-128*x**2*ln(x)-112*x**3+16*x**2)/(((((-1+x)*ln(x)+x**2-x)*ln(x+ln(x))**2+((4*x**2-8*x)*ln(x)+4*x**2)*ln(x+ln(x))-16*x**2*ln(x)-16*x**3),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$x \log\left(\frac{-4x^2 + (x^2 - x) \log(x + \log(x))}{4x + \log(x + \log(x))}\right)^2 + 4x \log\left(\frac{-4x^2 + (x^2 - x) \log(x + \log(x))}{4x + \log(x + \log(x))}\right) + 4x$$

#### 44.49 Problem number 3156

$$\int \frac{(-5 - e)e^x + e^{2x} + (-e^x x \log(x) + (-5x - ex + e^x x) \log(5 + e - e^x) \log(x)) \log(\log(x))}{(e^{2x} x + e^x(-5x - ex)) \log(x) \log(\log(x))} dx$$

Optimal antiderivative

$$\ln(\ln(\ln(x))) - \ln(-e^x + e + 5) e^{-x}$$

command

```
integrate((((exp(x)*x-x*exp(1)-5*x)*ln(x)*ln(-exp(x)+exp(1)+5)-x*exp(x)*ln(x))*ln(ln(x))+exp(exp(1)-5)*exp(x))/(x*exp(x)**2+(-x*exp(1)-5*x)*exp(x))/ln(x)/ln(ln(x)),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\log(\log(\log(x))) - e^{-x} \log(-e^x + e + 5)$$

#### 44.50 Problem number 3383

$$\int \frac{(4 - 4e^4 + e^8) \log^2(x) + (-4 + 2e^4) \log^2(x) \log(4x^2) + \log^2(x) \log^2(4x^2) + e^{\frac{4}{-2+e^4+\log(4x^2)}} (4 - 4e^4 + e^8 + (4 + \dots))}{(4 - 4e^4 + e^8) \log^2(x) + (-4 + 2e^4) \log^2(x) \log(4x^2) + \log^2(x) \log^2(4x^2) + e^{\frac{4}{-2+e^4+\log(4x^2)}} (4 - 4e^4 + e^8 + (4 + \dots))} dx$$

Optimal antiderivative

$$\left( \frac{1-x}{x} - \frac{e^{\frac{4}{\ln(4x^2)+e^4-2}}}{\ln(x)} + 2 \right) x$$

command

```
integrate((((1-ln(x))*ln(4*x**2)**2+((-2*exp(4)+4)*ln(x)+2*exp(4)-4)*ln(4*x**2))+(-
exp(4)**2+4*exp(4)+4)*ln(x)+exp(4)**2-4*exp(4)+4)*exp(4/(ln(4*x**2)+exp(4)-2))+ln(x)**2*ln(4*
4)*ln(x)**2*ln(4*x**2)+(exp(4)**2-4*exp(4)+4)*ln(x)**2)/(ln(x)**2*ln(4*x**2)**2+(2*exp(4)-
4)*ln(x)**2*ln(4*x**2)+(exp(4)**2-4*exp(4)+4)*ln(x)**2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$- \frac{x e^{\frac{4}{2 \log(x) - 2 + \log(4) + e^4}}}{\log(x)} + x$$

#### 44.51 Problem number 3596

$$\int \frac{(12x + 18x^2 + e^x(12 + 12x + 6x^2)) + (6x + 6e^x x) \log(x)) \log(25e^x x^2 + 25x^3 + (50e^x x + 50x^2) \log(x) + (25e^x + \dots))}{2e^x x + 2x^2} dx$$

Optimal antiderivative

$$\frac{3x \ln(5(x + \ln(x))) (5x + 5 \ln(x)) (x + e^x))^2}{2}$$

command

```
integrate((((3*exp(x)+3*x)*ln(x)+3*exp(x)*x+3*x**2)*ln((25*exp(x)+25*x)*ln(x)**2+(50*exp(x)*x
+50*x**2)*ln(x))+((12*x+18*x**2+exp(x)*(12+12*x+6*x**2))+6*x*(1+exp(x))*ln(x))*ln(25*exp(x)*x**2+25*x**3+(50*exp(x)*x+50*x**2)*ln(x)+(25*exp(x)+25*x)*ln(x)**2+(50*exp(x)*x+50*x**2)*ln(x)), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{3x \log(25x^3 + 25x^2 e^x + (25x + 25e^x) \log(x)^2 + (50x^2 + 50x e^x) \log(x))^2}{2}$$

## 44.52 Problem number 3706

$$\int \frac{1 + 2x + x^2 + x^{\frac{-x-2x^2}{1+x}} (-1 - x + 2x^2 + 2x^3 + (x + 4x^2 + 2x^3) \log(x))}{1 + 2x + x^2} dx$$

Optimal antiderivative

$$-5 - x e^{\ln(x) \left( \frac{x^2}{-1-x} - x \right)} + x$$

command

```
integrate((((2*x**3+4*x**2+x)*ln(x)+2*x**3+2*x**2-x-1)*exp((-2*x**2-x)*ln(x)/(1+x))+x**2+2*x+
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$-x e^{\frac{(-2x^2-x) \log(x)}{x+1}} + x$$

## 44.53 Problem number 3715

$$\int \frac{-3e^x x + e^{2x} x^2 + (12 + 4e^x x^2) \log\left(\frac{9-6e^x x + e^{2x} x^2}{e^{8/5} x^2}\right) + (3x - e^x x^2) \log^2\left(\frac{9-6e^x x + e^{2x} x^2}{e^{8/5} x^2}\right)}{-3e^x x + e^{2x} x^2} dx$$

Optimal antiderivative

$$x + \ln\left(\left(e^x - \frac{3}{x}\right)^2 e^{-\frac{8}{5}}\right)^2 e^{-x}$$

command

```
integrate((((-exp(x)*x**2+3*x)*ln((exp(x)**2*x**2-6*exp(x)*x+9)/x**2/exp(4/5)**2)**2+(4*exp(x)+6*exp(x)*x+9)/x**2/exp(4/5)**2)+exp(x)**2*x**2-3*exp(x)*x)/(exp(x)**2*x**2-3*exp(x)*x), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$x + e^{-x} \log\left(\frac{x^2 e^{2x} - 6x e^x + 9}{x^2 e^{\frac{8}{5}}}\right)^2$$



## 44.54 Problem number 3737

$$\int \frac{e^{4-100e^{4-2x}-x} (e^{-4+2x}(-1+x) + 200x)}{x^2} dx$$

Optimal antiderivative

$$5 + \frac{e^x e^{-100e^{4-2x}}}{x}$$

command

```
integrate((( -1+x)*exp(-2+x)**2+200*x)*exp(x)/x**2/exp(-2+x)**2/exp(100/exp(-2+x)**2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: SystemError

Sympy 1.8 under Python 3.8.8 output

$$\frac{e^x e^{-100e^4 e^{-2x}}}{x}$$

## 44.55 Problem number 3939

$$\int \frac{e^{-2-e^{\frac{-x+(4-\log(3))\log(x^3)}{\log(x^3)}}} -x \left( e^{\frac{-x+(4-\log(3))\log(x^3)}{\log(x^3)}} (-3 + \log(x^3)) - \log^2(x^3) \right)}{\log^2(x^3)} dx$$

Optimal antiderivative

$$e^{-2-x-e^{4-\frac{x}{\ln(x^3)}-\ln(3)}}$$

command

```
integrate(((ln(x**3)-3)*exp((( -ln(3)+4)*ln(x**3)-x)/ln(x**3))-ln(x**3)**2)*exp(-exp((( -ln(3)+4)*ln(x**3)-x)/ln(x**3))-x-2)/ln(x**3)**2, x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$e^{-x-e^{\frac{-x+(4-\log(3))\log(x^3)}{\log(x^3)}}} -2$$

#### 44.56 Problem number 4051

$$\int e^{\frac{4x^2+x^2 \log(169)}{20 \log(50-x)}} \frac{(-4x^2 - x^2 \log(169) + (-400x + 8x^2 + (-100x + 2x^2) \log(169)) \log(50-x))}{(-1000 + 20x) \log^2(50-x)} dx$$

Optimal antiderivative

$$e^{\frac{x^2 \left( \frac{4}{5} + \frac{2 \ln(13)}{5} \right)}{4 \ln(-x+50)}}$$

command

```
integrate(((2*(2*x**2-100*x)*ln(13)+8*x**2-400*x)*ln(-x+50)-2*x**2*ln(13)-4*x**2)*exp(1/20*(2*x+50))/(20*x-1000)/ln(-x+50)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$e^{\frac{\frac{x^2}{5} + \frac{x^2 \log(13)}{10}}{\log(50-x)}}$$

#### 44.57 Problem number 4056

$$\int e^{\frac{\frac{1}{15} \left( \frac{2-16x+4x^2+4x \log(25+10x+x^2)}{12+5e^{-4x+x^2+x \log(25+10x+x^2)}} x \right) + \frac{2-16x+4x^2+4x \log(25+10x+x^2)}{-4x+x^2+x \log(25+10x+x^2)}}{\frac{(40+64x-28x^2-3x^3+x^4+(-10-42x+2x^2))}{240x-72x^2-9x^3+3x^4+(-120x+6x^2+6x^3) \log(25+10x+x^2)+(15x^2+10x+25) \log^2(25+10x+x^2)}}} dx$$

Optimal antiderivative

$$e^{\frac{x e^{\frac{4 - \frac{2}{x(5 - \ln((5+x)^2) - x) - x}}{3}}}{3}} + \frac{4}{5}$$

command

```
integrate(((x**2+5*x)*ln(x**2+10*x+25)**2+(2*x**3+2*x**2-42*x-10)*ln(x**2+10*x+25)+x**4-3*x**3-28*x**2+64*x+40)*exp((4*x*ln(x**2+10*x+25)+4*x**2-16*x+2)/(x*ln(x**2+10*x+25)+x**2-4*x))*exp(1/3*x*exp((4*x*ln(x**2+10*x+25)+4*x**2-16*x+2)/(x*ln(x**2+10*x+25)+x**2-4*x)))+4/5)/((3*x**2+15*x)*ln(x**2+10*x+25)**2+(6*x**3+6*x**2-120*x)*ln(x**2+10*x+25)+3*x**4-9*x**3-72*x**2+240*x),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$e^{\frac{x e^{\frac{4x^2+4x \log(x^2+10x+25)-16x+2}{x^2+x \log(x^2+10x+25)-4x}}}{3}} + \frac{4}{5}$$

## 44.58 Problem number 4088

$$\int \frac{(-2e^x x + 2x^2) \log^2(x) + ((e^x(-10 - 2x) + 10x + 2x^2) \log^2(x) + (10x + 12x^2 + 2x^3 + e^{2x}(10x + 2x^2) + e^x(-10 - 2x) + 2x^2) \log(x) + (10x + 12x^2 + 2x^3 + e^{2x}(10x + 2x^2) + e^x(-10 - 2x) + 2x^2) \log^2(x))}{(x^2 + 5x) \log(x)^3 \log(5+x)}$$

Optimal antiderivative

$$2x + \left( e^x - x - \frac{\ln(x \ln(5+x))}{\ln(x)} \right)^2 + 2$$

command

```
integrate((( -2*x-10)*ln(5+x)*ln(x*ln(5+x))**2+((( -2*x**2-10*x)*exp(x)+2*x**2+10*x)*ln(x)**2+
2*x**2-8*x+10)*ln(x))*ln(5+x)+2*x*ln(x))*ln(x*ln(5+x))+(((2*x**2+10*x)*exp(x)**2+(-
2*x**3-12*x**2-10*x)*exp(x)+2*x**3+12*x**2+10*x)*ln(x)**3+((-2*x-10)*exp(x)+2*x**2+10*x)*ln(x)
2*exp(x)*x+2*x**2)*ln(x)**2)/(x**2+5*x)/ln(x)**3/ln(5+x), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$x^2 + 2x + \frac{2x \log(x \log(x+5))}{\log(x)} + \frac{(-2x \log(x) - 2 \log(x \log(x+5))) e^x + e^{2x} \log(x)}{\log(x)} + \frac{\log(x \log(x+5))^2}{\log(x)^2}$$

## 44.59 Problem number 4109

$$\int \frac{30x^2 - 10x^4 + (30 - 10x^2 + 9x^5 - 6x^7 + x^9 - 60x^6 \log(3)) \log(x) + (18x^3 - 12x^5 + 2x^7 - 120x^4 \log(3)) \log^2(x)}{(9x^5 - 6x^7 + x^9) \log(x) + (18x^3 - 12x^5 + 2x^7) \log^2(x)}$$

Optimal antiderivative

$$x - \frac{10 \left( \frac{\ln(\ln(x))}{\ln(x)+x^2} - 3 \ln(3) \right)}{x^2 - 3}$$

command

```
integrate(((20*x**2*ln(x)**2+(40*x**4-50*x**2-30)*ln(x))*ln(ln(x))+(-60*x**2*ln(3)+x**5-
6*x**3+9*x)*ln(x)**3+(-120*x**4*ln(3)+2*x**7-12*x**5+18*x**3)*ln(x)**2+(-60*x**6*ln(3)+x**9-
6*x**7+9*x**5-10*x**2+30)*ln(x)-10*x**4+30*x**2)/((x**5-6*x**3+9*x)*ln(x)**3+(2*x**7-
12*x**5+18*x**3)*ln(x)**2+(x**9-6*x**7+9*x**5)*ln(x)), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$x - \frac{10 \log(\log(x))}{x^4 + x^2 \log(x) - 3x^2 - 3 \log(x)} + \frac{30 \log(3)}{x^2 - 3}$$

## 44.60 Problem number 4134

$$\int \frac{e^{\frac{-15+3x}{x+\log(x)}} (15x^3 + 12x^4 + 3x^5 + e^x(-15 - 12x - x^3)) + (-15x^2 - 12x^3 - 2x^4) \log(15) + (9x^4 + e^x(-3x - 2x^2))}{2x^3 + 4x^2 \log(x) + 2x \log^2(x)}$$

Optimal antiderivative

$$\frac{(x^2(x - \ln(15)) - e^x) e^{\frac{-5+x}{\frac{x}{3} + \frac{\ln(x)}{3}}}}{2}$$

command

```
integrate((( -exp(x)*x-2*x**2*ln(15)+3*x**3)*ln(x)**2+((-2*x**2-3*x)*exp(x)-7*x**3*ln(15)+9*x**3-12*x-15)*exp(x)+(-2*x**4-12*x**3-15*x**2)*ln(15)+3*x**5+12*x**4+15*x**3)*exp((3*x-15)/(x+ln(x)))/(2*x*ln(x)**2+4*x**2*ln(x)+2*x**3), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{(x^3 - x^2 \log(15) - e^x) e^{\frac{3x-15}{x+\log(x)}}}{2}$$

## 44.61 Problem number 4177

$$\int \frac{e^{-\frac{180x^3}{(-1+x)\log(x)}} (-180x^2 + 180x^3 + (540x^2 - 360x^3) \log(x))}{(1 - 2x + x^2) \log^2(x)} dx$$

Optimal antiderivative

$$e^{\frac{180x^3}{\ln(x)(1-x)}}$$

command

```
integrate((( -360*x**3+540*x**2)*ln(x)+180*x**3-180*x**2)*exp(-180*x**3/(-1+x)/ln(x))/(x**2-2*x+1)/ln(x)**2, x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$e^{-\frac{180x^3}{(x-1)\log(x)}}$$

#### 44.62 Problem number 4204

$$\int \frac{(-128x^3 + 256x^4 + (128x^2 - 256x^3) \log(2) + (-128x^2 + 256x^3 + (128x - 256x^2) \log(2)) \log(x)) \log(-12 - x)}{-12x^3 - x^4 + x^5 + (-24x^2 - 2x^3 +$$

Optimal antiderivative

$$\frac{64 \ln((3 + x)(-4 + x))^2 (x - \ln(2))}{x + \ln(x)}$$

command

```
integrate((((64*x**3-64*x**2-768*x)*ln(x)+(64*x**3-832*x-768)*ln(2)-64*x**3+64*x**2+768*x)*ln(x-12)**2+((-256*x**2+128*x)*ln(2)+256*x**3-128*x**2)*ln(x)+(-256*x**3+128*x**2)*ln(2)+256*x**3-128*x**3)*ln(x**2-x-12))/((x**3-x**2-12*x)*ln(x)**2+(2*x**4-2*x**3-24*x**2)*ln(x)+x**5-x**4-12*x**3), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{(64x - 64 \log(2)) \log(x^2 - x - 12)^2}{x + \log(x)}$$

#### 44.63 Problem number 4469

$$\int \frac{12x + 16x^2 + (-14x - 8x^2) \log(x) + (3x + 10x^2 + 8x^3) \log^3(x) + (12 + 16x) \log(3 + 4x)}{(3x + 4x^2) \log^3(x)} dx$$

Optimal antiderivative

$$x + x^2 - \frac{2 \ln(3 + 4x) + 2x}{\ln(x)^2} - 2$$

command

```
integrate(((16*x+12)*ln(3+4*x)+(8*x**3+10*x**2+3*x)*ln(x)**3+(-8*x**2-14*x)*ln(x)+16*x**2+12*
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$x^2 + x - \frac{2x}{\log(x)^2} - \frac{2 \log(4x + 3)}{\log(x)^2}$$

## 44.64 Problem number 4629

$$\int \frac{81x^2 + 18x^4 + x^6 + (162x + 18x^3) \log(2) + 81 \log^2(2) + (-81x^2 - 18x^3 - 18x^4 - 2x^5 - x^6 + (-162x - 18x^2 -$$

Optimal antiderivative

$$\frac{\ln(x) - e^{\frac{1}{3x + \frac{x^2(x - \ln(x))}{3} + 3 \ln(2)}}}{x}$$

command

```
integrate(((x**4*ln(x)**2+(-18*x**2*ln(2)-2*x**5-18*x**3-6*x**2)*ln(x)+81*ln(2)**2+(18*x**3+1
3/(x**2*ln(x)-9*ln(2)-x**3-9*x))-x**4*ln(x)**3+(18*x**2*ln(2)+2*x**5+x**4+18*x**3)*ln(x)**2+(
81*ln(2)**2+(-18*x**3-18*x**2-162*x)*ln(2)-x**6-2*x**5-18*x**4-18*x**3-81*x**2)*ln(x)+81*ln(2
18*x**4*ln(2)-2*x**7-18*x**5)*ln(x)+81*x**2*ln(2)**2+(18*x**5+162*x**3)*ln(2)+x**8+18*x**6+81
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{\log(x)}{x} - \frac{e^{-\frac{3}{-x^3+x^2 \log(x)-9x-9 \log(2)}}}{x}$$

## 44.65 Problem number 4683

$$\int \frac{-3x^2 + 6x^3 + e^x(2x^2 + 6x^3 - 2x^4) + (-6x^2 + e^x(-8x - 4x^2 + 4x^3)) \log\left(\frac{1}{3}e^{-2x}(-3x + e^x(-4 - 2x + 2x^2))\right)}{(-3x + e^x(-4 - 2x + 2x^2)) \log\left(\frac{1}{3}e^{-2x}(-3x + e^x(-4 - 2x + 2x^2))\right)}$$

Optimal antiderivative

$$x^2 \ln\left(\ln\left(\frac{(2x^2 - 4 - x(3e^{-x} + 2))e^{-x}}{3}\right)\right)$$

command

```
integrate((((4*x**3-4*x**2-8*x)*exp(x)-6*x**2)*ln(1/3*((2*x**2-2*x-4)*exp(x)-3*x)/exp(x)**2)*
2*x-4)*exp(x)-3*x)/exp(x)**2))+(-2*x**4+6*x**3+2*x**2)*exp(x)+6*x**3-3*x**2)/((2*x**2-
2*x-4)*exp(x)-3*x)/ln(1/3*((2*x**2-2*x-4)*exp(x)-3*x)/exp(x)**2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$x^2 \log\left(\log\left(\left(-x + \frac{(2x^2 - 2x - 4)e^x}{3}\right)e^{-2x}\right)\right)$$

#### 44.66 Problem number 4944

$$\int \frac{(6x^2 + 2x^3) \log(1-x) + (3+x-4x^2) \log^2(1-x) + ((6x+2x^2) \log(1-x) + (-x+x^2) \log^2(1-x)) \log(x)}{-x^3 + x^4 + (-2x^2 + 2x^3) \log(x) + (-x+x^2) \log^2(x)} dx$$

Optimal antiderivative

$$\frac{\ln(1-x)^2(3+x)}{x + \ln(x)}$$

command

```
integrate((((x**2-x)*ln(1-x)**2+(2*x**2+6*x)*ln(1-x))*ln(x)+(-4*x**2+x+3)*ln(1-x)**2+(2*x**3+x)))/((x**2-x)*ln(x)**2+(2*x**3-2*x**2)*ln(x)+x**4-x**3),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{(x+3) \log(1-x)^2}{x + \log(x)}$$

#### 44.67 Problem number 5196

$$\int \frac{16x^5 + 4x^6 + e^{\frac{3(-1+2x)}{x}}(-1296x - 756x^2) + e^{\frac{4(-1+2x)}{x}}(1296 + 324x - 108x^2) + e^{\frac{2(-1+2x)}{x}}(432x^2 + 540x^3 + 72x^4)}{dx}$$

Optimal antiderivative

$$\left(e^{2-\frac{1}{x}}(3 - \ln(4+x)) - x\right)^4$$

command

```
integrate(((4*x+16)*exp((-1+2*x)/x)**4*ln(4+x)**4+((4*x**2-48*x-192)*exp((-1+2*x)/x)**4+(4*x**1+2*x)/x)**3)*ln(4+x)**3+((-36*x**2+216*x+864)*exp((-1+2*x)/x)**4+(-24*x**3-252*x**2-432*x)*exp((-1+2*x)/x)**3+(12*x**4+60*x**3+48*x**2)*exp((-1+2*x)/x)**2)*ln(4+x)**2+((108*x**2+432*x-1728)*exp((-1+2*x)/x)**4+(36*x**3+756*x**2+1296*x)*exp((-1+2*x)/x)**3+(-60*x**4-360*x**3-288*x**2)*exp((-1+2*x)/x)**2+(12*x**5+52*x**4+16*x**3)*exp((-1+2*x)/x))*ln(4+x)+(-108*x**2+324*x+1296)*exp((-1+2*x)/x)**4+(-756*x**2-1296*x)*exp((-1+2*x)/x)**3+(72*x**4+540*x**1+2*x)/x)**2+(-32*x**5-156*x**4-48*x**3)*exp((-1+2*x)/x)+4*x**6+16*x**5)/(x**3+4*x**2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\begin{aligned} & x^4 + (4x^3 \log(x+4) - 12x^3) e^{\frac{2x-1}{x}} + (6x^2 \log(x+4)^2 - 36x^2 \log(x+4) + 54x^2) e^{\frac{2(2x-1)}{x}} \\ & + (4x \log(x+4)^3 - 36x \log(x+4)^2 + 108x \log(x+4) - 108x) e^{\frac{3(2x-1)}{x}} \\ & + (\log(x+4)^4 - 12 \log(x+4)^3 + 54 \log(x+4)^2 - 108 \log(x+4) + 81) e^{\frac{4(2x-1)}{x}} \end{aligned}$$

#### 44.68 Problem number 5357

$$\int \frac{e^x x \log(x) + e^x x^2 \log^2(x) + e^{e^{3e^{-x}}} (-3e^{3e^{-x}} x \log(x) + e^x x \log^2(x)) + (-e^{e^{3e^{-x}+x}} - e^x x) \log(e^{e^{3e^{-x}}} + x)}{e^{e^{3e^{-x}+x}} x \log^2(x) + e^x x^2 \log^2(x)} dx$$

Optimal antiderivative

$$\frac{\ln(e^{e^{3e^{-x}}} + x)}{\ln(x)} + x$$

command

```
integrate((( -exp(x)*exp(exp(3/exp(x))) - exp(x)*x)*ln(exp(exp(3/exp(x))))+x)+(-3*x*ln(x)*exp(3/e
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$x + \frac{\log(x + e^{e^{3e^{-x}}})}{\log(x)}$$

#### 44.69 Problem number 5388

$$\int \frac{(-8x + 8x^2 + (3x - 4x^2 + x^3) \log(x)) \log\left(\frac{1}{3}(8 + (-3 + x) \log(x))\right) + (-6 + 8x - 2x^2 + (2x - 2x^2) \log(x)) \log\left(\frac{1}{3}(8 + (-3 + x) \log(x))\right)}{2}$$

Optimal antiderivative

$$\frac{\left(1 + \ln\left(\ln\left(\frac{\ln(x)(-3+x)}{3} + \frac{8}{3}\right)\right)^2 - x\right) \left(\frac{1}{2} + \frac{\ln\left(\ln\left(\frac{\ln(x)(-3+x)}{3} + \frac{8}{3}\right)\right)^2}{2} - \frac{x}{2}\right)}{2}$$

command



```
integrate(((2*x*ln(x)+2*x-6)*ln(ln(1/3*ln(x)*(-3+x)+8/3))**3+((-x**2+3*x)*ln(x)-8*x)*ln(1/3*ln(
3+x)+8/3)*ln(ln(1/3*ln(x)*(-3+x)+8/3))**2+((-2*x**2+2*x)*ln(x)-2*x**2+8*x-6)*ln(ln(1/3*ln(x)*
3+x)+8/3))+((x**3-4*x**2+3*x)*ln(x)+8*x**2-8*x)*ln(1/3*ln(x)*(-3+x)+8/3))/((2*x**2-
6*x)*ln(x)+16*x)/ln(1/3*ln(x)*(-3+x)+8/3), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{x^2}{4} - \frac{x}{2} + \left(\frac{1}{2} - \frac{x}{2}\right) \log\left(\log\left(\frac{(x-3)\log(x)}{3} + \frac{8}{3}\right)\right)^2 + \frac{\log\left(\log\left(\frac{(x-3)\log(x)}{3} + \frac{8}{3}\right)\right)^4}{4}$$

#### 44.70 Problem number 5539

$$\int \frac{e^{-\frac{1250}{x \log(2x)}}(-2500 - 2500 \log(2x)) + e^{-\frac{2500}{x \log(2x)}}(2500 + 2500 \log(2x))}{x^2 \log^2(2x)} dx$$

Optimal antiderivative

$$\left(1 - e^{-\frac{1250}{x \ln(2x)}}\right)^2$$

command

```
integrate(((2500*ln(2*x)+2500)*exp(-1250/x/ln(2*x))**2+(-2500*ln(2*x)-2500)*exp(-1250/x/ln(2*
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$-2e^{-\frac{1250}{x \log(2x)}} + e^{-\frac{2500}{x \log(2x)}}$$

#### 44.71 Problem number 5630

$$\int \frac{5^{\frac{5}{(x+x^2 \log(5)) \log(x)}} e^{-4+5^{\frac{5}{(x+x^2 \log(5)) \log(x)}}} (-5 \log(5) - 5x \log^2(5) + (-5 \log(5) - 10x \log^2(5)) \log(x))}{(x^2 + 2x^3 \log(5) + x^4 \log^2(5)) \log^2(x)} dx$$

Optimal antiderivative

$$e^{-4+e^{\frac{5}{\left(x^2 + \frac{x}{\ln(5)}\right) \ln(x)}}$$

command

```
integrate((((-10*x*ln(5)**2-5*ln(5))*ln(x)-5*x*ln(5)**2-5*ln(5))*exp(5*ln(5)/(x**2*ln(5)+x)/ln(4)/(x**4*ln(5)**2+2*x**3*ln(5)+x**2)/ln(x)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$e^{e^{\frac{5 \log(5)}{(x^2 \log(5) + x) \log(x)} - 4}}$$

## 44.72 Problem number 5647

$$\int \frac{(-30 - 12x^2 - 6x^3 + 6x^4) \log^2\left(\frac{5+2x^2+x^3-x^4}{x^2}\right) + \log\left(\frac{2}{x}\right) \left( (60 - 6x^3 + 12x^4) \log\left(\frac{5+2x^2+x^3-x^4}{x^2}\right) + (30 + 12x^2 + (-5x^3 - 2x^5 - x^6 + x^7) \log^3\left(\frac{2}{x}\right) \right)}{(-5x^3 - 2x^5 - x^6 + x^7) \log^3\left(\frac{2}{x}\right)}$$

Optimal antiderivative

$$\frac{3 \ln\left(2 + x + \frac{5}{x^2} - x^2\right)^2}{\ln\left(\frac{2}{x}\right)^2 x^2}$$

command

```
integrate(((((-6*x**4+6*x**3+12*x**2+30)*ln((-x**4+x**3+2*x**2+5)/x**2)**2+(12*x**4-6*x**3+60)*ln((-x**4+x**3+2*x**2+5)/x**2))*ln(2/x)+(6*x**4-6*x**3-12*x**2-30)*ln((-x**4+x**3+2*x**2+5)/x**2)**2)/(x**7-x**6-2*x**5-5*x**3)/ln(2/x)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{3 \log\left(\frac{-x^4+x^3+2x^2+5}{x^2}\right)^2}{x^2 \log\left(\frac{2}{x}\right)^2}$$

#### 44.73 Problem number 5655

$$\int \frac{e^{\frac{1}{3}(-4-15x)} \left( -75 \log^2(x) + e^{\frac{2}{3} \left( -1 + e^{\frac{x}{\log(x)}} \right)} \left( -75 \log^2(x) + e^{\frac{x}{\log(x)}} (-10 + 10 \log(x)) \right) \right)}{\log^2(x)} dx$$

Optimal antiderivative

$$15 \left( e^{\frac{2e^{\frac{x}{\ln(x)}}}{3} - \frac{2}{3}} + 1 \right) e^{-5x - \frac{4}{3}}$$

command

```
integrate((((10*ln(x)-10)*exp(x/ln(x))-75*ln(x)**2)*exp(1/3*exp(x/ln(x))-1/3)**2-75*ln(x)**2)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$15e^{-5x - \frac{4}{3}} e^{\frac{2e^{\frac{x}{\log(x)}}}{3} - \frac{2}{3}} + 15e^{-5x - \frac{4}{3}}$$

#### 44.74 Problem number 5656

$$\int \frac{(-250x^2 + 50x^3) \log(x) \log(3+x) + (375x + 50x^2 - 25x^3 + (-750x - 250x^2) \log(x)) \log^2(3+x)}{(-54000 + 14400x + 4320x^2 - 1728x^3 + 144x^4) \log^2(x)} dx$$

Optimal antiderivative

$$\frac{25x^2 \ln(3+x)^2}{144(5-x)^2 \ln(x)}$$

command

```
integrate(((((-250*x**2-750*x)*ln(x)-25*x**3+50*x**2+375*x)*ln(3+x)**2+(50*x**3-250*x**2)*ln(x)-1728*x**3+4320*x**2+14400*x-54000)/ln(x)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{25x^2 \log(x+3)^2}{144x^2 \log(x) - 1440x \log(x) + 3600 \log(x)}$$

## 44.75 Problem number 5688

$$\int \frac{e^{\frac{x}{4\log(x)}} \left( -e^{\frac{625-1000x+100x^2+440x^3-71x^4-88x^5+4x^6+8x^7+x^8}{x^4}} x^5 - x^6 - x^7 + \left( e^{\frac{625-1000x+100x^2+440x^3-71x^4-88x^5+4x^6+8x^7+x^8}{x^4}} x^5 + \right. \right.}{\left. \left. \right. \right.}$$

Optimal antiderivative

$$\left( x + e^{(2+x-\frac{5}{x})^4+3} + x^2 \right) e^{\frac{x}{4\ln(x)}}$$

command

```
integrate(1/4*((16*x**8+96*x**7+32*x**6-352*x**5-1760*x**3-800*x**2+12000*x-10000)*exp((x**8+88*x**5-71*x**4+440*x**3+100*x**2-1000*x+625)/x**4)+8*x**6+4*x**5)*ln(x)**2+(x**5*exp((x**8+88*x**5-71*x**4+440*x**3+100*x**2-1000*x+625)/x**4)+x**7+x**6)*ln(x)-x**5*exp((x**8+8*x**7+4*88*x**5-71*x**4+440*x**3+100*x**2-1000*x+625)/x**4)-x**7-x**6)*exp(1/4*x/ln(x))/x**5/ln(x)**2
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$(x^2 + x) e^{\frac{x}{4\log(x)}} + e^{\frac{x^8+8x^7+4x^6-88x^5-71x^4+440x^3+100x^2-1000x+625}{x^4}} e^{\frac{x}{4\log(x)}}$$

## 44.76 Problem number 5709

$$\int \frac{(3 + 12x^2) \log^2(x) + e^{\frac{4+4x \log(x)}{x \log(x)}} (-80x - 80x \log(x) + 60x^2 \log^2(x)) + e^{\frac{2(4+4x \log(x))}{x \log(x)}} (-200x - 200x \log(x) + 75x^2)}{\log^2(x)}$$

Optimal antiderivative

$$x \left( 3 + \left( 2 + 5 e^{\frac{4x^2 + \frac{4x}{\ln(x)}}{x^2}} \right)^2 x^2 \right)$$

command

```
integrate(((75*x**2*ln(x)**2-200*x*ln(x)-200*x)*exp((4*x*ln(x)+4)/x/ln(x))**2+(60*x**2*ln(x)*80*x*ln(x)-80*x)*exp((4*x*ln(x)+4)/x/ln(x))+(12*x**2+3)*ln(x)**2)/ln(x)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$25x^3 e^{\frac{2(4x \log(x)+4)}{x \log(x)}} + 20x^3 e^{\frac{4x \log(x)+4}{x \log(x)}} + 4x^3 + 3x$$

## 44.77 Problem number 5746

$$\int \frac{e^{\frac{x^3}{4+2x+\log(2+x)}} (16 + 16x + 28x^2 + 19x^3 + 4x^4 + (8 + 4x + 6x^2 + 3x^3) \log(2+x) + \log^2(2+x))}{16 + 16x + 4x^2 + (8 + 4x) \log(2+x) + \log^2(2+x)} dx$$

Optimal antiderivative

$$(2+x) e^{\frac{x^3}{\ln(2+x)+2x+4}}$$

command

```
integrate((ln(2+x)**2+(3*x**3+6*x**2+4*x+8)*ln(2+x)+4*x**4+19*x**3+28*x**2+16*x+16)*exp(x**3/
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$(x+2) e^{\frac{x^3}{2x+\log(x+2)+4}}$$

## 44.78 Problem number 5929

$$\int e^{-x^2} \left( -4e^{x^2} x^3 + e^{2e^{-x^2}(3e^{x^2}+x)} (-2e^{x^2} x - 2x^2 + 4x^4) + e^{e^{-x^2}(3e^{x^2}+x)} (6e^{x^2} x^2 + 2x^3 - 4x^5) \right) dx$$

Optimal antiderivative

$$6 - x^2 \left( e^{x e^{-x^2} + 3} - x \right)^2$$

command

```
integrate((( -2*exp(x**2)*x+4*x**4-2*x**2)*exp((3*exp(x**2)+x)/exp(x**2))**2+(6*x**2*exp(x**2)+4*x**5+2*x**3)*exp((3*exp(x**2)+x)/exp(x**2))-4*x**3*exp(x**2))/exp(x**2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$-x^4 + 2x^3 e^{(x+3e^{x^2})e^{-x^2}} - x^2 e^{2(x+3e^{x^2})e^{-x^2}}$$

## 44.79 Problem number 5942

$$\int \frac{-8x + 8x^2 + (8x - 16x^2) \log(x) + \frac{e^{-3+x^2}(-2x+2x^2+(6x-4x^2-8x^3)\log(x))}{x} + \left(16x - 16x \log(x) + \frac{e^{-3+x^2}(4x+(4-4x-8x^2)\log(x))}{x}\right)}{e^{-3+x^2} \log^2(x) + 4x \log^2(x)}$$

Optimal antiderivative

$$\frac{2x - 2 \left( \ln \left( \left( 4 + e^{-\ln(x)+x^2-3} \right) \ln(5) \right) + x \right)^2}{\ln(x)}$$

command

```
integrate(((2*exp(-ln(x)+x**2-3)+8)*ln(ln(5)*exp(-ln(x)+x**2-3)+4*ln(5))**2+((-8*x**2-4*x+4)*ln(x)+4*x)*exp(-ln(x)+x**2-3)-16*x*ln(x)+16*x)*ln(ln(5)*exp(-ln(x)+x**2-3)+4*ln(5))+((8*x**3-4*x**2+6*x)*ln(x)+2*x**2-2*x)*exp(-ln(x)+x**2-3)+(-16*x**2+8*x)*ln(x)+8*x**2-8*x)/(x*ln(x)**2*exp(-ln(x)+x**2-3)+4*x*ln(x)**2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$-\frac{4x \log \left( 4 \log(5) + \frac{e^{x^2-3} \log(5)}{x} \right)}{\log(x)} + \frac{-2x^2 + 2x}{\log(x)} - \frac{2 \log \left( 4 \log(5) + \frac{e^{x^2-3} \log(5)}{x} \right)^2}{\log(x)}$$

## 44.80 Problem number 5958

$$\int \frac{\left( 2e^x x^2 + e^{\frac{5e^{-x}}{x}} (-10 - 10x) \log(18) \right) \log \left( \frac{x + e^{\frac{5e^{-x}}{x}} \log(18)}{\log(18)} \right) + \left( e^x x^2 + e^{\frac{5e^{-x}}{x} + x} x \log(18) \right) \log^2 \left( \frac{x + e^{\frac{5e^{-x}}{x}} \log(18)}{\log(18)} \right)}{2e^x x^2 + 2e^{\frac{5e^{-x}}{x} + x} x \log(18)}$$

Optimal antiderivative

$$\frac{\ln \left( e^{\frac{5e^{-x}}{x}} + \frac{x}{\ln(18)} \right)^2}{2} x$$

command

```
integrate(((x*ln(18)*exp(x)*exp(5/exp(x)/x)+exp(x)*x**2)*ln((ln(18)*exp(5/exp(x)/x)+x)/ln(18)))+(2*x**2-2*x)*exp(x)*x**2)*ln((ln(18)*exp(5/exp(x)/x)+x)/ln(18)))/(2*x*ln(18)*exp(5/exp(x)/x)+2*x*ln(18))
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{x \log \left( \frac{x + e^{\frac{5e^{-x}}{\log(18)}} \log(18)}{\log(18)} \right)^2}{2}$$

#### 44.81 Problem number 5964

$$\int \frac{e(-30x^4 - 5x^5) + (x^3 + 30x^5 + 5x^6) \log^2(x) + (e(60x^2 + 10x^3) + (-x - 60x^3 - 10x^4) \log^2(x)) \log(2x) + (e(-30x^5 + 5x^6) \log^2(x) + (-60x^3 - 10x^4) \log^2(x) \log(2x) +$$

Optimal antiderivative

$$\frac{e}{\ln(x)} + \frac{\ln(6+x)}{5x^2 - 5\ln(2x)} + x$$

command

```
integrate((((5*x**2+30*x)*ln(x)**2+(-5*x-30)*exp(1))*ln(2*x)**2+((-10*x**4-60*x**3-x)*ln(x)**2+(10*x**3+60*x**2)*exp(1))*ln(2*x)+(-2*x**3-12*x**2+x+6)*ln(x)**2*ln(6+x)+(5*x**6+5*x**5-30*x**4)*exp(1))/((5*x**2+30*x)*ln(x)**2*ln(2*x)**2+(-10*x**4-60*x**3)*ln(x)**2*ln(2*x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$x + \frac{e}{\log(x)} + \frac{\log(x+6)}{5x^2 - 5\log(x) - 5\log(2)}$$

#### 44.82 Problem number 6025

$$\int \frac{\left(\frac{16}{e^{25}} + x\right) \log\left(\frac{16}{e^{25}} + x\right) + \left(\frac{16x}{e^{25}} + x^2\right) \log(x) \log^2\left(\frac{16}{e^{25}} + x\right) + \left(-x \log(x) + \left(-\frac{16}{e^{25}} - x\right) \log(x) \log\left(\frac{16}{e^{25}} + x\right)\right) \log\left(\frac{16}{e^{25}} + x\right)}{\left(\frac{16x^2}{e^{25}} + x^3\right) \log(x) \log^2\left(\frac{16}{e^{25}} + x\right)}$$

Optimal antiderivative

$$\ln\left(x e^{\frac{\ln(\ln(x))}{x \ln\left(e^{4 \ln(2)} - 25 + x\right)}}\right)$$

command

```
integrate((((-exp(4*ln(2)-25)-x)*ln(x)*ln(exp(4*ln(2)-25)+x)-x*ln(x))*ln(ln(x))+(x*exp(4*ln(2)-25)+x**2)*ln(x)*ln(exp(4*ln(2)-25)+x)**2+(exp(4*ln(2)-25)+x)*ln(exp(4*ln(2)-25)+x))/(x**2*exp(25)+x**3)/ln(x)/ln(exp(4*ln(2)-25)+x)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\log(x) + \frac{\log(\log(x))}{x \log\left(x + \frac{16}{e^{25}}\right)}$$

#### 44.83 Problem number 6249

$$\int \frac{(8x^2 - 50x^3 - 2x^4 - 16x^5 + 100x^6 + 4x^7) \log^2\left(\frac{1}{x}\right) + (8 - 50x - 2x^2) \log(4 - 25x - x^2) + \log\left(\frac{1}{x}\right) (-50x - 4x^2)}{(-4x^2 + 25x^3 + x^4) \log^2\left(\frac{1}{x}\right)} dx$$

Optimal antiderivative

$$x^4 - 2x - \frac{2 \ln(4 - x(x + 25))}{x \ln\left(\frac{1}{x}\right)}$$

command

```
integrate((((4*x**7+100*x**6-16*x**5-2*x**4-50*x**3+8*x**2)*ln(1/x)**2+((2*x**2+50*x-8)*ln(-x**2-25*x+4)-4*x**2-50*x)*ln(1/x)+(-2*x**2-50*x+8)*ln(-x**2-25*x+4))/(x**4+25*x**3-4*x**2)/ln(1/x)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$x^4 - 2x - \frac{2 \log(-x^2 - 25x + 4)}{x \log\left(\frac{1}{x}\right)}$$



## 44.84 Problem number 6406

$$\int \frac{\log(x) \log^2(\log(x)) + e^{\frac{3x}{\log(\log(x))}} (-3 + 3 \log(x) \log(\log(x)))}{\log(x) \log^2(\log(x))} dx$$

Optimal antiderivative

$$e^{\frac{3x}{\ln(\ln(x))}} + x$$

command

```
integrate(((3*ln(x)*ln(ln(x))-3)*exp(3/2*x/ln(ln(x)))**2+ln(x)*ln(ln(x))**2)/ln(x)/ln(ln(x))*
SymPy 1.10.1 under Python 3.10.4 output
```

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$x + e^{\frac{3x}{\log(\log(x))}}$$

## 44.85 Problem number 6477

$$\int \frac{2x \log(x) + (-216x^3 + 216x^4 - 72x^5 + 8x^6) \log^3(x) + (6 - 2x + (6 - 4x) \log(x)) \log(9 - 6x + x^2)}{(-27x^3 + 27x^4 - 9x^5 + x^6) \log^3(x)} dx$$

Optimal antiderivative

$$\frac{\ln\left((-3 + x)^2\right)}{\ln(x)^2 x^2 (-3 + x)^2} + 8x$$

command

```
integrate((((6-4*x)*ln(x)+6-2*x)*ln(x**2-6*x+9)+(8*x**6-72*x**5+216*x**4-216*x**3)*ln(x)**3+2
9*x**5+27*x**4-27*x**3)/ln(x)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$8x + \frac{\log(x^2 - 6x + 9)}{x^4 \log(x)^2 - 6x^3 \log(x)^2 + 9x^2 \log(x)^2}$$

#### 44.86 Problem number 6591

$$\int \frac{(16x - 16x^2 + (96x - 48x^2 + (-32x + 16x^2) \log(x)) \log(-3 + \log(x))) \log^3\left(-\frac{x^2 \log(3)}{(-2+2x) \log(-3+\log(x))}\right) + (24x - \dots)}{(3 - 3x + (-1 + x) \log(x)) \log(-3 + \log(x))}$$

Optimal antiderivative

$$4 \ln\left(\frac{x^2 \ln(3)}{2(1-x) \ln(\ln(x) - 3)}\right)^4 x^2$$

command

```
integrate((((8*x**2-8*x)*ln(x)-24*x**2+24*x)*ln(ln(x)-3)*ln(-x**2*ln(3)/(-2+2*x)/ln(ln(x)-3))**4+(((16*x**2-32*x)*ln(x)-48*x**2+96*x)*ln(ln(x)-3)-16*x**2+16*x)*ln(-x**2*ln(3)/(-2+2*x)/ln(ln(x)-3))**3)/((-1+x)*ln(x)-3*x+3)/ln(ln(x)-3), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$4x^2 \log\left(-\frac{x^2 \log(3)}{(2x-2) \log(\log(x)-3)}\right)^4$$

#### 44.87 Problem number 6634

$$\int e^{\frac{-8+x-2 \log(x)+10x \log(4-2x-\log(5))}{2x \log(4-2x-\log(5))}} \frac{(8x - x^2 + (-12 + 6x + 3 \log(5)) \log(4 - 2x - \log(5)) + \log(x)(2x + (-4 + 2x + \dots))}{(-4x^2 + 2x^3 + x^2 \log(5)) \log^2(4 - 2x - \log(5))}$$

Optimal antiderivative

$$e^{4 + \frac{\frac{x}{2} - \ln(x) - 4}{\ln(-\ln(5) + 4 - 2x)} + x}$$

command

```
integrate((((ln(5)+2*x-4)*ln(-ln(5)+4-2*x)+2*x)*ln(x)+(3*ln(5)+6*x-12)*ln(-ln(5)+4-2*x)-x**2+8*x)*exp(1/2*(-2*ln(x)+10*x*ln(-ln(5)+4-2*x)-8+x)/x/ln(-ln(5)+4-2*x))/(x**2*ln(5)+24*x**2)/ln(-ln(5)+4-2*x)**2, x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$e^{\frac{5x \log(-2x - \log(5) + 4) + \frac{x}{2} - \log(x) - 4}{x \log(-2x - \log(5) + 4)}}$$

#### 44.88 Problem number 6685

$$\int \frac{2496x \log(1-2x) + (240-480x) \log^2(1-2x) + (-768x \log(1-2x) + (-192+384x) \log^2(1-2x)) \log(x^2)}{-169x^2 + 338x^3 + (104x^2 - 208x^3) \log(x^2) + (-16x^2 + 32x^3) \log^2(x^2)} dx$$

Optimal antiderivative

$$\frac{16 \ln(1-2x)^2}{x \left( \frac{16}{3} - \frac{4 \ln(x^2)}{3} \right) - x}$$

command

```
integrate((((384*x-192)*ln(1-2*x)**2-768*x*ln(1-2*x))*ln(x**2)+(-480*x+240)*ln(1-2*x)**2+2496*2*x))/((32*x**3-16*x**2)*ln(x**2)**2+(-208*x**3+104*x**2)*ln(x**2)+338*x**3-169*x**2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{48 \log(1-2x)^2}{4x \log(x^2) - 13x}$$

#### 44.89 Problem number 6772

$$\int \frac{(-3+x) \log(3-x) + \frac{e^{e^5} + \frac{e^{e^5}(e^3+x)}{\log(3-x)}}{e^{e^5} + \frac{e^{e^5}(e^3+x)}{\log(3-x)}} (-e^3x - x^2 + (-3x+x^2) \log(3-x))}{(-3x+x^2) \log(3-x)} dx$$

Optimal antiderivative

$$2 + e^{(e^3+x)e^{-\ln(\ln(3-x))+e^5}} + \ln(x)$$

command

```
integrate((((x**2-3*x)*ln(3-x)-x*exp(3)-x**2)*exp(-ln(ln(3-x))+exp(exp(5)))*exp((exp(3)+x)*exp(ln(ln(3-x))+exp(exp(5)))))+(-3+x)*ln(3-x))/(x**2-3*x)/ln(3-x),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$e^{\frac{(x+e^3)e^{e^5}}{\log(3-x)}} + \log(x)$$

#### 44.90 Problem number 6869

$$\int \frac{e^5(-10 - 2x + 10x^2 + 2x^3) + e^{10}(10 - 8x - 2x^2) \log^2(5 + x) + \log\left(\frac{3e^x}{x}\right) (e^5(-10x - 2x^2) + 4e^{10}x \log(5 + x))}{5x + 11x^2 + 7x^3 + x^4 + e^5(-10x - 12x^2 - 2x^3) \log^2(5 + x) + e^{10}(5x + x^2) \log^4(5 + x)}$$

Optimal antiderivative

$$\frac{2 \ln\left(\frac{e^{\ln(3)+x}}{x}\right)}{(1+x)e^{-5} - \ln(5+x)^2}$$

command

```
integrate((((4*x*exp(5)**2*ln(5+x)+(-2*x**2-10*x)*exp(5))*ln(exp(ln(3)+x)/x)+(-2*x**2-8*x+10)*exp(5)**2*ln(5+x)**2+(2*x**3+10*x**2-2*x-10)*exp(5))/((x**2+5*x)*exp(5)**2*ln(5+x)**4+2*x**3-12*x**2-10*x)*exp(5)*ln(5+x)**2+x**4+7*x**3+11*x**2+5*x), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{2e^5 \log\left(\frac{3e^x}{x}\right)}{x - e^5 \log(x+5)^2 + 1}$$

#### 44.91 Problem number 7164

$$\int \frac{(-20x + e^{3+2x}(16x + 16x^2 + 4x^3)) \log(x) \log\left(\frac{-5x + e^{3+2x}(2+x)}{2+x}\right) + (20x + 10x^2 + e^{3+2x}(-8 - 8x - 2x^2) + (-10x - 5x^2 + e^{3+2x}(4 + 4x + x^2)) \log^3(x))}{(-10x - 5x^2 + e^{3+2x}(4 + 4x + x^2)) \log^3(x)}$$

Optimal antiderivative

$$\frac{x \ln\left(e^{3+2x} - \frac{5x}{2+x}\right)^2}{\ln(x)^2}$$

command

```
integrate((((((x**2+4*x+4)*exp(3+2*x)-5*x**2-10*x)*ln(x)+(-2*x**2-8*x-8)*exp(3+2*x)+10*x**2+20*5*x)/(2+x))**2+((4*x**3+16*x**2+16*x)*exp(3+2*x)-20*x)*ln(x)*ln(((2+x)*exp(3+2*x)-5*x)/(2+x)))/((x**2+4*x+4)*exp(3+2*x)-5*x**2-10*x)/ln(x)**3, x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{x \log\left(\frac{-5x+(x+2)e^{2x+3}}{x+2}\right)^2}{\log(x)^2}$$

#### 44.92 Problem number 7254

$$\int \frac{(7x + 14e^{2x}x) \log\left(-\frac{10}{x}\right) + (-7e^{2x} - 7x + (-7e^{2x}x - 7x^2) \log\left(-\frac{10}{x}\right)) \log(e^{2x} + x)}{3e^{3x}x + 3e^xx^2} dx$$

Optimal antiderivative

$$\frac{7 \ln(e^{2x} + x) \ln\left(-\frac{10}{x}\right) e^{-x}}{3}$$

command

```
integrate(((((-7*x*exp(x)**2-7*x**2)*ln(-10/x)-7*exp(x)**2-7*x)*ln(exp(x)**2+x)+(14*x*exp(x)**2+10/x))/(3*x*exp(x)**3+3*exp(x)*x**2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$\frac{7e^{-x} \log\left(-\frac{10}{x}\right) \log(x + e^{2x})}{3}$$

#### 44.93 Problem number 7265

$$\int \frac{9x + 12x^2 + 18x^3 - 6x^4 - 6x^2 \log(x) + (24x^2 - 12x^4 - 12x^2 \log(x) + (-36x + 18x^3 + 18x \log(x)) \log(-2 + x^2))}{4x - 2x^3 - 2x \log(x) + (-6 + 3x^2 + 3 \log(x)) \log(-2 + x^2 + \log(x))} dx$$

Optimal antiderivative

$$3x^2 \ln(3 \ln(\ln(x) + x^2 - 2) - 2x)$$

command

```
integrate((((18*x*ln(x)+18*x**3-36*x)*ln(ln(x)+x**2-2)-12*x**2*ln(x)-12*x**4+24*x**2)*ln(3*ln(2)-2*x)-6*x**2*ln(x)-6*x**4+18*x**3+12*x**2+9*x)/((3*ln(x)+3*x**2-6)*ln(ln(x)+x**2-2)-2*x*ln(x)-2*x**3+4*x),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$3x^2 \log(-2x + 3 \log(x^2 + \log(x) - 2))$$

#### 44.94 Problem number 7310

$$\int \frac{-108 + 144x^3 + 2x^6 + e^{-1+x}(-54 + 72x^3 + x^6) + (e^{-1+x}(9x - 24x^4 + 16x^7) + e^{-1+x}x^7 \log(x)) \log\left(\frac{9-24x^3+16x^6}{9x-24x^4+16x^7+x^7 \log(x)}\right)}{9x - 24x^4 + 16x^7 + x^7 \log(x)}$$

Optimal antiderivative

$$\ln\left(\ln(x) + \left(4 - \frac{3}{x^3}\right)^2\right) (2 + e^{-1+x})$$

command

```
integrate(((x**7*exp(-1+x)*ln(x)+(16*x**7-24*x**4+9*x)*exp(-1+x))*ln((x**6*ln(x)+16*x**6-24*x**3+9)/x**6)+(x**6+72*x**3-54)*exp(-1+x)+2*x**6+144*x**3-108)/(x**7*ln(x)+16*x**7-24*x**4+9*x), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$e^{x-1} \log\left(\frac{x^6 \log(x) + 16x^6 - 24x^3 + 9}{x^6}\right) + 2 \log\left(\log(x) + \frac{16x^6 - 24x^3 + 9}{x^6}\right)$$

#### 44.95 Problem number 7313

$$\int e^{\frac{20+5x+x \log(x)+5 \log^2(x)}{4+x+\log^2(x)}} \frac{(-128 - 56x - 6x^2 + 8x \log(x) + (-64 - 18x) \log^2(x) + 2x \log^3(x) - 8 \log^4(x))}{16x^5 + 8x^6 + x^7 + (8x^5 + 2x^6) \log^2(x) + x^5 \log^4(x)} dx$$

Optimal antiderivative

$$\frac{2e^{\frac{x}{4+x+\ln(x)}+5}}{x^4}$$

command

```
integrate((-8*ln(x)**4+2*x*ln(x)**3+(-18*x-64)*ln(x)**2+8*x*ln(x)-6*x**2-56*x-128)*exp((5*ln(x)+5)/x), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{2e^{\frac{x \log(x)+5x+5 \log(x)^2+20}{x+\log(x)^2+4}}}{x^4}$$

## 44.96 Problem number 7350

$$\int \frac{((-320 - 16x - 1280x^2 - 64x^3) \log\left(\frac{20+x}{4}\right) + (1280x^2 + 64x^3) \log(x^2) \log\left(\frac{20+x}{4}\right)) \log\left(\frac{16+64x^2}{\log(x^2)}\right) + (-4x - 16x^2)}{(20x + x^2 + 80x^3 + 4x^4) \log(x^2) \log^2\left(\frac{20+x}{4}\right)} dx$$

Optimal antiderivative

$$\frac{4 \ln\left(\frac{64x^2+16}{\ln(x^2)}\right)^2}{\ln\left(5 + \frac{x}{4}\right)}$$

command

```
integrate((( -16*x**3-4*x)*ln(x**2)*ln((64*x**2+16)/ln(x**2))**2+((64*x**3+1280*x**2)*ln(5+1/4*x)-64*x**3-1280*x**2-16*x-320)*ln(5+1/4*x))*ln((64*x**2+16)/ln(x**2)))/(4*x**4+80*x**3+x**2+20*x
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{4 \log\left(\frac{64x^2+16}{\log(x^2)}\right)^2}{\log\left(\frac{x}{4} + 5\right)}$$

## 44.97 Problem number 7422

$$\int e^{\frac{12}{(-15+20x) \log(x \log^2(2))}} \frac{(72 - 96x - 96x \log(x \log^2(2)))}{(45x - 120x^2 + 80x^3) \log^2(x \log^2(2))} dx$$

Optimal antiderivative

$$2e^{\frac{3}{5 \ln(x \ln(2)^2) \left(x - \frac{3}{4}\right)}} - 5$$

command

```
integrate((-96*x*ln(x*ln(2)**2)-96*x+72)*exp(12/(20*x-15)/ln(x*ln(2)**2))/(80*x**3-120*x**2+45*x)/ln(x*ln(2)**2)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$2e^{\frac{12}{(20x-15) \log(x \log(2)^2)}}$$

#### 44.98 Problem number 7508

$$\int \frac{(-8x + 12x^2 + 72x^3 + 32x^4) \log(x) + (64 - 216x - 148x^2 - 44x^3 - 16x^4 + (224x + 284x^2 + 60x^3 + 32x^4) \log(x))}{(8x + x^2 + x^3) \log^2(x)}$$

Optimal antiderivative

$$\frac{4(2+x) \ln(x^2+x+8) (-1+4x)}{\ln(x)}$$

command

```
integrate((((32*x**4+60*x**3+284*x**2+224*x)*ln(x)-16*x**4-44*x**3-148*x**2-216*x+64)*ln(x**2+8*x)*ln(x))/(x**3+x**2+8*x)/ln(x)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{(16x^2 + 28x - 8) \log(x^2 + x + 8)}{\log(x)}$$

#### 44.99 Problem number 7742

$$\int e^{\frac{x-x^3 \log(x)}{\log^2(x)}} \frac{(2 + (-1 - x^2) \log(x) + 3x^2 \log^2(x))}{\log^3(x)} dx$$

Optimal antiderivative

$$e^{4-e^4} + 9 - e^{\frac{\frac{x}{\ln(x)} - x^3}{\ln(x)}}$$

command

```
integrate((3*x**2*ln(x)**2+(-x**2-1)*ln(x)+2)*exp((-x**3*ln(x)+x)/ln(x)**2)/ln(x)**3,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$-e^{\frac{-x^3 \log(x)+x}{\log(x)^2}}$$



## 44.100 Problem number 7758

$$\int \frac{e^{\frac{-e+\log(3)}{-20-x+x\log(x)}} (e - \log(3)) \log(x)}{400 + 40x + x^2 + (-40x - 2x^2) \log(x) + x^2 \log^2(x)} dx$$

Optimal antiderivative

$$e^{\frac{-\ln(3)+e}{20+x-x\ln(x)}}$$

command

```
integrate((-ln(3)+exp(1))*ln(x)*exp((ln(3)-exp(1))/(x*ln(x)-x-20))/(x**2*ln(x)**2+(-2*x**2-40*x)*ln(x)+x**2+40*x+400), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$e^{\frac{-e+\log(3)}{x\log(x)-x-20}}$$

## 44.101 Problem number 7785

$$\int \frac{e^{-4x^2} \left( e^{4x^2} \log^2(x) + e^{\frac{e^{-4x^2}x}{\log(x)}} \left( 31 + 5x + (-31 - 5x + 248x^2 + 40x^3) \log(x) - 5e^{4x^2} \log^2(x) \right) \right)}{\log^2(x)} dx$$

Optimal antiderivative

$$x + e^{\frac{x e^{-4x^2}}{\ln(x)}} (-31 - 5x)$$

command

```
integrate((((-5*exp(4*x**2))*ln(x)**2+(40*x**3+248*x**2-5*x-31)*ln(x)+5*x+31)*exp(x/exp(4*x**2))
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: SystemError

Sympy 1.8 under Python 3.8.8 output

$$x + (-5x - 31) e^{\frac{x e^{-4x^2}}{\log(x)}}$$

## 44.102 Problem number 7935

$$\int \frac{e^2(2x + 2x^2) + e(52x + 4x^2) \log(x) + (50x + 2x^2) \log^2(x) + (e(96 + 48x) + e^2(2x + x^2) + e(4x + 2x^2) \log(x))}{e^2(2x + x^2) + e(4x + 2x^2) \log(x) + (2x + x^2) \log^2(x)}$$

Optimal antiderivative

$$x - \ln(2 + x) \left( -x - \frac{48 \ln(x)}{e + \ln(x)} \right)$$

command

```
integrate(((x**2+2*x)*ln(x)**2+(2*x**2+4*x)*exp(1)*ln(x)+(x**2+2*x)*exp(1)**2+(48*x+96)*exp(1)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$x + 48 \log(x + 2) + \frac{(x \log(x) + ex - 48e) \log(x + 2)}{\log(x) + e}$$

## 44.103 Problem number 8068

$$\int \frac{e^x(10x^3 - 10x^4 + e^3(-10x^2 + 10x^3)) + e^x(20e^3x^2 - 20x^3) \log(e^3 - 9)}{9x^5 - 18x^6 + 9x^7 + e^3(-9x^4 + 18x^5 - 9x^6) + e^x(-5x^5 + 10x^6 - 5x^7 + e^3(5x^4 - 10x^5 + 5x^6)) + (-36x^5 + 36x^6)}$$

Optimal antiderivative

$$\frac{\ln(5e^x - 9)}{x \left( \frac{(\ln(-x + e^3) + x)^2}{2} - \frac{x}{2} \right)}$$

command

```
integrate(((((-10*exp(3)+10*x)*exp(x)+18*exp(3)-18*x)*ln(-x+exp(3))**2+((-40*x*exp(3)+40*x**2-72*x**2-36*x)*ln(-x+exp(3))+((-30*x**2+20*x)*exp(3)+30*x**3)*exp(x)+(54*x**2-36*x)*exp(3)-54*x**3)*ln(5*exp(x)-9)+(10*x*exp(3)-10*x**2)*exp(x)*ln(-x+exp(3))**2+(20*x**2*exp(3)-20*x**3)*exp(x)*ln(-x+exp(3))+((10*x**3-10*x**2)*exp(3)-10*x**4+10*x**3)*exp(x))/((5*x**2*exp(3)+5*x**3)*exp(x)-9*x**2*exp(3)+9*x**3)*ln(-x+exp(3))**4+((20*x**3*exp(3)-20*x**4)*exp(x)-36*x**3*exp(3)+36*x**4)*ln(-x+exp(3))**3+(((30*x**4-10*x**3)*exp(3)-30*x**5+10*x**4)*exp(x)+(54*x**4+18*x**3)*exp(3)+54*x**5-18*x**4)*ln(-x+exp(3))**2+(((20*x**5-20*x**4)*exp(3)-20*x**6+20*x**5)*exp(x)+(-36*x**5+36*x**4)*exp(3)+36*x**6-36*x**5)*ln(-x+exp(3))+((5*x**6-10*x**5+5*x**4)*exp(3)-5*x**7+10*x**6-5*x**5)*exp(x)+(-9*x**6+18*x**5-9*x**4)*exp(3)+9*x**7-18*x**6+9*x**5),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{2 \log(5e^x - 9)}{x^3 + 2x^2 \log(-x + e^3) - x^2 + x \log(-x + e^3)^2}$$

## 44.104 Problem number 8168

$$\int \frac{(-6x + 3 \log(4)) \log\left(\frac{x}{2}\right) + (-2x^2 + x \log(4)) \log^2\left(\frac{x}{2}\right) + (3x - 3 \log(4) + (-x^2 + x \log(4)) \log^2\left(\frac{x}{2}\right)) \log(-x^2 + x \log(4))}{(-x^2 + x \log(4)) \log^2\left(\frac{x}{2}\right)}$$

Optimal antiderivative

$$\ln(x(2 \ln(2) - x)) \left( \frac{3}{\ln\left(\frac{x}{2}\right)} + x \right)$$

command

```
integrate((((2*x*ln(2)-x**2)*ln(1/2*x)**2-6*ln(2)+3*x)*ln(2*x*ln(2)-x**2)+(2*x*ln(2)-2*x**2)*ln(1/2*x)**2+(6*ln(2)-6*x)*ln(1/2*x))/(2*x*ln(2)-x**2)/ln(1/2*x)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{(x \log\left(\frac{x}{2}\right) + 3) \log(-x^2 + 2x \log(2))}{\log\left(\frac{x}{2}\right)}$$

## 44.105 Problem number 8246

$$\int \frac{(486x - 324x^2 - 108x^3 + 144x^4 - 42x^5 + 4x^6 + e^x(-162x + 216x^2 - 108x^3 + 24x^4 - 2x^5)) + (324 - 108x - 216x^2)}{(486x - 324x^2 - 108x^3 + 144x^4 - 42x^5 + 4x^6 + e^x(-162x + 216x^2 - 108x^3 + 24x^4 - 2x^5)) + (324 - 108x - 216x^2)}$$

Optimal antiderivative

$$(3 - x)^4 \ln\left(e^x + \ln(5) - (x + \ln(x))^2 - x\right)^2 e^{-2x}$$

command

```
integrate(((((-2*x**5+28*x**4-144*x**3+324*x**2-270*x)*ln(x)**2+(-4*x**6+56*x**5-288*x**4+648*540*x**2)*ln(x)+(2*x**5-28*x**4+144*x**3-324*x**2+270*x)*exp(x)+(2*x**5-28*x**4+144*x**3-324*x**2+270*x)*ln(5)-2*x**7+26*x**6-116*x**5+180*x**4+54*x**3-270*x**2)*ln(-ln(x)**2-2*x*ln(x)+exp(x)+ln(5)-x**2-x)**2+((4*x**5-44*x**4+168*x**3-216*x**2-108*x+324)*ln(x)+(-2*x**5+24*x**4-108*x**3+216*x**2-162*x)*exp(x)+4*x**6-42*x**5+144*x**4-108*x**3-324*x**2+486*ln(x)**2-2*x*ln(x)+exp(x)+ln(5)-x**2-x))/(x*exp(x)**2*ln(x)**2+2*x**2*exp(x)**2*ln(x)-x*exp(x)**3+(-x*ln(5)+x**3+x**2)*exp(x)**2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$(x^4 - 12x^3 + 54x^2 - 108x + 81) e^{-2x} \log\left(-x^2 - 2x \log(x) - x + e^x - \log(x)^2 + \log(5)\right)^2$$

## 44.106 Problem number 8247

$$\int \frac{(1 + 9x) \log(x) + (-22 - 9x - \log(x)) \log\left(\frac{1}{2}(22 + 9x + \log(x))\right)}{(22x + 9x^2) \log^2(x) + x \log^3(x)} dx$$

Optimal antiderivative

$$\frac{\ln\left(\frac{\ln(x)}{2} + \frac{9x}{2} + 11\right)}{\ln(x)} + 4$$

command

```
integrate((-ln(x)-9*x-22)*ln(1/2*ln(x)+9/2*x+11)+(9*x+1)*ln(x))/(x*ln(x)**3+(9*x**2+22*x)*ln
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{\log\left(\frac{9x}{2} + \frac{\log(x)}{2} + 11\right)}{\log(x)}$$

## 44.107 Problem number 8305

$$\int \frac{e^{\frac{9x}{6+e^3+\log\left(\frac{5}{3}\right)+3\log(x)}} + \frac{9x}{6+e^3+\log\left(\frac{5}{3}\right)+3\log(x)} (27 + 9e^3 + 9 \log\left(\frac{5}{3}\right) + 27 \log(x))}{36 + 12e^3 + e^6 - (-12 - 2e^3) \log\left(\frac{5}{3}\right) + \log^2\left(\frac{5}{3}\right) + (36 + 6e^3 + 6 \log\left(\frac{5}{3}\right)) \log(x) + 9 \log^2(x)} dx$$

Optimal antiderivative

$$e^{e^{\ln(x)+2-\frac{3x}{\ln\left(\frac{3}{5}\right)+\frac{e^3}{3}}}}$$

command

```
integrate((27*ln(x)-9*ln(3/5)+9*exp(3)+27)*exp(9*x/(3*ln(x)-ln(3/5)+exp(3)+6))*exp(exp(9*x/(3*ln(3/5)+exp(3)+6)))/(9*ln(x)**2+(-6*ln(3/5)+6*exp(3)+36)*ln(x)+ln(3/5)**2+(-2*exp(3)-12)*ln(3/5)+exp(3)**2+12*exp(3)+36), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$e^{\frac{9x}{3\log(x)-\log\left(\frac{3}{5}\right)+6+e^3}}$$

## 44.108 Problem number 8314

$$\int \frac{(-16x + 28x^2 - 8x^3) \log(x) + (8x - 4x^2 + (8x - 28x^2 + 12x^3) \log(x)) \log(e^{-2x}(-2x^2 + x^3)) + (-16x + 8x^2)}{(-2 + x) \log(x) \log(e^{-2x}(-2x^2 + x^3))} dx$$

Optimal antiderivative

$$4x^2 \left( x + \ln \left( \frac{\ln(x^2 e^{-2x}(-2 + x))}{x \ln(x)} \right) \right)$$

command

```
integrate(((8*x**2-16*x)*ln(x)*ln((x**3-2*x**2)/exp(x)**2)*ln(ln((x**3-2*x**2)/exp(x)**2)/x/ln(x)) + (8*x-4*x**2+8*x)*ln(x)-4*x**2+8*x)*ln((x**3-2*x**2)/exp(x)**2)+(-8*x**3+28*x**2-16*x)*ln(x))/(-2+x)/ln(x)/ln((x**3-2*x**2)/exp(x)**2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$4x^3 + \left(4x^2 - \frac{8}{3}\right) \log \left( \frac{\log((x^3 - 2x^2) e^{-2x})}{x \log(x)} \right) - \frac{8 \log(x)}{3} - \frac{8 \log(\log(x))}{3} + \frac{8 \log(\log((x^3 - 2x^2) e^{-2x}))}{3}$$

## 44.109 Problem number 8325

$$\int \frac{(600 - 160x - 136x^2 - 16x^3) \log^2(2x) + e^{\frac{x}{(40+8x) \log(2x)}} (-5x - x^2 + 5x \log(2x) + (200 + 80x + 8x^2) \log^2(2x))}{(200 + 80x + 8x^2) \log^2(2x)} dx$$

Optimal antiderivative

$$\left( 3 + e^{\frac{x}{8(5+x) \ln(2x)}} - x \right) x$$

command

```
integrate((((8*x**2+80*x+200)*ln(2*x)**2+5*x*ln(2*x)-x**2-5*x)*exp(x/(8*x+40)/ln(2*x)))+(-16*x**3-136*x**2-160*x+600)*ln(2*x)**2)/(8*x**2+80*x+200)/ln(2*x)**2, x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$-x^2 + x e^{\frac{x}{(8x+40) \log(2x)}} + 3x$$

## 44.110 Problem number 8348

$$\int e^{\frac{50}{x}+2x} (2400x^6 - 384x^7 - 96x^8) dx$$

Optimal antiderivative

$$-2 - 48x^8 e^{2x} e^{\frac{50}{x}}$$

command

```
integrate((-96*x**8-384*x**7+2400*x**6)*exp(25/x)**2*exp(x)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$-48x^8 e^{\frac{50}{x}} e^{2x}$$

## 44.111 Problem number 8392

$$\int \frac{-720 + 540x + 3e^{4+x}x^3 + (180x - 6e^{4+x}x^2) \log\left(\frac{x}{\log(5)}\right) + 3e^{4+x}x \log^2\left(\frac{x}{\log(5)}\right) + (-720x - 180x^2 + 3e^{4+x}x^2 + 240x - 60x^2 + e^{4+x}x^2 + (240 + 60x - 2e^{4+x}x^2))}{-240x - 60x^2 + e^{4+x}x^2 + (240 + 60x - 2e^{4+x}x^2)}$$

Optimal antiderivative

$$3 \ln\left(\frac{60 + 15x}{\frac{x}{4} - \frac{\ln\left(\frac{x}{\ln(5)}\right)}{4}} - e^{4+x}\right) x$$

command

```
integrate(((3*exp(4+x)*ln(x/ln(5))**2+(-6*x*exp(4+x)+180*x+720)*ln(x/ln(5))+3*x**2*exp(4+x)-180*x**2-720*x)*ln((-exp(4+x)*ln(x/ln(5))+x*exp(4+x)-60*x-240)/(ln(x/ln(5))-x))+3*x*exp(4+x)*6*x**2*exp(4+x)+180*x)*ln(x/ln(5))+3*x**3*exp(4+x)+540*x-720)/(exp(4+x)*ln(x/ln(5))**2+(-2*x*exp(4+x)+60*x+240)*ln(x/ln(5))+x**2*exp(4+x)-60*x**2-240*x),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$3x \log\left(\frac{xe^{x+4} - 60x - e^{x+4} \log\left(\frac{x}{\log(5)}\right) - 240}{-x + \log\left(\frac{x}{\log(5)}\right)}\right)$$

#### 44.112 Problem number 8435

$$\int \frac{75 + 90x + 3x^2 + (120x - 24x^2 + (75 - 30x + 3x^2) \log(\frac{x}{e})) \log\left(\frac{-8x + (-5+x) \log(\frac{x}{e})}{-5+x}\right) \log\left(-\log\left(\frac{-8x + (-5+x) \log(\frac{x}{e})}{-5+x}\right)\right)}{(40x - 8x^2 + (25 - 10x + x^2) \log(\frac{x}{e})) \log\left(\frac{-8x + (-5+x) \log(\frac{x}{e})}{-5+x}\right)}$$

Optimal antiderivative

$$3x \ln\left(-\ln\left(\ln(e^{-1}x) + \frac{8x}{5-x}\right)\right)$$

command

```
integrate((((3*x**2-30*x+75)*ln(x/exp(1))-24*x**2+120*x)*ln(((5+x)*ln(x/exp(1))-8*x)/(-5+x))*ln(-ln(((5+x)*ln(x/exp(1))-8*x)/(-5+x)))+3*x**2+90*x+75)/((x**2-10*x+25)*ln(x/exp(1))-8*x**2+40*x)/ln(((5+x)*ln(x/exp(1))-8*x)/(-5+x)), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\left(3x - \frac{5}{2}\right) \log\left(-\log\left(\frac{-8x + (x-5) \log(\frac{x}{e})}{x-5}\right)\right) + \frac{5 \log\left(\log\left(\frac{-8x + (x-5) \log(\frac{x}{e})}{x-5}\right)\right)}{2}$$

#### 44.113 Problem number 8519

$$\int \frac{4x - x^2 + e^x(-4x + x^2) + e^{e^x}(-x^2 + e^{2x}(-4x + x^2) + e^x(x + 4x^2 - x^3)) + (-2e^x + 2x) \log(\frac{x}{5}) + (e^x x - x^2)}{e^x x - x^2}$$

Optimal antiderivative

$$(-4 + x) (e^{e^x} + \ln(x - e^x)) - \ln\left(\frac{x}{5}\right)^2$$

command

```
integrate(((exp(x)*x-x**2)*ln(x-exp(x))+((x**2-4*x)*exp(x)**2+(-x**3+4*x**2+x)*exp(x)-x**2)*exp(exp(x))+(-2*exp(x)+2*x)*ln(1/5*x)+(x**2-4*x)*exp(x)-x**2+4*x)/(exp(x)*x-x**2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$x \log(x - e^x) + (x - 4) e^{e^x} - \log\left(\frac{x}{5}\right)^2 - 4 \log(-x + e^x)$$

## 44.114 Problem number 8523

$$\int \frac{(80 - 40x - 2x^2 + x^3) \log(-2 + x) + e^{3e^x+x}(-60x^2 + 30x^3) \log(-2 + x) + (-40x - 10e^{3e^x}x^2 - x^3) \log\left(\frac{40+10e^{3e^x}}{x}\right)}{e^{3e^x}(-20x^2 + 10x^3) \log^2(-2 + x) + (-80x + 40x^2 - 2x^3 + x^4) \log^2(-2 + x)}$$

Optimal antiderivative

$$\frac{\ln\left(10e^{3e^x} + \frac{40}{x} + x\right)}{\ln(-2 + x)} - 5$$

command

```
integrate((( -10*x**2*exp(3*exp(x)) - x**3 - 40*x) * ln((10*x*exp(3*exp(x)) + x**2 + 40)/x) + (30*x**3 - 60*x**2) * exp(x) * ln(-2+x) * exp(3*exp(x)) + (x**3 - 2*x**2 - 40*x + 80) * ln(-2+x)) / ((10*x**3 - 20*x**2) * ln(2+x)**2 * exp(3*exp(x)) + (x**4 - 2*x**3 + 40*x**2 - 80*x) * ln(-2+x)**2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{\log\left(\frac{x^2 + 10xe^{3e^x} + 40}{x}\right)}{\log(x - 2)}$$

## 44.115 Problem number 8610

$$\int \frac{36x^3 + 12x^4 + (-36x^3 - 9x^4) \log(x) + (-18 - 9x - x^2) \log^2(5) \log^3(x) + (-18x^3 - 6x^4 + (18x^3 + 6x^4) \log(x))}{(3x^2 + x^3) \log^2(5) \log^3(x)}$$

Optimal antiderivative

$$\frac{\left(\frac{3x^3}{\ln(5)^2 \ln(x)^2} - 3 - x\right) (\ln(3 + x) - 2)}{x}$$

command

```
integrate((((3*x+9)*ln(5)**2*ln(x)**3 + (6*x**4+18*x**3)*ln(x) - 6*x**4 - 18*x**3)*ln(3+x) + (-x**2 - 9*x - 18)*ln(5)**2*ln(x)**3 + (-9*x**4 - 36*x**3)*ln(x) + 12*x**4 + 36*x**3) / (x**3 + 3*x**2) / ln(5)**2)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$-\frac{6x^2}{\log(5)^2 \log(x)^2} - \log(x + 3) + \frac{(3x^3 - 3 \log(5)^2 \log(x)^2) \log(x + 3)}{x \log(5)^2 \log(x)^2} + \frac{6}{x}$$



## 44.116 Problem number 8617

$$\int \frac{(6x + 12x^2 + 4x^3) \log^2(3x) + e^{\frac{2(-x^2+x^3)}{\log(3x)}} (6x^2 + 2x^3 - 6x^4 - 2x^5 + (-12x^2 + 2x^3 + 20x^4 + 6x^5) \log(3x) + (3 + \log^2(3x))}{\log^2(3x)}$$

Optimal antiderivative

$$\left( x^2 + x e^{\frac{2x(x^2-x)}{\ln(3x)}} \right) \left( (2+x)^2 - 1 \right)$$

command

```
integrate((((3*x**2+8*x+3)*ln(3*x)**2+(6*x**5+20*x**4+2*x**3-12*x**2)*ln(3*x)-2*x**5-6*x**4+2*x**3+6*x**2)*exp((x**3-x**2)/ln(3*x))**2+(4*x**3+12*x**2+6*x)*ln(3*x)**2)/ln(3*x)**2
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$x^4 + 4x^3 + 3x^2 + (x^3 + 4x^2 + 3x) e^{\frac{2(x^3-x^2)}{\log(3x)}}$$

## 44.117 Problem number 8620

$$\int \frac{e^{-\frac{5}{x^4 \log(-3x+2x^5)}} (-45 + 150x^4 + (-180 + 120x^4) \log(-3x + 2x^5))}{(-3x^5 + 2x^9) \log^2(-3x + 2x^5)} dx$$

Optimal antiderivative

$$3e^{-\frac{5}{x^4 \ln(2x(x^4-2)+x)}}$$

command

```
integrate(((120*x**4-180)*ln(2*x**5-3*x)+150*x**4-45)/(2*x**9-3*x**5)/ln(2*x**5-3*x)**2/exp(5*3*x)),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$3e^{-\frac{5}{x^4 \log(2x^5-3x)}}$$

## 44.118 Problem number 8793

$$\int \frac{x \log(4) + \log^2(4) + e^x(-x^2 \log(4) - x \log^2(4)) + (-x \log(4) + \log^2(4) + e^x(x^3 \log(4) + x^2 \log^2(4))) \log\left(\frac{x}{4}\right) + (x^3 + x^2 \log(4)) \log^2\left(\frac{x}{4}\right)}{(x^3 + x^2 \log(4)) \log^2\left(\frac{x}{4}\right)}$$

Optimal antiderivative

$$\frac{2 \ln(2) \left( e^x - \frac{\ln((x+2\ln(2))^2)+1}{x} \right)}{\ln\left(\frac{x}{4}\right)}$$

command

```
integrate((((4*ln(2)**2+2*x*ln(2))*ln(1/4*x)+4*ln(2)**2+2*x*ln(2))*ln(4*ln(2)**2+4*x*ln(2)+x*2*x*ln(2))*ln(1/4*x)+(-4*x*ln(2)**2-2*x**2*ln(2))*exp(x)+4*ln(2)**2+2*x*ln(2))/(2*x**2*ln(2)+
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{2e^x \log(2)}{\log\left(\frac{x}{4}\right)} - \frac{2 \log(2) \log\left(x^2 + 4x \log(2) + 4 \log(2)^2\right)}{x \log\left(\frac{x}{4}\right)} - \frac{2 \log(2)}{x \log\left(\frac{x}{4}\right)}$$

## 44.119 Problem number 8800

$$\int \frac{-4 \log(5) \log^2\left(\frac{16}{x^2}\right) + e^{\frac{x}{\log\left(\frac{16}{x^2}\right)}} (8x - 8x^2 + (4x - 4x^2) \log\left(\frac{16}{x^2}\right) - 4 \log^2\left(\frac{16}{x^2}\right))}{3x^2 \log^2\left(\frac{16}{x^2}\right)} dx$$

Optimal antiderivative

$$\frac{\left( e^{\frac{x}{\ln\left(\frac{16}{x^2}\right)}} + \ln(5) \right) (4 - 4x)}{3x}$$

command

```
integrate(1/3*((-4*ln(16/x**2)**2+(-4*x**2+4*x)*ln(16/x**2)-8*x**2+8*x)*exp(x/ln(16/x**2))-4*ln(5)*ln(16/x**2)**2)/x**2/ln(16/x**2)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{(4 - 4x) e^{\frac{x}{\log\left(\frac{16}{x^2}\right)}}}{3x} + \frac{4 \log(5)}{3x}$$

## 44.120 Problem number 8869

$$\int \frac{e^{-\frac{1}{400x^5 + (2000x^4 + 400x^5) \log(x)}} (-5 + 4x + (-20 - 5x) \log(x))}{200x^7 + (-2000x^6 - 400x^7) \log(x) + (5000x^5 + 2000x^6 + 200x^7) \log^2(x)} dx$$

Optimal antiderivative

$$2e^{-\frac{1}{400x^4(x+(-x-5)\ln(x))}}$$

command

```
integrate((( -5*x-20)*ln(x)+4*x-5)/((200*x**7+2000*x**6+5000*x**5)*ln(x)**2+(-400*x**7-2000*x**6)*ln(x)+200*x**7)/exp(-1/((400*x**5+2000*x**4)*ln(x)-400*x**5)), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$2e^{-\frac{1}{400x^5 + (400x^5 + 2000x^4) \log(x)}}$$

## 44.121 Problem number 8893

$$\int \frac{4e^5 x^2 \log^2(x) + e^{\frac{3}{e^5 x^2 \log(x)}} \log(2 + e^2) (6 + 12 \log(x) - 4e^5 x^2 \log^2(x)) + e^{\frac{6}{e^5 x^2 \log(x)}} \log^2(2 + e^2) (-3 - 6 \log(x))}{2e^5 x \log^2(2 + e^2) \log^2(x)}$$

Optimal antiderivative

$$\left( \frac{x}{\ln(e^2 + 2)} - \frac{x e^{\frac{3e^{-5}}{x^2 \ln(x)}}}{2} \right)^2$$

command

```
integrate(1/2*((x**2*exp(5)*ln(x)**2-6*ln(x)-3)*ln(exp(2)+2)**2*exp(3/x**2/exp(5)/ln(x))**2+(4*x**2*exp(5)*ln(x)**2+12*ln(x)+6)*ln(exp(2)+2)*exp(3/x**2/exp(5)/ln(x))+4*x**2*exp(5)*ln(x)*
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{x^2}{\log(2 + e^2)^2} + \frac{x^2 e^{\frac{6}{x^2 e^5 \log(x)}} \log(2 + e^2) - 4x^2 e^{\frac{3}{x^2 e^5 \log(x)}}}{4 \log(2 + e^2)}$$

## 44.122 Problem number 8900

$$\int \frac{e^{\frac{x}{2+\log(-18-2x+\log(4)+3\log(25))}} (-36 - 2x + 2\log(4) + 6\log(25) + (-18 - 2x + \log(4) + 3\log(25)))}{-72 - 8x + 4\log(4) + 12\log(25) + (-72 - 8x + 4\log(4) + 12\log(25))\log(-18 - 2x + \log(4) + 3\log(25)) + (-18 - 2x + \log(4) + 3\log(25))^2} dx$$

Optimal antiderivative

$$e^{\frac{x}{\ln(6\ln(5)+2\ln(2)-2x-18)+2}}$$

command

```
integrate(((6*ln(5)+2*ln(2)-2*x-18)*ln(6*ln(5)+2*ln(2)-2*x-18)+12*ln(5)+4*ln(2)-2*x-36)*exp(x/(ln(6*ln(5)+2*ln(2)-2*x-18)+2))/((6*ln(5)+2*ln(2)-2*x-18)*ln(6*ln(5)+2*ln(2)-2*x-18)**2+(24*ln(5)+8*ln(2)-8*x-72)*ln(6*ln(5)+2*ln(2)-2*x-18)+24*ln(5)+8*ln(2)-8*x-72),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$e^{\frac{x}{\log(-2x-18+2\log(2)+6\log(5))+2}}$$

## 44.123 Problem number 9001

$$\int \frac{e^{\frac{2x}{\log(x)}} (-2x^2 + 2x^2 \log(x) + 2x \log^2(x) + e^{-30+2x-2x\log(2x)} (-2 + 2\log(x) - 2\log^2(x)\log(2x)) + e^{-15+x-x\log(2x)})}{5\log^2(x)} dx$$

Optimal antiderivative

$$\frac{e^{\frac{2x}{\ln(x)}} (x - e^{-x\ln(2x)+x-15})^2}{5}$$

command

```
integrate(1/5*((-2*ln(x)**2*ln(2*x)+2*ln(x)-2)*exp(-x*ln(2*x)+x-15)**2+(2*x*ln(x)**2*ln(2*x)-2*ln(x)**2-4*x*ln(x)+4*x)*exp(-x*ln(2*x)+x-15)+2*x*ln(x)**2+2*x**2*ln(x)-2*x**2)*exp(x/ln(x))),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{x^2 e^{\frac{2x}{\log(x)}}}{5} - \frac{2x e^{\frac{2x}{\log(x)}} e^{-x(\log(x)+\log(2))+x-15}}{5} + \frac{e^{\frac{2x}{\log(x)}} e^{-2x(\log(x)+\log(2))+2x-30}}{5}$$

## 44.124 Problem number 9011

$$\int \frac{e^{\frac{3+e^{10}(-x^2-x^3)}{-e^{10}x^2+e^{10}x^2\log\left(\frac{x}{5\log(x)}\right)}} \left(3+e^{10}(-x^2-x^3) + (3+e^{10}(x^2+2x^3))\log(x) + (-6-e^{10}x^3)\log(x)\log\left(\frac{x}{5\log(x)}\right)\right)}{e^{10}x^3\log(x) - 2e^{10}x^3\log(x)\log\left(\frac{x}{5\log(x)}\right) + e^{10}x^3\log(x)\log^2\left(\frac{x}{5\log(x)}\right)} dx$$

Optimal antiderivative

$$e^{\frac{3e^{-10}-x-1}{x^2}} \frac{1}{\ln\left(\frac{x}{5\ln(x)}\right)^{-1}}$$

command

```
integrate((( -x**3*exp(5)**2-6)*ln(x)*ln(1/5*x/ln(x))+((2*x**3+x**2)*exp(5)**2+3)*ln(x)+(-x**3-x**2)*exp(5)**2+3)*exp((( -x**3-x**2)*exp(5)**2+3)/(x**2*exp(5)**2*ln(1/5*x/ln(x))-x**2*exp(5)**2)))/(x**3*exp(5)**2*ln(x)*ln(1/5*x/ln(x))**2-2*x**3*exp(5)**2*ln(x)*ln(1/5*x/ln(x)))
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$e^{\frac{(-x^3-x^2)e^{10}+3}{x^2e^{10}\log\left(\frac{x}{5\log(x)}\right)-x^2e^{10}}}$$

## 44.125 Problem number 9095

$$\int \frac{30+30x+e^4(10+25x+15x^2) + (3x+e^4(-15x-15x^2))\log(x) + e^4(15+15x)\log^2(x) + (-3-3x)\log(1+x)}{e^4(15x+15x^2)\log^2(x)} dx$$

Optimal antiderivative

$$\ln(x) + \frac{-\frac{2}{3} + \left(\frac{\ln(1+x)}{5} - 2\right)e^{-4} - x}{\ln(x)}$$

command

```
integrate((( -3*x-3)*ln(1+x)+(15*x+15)*exp(4)*ln(x)**2+((-15*x**2-15*x)*exp(4)+3*x)*ln(x)+(15*x+15)*exp(4)*log(1+x)))/(e^4*(15*x+15*x^2)*log(x)**2)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{-3xe^4 - 2e^4 - 6}{3e^4\log(x)} + \log(x) + \frac{\log(x+1)}{5e^4\log(x)}$$

#### 44.126 Problem number 9101

$$\int \frac{2x \log(x) + (-x + (x + x^2 - x^3) \log(x)) \log(2x^2) + (-2 \log(x) + (-1 + x^2) \log(x) \log(2x^2)) \log(\log(x)) + (2x - x^3 \log(5) \log(x) \log^2(2x^2) + x^2 \log(5) \log(x) \log^2(2x^2) \log(\log(x)))}{-x^3 \log(5) \log(x) \log^2(2x^2) + x^2 \log(5) \log(x) \log^2(2x^2) \log(\log(x))}$$

Optimal antiderivative

$$\frac{\ln\left(\frac{e^x}{\ln(\ln(x))-x}\right) + \frac{1}{x}}{\ln(2x^2) \ln(5)}$$

command

```
integrate((( -2*x*ln(x)*ln(ln(x))+2*x**2*ln(x))*ln(exp(x)/(ln(ln(x))-x))+((x**2-1)*ln(x)*ln(2*x*ln(x))*ln(ln(x)))+( (-x**3+x**2+x)*ln(x)-x)*ln(2*x**2)+2*x*ln(x))/(x**2*ln(5)*ln(x)*ln(2*x**2*x**3*ln(5)*ln(x)*ln(2*x**2)**2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{1}{2x \log(5) \log(x) + x \log(2) \log(5)} + \frac{\log\left(\frac{e^x}{-x + \log(\log(x))}\right)}{2 \log(5) \log(x) + \log(2) \log(5)}$$

#### 44.127 Problem number 9111

$$\int \frac{-36x + 36x^2 + (3x - 3x^2) \log(x) + (33x - 108x^2 + (-3x + 9x^2) \log(x)) \log(2x) + (-36x + 3x \log(x) + (72x - (-12 + \log(x)) \log^2(2x)))}{(-12 + \log(x)) \log^2(2x)}$$

Optimal antiderivative

$$\frac{3x^2 \left( x - 1 - \ln\left(\frac{\frac{\ln(x)}{4} - 3}{2x}\right) \right)}{\ln(2x)}$$

command

```
integrate(((( -6*x*ln(x)+72*x)*ln(2*x)+3*x*ln(x)-36*x)*ln(1/8*(ln(x)-12)/x))+((9*x**2-3*x)*ln(x)-108*x**2+33*x)*ln(2*x)+(-3*x**2+3*x)*ln(x)+36*x**2-36*x)/(ln(x)-12)/ln(2*x)**2, x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$-\frac{3x^2 \log\left(\frac{\frac{\log(x)}{8} - \frac{3}{2}}{x}\right)}{\log(x) + \log(2)} + \frac{3x^3 - 3x^2}{\log(x) + \log(2)}$$

#### 44.128 Problem number 9117

$$\int \frac{3 - x + e^x(-3 - 5x - x^2 + x^3) \log(16) + (-x + e^x(1 + x^2) \log(16)) \log(x + e^x(-1 - x^2) \log(16)) \log(\log(x + e^x(-1 - x^2) \log(16)))}{(-x + e^x(1 + x^2) \log(16)) \log(x + e^x(-1 - x^2) \log(16))}$$

Optimal antiderivative

$$(-3 + x) \ln(\ln(x - 4 \ln(2) (e^x x^2 + e^x)))$$

command

```
integrate(((4*(x**2+1)*ln(2)*exp(x)-x)*ln(4*(-x**2-1)*ln(2)*exp(x)+x)*ln(ln(4*(-x**2-1)*ln(2)*exp(x)+x))+4*(x**3-x**2-5*x-3)*ln(2)*exp(x)+3-x)/(4*(x**2+1)*ln(2)*exp(x)-x)/ln(4*(-x**2-1)*ln(2)*exp(x)+x), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$(x - 1) \log(\log(x + (-4x^2 - 4) e^x \log(2))) - 2 \log(\log(x + (-4x^2 - 4) e^x \log(2)))$$

#### 44.129 Problem number 9277

$$\int \frac{e^x x^3 + (324 + 324x + e^{4x}(324 + 324x)) \log(1 + e^{4x}) + 2e^x x^2 \log(x) + e^x x \log^2(x) + e^{4x}(-1296e^{4x}x^2 + e^x x^3)}{x^3 + 2x^2 \log(x) + x \log^2(x) + e^{4x}(x^3 + 2x^2 \log(x) + x \log^2(x))}$$

Optimal antiderivative

$$e^x - \frac{324 \ln(e^{4x} + 1)}{x + \ln(x)}$$

command

```
integrate((((324*x+324)*exp(exp(4*x))+324*x+324)*ln(exp(exp(4*x))+1)+(x*exp(x)*ln(x)**2+(-1296*x*exp(4*x)+2*exp(x)*x**2)*ln(x)-1296*x**2*exp(4*x)+exp(x)*x**3)*exp(exp(4*x))+x*exp(x)*ln(x)), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$e^x - \frac{324 \log(e^{4x} + 1)}{x + \log(x)}$$

## 44.130 Problem number 9310

$$\int \frac{(8 + 2x - 2x^2 + e(16x + 4x^2 - 4x^3)) \log(x) + (-4 + 8x + e(-4x + 8x^2)) \log^2(x) \log\left(\frac{x+ex^2}{e}\right) + (-8 - 2x + 2x^2) \log^3(x)}{(1 + ex) \log^2(x) \log\left(\frac{x+ex^2}{e}\right)}$$

Optimal antiderivative

$$\left( \frac{2 \ln(\ln(x^2 + e^{-1}x)) x}{\ln(x)} - 4 \right) (-x^2 + x + 4)$$

command

```
integrate(((((-6*x**3+4*x**2+8*x)*exp(1)-6*x**2+4*x+8)*ln(x)+(2*x**3-2*x**2-8*x)*exp(1)+2*x**2*x-8)*ln((x**2*exp(1)+x)/exp(1))*ln(ln((x**2*exp(1)+x)/exp(1)))+(8*x**2-4*x)*exp(1)+8*x-4)*ln(x)**2*ln((x**2*exp(1)+x)/exp(1))+((-4*x**3+4*x**2+16*x)*exp(1)-2*x**2+2*x+8)*ln(x))/(x**2+1)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$4x^2 - 4x + \frac{(-2x^3 + 2x^2 + 8x) \log\left(\log\left(\frac{ex^2+x}{e}\right)\right)}{\log(x)}$$

## 44.131 Problem number 9541

$$\int \frac{e^{\frac{9+123x+30x^2}{10 \log\left(\frac{2+x^2}{x}\right)}} \left(18 + 246x + 51x^2 - 123x^3 - 30x^4 + (246x + 120x^2 + 123x^3 + 60x^4) \log\left(\frac{2+x^2}{x}\right) + (20 + 10x^2) \log^2\left(\frac{2+x^2}{x}\right)\right)}{(20 + 10x^2) \log^2\left(\frac{2+x^2}{x}\right)}$$

Optimal antiderivative

$$x e^{\frac{3x+3(3+x)\left(\frac{1}{10}+x\right)}{\ln\left(x+\frac{2}{x}\right)}}$$

command

```
integrate((((10*x**2+20)*ln((x**2+2)/x)**2+(60*x**4+123*x**3+120*x**2+246*x)*ln((x**2+2)/x)-30*x**4-123*x**3+51*x**2+246*x+18)*exp(1/10*(30*x**2+123*x+9)/ln((x**2+2)/x))/(10*x**2+20)/ln((x**2+2)/x))
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$x e^{\frac{3x^2 + \frac{123x}{10} + \frac{9}{10}}{\log\left(\frac{x^2+2}{x}\right)}}$$



## 44.132 Problem number 9749

$$\int \frac{-50x - 20e^4x - 2e^8x + (-20x - 4e^4x) \log(x) - 2x \log^2(x) + e^{\frac{x^3+x^2 \log(4)}{5+e^4+\log(x)}} (25 + e^8 + 14x^3 + e^4(10 + 3x^3)) + (9x^3 + 25 + e^8 + 14x^3 + e^4(10 + 3x^3)) \log(x) + \log^2(x)}{25 + 10e^4 + e^8 + (10 + 2e^4) \log(x) + \log^2(x)}$$

Optimal antiderivative

$$x \left( e^{\frac{(x+2 \ln(2))x^2}{\ln(x)+5+e^4}} - x \right)$$

command

```
integrate(((ln(x)**2+(4*x**2*ln(2)+2*exp(4)+3*x**3+10)*ln(x)+2*(2*x**2*exp(4)+9*x**2)*ln(2)+2*x*ln(x)**2+(-4*x*exp(4)-20*x)*ln(x)-2*x*exp(4)**2-20*x*exp(4)-50*x)/(ln(x)**2+(2*exp(4)+10)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$-x^2 + x e^{\frac{x^3+2x^2 \log(2)}{\log(x)+5+e^4}}$$

## 44.133 Problem number 9775

$$\int \frac{(e^{10} - e^5x) \log^2(e^{10} - 2e^5x + x^2) + e^{\frac{4x^3}{e^5 \log(e^{10} - 2e^5x + x^2)}} (8x^3 + (12e^5x^2 - 12x^3) \log(e^{10} - 2e^5x + x^2))}{(e^{10} - e^5x) \log^2(e^{10} - 2e^5x + x^2)} dx$$

Optimal antiderivative

$$1 + e^{\frac{4x^3 e^{-5}}{\ln((e^5 - x)^2)}} + x$$

command

```
integrate((((12*x**2*exp(5)-12*x**3)*ln(exp(5)**2-2*x*exp(5)+x**2)+8*x**3)*exp(4*x**3/exp(5))/2*x*exp(5)+x**2)+(exp(5)**2-x*exp(5))*ln(exp(5)**2-2*x*exp(5)+x**2)**2)/(exp(5)**2-x*exp(5))/ln(exp(5)**2-2*x*exp(5)+x**2)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$x + e^{\frac{4x^3}{e^5 \log(x^2 - 2xe^5 + e^{10})}}$$

## 44.134 Problem number 9947

$$\int e^{\frac{3x^2+5x^3+2x^4}{(4+8x)\log(x^2)}} \frac{(-6x^2 - 22x^3 - 24x^4 - 8x^5 + (6x^2 + 21x^3 + 28x^4 + 12x^5)\log(x^2) + (-8 - 32x - 32x^2)\log^2(x^2))}{(4x^3 + 16x^4 + 16x^5)\log^2(x^2)} dx$$

Optimal antiderivative

$$\frac{e^{\frac{x\left(\frac{x}{1+2x} + x^2 + 2x\right)}{4\ln(x^2)}}}{x^2}$$

command

```
integrate((( -32*x**2-32*x-8)*ln(x**2)**2+(12*x**5+28*x**4+21*x**3+6*x**2)*ln(x**2)-
8*x**5-24*x**4-22*x**3-6*x**2)*exp((2*x**4+5*x**3+3*x**2)/(8*x+4)/ln(x**2))/(16*x**5+16*x**4+
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{e^{\frac{2x^4+5x^3+3x^2}{(8x+4)\log(x^2)}}}{x^2}$$

## 44.135 Problem number 10034

$$\int \frac{e^{9e^{\frac{2(-4x-x^2)}{6-3x+3\log(x)}}} + e^{\frac{2(-4x-x^2)}{6-3x+3\log(x)}}}{4-4x+x^2+(4-2x)\log(x)+\log^2(x)} dx$$

Optimal antiderivative

$$e^9 e^{\frac{2(4+x)x}{3x-3\ln(x)-6}}$$

command

```
integrate((( -12*x-24)*ln(x)+6*x**2-18*x-24)*exp((-x**2-4*x)/(3*ln(x)-3*x+6))**2*exp(9*exp((-
x**2-4*x)/(3*ln(x)-3*x+6))**2)/(ln(x)**2+(4-2*x)*ln(x)+x**2-4*x+4), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$e^9 e^{\frac{2(-x^2-4x)}{-3x+3\log(x)+6}}$$

## 44.136 Problem number 10123

$$\int \frac{7x^2 + (12 + 4x) \log^3(5x) \log(3 + x) + \log^4(5x)(-x + (-3 - x) \log(3 + x))}{147x^2 + 49x^3 + (-42x - 14x^2) \log^4(5x) + (3 + x) \log^8(5x)} dx$$

Optimal antiderivative

$$\frac{\ln(3 + x) x}{7x - \ln(5x)^4}$$

command

```
integrate(((((-3-x)*ln(3+x)-x)*ln(5*x)**4+(4*x+12)*ln(3+x)*ln(5*x)**3+7*x**2)/((3+x)*ln(5*x)**4*x**2-42*x)*ln(5*x)**4+49*x**3+147*x**2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{x \log(x + 3)}{7x - \log(5x)^4}$$

## 44.137 Problem number 10148

$$\int \frac{e^{-\frac{-5x+e^5x+(-5+e^5)\log(x)+\log\left(\frac{x}{\log(x)}\right)}{x+\log(x)}}}{e^{-\frac{-5x+e^5x+(-5+e^5)\log(x)+\log\left(\frac{x}{\log(x)}\right)}{x+\log(x)}}} \left( x^2 \log(x) + 2x \log^2(x) + \log^3(x) \right) + e^{-\frac{-5x+e^5x+(-5+e^5)\log(x)+\log\left(\frac{x}{\log(x)}\right)}{x+\log(x)}}}{e^{-\frac{-5x+e^5x+(-5+e^5)\log(x)+\log\left(\frac{x}{\log(x)}\right)}{x+\log(x)}}} \left( x^2 \log(x) + 2x \log^2(x) + \log^3(x) \right) + e^{-\frac{-5x+e^5x+(-5+e^5)\log(x)+\log\left(\frac{x}{\log(x)}\right)}{x+\log(x)}}}$$

Optimal antiderivative

$$e^{-2+x} e^{-\frac{\ln\left(\frac{x}{\ln(x)}\right)}{x+\ln(x)} - e^5 + 5} + x$$

command

```
integrate((((1+x)*ln(x)*ln(x/ln(x))+ln(x)**3+(-1+2*x)*ln(x)**2+(x**2-x+1)*ln(x)+x)*exp((-2*exp((ln(x/ln(x)))+(exp(5)-5)*ln(x)+x*exp(5)-5*x)/(x+ln(x)))+x)/exp((ln(x/ln(x)))+(exp(5)-5)*ln(x)+x*exp(5)-5*x)/(x+ln(x)))+(ln(x)**3+2*x*ln(x)**2+x**2*ln(x))*exp((ln(x/ln(x)))+(exp(5)-5)*ln(x)+x*exp(5)-5*x)/(x+ln(x)))/(ln(x)**3+2*x*ln(x)**2+x**2*ln(x))/exp((ln(x/ln(x)))+(exp(5)-5)*ln(x)+x*exp(5)-5*x)/(x+ln(x))), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$x + e \left( x - 2e \frac{-5x + xe^5 + (-5 + e^5) \log(x) + \log\left(\frac{x}{\log(x)}\right)}{x + \log(x)} \right) e^{-\frac{-5x + xe^5 + (-5 + e^5) \log(x) + \log\left(\frac{x}{\log(x)}\right)}{x + \log(x)}}$$

#### 44.138 Problem number 10161

$$\int \frac{128 - 32x + 64x^2 - 16x^3 + 8x^4 - 2x^5 + (40 - 530x + 280x^2 - 545x^3 + 258x^4 - 128x^5 + 48x^6 - 6x^7) \log(x) + \dots}{\dots} dx$$

Optimal antiderivative

$$\left( (x^2 + 4)^2 + \ln(x) \right) x \left( \frac{\ln(\ln(x)^2)}{\ln(x) - x + 4} - x \right)$$

command

```
integrate(((ln(x)**3+(5*x**4+24*x**2+20)*ln(x)**2+(-4*x**5+19*x**4-16*x**3+88*x**2-x+52)*ln(x))*ln(ln(x)**2)-2*x*ln(x)**4+(-6*x**5-32*x**3+4*x**2-49*x)*ln(x)**3+(12*x**6-48*x**5+64*x**4-258*x**3+82*x**2-296*x+2)*ln(x)**2+(-6*x**7+48*x**6-128*x**5+258*x**4-545*x**3+280*x**2-530*x+40)*ln(x)-2*x**5+8*x**4-16*x**3+64*x**2-32*x+128)/(ln(x)**3+(-2*x+8)*ln(x)**2+(x**2-8*x+16)*ln(x)), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$-x^6 - 8x^4 - x^2 \log(x) - 16x^2 + \frac{(-x^5 - 8x^3 - x \log(x) - 16x) \log(\log(x)^2)}{x - \log(x) - 4}$$

#### 44.139 Problem number 10182

$$\int \frac{e^x(1-x) + (-4e^{2x} + e^x x) \log(e^{-x}(-4e^x + x)) \log(\log(e^{-x}(-4e^x + x))) + (8e^{2x} - 2e^x x) \log(e^{-x}(-4e^x + x))}{(8e^x - 2x) \log(e^{-x}(-4e^x + x))} dx$$

Optimal antiderivative

$$\left( \ln(\ln(4 \ln(2))) - \frac{\ln(\ln(x e^{-x} - 4))}{2} \right) e^x$$

command

```
integrate((( -4*exp(x)**2+exp(x)*x)*ln((-4*exp(x)+x)/exp(x))*ln(ln((-4*exp(x)+x)/exp(x)))+(8*exp(x)*x)*ln((-4*exp(x)+x)/exp(x))*ln(ln(4*ln(2)))+(1-x)*exp(x))/(8*exp(x)-2*x)/ln((-4*exp(x)+x)/exp(x)), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Timed out

Sympy 1.8 under Python 3.8.8 output

$$-\frac{e^x \log(\log((x - 4e^x) e^{-x}))}{2} + e^x \log(\log(\log(2)) + 2 \log(2))$$

#### 44.140 Problem number 10263

$$\int \frac{-507x^2 + 481x^3 - 152x^4 + 16x^5 + (-234x^2 + 150x^3 - 24x^4) \log(81 - 108x + 54x^2 - 12x^3 + x^4) + (-27x^2 + -507x^2 + 481x^3 - 152x^4 + 16x^5 + (-234x^2 + 150x^3 - 24x^4))}{-507x^2 + 481x^3 - 152x^4 + 16x^5 + (-234x^2 + 150x^3 - 24x^4)}$$

Optimal antiderivative

$$x + e^{\frac{1}{x(-4x+13+3\ln((-3+x)^4))}}$$

command

```
integrate(((( -3*x+9)*ln(x**4-12*x**3+54*x**2-108*x+81)+8*x**2-49*x+39)*exp(1/(3*x*ln(x**4-12*x**3+54*x**2-108*x+81)-4*x**2+13*x)))+(9*x**3-27*x**2)*ln(x**4-12*x**3+54*x**2-108*x+81)**2+24*x**4+150*x**3-234*x**2)*ln(x**4-12*x**3+54*x**2-108*x+81)+16*x**5-152*x**4+481*x**3-507*x**2)/((9*x**3-27*x**2)*ln(x**4-12*x**3+54*x**2-108*x+81)**2+(-24*x**4+150*x**3-234*x**2)*ln(x**4-12*x**3+54*x**2-108*x+81)+16*x**5-152*x**4+481*x**3-507*x**2), x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$x + e^{\frac{1}{-4x^2+3x \log(x^4-12x^3+54x^2-108x+81)+13x}}$$

## 44.141 Problem number 10318

$$\int \frac{(-208x - 60x^2 + 108x^3 - 36x^4 + 4x^5) \log(2x) + (-60 + 164x - 57x^2 - 15x^3 + 9x^4 - x^5) \log\left(\frac{400 - 1920x + 2424x^2}{81}\right)}{(60x - 164x^2 + 57x^3 + 15x^4 - 9x^5 + x^6) \log^2(2x)}$$

Optimal antiderivative

$$\frac{\ln\left(\left(\left(x + \frac{4}{3-x}\right)^2 - 4\right)^2\right)}{\ln(2x)}$$

command

```
integrate(((x**5+9*x**4-15*x**3-57*x**2+164*x-60)*ln((x**8-12*x**7+30*x**6+132*x**5-607*x**4-48*x**3+2424*x**2-1920*x+400)/(x**4-12*x**3+54*x**2-108*x+81)))+(4*x**5-36*x**4+108*x**3-60*x**2-208*x)*ln(2*x))/(x**6-9*x**5+15*x**4+57*x**3-164*x**2+60*x)/ln(2*x)**2,x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{\log\left(\frac{x^8 - 12x^7 + 30x^6 + 132x^5 - 607x^4 - 48x^3 + 2424x^2 - 1920x + 400}{x^4 - 12x^3 + 54x^2 - 108x + 81}\right)}{\log(2x)}$$

## 44.142 Problem number 10323

$$\int \frac{5 - 5e^x + 5x - 5x^2 + (5 + 4x^2 + x^3 + e^x(-5 + 6x)) \log(x) + (-x^3 + e^x(x - x^2)) \log^2(x) + (5 + (5 - x) \log(x)) \log^3(x)}{-5x^2 \log^2(x) + x^3 \log^3(x)}$$

Optimal antiderivative

$$\frac{x - e^x + 1 - x^2 + \ln\left(\frac{x \log(x)}{2} - \frac{5}{2}\right)}{\ln(x) x}$$

command

```
integrate(((x*ln(x)**2+ln(x)*(5-x)+5)*ln(1/2*x*ln(x)-5/2)+((-x**2+x)*exp(x)-x**3)*ln(x)**2+(5)*exp(x)+x**3+4*x**2+5)*ln(x)-5*exp(x)-5*x**2+5*x+5)/(x**3*ln(x)**3-5*x**2*ln(x)**2),x)
```

Sympy 1.10.1 under Python 3.10.4 output

Exception raised: TypeError

Sympy 1.8 under Python 3.8.8 output

$$\frac{-x^2 + x + 1}{x \log(x)} - \frac{e^x}{x \log(x)} + \frac{\log\left(\frac{x \log(x)}{2} - \frac{5}{2}\right)}{x \log(x)}$$