CAS integration tests regression report
Mathematica 13.1 vs. Mathematica 12.3

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September 21, 2022 Compiled on September 21, 2022 at 8:15pm

Contents

1 Summary of regression test table

Table 1: Summary table of regression tests

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
1	16	30	0 (not solved)	1 (pass)
2	33	1427	0 (not solved)	1 (pass)
3	33	1428	0 (not solved)	1 (pass)
4	35	810	0 (not solved)	1 (pass)
5	35	811	0 (not solved)	1 (pass)
6	35	812	0 (not solved)	1 (pass)
7	35	923	0 (not solved)	1 (pass)
8	35	928	0 (not solved)	1 (pass)
9	38	371	0 (not solved)	1 (pass)
10	74	1436	-1 (time out)	1 (pass)
11	103	312	0 (not solved)	1 (pass)
12	103	789	0 (not solved)	1 (pass)
13	103	1319	0 (not solved)	1 (pass)
14	104	207	0 (not solved)	1 (pass)
15	125	82	-1 (time out)	1 (pass)
16	144	173	0 (not solved)	1 (pass)
17	149	12	0 (not solved)	1 (pass)
18	149	19	0 (not solved)	1 (pass)
19	150	141	0 (not solved)	1 (pass)
20	150	142	0 (not solved)	1 (pass)
21	150	143	0 (not solved)	1 (pass)
22	150	144	0 (not solved)	1 (pass)
23	150	145	0 (not solved)	1 (pass)
24	150	146	0 (not solved)	1 (pass)
25	150	147	0 (not solved)	1 (pass)
26	150	1228	0 (not solved)	1 (pass)
27	151	62	-1 (time out)	1 (pass)

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#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
28	154	113	-1 (time out)	1 (pass)
29	154	145	0 (not solved)	1 (pass)
30	187	621	0 (not solved)	1 (pass)
31	188	22	0 (not solved)	1 (pass)
32	188	94	0 (not solved)	1 (pass)
33	188	208	0 (not solved)	1 (pass)
34	188	214	0 (not solved)	1 (pass)
35	188	220	0 (not solved)	1 (pass)
36	190	519	0 (not solved)	1 (pass)
37	190	520	0 (not solved)	1 (pass)
38	190	521	0 (not solved)	1 (pass)
39	191	178	0 (not solved)	1 (pass)
40	197	4	0 (not solved)	1 (pass)
41	201	18	-1 (time out)	1 (pass)
42	201	58	0 (not solved)	1 (pass)
43	209	3046	0 (not solved)	1 (pass)
44	210	8	-1 (time out)	1 (pass)
45	210	19	0 (not solved)	1 (pass)
46	210	38	0 (not solved)	1 (pass)
47	210	91	0 (not solved)	1 (pass)
48	210	294	0 (not solved)	1 (pass)
49	210	300	-1 (time out)	1 (pass)
50	210	358	0 (not solved)	1 (pass)
51	210	379	0 (not solved)	1 (pass)
52	210	414	0 (not solved)	1 (pass)
53	210	578	0 (not solved)	1 (pass)
54	210	760	-1 (time out)	1 (pass)
55	210	1020	0 (not solved)	1 (pass)
56	210	2052	-1 (time out)	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
57	210	2234	0 (not solved)	1 (pass)
58	210	2451	0 (not solved)	1 (pass)
59	210	2680	0 (not solved)	1 (pass)
60	210	2812	0 (not solved)	1 (pass)
61	210	2856	0 (not solved)	1 (pass)
62	210	2913	0 (not solved)	1 (pass)
63	210	3121	0 (not solved)	1 (pass)
64	210	3124	0 (not solved)	1 (pass)
65	210	3351	0 (not solved)	1 (pass)
66	210	3670	0 (not solved)	1 (pass)
67	210	3955	0 (not solved)	1 (pass)
68	210	4166	0 (not solved)	1 (pass)
69	210	4890	0 (not solved)	1 (pass)
70	210	4908	0 (not solved)	1 (pass)
71	210	5022	0 (not solved)	1 (pass)
72	210	5264	-1 (time out)	1 (pass)
73	210	5265	0 (not solved)	1 (pass)
74	210	5410	-1 (time out)	1 (pass)
75	210	5820	0 (not solved)	1 (pass)
76	210	5856	0 (not solved)	1 (pass)
77	210	6066	0 (not solved)	1 (pass)
78	210	6250	0 (not solved)	1 (pass)
79	210	6633	0 (not solved)	1 (pass)
80	210	6675	0 (not solved)	1 (pass)
81	210	6680	0 (not solved)	1 (pass)
82	210	6838	0 (not solved)	1 (pass)
83	210	7310	-1 (time out)	1 (pass)
84	210	7387	0 (not solved)	1 (pass)
85	210	7414	0 (not solved)	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Mathematica 13.1	Mathematica 12.3
86	210	7882	-1 (time out)	1 (pass)
87	210	8195	0 (not solved)	1 (pass)
88	210	8361	0 (not solved)	1 (pass)
89	210	8653	0 (not solved)	1 (pass)
90	210	8762	0 (not solved)	1 (pass)
91	210	8818	0 (not solved)	1 (pass)
92	210	9331	0 (not solved)	1 (pass)
93	210	9462	0 (not solved)	1 (pass)
94	210	9582	0 (not solved)	1 (pass)
95	210	9617	0 (not solved)	1 (pass)
96	210	9713	0 (not solved)	1 (pass)
97	210	10072	0 (not solved)	1 (pass)
98	210	10272	0 (not solved)	1 (pass)

2 Test file number 16

Test folder name:

`test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.1_Linear/16_1.1.1.5_P-x-a+b_x-^m-c+d_x-^n`

2.1 Problem number 30

$$\int \frac{(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx$$

Optimal antiderivative

$$\frac{(bCd-2Dad-Dbc)\,(dx+c)^{1+n}}{b^3d^2\,(1+n)}-\frac{\left(A-\frac{a(B\,b^2-Cab+Da^2)}{b^3}\right)(dx+c)^{1+n}}{(-ad+bc)\,(bx+a)}+\frac{D(dx+c)^{2+n}}{b^2d^2\,(2+n)}$$
$$+\frac{\left(a^3dD(3+n)-b^3(Adn+Bc)+a\,b^2(2cC+Bd(1+n))-a^2b(3cD+Cd(2+n))\right)(dx+c)^{1+n}\,\mathrm{hypergeom}\Big([1,1\cdots}{b^3\,(-ad+bc)^2\,(1+n)}$$

command

```
Integrate[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^2,x]
```

Mathematica 13.1 output

$$\int \frac{(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx$$

Mathematica 12.3 output

$$\frac{(c+dx)^{n+1}\left(\frac{d\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)\,{}_2F_1\left(2,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{(n+1)(bc-ad)^2}-\frac{\left(3a^2D-2abC+b^2B\right)\,{}_2F_1\left(1,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{(n+1)(bc-ad)}+\frac{-2adD-bcD+bC\cdots}{d^2(n+1)}\right.}{b^3}$$

3 Test file number 33

Test folder name:

`test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.1_Quadratic/33_1.2.1.2-d+e_x-^m-a+b_x+c_x^2-^p`

3.1 Problem number 1427

$$\int \frac{(bd + 2cdx)^m}{(a + bx + cx^2)^2} \, dx$$

Optimal antiderivative

$$\frac{8c(d(2cx + b))^{1+m} \operatorname{hypergeom}\left(\left[2, \frac{1}{2} + \frac{m}{2}\right], \left[\frac{3}{2} + \frac{m}{2}\right], \frac{(2cx+b)^2}{-4ac+b^2}\right)}{(-4ac + b^2)^2 \, d\,(1 + m)}$$

command

```
Integrate[(b*d + 2*c*d*x)^m/(a + b*x + c*x^2)^2,x]
```

Mathematica 13.1 output

$$\int \frac{(bd + 2cdx)^m}{(a + bx + cx^2)^2} \, dx$$

Mathematica 12.3 output

$$\frac{8c(b + 2cx)(d(b + 2cx))^m \, {}_2F_1\left(2, \frac{m+1}{2}; \frac{m+3}{2}; \frac{(b+2cx)^2}{b^2-4ac}\right)}{(m + 1)\,(b^2 - 4ac)^2}$$

3.2 Problem number 1428

$$\int \frac{(bd + 2cdx)^m}{(a + bx + cx^2)^3} \, dx$$

Optimal antiderivative

$$-\frac{32c^2(d(2cx + b))^{1+m} \operatorname{hypergeom}\left(\left[3, \frac{1}{2} + \frac{m}{2}\right], \left[\frac{3}{2} + \frac{m}{2}\right], \frac{(2cx+b)^2}{-4ac+b^2}\right)}{(-4ac + b^2)^3 \, d\,(1 + m)}$$

command

```
Integrate[(b*d + 2*c*d*x)^m/(a + b*x + c*x^2)^3,x]
```

Mathematica 13.1 output

$$\int \frac{(bd + 2cdx)^m}{(a + bx + cx^2)^3} \, dx$$

Mathematica 12.3 output

$$-\frac{32c^2(b + 2cx)(d(b + 2cx))^m \, {}_2F_1\left(3, \frac{m+1}{2}; \frac{m+3}{2}; \frac{(b+2cx)^2}{b^2-4ac}\right)}{(m + 1)\,(b^2 - 4ac)^3}$$

4 Test file number 35

Test folder name:

`test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.1_Quadratic/35_1.2.1.4-d+e_x-^m-f+g_x-^n-a+b_x+c_x^2-^p`

4.1 Problem number 810

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^2}\,dx$$

Optimal antiderivative

$$\frac{c(gx+f)^{1+n}}{eg\,(1+n)}-\frac{(c\,d^2-ae)\,g(gx+f)^{1+n}\,\mathrm{hypergeom}\left([2,1+n]\,,[2+n]\,,\frac{e(gx+f)}{-dg+ef}\right)}{e\,(-dg+ef)^2\,(1+n)}$$

command

```
Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^2,x]
```

Mathematica 13.1 output

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^2}\,dx$$

Mathematica 12.3 output

$$\frac{(f+gx)^{n+1}\left(g^2(ae-cd^2)\ {}_2F_1\left(2,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)+c(ef-dg)^2\right)}{eg(n+1)(ef-dg)^2}$$

4.2 Problem number 811

$$\int \frac{(f+gx)^n\,(a+2cdx+cex^2)}{(d+ex)^3}\,dx$$

Optimal antiderivative

$$-\frac{\left(a-\frac{c\,d^2}{e}\right)(gx+f)^{1+n}}{2\,(-dg+ef)\,(ex+d)^2}-\frac{(c\,d^2-ae)\,g(1-n)\,(gx+f)^{1+n}}{2e\,(-dg+ef)^2\,(ex+d)}$$
$$+\frac{\left(ae\,g^2(1-n)\,n-c\left(2e^2f^2-4defg+d^2g^2\left(-n^2+n+2\right)\right)\right)(gx+f)^{1+n}\,\mathrm{hypergeom}\left([1,1+n]\,,[2+n]\,,\frac{e(gx+f)}{-dg+ef}\right)}{2e\,(-dg+ef)^3\,(1+n)}$$

command

```
Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^3,x]
```

Mathematica 13.1 output

$$\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^3}\,dx$$

Mathematica 12.3 output

$$-\frac{(f+gx)^{n+1}\left(g^2\left(ae-cd^2\right)\,{}_2F_1\left(3,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)+c(ef-dg)^2\,{}_2F_1\left(1,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)\right)}{e(n+1)(ef-dg)^3}$$

4.3 Problem number 812

$$\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^4}\,dx$$

Optimal antiderivative

$$-\frac{\left(a-\frac{c\,d^2}{e}\right)(gx+f)^{1+n}}{3\,(-dg+ef)\,(ex+d)^3}-\frac{\left(c\,d^2-ae\right)g(2-n)\,(gx+f)^{1+n}}{6e\,(-dg+ef)^2\,(ex+d)^2}$$
$$+\frac{g\left(ae\,g^2\left(n^2-3n+2\right)+c\left(6e^2f^2-12defg+d^2g^2\left(-n^2+3n+4\right)\right)\right)(gx+f)^{1+n}\,\mathrm{hypergeom}\left([2,1+n]\,,[2+n]\,,\ldots\right.}{6e\,(-dg+ef)^4\,(1+n)}$$

command

```
Integrate[((f + g*x)^n*(a + 2*c*d*x + c*e*x^2))/(d + e*x)^4,x]
```

Mathematica 13.1 output

$$\int \frac{(f+gx)^n\left(a+2cdx+cex^2\right)}{(d+ex)^4}\,dx$$

Mathematica 12.3 output

$$\frac{g(f+gx)^{n+1}\left(g^2\left(ae-cd^2\right)\,{}_2F_1\left(4,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)+c(ef-dg)^2\,{}_2F_1\left(2,n+1;n+2;\frac{e(f+gx)}{ef-dg}\right)\right)}{e(n+1)(ef-dg)^4}$$

4.4 Problem number 923

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^2}\,dx$$

Optimal antiderivative

$$\frac{c(ex+d)^{1+m}}{e\,g^2\,(1+m)}+\frac{\left(a+\frac{f(-bg+cf)}{g^2}\right)(ex+d)^{1+m}}{(-dg+ef)\,(gx+f)}$$
$$+\frac{(cf(2dg-ef(2+m))-g(aegm+b(dg-ef(1+m))))\,(ex+d)^{1+m}\,\mathrm{hypergeom}\left([1,1+m],[2+m],-\frac{g(ex+d)}{-dg+ef}\right)}{g^2\,(-dg+ef)^2\,(1+m)}$$

command

```
Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(f + g*x)^2,x]
```

Mathematica 13.1 output

$$\int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^2}\,dx$$

Mathematica 12.3 output

$$\frac{(d+ex)^{m+1}\left(e^2\left(g(ag-bf)+cf^2\right)\,{}_2F_1\left(2,m+1;m+2;\frac{g(d+ex)}{dg-ef}\right)-e(2cf-bg)(ef-dg)\,{}_2F_1\left(1,m+1;m+2;\frac{g(\cdots}{dg\cdots}\right.\right.}{eg^2(m+1)(ef-dg)^2}$$

4.5 Problem number 928

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^2}\,dx$$

Optimal antiderivative

$$\frac{\left(b^2e^2g^2+c^2\left(d^2g^2+2defg+3e^2f^2\right)+2ceg(aeg-b(dg+2ef))\right)(ex+d)^{1+m}}{e^3g^4\,(1+m)}$$
$$-\frac{2c(-beg+cdg+cef)\,(ex+d)^{2+m}}{e^3g^3\,(2+m)}+\frac{c^2(ex+d)^{3+m}}{e^3g^2\,(3+m)}+\frac{\left(a\,g^2-bfg+c\,f^2\right)^2(ex+d)^{1+m}}{g^4\,(-dg+ef)\,(gx+f)}$$
$$+\frac{\left(a\,g^2-bfg+c\,f^2\right)(cf(4dg-ef(4+m))-g(aegm+b(2dg-ef(2+m))))\,(ex+d)^{1+m}\,\mathrm{hypergeom}\left([1,1+m],\ldots\right.}{g^4\,(-dg+ef)^2\,(1+m)}$$

command

```
Integrate[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^2,x]
```

Mathematica 13.1 output

$$\int \frac{(d+ex)^m \left(a+bx+cx^2\right)^2}{(f+gx)^2}\,dx$$

Mathematica 12.3 output

$$\frac{(d+ex)^{m+1}\left(\frac{2ceg(aeg-b(dg+2ef))+b^2e^2g^2+c^2\left(d^2g^2+2defg+3e^2f^2\right)}{e^3(m+1)}+\frac{e\left(g(ag-bf)+cf^2\right)^2 {}_2F_1\left(2,m+1;m+2;\frac{g(d+ex)}{dg-ef}\right)}{(m+1)(ef-dg)^2}-\frac{2(2cf-bg)(g(\cdots}{\cdots}\right.}{g^4}$$

5 Test file number 38

Test folder name:

test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.1_Quadratic/38_1.2.1.9_P-x-d+e_x-^m-a+b_x+c_x^2-^p

5.1 Problem number 371

$$\int \frac{(d+ex)^m \left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$$

Optimal antiderivative

$$\frac{4(ex+d)^{1+m}}{25e\,(1+m)}-\frac{(1367d-293e+(423d-1367e)\,x)\,(ex+d)^{1+m}}{700\,(5d^2-2de+3e^2)\,(5x^2+2x+3)}$$
$$+\frac{(ex+d)^{1+m}\,\mathrm{hypergeom}\left([1,1+m],[2+m],\frac{5ex+5d}{5d-e\left(1+\mathrm{I}\sqrt{14}\right)}\right)\left(80360d^2-32144de+48216e^2-5922dem+191\cdots\right.}{19600\,(5d^2-2de+3e^2)\,(1+m)\left(5d-e\left(1+\mathrm{I}\sqrt{1\cdots}\right.\right.}$$
$$+\frac{(ex+d)^{1+m}\,\mathrm{hypergeom}\left([1,1+m],[2+m],\frac{5ex+5d}{5d-e+\mathrm{I}\sqrt{14}\,e}\right)\left(80360d^2-32144de+48216e^2-5922dem+19138\cdots\right.}{19600\,(5d^2-2de+3e^2)\,(1+m)\left(5d+\mathrm{I}e\left(\mathrm{I}+\sqrt{14}\cdots\right.\right.}$$

command

```
Integrate[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

Mathematica 13.1 output

$$\int \frac{(d+ex)^m \left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$$

Mathematica 12.3 output

$$(d+ex)^{m+1}\left(-\frac{\sqrt{14}\left(\frac{\left(2115d^2+de\left(-846+\left(-6412+423i\sqrt{14}\right)m\right)+e^2\left(1269+\left(98-1367i\sqrt{14}\right)m\right)\right){}_2F_1\left(1,m+1;m+2;\frac{5(d+ex)}{5d+\left(-1-i\sqrt{14}\right)e}\right)}{5id+\left(\sqrt{14}-i\right)e}-\cdots\right.}{(m+1)(5d^2-2de\cdots}\right.$$

6 Test file number 74

Test folder name:

`test_cases/4_Trig_functions/4.1_Sine/74_4.1.2.2-g_cos-^p-a+b_sin-^m-c+d_sin-^n`

6.1 Problem number 1436

$$\int \frac{(d\sin(e+fx))^{5/2}}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$$

Optimal antiderivative

$$\frac{a^2 d^{\frac{5}{2}} \arctan\left(-1+\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(fx+e)}}{\sqrt{g}\,\sqrt{d\sin(fx+e)}}\right)\sqrt{2}}{2b\,(a^2-b^2)\,f\,g^{\frac{3}{2}}} - \frac{b\,d^{\frac{5}{2}} \arctan\left(-1+\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(fx+e)}}{\sqrt{g}\,\sqrt{d\sin(fx+e)}}\right)\sqrt{2}}{2\,(a^2-b^2)\,f\,g^{\frac{3}{2}}} + \frac{a^2 d^{\frac{5}{2}} \arctan\left(1+\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(fx+e)}}{\sqrt{g}\,\sqrt{d\sin(fx+e)}}\right)\sqrt{2}}{2b\,(a^2-b^2)\,f\,g^{\frac{3}{2}}} - \frac{b\,d^{\frac{5}{2}} \arctan\left(1+\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(fx+e)}}{\sqrt{g}\,\sqrt{d\sin(fx+e)}}\right)\sqrt{2}}{2\,(a^2-b^2)\,f\,g^{\frac{3}{2}}} + \frac{a^2 d^{\frac{5}{2}} \ln\left(\sqrt{g}+\cot(fx+e)\sqrt{g}-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(fx+e)}}{\sqrt{d\sin(fx+e)}}\right)\sqrt{2}}{4b\,(a^2-b^2)\,f\,g^{\frac{3}{2}}} - \frac{b\,d^{\frac{5}{2}} \ln\left(\sqrt{g}+\cot(fx+e)\sqrt{g}-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(fx+e)}}{\sqrt{d\sin(fx+e)}}\right)\sqrt{2}}{4\,(a^2-b^2)\,f\,g^{\frac{3}{2}}} - \frac{a^2 d^{\frac{5}{2}} \ln\left(\sqrt{g}+\cot(fx+e)\sqrt{g}+\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(fx+e)}}{\sqrt{d\sin(fx+e)}}\right)\sqrt{2}}{4b\,(a^2-b^2)\,f\,g^{\frac{3}{2}}} + \frac{b\,d^{\frac{5}{2}} \ln\left(\sqrt{g}+\cot(fx+e)\sqrt{g}+\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(fx+e)}}{\sqrt{d\sin(fx+e)}}\right)\sqrt{2}}{4\,(a^2-b^2)\,f\,g^{\frac{3}{2}}} + \frac{2ad(d\sin(fx+e))^{\frac{3}{2}}}{(a^2-b^2)\,fg\sqrt{g\cos(fx+e)}} - \frac{2a^3d^3\,\mathrm{EllipticPi}\left(\frac{\sqrt{g\cos(fx+e)}}{\sqrt{g}\,\sqrt{1+\sin(fx+e)}},-\frac{\sqrt{-a+b}}{\sqrt{a+b}},\mathrm{I}\right)\sqrt{2}\,\left(\sqrt{\sin}\,(fx+e)\right)}{b\,(-a+b)^{\frac{3}{2}}\,(a+b)^{\frac{3}{2}}\,f\,g^{\frac{3}{2}}\sqrt{d\sin(fx+e)}} + \frac{2a^3d^3\,\mathrm{EllipticPi}\left(\frac{\sqrt{g\cos(fx+e)}}{\sqrt{g}\,\sqrt{1+\sin(fx+e)}},\frac{\sqrt{-a+b}}{\sqrt{a+b}},\mathrm{I}\right)\sqrt{2}\,\left(\sqrt{\sin}\,(fx+e)\right)}{b\,(-a+b)^{\frac{3}{2}}\,(a+b)^{\frac{3}{2}}\,f\,g^{\frac{3}{2}}\sqrt{d\sin(fx+e)}} - \frac{2b\,d^2\sqrt{d\sin(fx+e)}}{(a^2-b^2)\,fg\sqrt{g\cos(fx+e)}} + \frac{2a\,d^2\sqrt{\frac{1}{2}+\frac{\sin(2fx+2e)}{2}}\,\mathrm{EllipticE}\left(\cos\left(e+\frac{\pi}{4}+fx\right),\sqrt{2}\right)\sqrt{g\cos(fx+e)}\,\sqrt{d\sin(fx+e)}}{\sin\left(e+\frac{\pi}{4}+fx\right)(a^2-b^2)\,f\,g^2\sqrt{\sin(2fx+2e)}}$$

command

```
Integrate[(d*Sin[e + f*x])^(5/2)/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

Mathematica 13.1 output

$$\$Aborted$$

Mathematica 12.3 output

$$\frac{2\cot(e+fx)\csc(e+fx)(d\sin(e+fx))^{5/2}(a\sin(e+fx)-b)}{(a^2-b^2)\,f(g\cos(e+fx))^{3/2}}$$

$$-\frac{\cos^{\frac{3}{2}}(e+fx)(d\sin(e+fx))^{5/2}\left(-\frac{2(3a^2-b^2)\left(aF_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)-bF_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)}{3(a^2-b^2)(1-\cos^2(e+fx))^{3/4}(a+b\sin(e+f}\right.}{}$$

7 Test file number 103

Test folder name:

`test_cases/4_Trig_functions/4.3_Tangent/103_4.3.2.1-a+b_tan-^m-c+d_tan-^n`

7.1 Problem number 312

$$\int (d\tan(e+fx))^n(a+ia\tan(e+fx))^2\,dx$$

Optimal antiderivative

$$-\frac{a^2(d\tan(fx+e))^{1+n}}{df\,(1+n)}+\frac{2a^2\,\mathrm{hypergeom}([1,1+n]\,,[2+n]\,,\mathrm{I}\tan(fx+e))\,(d\tan(fx+e))^{1+n}}{df\,(1+n)}$$

command

```
Integrate[(d*Tan[e + f*x])^n*(a + I*a*Tan[e + f*x])^2,x]
```

Mathematica 13.1 output

$$\int (d\tan(e+fx))^n(a+ia\tan(e+fx))^2\,dx$$

Mathematica 12.3 output

$$\frac{e^{-2ie}2^{-n}\left(-\frac{i(-1+e^{2i(e+fx)})}{1+e^{2i(e+fx)}}\right)^{n+1}\cos^2(e+fx)(a+ia\tan(e+fx))^2\left(-2^n+\left(1+e^{2i(e+fx)}\right)^{n+1}\,{}_2F_1(n+1,n+1;n+}{f(n+1)(\cos(fx)+i\sin(fx))^2}$$

7.2 Problem number 789

$$\int (d\cot(e+fx))^n(a+ia\tan(e+fx))^2\,dx$$

Optimal antiderivative

$$\frac{a^2d(d\cot(fx+e))^{-1+n}}{f\,(1-n)}-\frac{2a^2d(d\cot(fx+e))^{-1+n}\,\text{hypergeom}([1,-1+n],[n],-\text{I}\cot(fx+e))}{f\,(1-n)}$$

command

```
Integrate[(d*Cot[e + f*x])^n*(a + I*a*Tan[e + f*x])^2,x]
```

Mathematica 13.1 output

$$\int (d\cot(e+fx))^n(a+ia\tan(e+fx))^2\,dx$$

Mathematica 12.3 output

$$-\frac{e^{-2ie}\left(1+e^{2i(e+fx)}\right)^{-n}\left(\frac{i\left(1+e^{2i(e+fx)}\right)}{-1+e^{2i(e+fx)}}\right)^{n-1}\cos^2(e+fx)(a+ia\tan(e+fx))^2\left(2^n\left(1+e^{2i(e+fx)}\right)\,{}_2F_1\left(1-n,1-n;\right.\right.}{f(n-1)(\cos(fx)+i\sin(fx))^2}$$

7.3 Problem number 1319

$$\int \left(c(d\tan(e+fx))^p\right)^n\left(a+ia\tan(e+fx)\right)^2\,dx$$

Optimal antiderivative

$$-\frac{a^2\tan(fx+e)\left(c(d\tan(fx+e))^p\right)^n}{f\,(np+1)}$$
$$+\frac{2a^2\,\text{hypergeom}([1,np+1],[np+2],\text{I}\tan(fx+e))\tan(fx+e)\left(c(d\tan(fx+e))^p\right)^n}{f\,(np+1)}$$

command

```
Integrate[(c*(d*Tan[e + f*x])^p)^n*(a + I*a*Tan[e + f*x])^2,x]
```

Mathematica 13.1 output

$$\int \left(c(d\tan(e+fx))^p\right)^n\left(a+ia\tan(e+fx)\right)^2\,dx$$

Mathematica 12.3 output

$$\frac{a^2e^{-2ie}2^{-np}\left(-\frac{i\left(-1+e^{2i(e+fx)}\right)}{1+e^{2i(e+fx)}}\right)^{np+1}(\cos(e+fx)+i\sin(e+fx))^2\left(-2^{np}+\left(1+e^{2i(e+fx)}\right)^{np+1}\,{}_2F_1\left(np+1,np+1;\right.\right.}{(fnp+f)(\cos(fx)+i\sin(fx))^2}$$

8 Test file number 104

Test folder name:

`test_cases/4_Trig_functions/4.3_Tangent/104_4.3.3.1-a+b_tan-^m-c+d_tan-^n-A+B_tan-`

8.1 Problem number 207

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$

Optimal antiderivative

$$\frac{\mathrm{I}aB\left(\tan^{1+m}(dx+c)\right)}{d(1+m)}+\frac{a(A-\mathrm{I}B)\,\mathrm{hypergeom}([1,1+m],[2+m],\mathrm{I}\tan(dx+c))\left(\tan^{1+m}(dx+c)\right)}{d(1+m)}$$

command

```
Integrate[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

Mathematica 13.1 output

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$

Mathematica 12.3 output

$$-\frac{iae^{-ic}2^{-m-1}\left(-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}\right)^{m+1}\cos^2(c+dx)(1+i\tan(c+dx))(A+B\tan(c+dx))\left(-B2^{m+1}+(B+iA)\,(1\right.}{d(m+1)(\cos(dx)+i\sin(dx))(A\cos(c+dx)+B\sin(c+}$$

9 Test file number 125

Test folder name:

`test_cases/4_Trig_functions/4.5_Secant/125_4.5.4.2-a+b_sec-^m-d_sec-^n-A+B_sec+C_sec^2-`

9.1 Problem number 82

$$\int \frac{(b\sec(c+dx))^n\left(A+B\sec(c+dx)+C\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Optimal antiderivative

$$-\frac{2C(b\sec(dx+c))^n\sin(dx+c)}{d(1-2n)\sqrt{\sec(dx+c)}}$$
$$-\frac{4(A+C(3-2n)-2An)\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{5}{4}-\frac{n}{2}\right],\left[\frac{9}{4}-\frac{n}{2}\right],\cos^2(dx+c)\right)(b\sec(dx+c))^n\sin(dx+c)}{d(4n^2-12n+5)\sec(dx+c)^{\frac{5}{2}}\sqrt{2-2\cos(2dx+2c)}}$$
$$-\frac{4B\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}-\frac{n}{2}\right],\left[\frac{7}{4}-\frac{n}{2}\right],\cos^2(dx+c)\right)(b\sec(dx+c))^n\sin(dx+c)}{d(3-2n)\sec(dx+c)^{\frac{3}{2}}\sqrt{2-2\cos(2dx+2c)}}$$

command

```
Integrate[((b*Sec[c + d*x])^n*(A + B*Sec[c + d*x] + C*Sec[c + d*x]^2))/Sec[c + d*x]^(3/2),x]
```

Mathematica 13.1 output

$$\$\text{Aborted}$$

Mathematica 12.3 output

$$-\frac{i2^{n+\frac{1}{2}}e^{-\frac{1}{2}i(4c+d(2n+1)x)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{n+\frac{1}{2}}\left(1+e^{2i(c+dx)}\right)^{n+\frac{1}{2}}\sec^{-n-2}(c+dx)(b\sec(c+dx))^n\left(A+B\sec(c+dx)+\right.}{}$$

10 Test file number 144

Test folder name:

`test_cases/5_Inverse_trig_functions/5.1_Inverse_sine/144_5.1.5_Inverse_sine_functions`

10.1 Problem number 173

$$\int x^2(a+b\text{ArcSin}(c+dx))^n\,dx$$

Optimal antiderivative

$$\begin{aligned}
&-\frac{\mathrm{I}(a+b\arcsin(dx+c))^n\,\Gamma\left(1+n,\frac{-\mathrm{I}(a+b\arcsin(dx+c))}{b}\right)\mathrm{e}^{\frac{-\mathrm{I}a}{b}}\left(\frac{-\mathrm{I}(a+b\arcsin(dx+c))}{b}\right)^{-n}}{8d^3}\\
&-\frac{\mathrm{I}c^2(a+b\arcsin(dx+c))^n\,\Gamma\left(1+n,\frac{-\mathrm{I}(a+b\arcsin(dx+c))}{b}\right)\mathrm{e}^{\frac{-\mathrm{I}a}{b}}\left(\frac{-\mathrm{I}(a+b\arcsin(dx+c))}{b}\right)^{-n}}{2d^3}\\
&+\frac{\mathrm{I}\,\mathrm{e}^{\frac{\mathrm{I}a}{b}}(a+b\arcsin(dx+c))^n\,\Gamma\left(1+n,\frac{\mathrm{I}(a+b\arcsin(dx+c))}{b}\right)\left(\frac{\mathrm{I}(a+b\arcsin(dx+c))}{b}\right)^{-n}}{8d^3}\\
&+\frac{\mathrm{I}c^2\mathrm{e}^{\frac{\mathrm{I}a}{b}}(a+b\arcsin(dx+c))^n\,\Gamma\left(1+n,\frac{\mathrm{I}(a+b\arcsin(dx+c))}{b}\right)\left(\frac{\mathrm{I}(a+b\arcsin(dx+c))}{b}\right)^{-n}}{2d^3}\\
&+\frac{2^{-2-n}c(a+b\arcsin(dx+c))^n\,\Gamma\left(1+n,\frac{-2\,\mathrm{I}(a+b\arcsin(dx+c))}{b}\right)\mathrm{e}^{\frac{-2\,\mathrm{I}a}{b}}\left(\frac{-\mathrm{I}(a+b\arcsin(dx+c))}{b}\right)^{-n}}{d^3}\\
&+\frac{2^{-2-n}c\,\mathrm{e}^{\frac{2\,\mathrm{I}a}{b}}(a+b\arcsin(dx+c))^n\,\Gamma\left(1+n,\frac{2\,\mathrm{I}(a+b\arcsin(dx+c))}{b}\right)\left(\frac{\mathrm{I}(a+b\arcsin(dx+c))}{b}\right)^{-n}}{d^3}\\
&+\frac{\mathrm{I}\,3^{-1-n}(a+b\arcsin(dx+c))^n\,\Gamma\left(1+n,\frac{-3\,\mathrm{I}(a+b\arcsin(dx+c))}{b}\right)\mathrm{e}^{\frac{-3\,\mathrm{I}a}{b}}\left(\frac{-\mathrm{I}(a+b\arcsin(dx+c))}{b}\right)^{-n}}{8d^3}\\
&-\frac{\mathrm{I}\,3^{-1-n}\mathrm{e}^{\frac{3\,\mathrm{I}a}{b}}(a+b\arcsin(dx+c))^n\,\Gamma\left(1+n,\frac{3\,\mathrm{I}(a+b\arcsin(dx+c))}{b}\right)\left(\frac{\mathrm{I}(a+b\arcsin(dx+c))}{b}\right)^{-n}}{8d^3}
\end{aligned}$$

command

```
Integrate[x^2*(a + b*ArcSin[c + d*x])^n,x]
```

Mathematica 13.1 output

$$\int x^2(a+b\text{ArcSin}(c+dx))^n\,dx$$

Mathematica 12.3 output

$$2^{-n-3}3^{-n-1}e^{-\frac{3ia}{b}}\left(a+b\sin^{-1}(c+dx)\right)^n\left(\frac{\left(a+b\sin^{-1}(c+dx)\right)^2}{b^2}\right)^{-n}\left(i\left(4c^2+1\right)2^n3^{n+1}e^{\frac{4ia}{b}}\left(-\frac{i\left(a+b\sin^{-1}(c+dx)\right)}{b}\right)^n\Gamma\left(n\right.\right.$$

11 Test file number 149

Test folder name:

`test_cases/5_Inverse_trig_functions/5.3_Inverse_tangent/149_5.3.3-d+e_x-^m-a+b_arctan-c_x^n-^p`

11.1 Problem number 12

$$\int\frac{(a+b\text{ArcTan}(cx))^2}{d+ex}\,dx$$

Optimal antiderivative

$$-\frac{(a+b\arctan(cx))^2\ln\left(\frac{2}{1-\text{I}cx}\right)}{e}+\frac{(a+b\arctan(cx))^2\ln\left(\frac{2c(ex+d)}{(cd+\text{I}e)(1-\text{I}cx)}\right)}{e}$$
$$+\frac{\text{I}b(a+b\arctan(cx))\,\text{polylog}\left(2,1-\frac{2}{1-\text{I}cx}\right)}{e}$$
$$-\frac{\text{I}b(a+b\arctan(cx))\,\text{polylog}\left(2,1-\frac{2c(ex+d)}{(cd+\text{I}e)(1-\text{I}cx)}\right)}{e}$$
$$-\frac{b^2\,\text{polylog}\left(3,1-\frac{2}{1-\text{I}cx}\right)}{2e}+\frac{b^2\,\text{polylog}\left(3,1-\frac{2c(ex+d)}{(cd+\text{I}e)(1-\text{I}cx)}\right)}{2e}$$

command

```
Integrate[(a + b*ArcTan[c*x])^2/(d + e*x),x]
```

Mathematica 13.1 output

$$\int\frac{(a+b\text{ArcTan}(cx))^2}{d+ex}\,dx$$

Mathematica 12.3 output

$$\frac{6a^2cd\log(d+ex)+12abcd\Big(\tan^{-1}(cx)\left(\frac{1}{2}\log\left(c^2x^2+1\right)+\log\left(\sin\left(\tan^{-1}\left(\frac{cd}{e}\right)+\tan^{-1}(cx)\right)\right)\right)+\frac{1}{2}\Big(-\log\Big(\frac{2}{\sqrt{c^2x^2}}}{}$$

11.2 Problem number 19

$$\int\frac{(a+b\text{ArcTan}(cx))^3}{(d+ex)^2}\,dx$$

Optimal antiderivative

$$\begin{aligned}&\frac{\text{I}c(a+b\arctan(cx))^3}{c^2d^2+e^2}+\frac{c^2d(a+b\arctan(cx))^3}{e\,(c^2d^2+e^2)}\\&-\frac{(a+b\arctan(cx))^3}{e\,(ex+d)}-\frac{3bc(a+b\arctan(cx))^2\ln\left(\frac{2}{1-\text{I}cx}\right)}{c^2d^2+e^2}\\&+\frac{3bc(a+b\arctan(cx))^2\ln\left(\frac{2}{1+\text{I}cx}\right)}{c^2d^2+e^2}+\frac{3bc(a+b\arctan(cx))^2\ln\left(\frac{2c(ex+d)}{(cd+\text{I}e)(1-\text{I}cx)}\right)}{c^2d^2+e^2}\\&+\frac{3\,\text{I}b^2c(a+b\arctan(cx))\operatorname{polylog}\left(2,1-\frac{2}{1-\text{I}cx}\right)}{c^2d^2+e^2}\\&+\frac{3\,\text{I}b^2c(a+b\arctan(cx))\operatorname{polylog}\left(2,1-\frac{2}{1+\text{I}cx}\right)}{c^2d^2+e^2}\\&-\frac{3\,\text{I}b^2c(a+b\arctan(cx))\operatorname{polylog}\left(2,1-\frac{2c(ex+d)}{(cd+\text{I}e)(1-\text{I}cx)}\right)}{c^2d^2+e^2}-\frac{3b^3c\operatorname{polylog}\left(3,1-\frac{2}{1-\text{I}cx}\right)}{2\,(c^2d^2+e^2)}\\&+\frac{3b^3c\operatorname{polylog}\left(3,1-\frac{2}{1+\text{I}cx}\right)}{2\,(c^2d^2+e^2)}+\frac{3b^3c\operatorname{polylog}\left(3,1-\frac{2c(ex+d)}{(cd+\text{I}e)(1-\text{I}cx)}\right)}{2\,(c^2d^2+e^2)}\end{aligned}$$

command

```
Integrate[(a + b*ArcTan[c*x])^3/(d + e*x)^2,x]
```

Mathematica 13.1 output

$$\int\frac{(a+b\text{ArcTan}(cx))^3}{(d+ex)^2}\,dx$$

Mathematica 12.3 output

$$-\frac{a^3}{e(d+ex)}+\frac{3a^2bc^2d\tan^{-1}(cx)}{c^2d^2e+e^3}-\frac{3a^2bc\log\left(c^2x^2+1\right)}{2\left(c^2d^2+e^2\right)}+\frac{3a^2bc\log(d+ex)}{c^2d^2+e^2}-\frac{3a^2b\tan^{-1}(cx)}{e(d+ex)}$$

$$+\frac{3ab^2\left(-\frac{cd\left(-\frac{1}{2}\pi\log(c^2x^2+1)+i\mathrm{Li}_2\left(e^{2i\left(\tan^{-1}\left(\frac{cd}{e}\right)+\tan^{-1}(cx)\right)}\right)-i\tan^{-1}(cx)\left(\pi-2\tan^{-1}\left(\frac{cd}{e}\right)\right)-2\left(\tan^{-1}\left(\frac{cd}{e}\right)+\tan^{-1}(cx)\right)\log\left(1-e^{2i\left(\tan\cdots\right.}\right.\right.}{c^2d^2+e^2}\cdots\right.}{\cdots}$$

$$+\frac{b^3\left(\frac{2x\tan^{-1}(cx)^3}{d+ex}+\frac{2\tan^{-1}(cx)\left(\tan^{-1}(cx)^2\left(-2e\sqrt{\frac{c^2d^2}{e^2}+1}\,e^{i\tan^{-1}\left(\frac{cd}{e}\right)}+icd+e\right)+3cd\left(2\tan^{-1}\left(\frac{cd}{e}\right)\left(\log\left(\frac{e^{-i\tan^{-1}\left(\frac{cd}{e}\right)}\left((cx-i)e^{2\cdots}\right.}{2\sqrt{c^2x^2\cdots}}\right.\right.\right.\right.}{\cdots}\right.}{\cdots}$$

12 Test file number 150

Test folder name:

`test_cases/5_Inverse_trig_functions/5.3_Inverse_tangent/150_5.3.4_u-a+b_arctan-c_x-^p`

12.1 Problem number 141

$$\int \frac{x^3(a+b\text{ArcTan}(cx))^2}{d+ex}\,dx$$

Optimal antiderivative

$$\frac{abdx}{c\,e^2}+\frac{b^2x}{3c^2e}-\frac{b^2\arctan(cx)}{3c^3e}+\frac{b^2dx\arctan(cx)}{c\,e^2}-\frac{b\,x^2(a+b\arctan(cx))}{3ce}$$
$$+\frac{\mathrm{I}b^2d^2\operatorname{polylog}\left(2,1-\frac{2}{1+\mathrm{I}cx}\right)}{c\,e^3}-\frac{d(a+b\arctan(cx))^2}{2c^2e^2}$$
$$-\frac{\mathrm{I}b^2\operatorname{polylog}\left(2,1-\frac{2}{1+\mathrm{I}cx}\right)}{3c^3e}+\frac{d^2x(a+b\arctan(cx))^2}{e^3}-\frac{d\,x^2(a+b\arctan(cx))^2}{2e^2}$$
$$+\frac{x^3(a+b\arctan(cx))^2}{3e}+\frac{d^3(a+b\arctan(cx))^2\ln\left(\frac{2}{1-\mathrm{I}cx}\right)}{e^4}$$
$$+\frac{2b\,d^2(a+b\arctan(cx))\ln\left(\frac{2}{1+\mathrm{I}cx}\right)}{c\,e^3}-\frac{2b(a+b\arctan(cx))\ln\left(\frac{2}{1+\mathrm{I}cx}\right)}{3c^3e}$$
$$-\frac{d^3(a+b\arctan(cx))^2\ln\left(\frac{2c(ex+d)}{(cd+\mathrm{I}e)(1-\mathrm{I}cx)}\right)}{e^4}-\frac{b^2d\ln(c^2x^2+1)}{2c^2e^2}$$
$$+\frac{\mathrm{I}b\,d^3(a+b\arctan(cx))\operatorname{polylog}\left(2,1-\frac{2c(ex+d)}{(cd+\mathrm{I}e)(1-\mathrm{I}cx)}\right)}{e^4}-\frac{\mathrm{I}(a+b\arctan(cx))^2}{3c^3e}$$
$$+\frac{\mathrm{I}d^2(a+b\arctan(cx))^2}{c\,e^3}-\frac{\mathrm{I}b\,d^3(a+b\arctan(cx))\operatorname{polylog}\left(2,1-\frac{2}{1-\mathrm{I}cx}\right)}{e^4}$$
$$+\frac{b^2d^3\operatorname{polylog}\left(3,1-\frac{2}{1-\mathrm{I}cx}\right)}{2e^4}-\frac{b^2d^3\operatorname{polylog}\left(3,1-\frac{2c(ex+d)}{(cd+\mathrm{I}e)(1-\mathrm{I}cx)}\right)}{2e^4}$$

command

```
Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

Mathematica 13.1 output

$$\int\frac{x^3(a+b\mathrm{ArcTan}(cx))^2}{d+ex}\,dx$$

Mathematica 12.3 output

output too large to display

12.2 Problem number 142

$$\int \frac{x^2(a+b\text{ArcTan}(cx))^2}{d+ex}\,dx$$

Optimal antiderivative

$$-\frac{abx}{ce}-\frac{b^2x\arctan(cx)}{ce}-\frac{\mathrm{I}d(a+b\arctan(cx))^2}{c\,e^2}+\frac{(a+b\arctan(cx))^2}{2c^2e}$$
$$-\frac{dx(a+b\arctan(cx))^2}{e^2}+\frac{x^2(a+b\arctan(cx))^2}{2e}-\frac{d^2(a+b\arctan(cx))^2\ln\left(\frac{2}{1-\mathrm{I}cx}\right)}{e^3}$$
$$-\frac{2bd(a+b\arctan(cx))\ln\left(\frac{2}{1+\mathrm{I}cx}\right)}{c\,e^2}+\frac{d^2(a+b\arctan(cx))^2\ln\left(\frac{2c(ex+d)}{(cd+\mathrm{I}e)(1-\mathrm{I}cx)}\right)}{e^3}$$
$$+\frac{b^2\ln(c^2x^2+1)}{2c^2e}+\frac{\mathrm{I}b\,d^2(a+b\arctan(cx))\,\text{polylog}\left(2,1-\frac{2}{1-\mathrm{I}cx}\right)}{e^3}$$
$$-\frac{\mathrm{I}b^2d\,\text{polylog}\left(2,1-\frac{2}{1+\mathrm{I}cx}\right)}{c\,e^2}-\frac{\mathrm{I}b\,d^2(a+b\arctan(cx))\,\text{polylog}\left(2,1-\frac{2c(ex+d)}{(cd+\mathrm{I}e)(1-\mathrm{I}cx)}\right)}{e^3}$$
$$-\frac{b^2d^2\,\text{polylog}\left(3,1-\frac{2}{1-\mathrm{I}cx}\right)}{2e^3}+\frac{b^2d^2\,\text{polylog}\left(3,1-\frac{2c(ex+d)}{(cd+\mathrm{I}e)(1-\mathrm{I}cx)}\right)}{2e^3}$$

command

```
Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

Mathematica 13.1 output

$$\int \frac{x^2(a+b\text{ArcTan}(cx))^2}{d+ex}\,dx$$

Mathematica 12.3 output

output too large to display

12.3 Problem number 143

$$\int \frac{x(a+b\text{ArcTan}(cx))^2}{d+ex}\,dx$$

Optimal antiderivative

$$\frac{\mathrm{I}(a+b\arctan(cx))^2}{ce}+\frac{x(a+b\arctan(cx))^2}{e}+\frac{d(a+b\arctan(cx))^2\ln\left(\frac{2}{1-\mathrm{I}cx}\right)}{e^2}+\frac{2b(a+b\arctan(cx))\ln\left(\frac{2}{1+\mathrm{I}cx}\right)}{ce}-\frac{d(a+b\arctan(cx))^2\ln\left(\frac{2c(ex+d)}{(cd+\mathrm{I}e)(1-\mathrm{I}cx)}\right)}{e^2}-\frac{\mathrm{I}bd(a+b\arctan(cx))\operatorname{polylog}\left(2,1-\frac{2}{1-\mathrm{I}cx}\right)}{e^2}+\frac{\mathrm{I}b^2\operatorname{polylog}\left(2,1-\frac{2}{1+\mathrm{I}cx}\right)}{ce}+\frac{\mathrm{I}bd(a+b\arctan(cx))\operatorname{polylog}\left(2,1-\frac{2c(ex+d)}{(cd+\mathrm{I}e)(1-\mathrm{I}cx)}\right)}{e^2}+\frac{b^2d\operatorname{polylog}\left(3,1-\frac{2}{1-\mathrm{I}cx}\right)}{2e^2}-\frac{b^2d\operatorname{polylog}\left(3,1-\frac{2c(ex+d)}{(cd+\mathrm{I}e)(1-\mathrm{I}cx)}\right)}{2e^2}$$

command

```
Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

Mathematica 13.1 output

$$\int\frac{x(a+b\mathrm{ArcTan}(cx))^2}{d+ex}\,dx$$

Mathematica 12.3 output

$$-\frac{-4b^2\sqrt{\frac{c^2d^2}{e^2}+1}\,ee^{i\tan^{-1}\left(\frac{cd}{e}\right)}\tan^{-1}(cx)^3+4ib^2cd\tan^{-1}(cx)^3+4b^2e\tan^{-1}(cx)^3-6ab\sqrt{\frac{c^2d^2}{e^2}+1}\,ee^{i\tan^{-1}\left(\frac{cd}{e}\right)}\mathrm{ta}}{}$$

12.4 Problem number 144

$$\int\frac{(a+b\mathrm{ArcTan}(cx))^2}{d+ex}\,dx$$

Optimal antiderivative

$$-\frac{(a+b\arctan(cx))^2\ln\left(\frac{2}{1-\mathrm{I}cx}\right)}{e}+\frac{(a+b\arctan(cx))^2\ln\left(\frac{2c(ex+d)}{(cd+\mathrm{I}e)(1-\mathrm{I}cx)}\right)}{e}+\frac{\mathrm{I}b(a+b\arctan(cx))\operatorname{polylog}\left(2,1-\frac{2}{1-\mathrm{I}cx}\right)}{e}-\frac{\mathrm{I}b(a+b\arctan(cx))\operatorname{polylog}\left(2,1-\frac{2c(ex+d)}{(cd+\mathrm{I}e)(1-\mathrm{I}cx)}\right)}{e}-\frac{b^2\operatorname{polylog}\left(3,1-\frac{2}{1-\mathrm{I}cx}\right)}{2e}+\frac{b^2\operatorname{polylog}\left(3,1-\frac{2c(ex+d)}{(cd+\mathrm{I}e)(1-\mathrm{I}cx)}\right)}{2e}$$

command

```
Integrate[(a + b*ArcTan[c*x])^2/(d + e*x),x]
```

Mathematica 13.1 output

$$\int \frac{(a+b\text{ArcTan}(cx))^2}{d+ex}\,dx$$

Mathematica 12.3 output

$$6a^2cd\log(d+ex)+12abcd\left(\tan^{-1}(cx)\left(\frac{1}{2}\log\left(c^2x^2+1\right)+\log\left(\sin\left(\tan^{-1}\left(\frac{cd}{e}\right)+\tan^{-1}(cx)\right)\right)\right)+\frac{1}{2}\left(-\log\left(\frac{2}{\sqrt{c^2x^2}}\right.\right.\right.$$

12.5 Problem number 145

$$\int \frac{(a+b\text{ArcTan}(cx))^2}{x(d+ex)}\,dx$$

Optimal antiderivative

$$-\frac{2(a+b\arctan(cx))^2\operatorname{arctanh}\left(-1+\frac{2}{1+\text{I}cx}\right)}{d}+\frac{(a+b\arctan(cx))^2\ln\left(\frac{2}{1-\text{I}cx}\right)}{d}$$
$$-\frac{(a+b\arctan(cx))^2\ln\left(\frac{2c(ex+d)}{(cd+\text{I}e)(1-\text{I}cx)}\right)}{d}-\frac{\text{I}b(a+b\arctan(cx))\operatorname{polylog}\left(2,1-\frac{2}{1-\text{I}cx}\right)}{d}$$
$$-\frac{\text{I}b(a+b\arctan(cx))\operatorname{polylog}\left(2,1-\frac{2}{1+\text{I}cx}\right)}{d}$$
$$+\frac{\text{I}b(a+b\arctan(cx))\operatorname{polylog}\left(2,-1+\frac{2}{1+\text{I}cx}\right)}{d}$$
$$+\frac{\text{I}b(a+b\arctan(cx))\operatorname{polylog}\left(2,1-\frac{2c(ex+d)}{(cd+\text{I}e)(1-\text{I}cx)}\right)}{d}$$
$$+\frac{b^2\operatorname{polylog}\left(3,1-\frac{2}{1-\text{I}cx}\right)}{2d}-\frac{b^2\operatorname{polylog}\left(3,1-\frac{2}{1+\text{I}cx}\right)}{2d}$$
$$+\frac{b^2\operatorname{polylog}\left(3,-1+\frac{2}{1+\text{I}cx}\right)}{2d}-\frac{b^2\operatorname{polylog}\left(3,1-\frac{2c(ex+d)}{(cd+\text{I}e)(1-\text{I}cx)}\right)}{2d}$$

command

```
Integrate[(a + b*ArcTan[c*x])^2/(x*(d + e*x)),x]
```

Mathematica 13.1 output

$$\int \frac{(a + b\text{ArcTan}(cx))^2}{x(d + ex)}\,dx$$

Mathematica 12.3 output

$$24cd\log(x)a^2 - 24cd\log(d+ex)a^2 - 24b\left(-\sqrt{\frac{c^2d^2}{e^2}+1}\,ee^{i\tan^{-1}\left(\frac{cd}{e}\right)}\tan^{-1}(cx)^2 + icd\tan^{-1}(cx)^2 + e\tan^{-1}(cx)^2 -$$

12.6 Problem number 146

$$\int \frac{(a + b\text{ArcTan}(cx))^2}{x^2(d + ex)}\,dx$$

Optimal antiderivative

$$\begin{aligned}
&-\frac{\text{I}c(a+b\arctan(cx))^2}{d} - \frac{(a+b\arctan(cx))^2}{dx} \\
&+\frac{2e(a+b\arctan(cx))^2\operatorname{arctanh}\left(-1+\frac{2}{1+\text{I}cx}\right)}{d^2} - \frac{e(a+b\arctan(cx))^2\ln\left(\frac{2}{1-\text{I}cx}\right)}{d^2} \\
&+\frac{e(a+b\arctan(cx))^2\ln\left(\frac{2c(ex+d)}{(cd+\text{I}e)(1-\text{I}cx)}\right)}{d^2} + \frac{2bc(a+b\arctan(cx))\ln\left(2-\frac{2}{1-\text{I}cx}\right)}{d} \\
&+\frac{\text{I}be(a+b\arctan(cx))\operatorname{polylog}\left(2,1-\frac{2}{1-\text{I}cx}\right)}{d^2} \\
&-\frac{\text{I}b^2c\operatorname{polylog}\left(2,-1+\frac{2}{1-\text{I}cx}\right)}{d} + \frac{\text{I}be(a+b\arctan(cx))\operatorname{polylog}\left(2,1-\frac{2}{1+\text{I}cx}\right)}{d^2} \\
&-\frac{\text{I}be(a+b\arctan(cx))\operatorname{polylog}\left(2,-1+\frac{2}{1+\text{I}cx}\right)}{d^2} \\
&-\frac{\text{I}be(a+b\arctan(cx))\operatorname{polylog}\left(2,1-\frac{2c(ex+d)}{(cd+\text{I}e)(1-\text{I}cx)}\right)}{d^2} \\
&-\frac{b^2e\operatorname{polylog}\left(3,1-\frac{2}{1-\text{I}cx}\right)}{2d^2} + \frac{b^2e\operatorname{polylog}\left(3,1-\frac{2}{1+\text{I}cx}\right)}{2d^2} \\
&-\frac{b^2e\operatorname{polylog}\left(3,-1+\frac{2}{1+\text{I}cx}\right)}{2d^2} + \frac{b^2e\operatorname{polylog}\left(3,1-\frac{2c(ex+d)}{(cd+\text{I}e)(1-\text{I}cx)}\right)}{2d^2}
\end{aligned}$$

command

```
Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x)),x]
```

Mathematica 13.1 output

$$\int \frac{(a + b\text{ArcTan}(cx))^2}{x^2(d + ex)}\,dx$$

Mathematica 12.3 output

output too large to display

12.7 Problem number 147

$$\int \frac{(a + b\text{ArcTan}(cx))^2}{x^3(d + ex)}\,dx$$

Optimal antiderivative

$$
\begin{aligned}
&-\frac{bc(a+b\arctan(cx))}{dx}-\frac{c^2(a+b\arctan(cx))^2}{2d} \\
&-\frac{Ib\,e^2(a+b\arctan(cx))\operatorname{polylog}\left(2,1-\frac{2}{1-Icx}\right)}{d^3}-\frac{(a+b\arctan(cx))^2}{2d\,x^2} \\
&+\frac{e(a+b\arctan(cx))^2}{d^2x}-\frac{2e^2(a+b\arctan(cx))^2\operatorname{arctanh}\left(-1+\frac{2}{1+Icx}\right)}{d^3}+\frac{b^2c^2\ln(x)}{d} \\
&+\frac{e^2(a+b\arctan(cx))^2\ln\left(\frac{2}{1-Icx}\right)}{d^3}-\frac{e^2(a+b\arctan(cx))^2\ln\left(\frac{2c(ex+d)}{(cd+Ie)(1-Icx)}\right)}{d^3} \\
&-\frac{b^2c^2\ln(c^2x^2+1)}{2d}-\frac{2bce(a+b\arctan(cx))\ln\left(2-\frac{2}{1-Icx}\right)}{d^2} \\
&+\frac{Ib\,e^2(a+b\arctan(cx))\operatorname{polylog}\left(2,-1+\frac{2}{1+Icx}\right)}{d^3} \\
&+\frac{Ib\,e^2(a+b\arctan(cx))\operatorname{polylog}\left(2,1-\frac{2c(ex+d)}{(cd+Ie)(1-Icx)}\right)}{d^3} \\
&-\frac{Ib\,e^2(a+b\arctan(cx))\operatorname{polylog}\left(2,1-\frac{2}{1+Icx}\right)}{d^3}+\frac{Ib^2ce\operatorname{polylog}\left(2,-1+\frac{2}{1-Icx}\right)}{d^2} \\
&+\frac{Ice(a+b\arctan(cx))^2}{d^2}+\frac{b^2e^2\operatorname{polylog}\left(3,1-\frac{2}{1-Icx}\right)}{2d^3}-\frac{b^2e^2\operatorname{polylog}\left(3,1-\frac{2}{1+Icx}\right)}{2d^3} \\
&+\frac{b^2e^2\operatorname{polylog}\left(3,-1+\frac{2}{1+Icx}\right)}{2d^3}-\frac{b^2e^2\operatorname{polylog}\left(3,1-\frac{2c(ex+d)}{(cd+Ie)(1-Icx)}\right)}{2d^3}
\end{aligned}
$$

command

```
Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x)),x]
```

Mathematica 13.1 output

$$\int \frac{(a + b\text{ArcTan}(cx))^2}{x^3(d + ex)}\,dx$$

Mathematica 12.3 output

output too large to display

12.8 Problem number 1228

$$\int x^m \left(d+ex^2\right)^3 (a+b\text{ArcTan}(cx))\,dx$$

Optimal antiderivative

$$-\frac{be\left(e^2\left(m^2+8m+15\right)-3c^2de\left(m^2+10m+21\right)+3c^4d^2\left(m^2+12m+35\right)\right)x^{2+m}}{c^5\,(2+m)\,(7+m)\,(m^2+8m+15)}$$
$$+\frac{b\,e^2\left(e(5+m)-3c^2d(7+m)\right)x^{4+m}}{c^3\,(4+m)\,(5+m)\,(7+m)}-\frac{b\,e^3x^{6+m}}{c\,(6+m)\,(7+m)}+\frac{d^3x^{1+m}(a+b\arctan(cx))}{1+m}$$
$$+\frac{3d^2e\,x^{3+m}(a+b\arctan(cx))}{3+m}+\frac{3d\,e^2x^{5+m}(a+b\arctan(cx))}{5+m}+\frac{e^3x^{7+m}(a+b\arctan(cx))}{7+m}$$
$$+\frac{b\left(e^3\left(m^3+9m^2+23m+15\right)-3c^2d\,e^2\left(m^3+11m^2+31m+21\right)+3c^4d^2e\left(m^3+13m^2+47m+35\right)-c^6d^3\left(m^3-\right.\right.}{c^5\,(m^2+12m+35)\,(m^3+6m^2+11m+6)}$$

command

```
Integrate[x^m*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

Mathematica 13.1 output

$$\int x^m \left(d+ex^2\right)^3 (a+b\text{ArcTan}(cx))\,dx$$

Mathematica 12.3 output

$$x^{m+1}\left(\frac{d^3\left(a+b\tan^{-1}(cx)\right)}{m+1}+\frac{3d^2ex^2\left(a+b\tan^{-1}(cx)\right)}{m+3}+\frac{3de^2x^4\left(a+b\tan^{-1}(cx)\right)}{m+5}+\frac{e^3x^6\left(a+b\tan^{-1}(cx)\right)}{m+7}-\frac{bcd^3x\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-c^2x^2\right)}{m^2+3m+2}-\frac{3bcd^2ex^3\,{}_2F_1\left(1,\frac{m+4}{2};\frac{m+6}{2};-c^2x^2\right)}{m^2+7m+12}-\frac{3bcde^2x^5\,{}_2F_1\left(1,\frac{m+6}{2};\frac{m+8}{2};-c^2x^2\right)}{(m+5)(m+6)}-\frac{bce^3x^7\,{}_2F_1\left(1,\frac{m}{2}+4;\frac{m}{2}+5;-c^2x^2\right)}{(m+7)(m+8)}\right)$$

13 Test file number 151

Test folder name:

`test_cases/5_Inverse_trig_functions/5.3_Inverse_tangent/151_5.3.5_u-a+b_arctan-c+d_x-^p`

13.1 Problem number 62

$$\int \frac{\mathrm{ArcTan}(d+ex)}{a+bx+cx^2}\,dx$$

Optimal antiderivative

$$\frac{\arctan(ex+d)\ln\left(\frac{2e\left(b+2cx-\sqrt{-4ac+b^2}\right)}{(1-\mathrm{I}(ex+d))\left(2c(\mathrm{I}-d)+e\left(b-\sqrt{-4ac+b^2}\right)\right)}\right)}{\sqrt{-4ac+b^2}} - \frac{\arctan(ex+d)\ln\left(\frac{2e\left(b+2cx+\sqrt{-4ac+b^2}\right)}{(1-\mathrm{I}(ex+d))\left(2c(\mathrm{I}-d)+e\left(b+\sqrt{-4ac+b^2}\right)\right)}\right)}{\sqrt{-4ac+b^2}} - \frac{\mathrm{I}\,\mathrm{polylog}\left(2,1+\frac{4cd-4c(ex+d)-2e\left(b-\sqrt{-4ac+b^2}\right)}{(1-\mathrm{I}(ex+d))\left(2\,\mathrm{I}c-2cd+be-e\sqrt{-4ac+b^2}\right)}\right)}{2\sqrt{-4ac+b^2}} + \frac{\mathrm{I}\,\mathrm{polylog}\left(2,1+\frac{4cd-4c(ex+d)-2e\left(b+\sqrt{-4ac+b^2}\right)}{(1-\mathrm{I}(ex+d))\left(2c(\mathrm{I}-d)+e\left(b+\sqrt{-4ac+b^2}\right)\right)}\right)}{2\sqrt{-4ac+b^2}}$$

command

```
Integrate[ArcTan[d + e*x]/(a + b*x + c*x^2),x]
```

Mathematica 13.1 output

$Aborted

Mathematica 12.3 output

$$i\left(-\mathrm{Li}_2\left(\frac{2c(d+ex-i)}{2c(d-i)+\left(\sqrt{b^2-4ac}-b\right)e}\right)+\mathrm{Li}_2\left(\frac{2c(d+ex-i)}{2c(d-i)-\left(b+\sqrt{b^2-4ac}\right)e}\right)+\mathrm{Li}_2\left(\frac{2c(d+ex+i)}{2c(d+i)+\left(\sqrt{b^2-4ac}-b\right)e}\right)-\mathrm{Li}_2\right.$$

14 Test file number 154

Test folder name:

`test_cases/5_Inverse_trig_functions/5.4_Inverse_cotangent/154_5.4.1_Inverse_cotangent_functio`

14.1 Problem number 113

$$\int \frac{\cot^{-1}(d+ex)}{a+bx+cx^2}\,dx$$

Optimal antiderivative

$$\frac{\operatorname{arccot}(ex+d)\ln\left(\frac{2e\left(b+2cx-\sqrt{-4ac+b^2}\right)}{(1-\mathrm{I}(ex+d))\left(2c(\mathrm{I}-d)+e\left(b-\sqrt{-4ac+b^2}\right)\right)}\right)}{\sqrt{-4ac+b^2}} - \frac{\operatorname{arccot}(ex+d)\ln\left(\frac{2e\left(b+2cx+\sqrt{-4ac+b^2}\right)}{(1-\mathrm{I}(ex+d))\left(2c(\mathrm{I}-d)+e\left(b+\sqrt{-4ac+b^2}\right)\right)}\right)}{\sqrt{-4ac+b^2}} + \frac{\mathrm{I}\,\mathrm{polylog}\left(2,1+\frac{4cd-4c(ex+d)-2e\left(b-\sqrt{-4ac+b^2}\right)}{(1-\mathrm{I}(ex+d))\left(2\,\mathrm{I}c-2cd+be-e\sqrt{-4ac+b^2}\right)}\right)}{2\sqrt{-4ac+b^2}} - \frac{\mathrm{I}\,\mathrm{polylog}\left(2,1+\frac{4cd-4c(ex+d)-2e\left(b+\sqrt{-4ac+b^2}\right)}{(1-\mathrm{I}(ex+d))\left(2c(\mathrm{I}-d)+e\left(b+\sqrt{-4ac+b^2}\right)\right)}\right)}{2\sqrt{-4ac+b^2}}$$

command

`Integrate[ArcCot[d + e*x]/(a + b*x + c*x^2),x]`

Mathematica 13.1 output

$$\$\text{Aborted}$$

Mathematica 12.3 output

$$-\frac{i\left(\mathrm{Li}_2\left(\frac{e\left(-b-2cx+\sqrt{b^2-4ac}\right)}{2c(d-i)+\left(\sqrt{b^2-4ac}-b\right)e}\right)-\mathrm{Li}_2\left(\frac{e\left(-b-2cx+\sqrt{b^2-4ac}\right)}{2c(d+i)+\left(\sqrt{b^2-4ac}-b\right)e}\right)-\mathrm{Li}_2\left(\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{\left(b+\sqrt{b^2-4ac}\right)e-2c(d-i)}\right)+\mathrm{Li}_2}{}$$

14.2 Problem number 145

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx$$

Optimal antiderivative

Expression too large to display

command

```
Integrate[(a + b*ArcCot[c + d*x])^3/(e + f*x)^2,x]
```

Mathematica 13.1 output

$$\int \frac{\left(a+b\cot^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx$$

Mathematica 12.3 output

output too large to display

15 Test file number 187

Test folder name:

```
test_cases/7_Inverse_hyperbolic_functions/7.1_Inverse_hyperbolic_sine/187_7.1.4-f_x-
^m-d+e_x^2-^p-a+b_arcsinh-c_x-^n
```

15.1 Problem number 621

$$\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx$$

Optimal antiderivative

$$\frac{\text{hyperbolicCosineIntegral}\left(\frac{a+b\,\text{arcsinh}(cx)}{b}\right)\cosh\left(\frac{a}{b}\right)}{bc}-\frac{\text{hyperbolicSineIntegral}\left(\frac{a+b\,\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc}$$

command

```
Integrate[(a + b*ArcSinh[c*x])^(-1),x]
```

Mathematica 13.1 output

$$\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx$$

Mathematica 12.3 output

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}$$

16 Test file number 188

Test folder name:

`test_cases/7_Inverse_hyperbolic_functions/7.1_Inverse_hyperbolic_sine/188_7.1.5_Inverse_hyper`

16.1 Problem number 22

$$\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx$$

Optimal antiderivative

$$\frac{\text{hyperbolicCosineIntegral}\left(\frac{a+b\,\text{arcsinh}(cx)}{b}\right)\cosh\left(\frac{a}{b}\right)}{bc}-\frac{\text{hyperbolicSineIntegral}\left(\frac{a+b\,\text{arcsinh}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc}$$

command

```
Integrate[(a + b*ArcSinh[c*x])^(-1),x]
```

Mathematica 13.1 output

$$\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx$$

Mathematica 12.3 output

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}$$

16.2 Problem number 94

$$\int x^2\left(a+b\sinh^{-1}(c+dx)\right)^n\,dx$$

Optimal antiderivative

$$\frac{3^{-1-n}(a+b\operatorname{arcsinh}(dx+c))^n\,\Gamma\Big(1+n,-\frac{3(a+b\operatorname{arcsinh}(dx+c))}{b}\Big)\,\mathrm{e}^{-\frac{3a}{b}}\left(\frac{-a-b\operatorname{arcsinh}(dx+c)}{b}\right)^{-n}}{8d^3}$$
$$-\frac{2^{-2-n}c(a+b\operatorname{arcsinh}(dx+c))^n\,\Gamma\Big(1+n,-\frac{2(a+b\operatorname{arcsinh}(dx+c))}{b}\Big)\,\mathrm{e}^{-\frac{2a}{b}}\left(\frac{-a-b\operatorname{arcsinh}(dx+c)}{b}\right)^{-n}}{d^3}$$
$$-\frac{(a+b\operatorname{arcsinh}(dx+c))^n\,\Gamma\Big(1+n,\frac{-a-b\operatorname{arcsinh}(dx+c)}{b}\Big)\,\mathrm{e}^{-\frac{a}{b}}\left(\frac{-a-b\operatorname{arcsinh}(dx+c)}{b}\right)^{-n}}{8d^3}$$
$$+\frac{c^2(a+b\operatorname{arcsinh}(dx+c))^n\,\Gamma\Big(1+n,\frac{-a-b\operatorname{arcsinh}(dx+c)}{b}\Big)\,\mathrm{e}^{-\frac{a}{b}}\left(\frac{-a-b\operatorname{arcsinh}(dx+c)}{b}\right)^{-n}}{2d^3}$$
$$+\frac{\mathrm{e}^{\frac{a}{b}}(a+b\operatorname{arcsinh}(dx+c))^n\,\Gamma\Big(1+n,\frac{a+b\operatorname{arcsinh}(dx+c)}{b}\Big)\left(\frac{a+b\operatorname{arcsinh}(dx+c)}{b}\right)^{-n}}{8d^3}$$
$$-\frac{c^2\mathrm{e}^{\frac{a}{b}}(a+b\operatorname{arcsinh}(dx+c))^n\,\Gamma\Big(1+n,\frac{a+b\operatorname{arcsinh}(dx+c)}{b}\Big)\left(\frac{a+b\operatorname{arcsinh}(dx+c)}{b}\right)^{-n}}{2d^3}$$
$$-\frac{2^{-2-n}c\,\mathrm{e}^{\frac{2a}{b}}(a+b\operatorname{arcsinh}(dx+c))^n\,\Gamma\Big(1+n,\frac{2a+2b\operatorname{arcsinh}(dx+c)}{b}\Big)\left(\frac{a+b\operatorname{arcsinh}(dx+c)}{b}\right)^{-n}}{d^3}$$
$$-\frac{3^{-1-n}\mathrm{e}^{\frac{3a}{b}}(a+b\operatorname{arcsinh}(dx+c))^n\,\Gamma\Big(1+n,\frac{3a+3b\operatorname{arcsinh}(dx+c)}{b}\Big)\left(\frac{a+b\operatorname{arcsinh}(dx+c)}{b}\right)^{-n}}{8d^3}$$

command

```
Integrate[x^2*(a + b*ArcSinh[c + d*x])^n,x]
```

Mathematica 13.1 output

$$\int x^2\left(a+b\sinh^{-1}(c+dx)\right)^n\,dx$$

Mathematica 12.3 output

$$2^{-n-3}3^{-n-1}e^{-\frac{3a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{b^2}\right)^{-n}\left(\left(4c^2-1\right)2^n3^{n+1}e^{\frac{2a}{b}}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)^n\Gamma\Big(\right.$$

16.3 Problem number 208

$$\int \frac{1}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$$

Optimal antiderivative

$$\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{erf}\left(\frac{\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}}{\sqrt{b}}\right)\sqrt{\pi}}{2d\sqrt{b}}+\frac{\mathrm{erfi}\left(\frac{\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}}{\sqrt{b}}\right)\sqrt{\pi}\;\mathrm{e}^{-\frac{a}{b}}}{2d\sqrt{b}}$$

command

```
Integrate[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

Mathematica 13.1 output

$$\int \frac{1}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$$

Mathematica 12.3 output

$$\frac{e^{-\frac{a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)}{2d\sqrt{a+b\sinh^{-1}(c+dx)}}$$

16.4 Problem number 214

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$$

Optimal antiderivative

$$-\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{erf}\left(\frac{\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}}{\sqrt{b}}\right)\sqrt{\pi}}{b^{\frac{3}{2}}d}+\frac{\mathrm{erfi}\left(\frac{\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}}{\sqrt{b}}\right)\sqrt{\pi}\;\mathrm{e}^{-\frac{a}{b}}}{b^{\frac{3}{2}}d}-\frac{2\sqrt{1+(dx+c)^{2}}}{bd\sqrt{a+b\,\mathrm{arcsinh}(dx+c)}}$$

command

```
Integrate[(a + b*ArcSinh[c + d*x])^(-3/2),x]
```

Mathematica 13.1 output

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$$

Mathematica 12.3 output

$$\frac{e^{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\left(-e^{a/b}\left(e^{2\sinh^{-1}(c+dx)}+1\right)+e^{\frac{2a}{b}+\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+e\right.}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

16.5 Problem number 220

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$$

Optimal antiderivative

$$\frac{2\,\mathrm{e}^{\frac{a}{b}}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(dx+c)}}{\sqrt{b}}\right)\sqrt{\pi}}{3b^{\frac{5}{2}}d}+\frac{2\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(dx+c)}}{\sqrt{b}}\right)\sqrt{\pi}\,\mathrm{e}^{-\frac{a}{b}}}{3b^{\frac{5}{2}}d}-\frac{2\sqrt{1+(dx+c)^2}}{3bd\left(a+b\operatorname{arcsinh}(dx+c)\right)^{\frac{3}{2}}}-\frac{4(dx+c)}{3b^2d\sqrt{a+b\operatorname{arcsinh}(dx+c)}}$$

command

```
Integrate[(a + b*ArcSinh[c + d*x])^(-5/2),x]
```

Mathematica 13.1 output

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$$

Mathematica 12.3 output

$$\frac{e^{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\left(-e^{a/b}\left(2a\left(e^{2\sinh^{-1}(c+dx)}-1\right)-2b\sinh^{-1}(c+dx)+be^{2\sinh^{-1}(c+dx)}\left(2\sinh^{-1}(c+dx)+1\right)+b\right)-\right.}{}$$

17 Test file number 190

Test folder name:

`test_cases/7_Inverse_hyperbolic_functions/7.2_Inverse_hyperbolic_cosine/190_7.2.4-f_x-^m-d+e_x^2-^p-a+b_arccosh-c_x-^n`

17.1 Problem number 519

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right) \, dx$$

Optimal antiderivative

$$\frac{d^3(fx)^{1+m}\left(a+b\,\mathrm{arccosh}(cx)\right)}{f\,(1+m)}+\frac{3d^2e(fx)^{3+m}\left(a+b\,\mathrm{arccosh}(cx)\right)}{f^3\,(3+m)}$$
$$+\frac{3d\,e^2(fx)^{5+m}\left(a+b\,\mathrm{arccosh}(cx)\right)}{f^5\,(5+m)}+\frac{e^3(fx)^{7+m}\left(a+b\,\mathrm{arccosh}(cx)\right)}{f^7\,(7+m)}$$
$$+\frac{be\left(3c^2de(7+m)^2\left(m^2+7m+12\right)+3c^4d^2\left(m^2+12m+35\right)^2+e^2\left(m^4+18m^3+119m^2+342m+360\right)\right)(fx)^{2+}}{c^5f^2\,(3+m)^2\,(5+m)^2\,(7+m)^2\,\sqrt{cx-1}\,\sqrt{cx+1}}$$
$$+\frac{b\,e^2\left(3c^2d(7+m)^2+e\left(m^2+11m+30\right)\right)(fx)^{4+m}\left(-c^2x^2+1\right)}{c^3f^4\,(5+m)^2\,(7+m)^2\,\sqrt{cx-1}\,\sqrt{cx+1}}$$
$$+\frac{b\,e^3(fx)^{6+m}\left(-c^2x^2+1\right)}{c\,f^6\,(7+m)^2\,\sqrt{cx-1}\,\sqrt{cx+1}}$$
$$-\frac{b\left(\frac{c^6d^3(3+m)(5+m)(7+m)}{1+m}+\frac{e(2+m)\left(3c^2de(7+m)^2\left(m^2+7m+12\right)+3c^4d^2\left(m^2+12m+35\right)^2+e^2\left(m^4+18m^3+119m^2+342m+360\right)\right)}{m^3+15m^2+71m+105}\right)(fx)^{}}{c^5f^2\,(2+m)\,(3+m)\,(5+m)\,(7+m)\,\sqrt{cx-1}\,\sqrt{cx+1}}$$

command

```
Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

Mathematica 13.1 output

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right) \, dx$$

Mathematica 12.3 output

$$x(fx)^m\left(\frac{d^3(a+b\cosh^{-1}(cx))}{m+1}+\frac{3d^2ex^2(a+b\cosh^{-1}(cx))}{m+3}+\frac{3de^2x^4(a+b\cosh^{-1}(cx))}{m+5}\right.$$
$$+\frac{e^3x^6(a+b\cosh^{-1}(cx))}{m+7}-\frac{bcd^3x\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{(m^2+3m+2)\sqrt{cx-1}\sqrt{cx+1}}$$
$$-\frac{3bcd^2ex^3\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};c^2x^2\right)}{(m^2+7m+12)\sqrt{cx-1}\sqrt{cx+1}}$$
$$-\frac{3bcde^2x^5\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+6}{2};\frac{m+8}{2};c^2x^2\right)}{(m+5)(m+6)\sqrt{cx-1}\sqrt{cx+1}}$$
$$\left.-\frac{bce^3x^7\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m}{2}+4;\frac{m}{2}+5;c^2x^2\right)}{(m+7)(m+8)\sqrt{cx-1}\sqrt{cx+1}}\right)$$

17.2 Problem number 520

$$\int (fx)^m\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)\,dx$$

Optimal antiderivative

$$\frac{d^2(fx)^{1+m}(a+b\,\text{arccosh}(cx))}{f(1+m)}+\frac{2de(fx)^{3+m}(a+b\,\text{arccosh}(cx))}{f^3(3+m)}+\frac{e^2(fx)^{5+m}(a+b\,\text{arccosh}(cx))}{f^5(5+m)}$$
$$+\frac{be\left(2c^2d(5+m)^2+e(m^2+7m+12)\right)(fx)^{2+m}(-c^2x^2+1)}{c^3f^2(3+m)^2(5+m)^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{b\,e^2(fx)^{4+m}(-c^2x^2+1)}{c\,f^4(5+m)^2\sqrt{cx-1}\sqrt{cx+1}}$$
$$-\frac{b\left(\frac{c^4d^2(3+m)(5+m)}{1+m}+\frac{e(2+m)\left(2c^2d(5+m)^2+e(m^2+7m+12)\right)}{(3+m)(5+m)}\right)(fx)^{2+m}\,\text{hypergeom}\left(\left[\frac{1}{2},1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right],c^2x^2\right)\sqrt{-c^2x^2}}{c^3f^2(2+m)(3+m)(5+m)\sqrt{cx-1}\sqrt{cx+1}}$$

command

```
Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

Mathematica 13.1 output

$$\int (fx)^m\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)\,dx$$

Mathematica 12.3 output

$$x(fx)^m\left(\frac{d^2(a+b\cosh^{-1}(cx))}{m+1}+\frac{2dex^2(a+b\cosh^{-1}(cx))}{m+3}+\frac{e^2x^4(a+b\cosh^{-1}(cx))}{m+5}\right.$$
$$-\frac{bcd^2x\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{(m^2+3m+2)\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bcdex^3\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};c^2x^2\right)}{(m^2+7m+12)\sqrt{cx-1}\sqrt{cx+1}}$$
$$\left.-\frac{bce^2x^5\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+6}{2};\frac{m+8}{2};c^2x^2\right)}{(m+5)(m+6)\sqrt{cx-1}\sqrt{cx+1}}\right)$$

17.3 Problem number 521

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\cosh^{-1}(cx)\right) \, dx$$

Optimal antiderivative

$$\frac{d(fx)^{1+m}\left(a+b\operatorname{arccosh}(cx)\right)}{f\,(1+m)}+\frac{e(fx)^{3+m}\left(a+b\operatorname{arccosh}(cx)\right)}{f^3\,(3+m)}-\frac{be(fx)^{2+m}\sqrt{cx-1}\,\sqrt{cx+1}}{c\,f^2\,(3+m)^2}$$
$$-\frac{b\Big(e(1+m)\,(2+m)+c^2d(3+m)^2\Big)\,(fx)^{2+m}\operatorname{hypergeom}\left(\left[\frac{1}{2},1+\frac{m}{2}\right],\left[2+\frac{m}{2}\right],c^2x^2\right)\sqrt{-c^2x^2+1}}{c\,f^2\,(1+m)\,(2+m)\,(3+m)^2\,\sqrt{cx-1}\,\sqrt{cx+1}}$$

command

```
Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

Mathematica 13.1 output

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\cosh^{-1}(cx)\right) \, dx$$

Mathematica 12.3 output

$$x(fx)^m\left(\frac{\frac{\left(d(m+3)+e(m+1)x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{m+1}-\frac{bcex^3\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};c^2x^2\right)}{(m+4)\sqrt{cx-1}\,\sqrt{cx+1}}}{m+3}\right.$$
$$\left.-\frac{bcdx\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{(m^2+3m+2)\sqrt{cx-1}\,\sqrt{cx+1}}\right)$$

18 Test file number 191

Test folder name:

test_cases/7_Inverse_hyperbolic_functions/7.2_Inverse_hyperbolic_cosine/191_7.2.5_Inverse_hyp

18.1 Problem number 178

$$\int \frac{1}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$$

Optimal antiderivative

$$-\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{erf}\left(\frac{\sqrt{a+b\,\mathrm{arccosh}\,(dx+c)}}{\sqrt{b}}\right)\sqrt{\pi}}{2d\sqrt{b}}+\frac{\mathrm{erfi}\left(\frac{\sqrt{a+b\,\mathrm{arccosh}\,(dx+c)}}{\sqrt{b}}\right)\sqrt{\pi}\,\mathrm{e}^{-\frac{a}{b}}}{2d\sqrt{b}}$$

command

```
Integrate[1/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

Mathematica 13.1 output

$$\int \frac{1}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$$

Mathematica 12.3 output

$$\frac{e^{-\frac{a}{b}}\left(e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)\right)}{2d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

19 Test file number 197

Test folder name:

test_cases/7_Inverse_hyperbolic_functions/7.3_Inverse_hyperbolic_tangent/197_7.3.7_Inverse_hy

19.1 Problem number 4

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x}\,dx$$

Optimal antiderivative

$$\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)\ln(x)-\frac{\operatorname{arcsinh}\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)^2\sqrt{d}\,\sqrt{1+\frac{e\,x^2}{d}}}{2\sqrt{e\,x^2+d}}$$
$$+\frac{\operatorname{arcsinh}\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)\ln\left(1-\left(\frac{x\sqrt{e}}{\sqrt{d}}+\sqrt{1+\frac{e\,x^2}{d}}\right)^2\right)\sqrt{d}\,\sqrt{1+\frac{e\,x^2}{d}}}{\sqrt{e\,x^2+d}}$$
$$-\frac{\operatorname{arcsinh}\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)\ln(x)\sqrt{d}\,\sqrt{1+\frac{e\,x^2}{d}}}{\sqrt{e\,x^2+d}}$$
$$+\frac{\operatorname{polylog}\left(2,\left(\frac{x\sqrt{e}}{\sqrt{d}}+\sqrt{1+\frac{e\,x^2}{d}}\right)^2\right)\sqrt{d}\,\sqrt{1+\frac{e\,x^2}{d}}}{2\sqrt{e\,x^2+d}}$$

command

```
Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x,x]
```

Mathematica 13.1 output

$$\int\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x}\,dx$$

Mathematica 12.3 output

$$\frac{\sqrt{e}\sqrt{\frac{ex^2}{d}+1}\left(-\mathrm{Li}_2\left(e^{-2\sinh^{-1}\left(\sqrt{\frac{e}{d}}\,x\right)}\right)-2\log(x)\log\left(\sqrt{\frac{ex^2}{d}+1}+x\sqrt{\frac{e}{d}}\right)+\sinh^{-1}\left(x\sqrt{\frac{e}{d}}\right)^2+2\sinh^{-1}\left(\right.\right.}{2\sqrt{\frac{e}{d}}\sqrt{d+ex^2}}$$
$$+\log(x)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

20 Test file number 201

Test folder name:

`test_cases/7_Inverse_hyperbolic_functions/7.5_Inverse_hyperbolic_secant/201_7.5.2_Inverse_hyp`

20.1 Problem number 18

$$\int \frac{\operatorname{sech}^{-1}(a+bx)^3}{x^2}\,dx$$

Optimal antiderivative

$$-\frac{b\,\mathrm{arcsech}(bx+a)^3}{a}-\frac{\mathrm{arcsech}(bx+a)^3}{x}+\frac{3b\,\mathrm{arcsech}(bx+a)^2\ln\left(1-\frac{a\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{1+\frac{1}{bx+a}}\right)}{1-\sqrt{-a^2+1}}\right)}{a\sqrt{-a^2+1}}-\frac{3b\,\mathrm{arcsech}(bx+a)^2\ln\left(1-\frac{a\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{1+\frac{1}{bx+a}}\right)}{1+\sqrt{-a^2+1}}\right)}{a\sqrt{-a^2+1}}+\frac{6b\,\mathrm{arcsech}(bx+a)\,\mathrm{polylog}\left(2,\frac{a\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{1+\frac{1}{bx+a}}\right)}{1-\sqrt{-a^2+1}}\right)}{a\sqrt{-a^2+1}}-\frac{6b\,\mathrm{arcsech}(bx+a)\,\mathrm{polylog}\left(2,\frac{a\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{1+\frac{1}{bx+a}}\right)}{1+\sqrt{-a^2+1}}\right)}{a\sqrt{-a^2+1}}-\frac{6b\,\mathrm{polylog}\left(3,\frac{a\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{1+\frac{1}{bx+a}}\right)}{1-\sqrt{-a^2+1}}\right)}{a\sqrt{-a^2+1}}+\frac{6b\,\mathrm{polylog}\left(3,\frac{a\left(\frac{1}{bx+a}+\sqrt{\frac{1}{bx+a}-1}\,\sqrt{1+\frac{1}{bx+a}}\right)}{1+\sqrt{-a^2+1}}\right)}{a\sqrt{-a^2+1}}$$

command

```
Integrate[ArcSech[a + b*x]^3/x^2,x]
```

Mathematica 13.1 output

$Aborted

Mathematica 12.3 output

output too large to display

20.2 Problem number 58

$$\int e^{\operatorname{sech}^{-1}(ax)} x^m \, dx$$

Optimal antiderivative

$$\frac{x^m}{am(1+m)} + \frac{\left(\frac{1}{ax} + \sqrt{\frac{1}{ax}-1}\,\sqrt{1+\frac{1}{ax}}\right) x^{1+m}}{1+m} + \frac{x^m \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{m}{2}\right], \left[1+\frac{m}{2}\right], a^2x^2\right) \sqrt{\frac{1}{ax+1}}\,\sqrt{ax+1}}{am(1+m)}$$

command

```
Integrate[E^ArcSech[a*x]*x^m,x]
```

Mathematica 13.1 output

$$\int e^{\operatorname{sech}^{-1}(ax)} x^m \, dx$$

Mathematica 12.3 output

$$-\frac{2^{m+1}x^m(ax)^{-m}e^{2\operatorname{sech}^{-1}(ax)}\left(\frac{e^{\operatorname{sech}^{-1}(ax)}}{e^{2\operatorname{sech}^{-1}(ax)}+1}\right)^m \left(e^{2\operatorname{sech}^{-1}(ax)}+1\right)^m \left((m+2)e^{2\operatorname{sech}^{-1}(ax)}\,{}_2F_1\left(\frac{m}{2}+2, m+2; \frac{m}{2}+3;\right.\right.}{a(m+2)(m+4)}$$

21 Test file number 209

Test folder name:

`test_cases/209_Blake_problems`

21.1 Problem number 3046

$$\int \frac{x^4}{\sqrt[4]{b+ax^4}\,(b+2ax^4+2x^8)}\,dx$$

Optimal antiderivative

$$\frac{\left(-1+(-1)^{\frac{1}{4}}\right)\arctan\left(\frac{(-1)^{\frac{7}{8}}\sqrt{2+\sqrt{2}}\,(a^2-2b)^{\frac{1}{8}}x(a\,x^4+b)^{\frac{1}{4}}}{(-1)^{\frac{3}{4}}(a^2-2b)^{\frac{1}{4}}x^2+\sqrt{a\,x^4+b}}\right)}{8\left(a^2-2b\right)^{\frac{5}{8}}}$$
$$+\frac{I\left(I\sqrt{2}+2+\sqrt{2}\right)\arctan\left(\frac{(-1)^{\frac{7}{8}}\left(-2+\sqrt{2}\right)(a^2-2b)^{\frac{1}{8}}x(a\,x^4+b)^{\frac{1}{4}}}{(-1)^{\frac{3}{4}}\sqrt{2-\sqrt{2}}\,(a^2-2b)^{\frac{1}{4}}x^2+\sqrt{2-\sqrt{2}}\,\sqrt{a\,x^4+b}}\right)}{16\left(a^2-2b\right)^{\frac{5}{8}}}$$
$$+\frac{\left(\sqrt{2}-I\left(2+\sqrt{2}\right)\right)\operatorname{arctanh}\left(\frac{(-1)^{\frac{7}{8}}(a^2-2b)^{\frac{1}{4}}x^2-(-1)^{\frac{1}{8}}\sqrt{a\,x^4+b}}{\sqrt{2-\sqrt{2}}\,(a^2-2b)^{\frac{1}{8}}x(a\,x^4+b)^{\frac{1}{4}}}\right)}{16\left(a^2-2b\right)^{\frac{5}{8}}}$$
$$+\frac{\left(-1+(-1)^{\frac{1}{4}}\right)\operatorname{arctanh}\left(\frac{(-1)^{\frac{7}{8}}(a^2-2b)^{\frac{1}{4}}x^2-(-1)^{\frac{1}{8}}\sqrt{a\,x^4+b}}{\sqrt{2+\sqrt{2}}\,(a^2-2b)^{\frac{1}{8}}x(a\,x^4+b)^{\frac{1}{4}}}\right)}{8\left(a^2-2b\right)^{\frac{5}{8}}}$$

command

```
Integrate[x^4/((b + a*x^4)^(1/4)*(b + 2*a*x^4 + 2*x^8)),x]
```

Mathematica 13.1 output

$$\int \frac{x^4}{\sqrt[4]{b+ax^4}\,(b+2ax^4+2x^8)}\,dx$$

Mathematica 12.3 output

$$\frac{-\frac{\sqrt[4]{a-\sqrt{a^2-2b}}\tan^{-1}\left(\frac{x\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{a-\sqrt{a^2-2b}}\sqrt[4]{ax^4+b}}\right)}{\sqrt[4]{-a\sqrt{a^2-2b}+a^2-2b}}+\frac{\sqrt[4]{\sqrt{a^2-2b}+a}\tan^{-1}\left(\frac{x\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}{\sqrt[4]{\sqrt{a^2-2b}+a}\sqrt[4]{ax^4+b}}\right)}{\sqrt[4]{a\sqrt{a^2-2b}+a^2-2b}}}{4}$$

22 Test file number 210

Test folder name:

`test_cases/210_Hebisch`

22.1 Problem number 8

$$\int \frac{(-8-3x)\log^2(x)+(-4-x)\log^2(x)\log\left(4x^2+x^3\right)+\left(-8-2x+(-8-2x)\log(x)+\left(20x^2+5x^3\right)\log^2(x)\right)\log}{(4x+x^2)\log^2(x)\log\left(4x^2+x^3\right)+\left((8x+2x^2)\log(x)+(8x^2+22x^3+5x^4)\log^2(x)\right)\log^2\left(4x^2+x^3\right)}$$

Optimal antiderivative

$$\ln\left(5x+\frac{\frac{2+\frac{\ln(x)}{\ln\left(x^2(4+x)\right)}}{\ln(x)}+2x}{x}\right)$$

command

```
Integrate[((-8 - 3*x)*Log[x]^2 + (-4 - x)*Log[x]^2*Log[4*x^2 + x^3] + (-8 - 2*x + (-
8 - 2*x)*Log[x] + (20*x^2 + 5*x^3)*Log[x]^2)*Log[4*x^2 + x^3]^2)/((4*x + x^2)*Log[x]^2*Log[4*
```

Mathematica 13.1 output

$$\$Aborted$$

Mathematica 12.3 output

$$\log(2+5x)-\log(x(2+5x))-\log(\log(x))-\log\left(\log\left(x^2(4+x)\right)\right)$$
$$+\log\left(\log(x)+2\log\left(x^2(4+x)\right)+2x\log(x)\log\left(x^2(4+x)\right)+5x^2\log(x)\log\left(x^2(4+x)\right)\right)$$

22.2 Problem number 19

$$\int \frac{}{-1728x^3+1728x^4-576x^5+64x^6+(864x^2-576x^3+96x^4)\log(4)+(-144x+48x^2)\log^2(4)+8\log^3(4)+e^{3x}}$$

Optimal antiderivative

$$\left(\frac{e^x}{(2\ln(2)-x(6-2x))(2+e^x)}+1\right)^2$$

command

```
Integrate[(E^(3*x)*(12 - 80*x + 72*x^2 - 16*x^3 + (12 - 8*x)*Log[4]) + E^(2*x)*(24 - 328*x + 
288*x + 576*x^2 - 256*x^3 + 32*x^4 + (48 - 128*x + 32*x^2)*Log[4] + 8*Log[4]^2))/(-
1728*x^3 + 1728*x^4 - 576*x^5 + 64*x^6 + (864*x^2 - 576*x^3 + 96*x^4)*Log[4] + (-144*x + 48*x
216*x^3 + 216*x^4 - 72*x^5 + 8*x^6 + (108*x^2 - 72*x^3 + 12*x^4)*Log[4] + (-18*x + 6*x^2)*Log
1296*x^3 + 1296*x^4 - 432*x^5 + 48*x^6 + (648*x^2 - 432*x^3 + 72*x^4)*Log[4] + (-108*x + 36*x
2592*x^3 + 2592*x^4 - 864*x^5 + 96*x^6 + (1296*x^2 - 864*x^3 + 144*x^4)*Log[4] + (-
216*x + 72*x^2)*Log[4]^2 + 12*Log[4]^3)),x]
```

Mathematica 13.1 output

$$\int \frac{}{-1728x^3+1728x^4-576x^5+64x^6+(864x^2-576x^3+96x^4)\log(4)+(-144x+48x^2)\log^2(4)+8\log^3(4)+e^{3x}}$$

Mathematica 12.3 output

$$-\frac{e^x\left(-e^x\left(-6x+2x^2+\log(4)\right)^2\left(1-12x+4x^2+\log(16)\right)-4\left(-72x^5+8x^6+\log^3(4)-4x^3(54+28\log(4)-5\log\right.\right.}{(2+e^x)^2\,(-}$$

22.3 Problem number 38

$$\int \frac{-x^3\log(x)+8e^{e^x+x}x^3\log(x)+8e^{2e^x+x}x^3\log(x)+\left(x^2-x^3+\left(-2x^2+2x^3\right)\log(x)+e^{e^x}\left(8x^2-16x^2\log(x)\right)+\right.}{x^5+8e^{e^x}x^5+4e^{2e^x}x^5-x^6+\left(4x^3+32e^{e^x}x^3+16e^{2e^x}x^3-4x^4\right)\log\left(\frac{1}{4}\left(-1-8\right.\right.}$$

Optimal antiderivative

$$\frac{\ln(x)}{2+\frac{x^2}{\ln\left(\frac{3}{4}+\frac{x}{4}-\left(e^{e^x}+1\right)^2\right)}}$$

command

```
Integrate[(-(x^3*Log[x]) + 8*E^(E^x + x)*x^3*Log[x] + 8*E^(2*E^x + x)*x^3*Log[x] + (x^2 - x^3
2*x^2 + 2*x^3)*Log[x] + E^E^x*(8*x^2 - 16*x^2*Log[x]) + E^(2*E^x)*(4*x^2 - 8*x^2*Log[x]))*Log
1 - 8*E^E^x - 4*E^(2*E^x) + x)/4] + (2 + 16*E^E^x + 8*E^(2*E^x) - 2*x)*Log[(-1 - 8*E^E^x - 4*
1 - 8*E^E^x - 4*E^(2*E^x) + x)/4] + (4*x + 32*E^E^x*x + 16*E^(2*E^x)*x - 4*x^2)*Log[(-
1 - 8*E^E^x - 4*E^(2*E^x) + x)/4]^2),x]
```

Mathematica 13.1 output

$$\int \frac{-x^3\log(x)+8e^{e^x+x}x^3\log(x)+8e^{2e^x+x}x^3\log(x)+\left(x^2-x^3+\left(-2x^2+2x^3\right)\log(x)+e^{e^x}\left(8x^2-16x^2\log(x)\right)+\right.}{x^5+8e^{e^x}x^5+4e^{2e^x}x^5-x^6+\left(4x^3+32e^{e^x}x^3+16e^{2e^x}x^3-4x^4\right)\log\left(\frac{1}{4}\left(-1-8\right.\right.}$$

Mathematica 12.3 output

$$\frac{\log(x)\left(2\left(8e^{e^x+x}+8e^{2e^x+x}+8e^{e^x}x+4e^{2e^x}x-x^2\right)\log\left(\frac{1}{4}\left(-1-8e^{e^x}-4e^{2e^x}+x\right)\right)-(-1+x)\left(\log(16)-2\log\left(-\right.\right.\right.}{2\left(-1+8e^{e^x+x}+8e^{2e^x+x}+x+8e^{e^x}x+4e^{2e^x}x-x^2\right)\left(x^2+2\log\left(\frac{1}{4}\left(-1-8e^{e^x}-4e^{2e^x}+x\right)\right.\right.}$$

22.4 Problem number 91

$$\int \frac{9-21x+6x^2+(3-6x)\log(4)+(-9+9x+(-3+3x)\log(4))\log\left(-x+x^2\right)+\left(-9+15x-7x^2+x^3+(-6\right.}{\left(-9+15x-7x^2+x^3+\left(-6+8x-2x^2\right)\log(4)+(-1+x)\log^2(4)\right)\log}$$

Optimal antiderivative

$$\frac{3x}{\ln(x^2-x)\left(2\ln(2)+3-x\right)}+x-1$$

command

```
Integrate[(9 - 21*x + 6*x^2 + (3 - 6*x)*Log[4] + (-9 + 9*x + (-3 + 3*x)*Log[4])*Log[-
x + x^2] + (-9 + 15*x - 7*x^2 + x^3 + (-6 + 8*x - 2*x^2)*Log[4] + (-1 + x)*Log[4]^2)*Log[-
x + x^2]^2)/((-9 + 15*x - 7*x^2 + x^3 + (-6 + 8*x - 2*x^2)*Log[4] + (-1 + x)*Log[4]^2)*Log[-
x + x^2]^2),x]
```

Mathematica 13.1 output

$$\int \frac{9-21x+6x^2+(3-6x)\log(4)+(-9+9x+(-3+3x)\log(4))\log\left(-x+x^2\right)+\left(-9+15x-7x^2+x^3+(-6\right.}{\left(-9+15x-7x^2+x^3+\left(-6+8x-2x^2\right)\log(4)+(-1+x)\log^2(4)\right)\log}$$

Mathematica 12.3 output

$$x-\frac{x(9-21x+6x^2-3x\log(16)+\log(64))}{(-1+2x)(-3+x-\log(4))^2\log((-1+x)x)}$$

22.5 Problem number 294

$$\int \frac{2e^x x-4x^3+\left(-4x+4x^2\right)\log(16)+\left(-4x^2+e^x(2x-2\log(16))+4x\log(16)\right)\log\left(\frac{e^{-x}(e^x-2x)}{-2x+2\log(16)}\right)}{2x^4-2x^3\log(16)+e^x\left(-x^3+x^2\log(16)\right)}\,dx$$

Optimal antiderivative

$$\frac{2\ln\left(\frac{\left(\frac{e^x}{2}-x\right)e^{-x}}{4\ln(2)-x}\right)}{x}$$

command

```
Integrate[(2*E^x*x - 4*x^3 + (-4*x + 4*x^2)*Log[16] + (-4*x^2 + E^x*(2*x - 2*Log[16]) + 4*x*L
2*x + 2*Log[16]))])/(2*x^4 - 2*x^3*Log[16] + E^x*(-x^3 + x^2*Log[16])),x]
```

Mathematica 13.1 output

$$\int \frac{2e^x x-4x^3+\left(-4x+4x^2\right)\log(16)+\left(-4x^2+e^x(2x-2\log(16))+4x\log(16)\right)\log\left(\frac{e^{-x}(e^x-2x)}{-2x+2\log(16)}\right)}{2x^4-2x^3\log(16)+e^x\left(-x^3+x^2\log(16)\right)}\,dx$$

Mathematica 12.3 output

$$\frac{2\log(x)}{\log(16)}-\frac{4\log(x)}{\log(256)}-\frac{2\log(x-\log(16))}{\log(16)}+\frac{2\log\left(-\frac{e^{-x}(e^x-2x)}{2x-\log(256)}\right)}{x}+\frac{4\log(2x-\log(256))}{\log(256)}$$

22.6 Problem number 300

$$\int \frac{-x^3 + e^{-2+x+\frac{e^{-2+x}\left(1+e^{5/x}x^2\right)}{x^2}}\left((2-x)\log(2) + e^{5/x}\left(5x - x^3\right)\log(2)\right)}{25x^3 + 10x^4 + x^5 + e^{\frac{e^{-2+x}\left(1+e^{5/x}x^2\right)}{x^2}}\left(10x^3 + 2x^4\right)\log(2) + e^{\frac{2e^{-2+x}\left(1+e^{5/x}x^2\right)}{x^2}}x^3\log^2(2)}\,dx$$

Optimal antiderivative

$$\frac{1}{5 + \ln(2)\,\mathrm{e}^{\left(\mathrm{e}^{\frac{5}{x}} + \frac{1}{x^2}\right)\mathrm{e}^{-2+x}} + x}$$

command

```
Integrate[(-x^3 + E^(-2 + x + (E^(-2 + x)*(1 + E^(5/x)*x^2))/x^2)*((2 - x)*Log[2] + E^(5/x)*(
2 + x)*(1 + E^(5/x)*x^2))/x^2)*(10*x^3 + 2*x^4)*Log[2] + E^((2*E^(-2 + x)*(1 + E^(5/x)*x^2))/
```

Mathematica 13.1 output

$Aborted

Mathematica 12.3 output

$$\frac{-e^4x^3\log(2) + e^{2+x}(5+x)(x\log(2) - \log(4)) + e^{2+\frac{5}{x}+x}x(5+x)\left(x^2\log(2) - \log(32)\right)}{e^2\left(-e^2x^3 + e^x\left(-10+3x+x^2\right) + e^{\frac{5}{x}+x}x\left(-25-5x+5x^2+x^3\right)\right)\log(2)\left(5+x+e^{e^{-2+x}\left(e^{5/x}+\frac{1}{x^2}\right)}\log(2)\right)}$$

22.7 Problem number 358

$$\int \frac{4e^{e^5}x + 2x^2 + \left(-10x^2 - 2x^3 - 5x^4 - x^5 + \left(-2x^2 - x^4\right)\log(3) + e^{2e^5}\left(-20x^2 - 4x^3 - 4x^2\log(3)\right) + e^{e^5}\left(-20x^3\right.\right.}{\left(5x^4 + x^5 + x^4\log(3) + e^{2e^5}\left(20x^2 + 4x^3 + 4x^2\log(3)\right) + e^{e^5}\left(20x^3 + 4x^4 + 4x^3\right.\right.}$$

Optimal antiderivative

$$-1 + \frac{x}{\left(\mathrm{e}^{\mathrm{e}^5} + \frac{x}{2}\right)\left(x - \ln(\ln(\ln(3) + 5 + x))\right)} - x$$

command

```
Integrate[(4*E^E^5*x + 2*x^2 + (-10*x^2 - 2*x^3 - 5*x^4 - x^5 + (-2*x^2 - x^4)*Log[3] + E^(2*
20*x^2 - 4*x^3 - 4*x^2*Log[3]) + E^E^5*(-20*x^3 - 4*x^4 - 4*x^3*Log[3]))*Log[5 + x + Log[3]]
20 - 4*x + 40*x^2 + 8*x^3 + (-4 + 8*x^2)*Log[3]))*Log[5 + x + Log[3]]*Log[Log[5 + x + Log[3]]
5*x^2 - x^3 + E^(2*E^5)*(-20 - 4*x - 4*Log[3]) - x^2*Log[3] + E^E^5*(-20*x - 4*x^2 - 4*x*Log[
10*x^3 - 2*x^4 - 2*x^3*Log[3] + E^(2*E^5)*(-40*x - 8*x^2 - 8*x*Log[3]) + E^E^5*(-40*x^2 - 8*x
```

Mathematica 13.1 output

$$\int \frac{4e^{e^5}x+2x^2+\left(-10x^2-2x^3-5x^4-x^5+\left(-2x^2-x^4\right)\log(3)+e^{2e^5}\left(-20x^2-4x^3-4x^2\log(3)\right)+e^{e^5}\left(-20x^3\right.\right.}{\left(5x^4+x^5+x^4\log(3)+e^{2e^5}\left(20x^2+4x^3+4x^2\log(3)\right)+e^{e^5}\left(20x^3+4x^4+4x^3\right.\right.}$$

Mathematica 12.3 output

$$x\left(-1+\frac{-2\left(2e^{e^5}+x\right)+\left(x(10+2x+\log(9))+e^{e^5}(20+4x+\log(81))\right)\log(5+x+\log(3))}{\left(2e^{e^5}+x\right)^2(-1+(5+x+\log(3))\log(5+x+\log(3)))(x-\log(\log(5+x+\log(3))))}\right)$$

22.8 Problem number 379

$$\int \frac{-4x^6+2x^6\log(2)+e^{4x^2}\left(-4x^2+2x^2\log(2)\right)+e^{3x^2}\left(-16x^3+8x^3\log(2)\right)+\left(-4x^3-3x^2\log(2)\right)\log(4)+e^{2x^2}\left(\right.}{16x^6+16x^7+4x^8+e^{4x^2}\left(16x^2+16x^3+4x^4\right)+e^{3x^2}\left(64x^3+64x^4+16x^5\right)+\left(8x^3+4x^4\right)}$$

Optimal antiderivative

$$\frac{-x-\ln(2)}{2x+\frac{2\ln(2)}{\left(e^{x^2}+x\right)^2x}+4}$$

command

```
Integrate[(-4*x^6 + 2*x^6*Log[2] + E^(4*x^2)*(-4*x^2 + 2*x^2*Log[2]) + E^(3*x^2)*(-
16*x^3 + 8*x^3*Log[2]) + (-4*x^3 - 3*x^2*Log[2])*Log[4] + E^(2*x^2)*(-24*x^4 + 12*x^4*Log[2]
2*x - 4*x^3 + (-1 - 4*x^2)*Log[2])*Log[4]) + E^x^2*(-16*x^5 + 8*x^5*Log[2] + (-6*x^2 - 4*x^4
4*x - 4*x^3)*Log[2])*Log[4]))/(16*x^6 + 16*x^7 + 4*x^8 + E^(4*x^2)*(16*x^2 + 16*x^3 + 4*x^4)
```

Mathematica 13.1 output

$$\int \frac{-4x^6+2x^6\log(2)+e^{4x^2}\left(-4x^2+2x^2\log(2)\right)+e^{3x^2}\left(-16x^3+8x^3\log(2)\right)+\left(-4x^3-3x^2\log(2)\right)\log(4)+e^{2x^2}\left(\right.}{16x^6+16x^7+4x^8+e^{4x^2}\left(16x^2+16x^3+4x^4\right)+e^{3x^2}\left(64x^3+64x^4+16x^5\right)+\left(8x^3+4x^4\right)}$$

Mathematica 12.3 output

$$\frac{2\log^3(4)+4x\log^3(4)+18x^2\log^3(4)-2e^{2x^2}x\log(4)\left(16x^7(-4+\log(4))+88x^8(-4+\log(4))+48x^9(-4+\log(4))\right.}{}$$

22.9 Problem number 414

$$\int \frac{e^{2e^x}\left(6-3e^4\right)+12x^2-3e^4x^2-3x^2\log(2)+\left(-16x+6e^4x+2x\log(2)\right)\log(3)+\left(6-3e^4\right)\log^2(3)+e^{e^x}\left(-16x\right.}{e^{2e^x}x^4+x^6-2x^5\log(3)+x^4\log^2(3)+e^{e^x}\left(-2x^5+2x^4\right.}$$

Optimal antiderivative

$$\frac{\frac{(2-\ln(2))x}{\ln(3)+e^{e^x}-x}+e^4-2}{x^3}$$

command

```
Integrate[(E^(2*E^x)*(6 - 3*E^4) + 12*x^2 - 3*E^4*x^2 - 3*x^2*Log[2] + (-16*x + 6*E^4*x + 2*x
16*x + 6*E^4*x + 2*x*Log[2] + E^x*(-2*x^2 + x^2*Log[2]) + (12 - 6*E^4)*Log[3]))/(E^(2*E^x)*x^
2*x^5 + 2*x^4*Log[3])),x]
```

Mathematica 13.1 output

$$\int \frac{e^{2e^x}(6-3e^4)+12x^2-3e^4x^2-3x^2\log(2)+(-16x+6e^4x+2x\log(2))\log(3)+(6-3e^4)\log^2(3)+e^{e^x}(-16x}{e^{2e^x}x^4+x^6-2x^5\log(3)+x^4\log^2(3)+e^{e^x}(-2x^5+2x^4}$$

Mathematica 12.3 output

$$\frac{-2+e^4-\frac{x(2-\log(2)+e^x(x(-2+\log(2))-\log(2)\log(3)+\log(9)))}{(-1+e^x(x-\log(3)))(e^{e^x}-x+\log(3))}}{x^3}$$

22.10 Problem number 578

$$\int \frac{((4ex^3-8x^4)\log(3)+(4ex-8x^2)\log^2(3))\log^3(x)+(2ex^3-4x^4)\log^2(-e+2x)+\log^2(x)(-4x^5-4x^3\log(3)}{(e-2x)\log^3(3)\log^3(x)+(-3ex+6x^2)\log^2(3)\,l}$$

Optimal antiderivative

$$\left(1-\frac{x}{\frac{\ln(-e+2x)}{\ln(x)}-\frac{\ln(3)}{x}}\right)^2$$

command

```
Integrate[(((4*E*x^3 - 8*x^4)*Log[3] + (4*E*x - 8*x^2)*Log[3]^2)*Log[x]^3 + (2*E*x^3 - 4*x^4)
E + 2*x]^2 + Log[x]^2*(-4*x^5 - 4*x^3*Log[3] + (-2*E*x^4 + 4*x^5 + (-6*E*x^2 + 12*x^3)*Log[3]
E + 2*x]) + Log[x]*((4*x^4 - 2*E*x^4 + 4*x^5 + (-2*E*x^2 + 4*x^3)*Log[3])*Log[-E + 2*x] + (2*
E + 2*x]^2))/((E - 2*x)*Log[3]^3*Log[x]^3 + (-3*E*x + 6*x^2)*Log[3]^2*Log[x]^2*Log[-
E + 2*x] + (3*E*x^2 - 6*x^3)*Log[3]*Log[x]*Log[-E + 2*x]^2 + (-(E*x^3) + 2*x^4)*Log[-
E + 2*x]^3),x]
```

Mathematica 13.1 output

$$\int \frac{((4ex^3-8x^4)\log(3)+(4ex-8x^2)\log^2(3))\log^3(x)+(2ex^3-4x^4)\log^2(-e+2x)+\log^2(x)(-4x^5-4x^3\log(3)}{(e-2x)\log^3(3)\log^3(x)+(-3ex+6x^2)\log^2(3)\,l}$$

Mathematica 12.3 output

$$x^2\log(x)\left(x^2\log(x)-\frac{\left(2(2x^2+e\log(3)-x\log(9))\right)^3+(-e^3\log(3)(2\log^2(3)+\log^2(9))+4x^3(x-\log(3))(-4\log^2(3)-\log(9)\log(81)+x\log(5314}{}\right.$$

22.11 Problem number 760

$$\int \frac{5-5x+e^{\frac{-x^3+\log(50x)}{x}}\left(-x^2+\left(1-x-2x^3+2x^4\right)\log(1-x)+(-1+x)\log(1-x)\log(50x)\right)}{-25+25x+e^{\frac{-x^3+\log(50x)}{x}}\left(10x-10x^2\right)\log(1-x)+e^{\frac{2\left(-x^3+\log(50x)\right)}{x}}\left(-x^2+x^3\right)\log^2(1-x)}\,dx$$

Optimal antiderivative

$$\frac{x}{x\ln(1-x)\,\mathrm{e}^{\frac{\ln(50x)}{x}-x^2}-5}$$

command

```
Integrate[(5 - 5*x + E^((-x^3 + Log[50*x])/x)*(-x^2 + (1 - x - 2*x^3 + 2*x^4)*Log[1 - x] + (-
1 + x)*Log[1 - x]*Log[50*x]))/(-25 + 25*x + E^((-x^3 + Log[50*x])/x)*(10*x - 10*x^2)*Log[1 - 
x^3 + Log[50*x]))/x)*(-x^2 + x^3)*Log[1 - x]^2),x]
```

Mathematica 13.1 output

$Aborted

Mathematica 12.3 output

$$-\frac{e^{x^2}x}{5e^{x^2}-50^{\frac{1}{x}}x^{1+\frac{1}{x}}\log(1-x)}$$

22.12 Problem number 1020

$$\int \frac{e^{\frac{x}{1+x}}\left(x^2+\left(-5-5x-6x^2\right)\log(2)\right)+e^{\frac{2x}{1+x}}\left(-2-4x-2x^2-243x^4-486x^5-243x^6+\left(2+4x+2x^2\right.\right.}{\left(x^2+2x^3+x^4\right)\log(2)+e^{\frac{x}{1+x}}\left(-4x-8x^2-4x^3+162x^5+324x^6+162x^7\right)\log(2)+e^{\frac{2x}{1+x}}\left(4+8x+4x^2-324x^4\right.}$$

Optimal antiderivative

$$\frac{\frac{x}{\ln(2)}+5-x}{x\,\mathrm{e}^{-\frac{x}{1+x}}-2+81x^4}$$

command

```
Integrate[(E^(x/(1 + x))*(x^2 + (-5 - 5*x - 6*x^2)*Log[2]) + E^((2*x)/(1 + x))*(-2 - 4*x - 2*
4*x - 8*x^2 - 4*x^3 + 162*x^5 + 324*x^6 + 162*x^7)*Log[2] + E^((2*x)/(1 + x))*(4 + 8*x + 4*x^
```

Mathematica 13.1 output

$$\int \frac{e^{\frac{x}{1+x}}\left(x^2+\left(-5-5x-6x^2\right)\log(2)\right)+e^{\frac{2x}{1+x}}\left(-2-4x-2x^2-243x^4-486x^5-243x^6+\left(2+4x+2x^2\right.\right.}{\left(x^2+2x^3+x^4\right)\log(2)+e^{\frac{x}{1+x}}\left(-4x-8x^2-4x^3+162x^5+324x^6+162x^7\right)\log(2)+e^{\frac{2x}{1+x}}\left(4+8x+4x^2-324x^4\right.}$$

Mathematica 12.3 output

$$3944312x+19785288\log(2)-1524858x\log(2)+72868\log(4)-595423x\log(4)-36434\log(16)+51192x\log(16)$$

22.13 Problem number 2052

$$\int \frac{(-384x-768x^2-384x^3)\log(x)+(-384x-1664x^2-2304x^3-1536x^4-512x^5)\log^2(x)+(512x^2+1536x^3}{-27+(108x+108x^2)\log(2x+e^5x)+(-144x^2-288x^3-144x^4)\log^2(2x+e^5x)+(64x^3+192x^4+}$$

Optimal antiderivative

$$\frac{4\ln(x)^2}{\left(\frac{3}{(4+4x)x}-\ln(x(e^5+2))\right)^2}$$

command

```
Integrate[((-384*x - 768*x^2 - 384*x^3)*Log[x] + (-384*x - 1664*x^2 - 2304*x^3 - 1536*x^4 - 5
27 + (108*x + 108*x^2)*Log[2*x + E^5*x] + (-144*x^2 - 288*x^3 - 144*x^4)*Log[2*x + E^5*x]^2 +
```

Mathematica 13.1 output

$Aborted

Mathematica 12.3 output

$$-\frac{4(-3+4x\log(2+e^5)+4x^2\log(2+e^5))(-3+4x\log(2+e^5)+4x^2\log(2+e^5)+8x(1+x)\log(x))}{(3-4x(1+x)\log((2+e^5)x))^2}$$

22.14 Problem number 2234

$$\int \frac{56x^2-8x^3+(-160x-8x^2-16x^3-32x\log(2))\log(x)+(100+20x+30x^2+(40+4x+6x^2)\log(2)+4\log^2}{16x^2+(-40x-8x\log(2))\log(x)+(25+10\log(2)+\log^2(2))\log^2(x)}$$

Optimal antiderivative

$$\frac{2x^2(1+x)}{\ln(2)+5-\frac{4x}{\ln(x)}}+4x$$

command

```
Integrate[(56*x^2 - 8*x^3 + (-160*x - 8*x^2 - 16*x^3 - 32*x*Log[2])*Log[x] + (100 + 20*x + 30
40*x - 8*x*Log[2])*Log[x] + (25 + 10*Log[2] + Log[2]^2)*Log[x]^2),x]
```

Mathematica 13.1 output

$$\int \frac{56x^2-8x^3+(-160x-8x^2-16x^3-32x\log(2))\log(x)+(100+20x+30x^2+(40+4x+6x^2)\log(2)+4\log^2}{16x^2+(-40x-8x\log(2))\log(x)+(25+10\log(2)+\log^2(2))\log^2(x)}$$

Mathematica 12.3 output

$$4x+\frac{x^2(10+\log(4))}{(5+\log(2))^2}+\frac{2x^3(15+\log(8))}{3(5+\log(2))^2} \\ +\frac{8x^3(25-15\log^2(2)-4x^2(5+\log(2))+\log^2(16)+x(5+\log^2(2)+\log(64))+\log(1024))}{(-5+4x-\log(2))(5+\log(2))^2(4x-(5+\log(2))\log(x))}$$

22.15 Problem number 2451

$$\int \frac{-32x^3 + e^x(3 - 3x - 80x^2)\log(2) - 50e^{2x}x\log^2(2) + e^{4x}(-64x^2 - 160e^x x\log(2) - 100e^{2x}\log^2(2))}{16x^2 + 40e^x x\log(2) + 25e^{2x}\log^2(2)}\,dx$$

Optimal antiderivative

$$4 - x^2 - \frac{3}{\frac{1}{1+\frac{\ln(2)e^x}{x}} - 5} - e^{4x}$$

command

```
Integrate[(-32*x^3 + E^x*(3 - 3*x - 80*x^2)*Log[2] - 50*E^(2*x)*x*Log[2]^2 + E^(4*x)*(-
64*x^2 - 160*E^x*x*Log[2] - 100*E^(2*x)*Log[2]^2))/(16*x^2 + 40*E^x*x*Log[2] + 25*E^(2*x)*Log
```

Mathematica 13.1 output

$$\int \frac{-32x^3 + e^x(3 - 3x - 80x^2)\log(2) - 50e^{2x}x\log^2(2) + e^{4x}(-64x^2 - 160e^x x\log(2) - 100e^{2x}\log^2(2))}{16x^2 + 40e^x x\log(2) + 25e^{2x}\log^2(2)}\,dx$$

Mathematica 12.3 output

$$-\frac{125e^{4x}\log^2(2)\log^4(32) + 125x^2\log^2(2)\log^4(32) - \frac{5x\log(8)\log^5(32)}{4x+e^x\log(32)} + 80e^{2x}(1 - 2x + 2x^2)\log^2(32)\,(75\log^2(2) - 20}{}$$

22.16 Problem number 2680

$$\int \frac{12x - 12x^2 - 12x^3 + 24x^4 - 12x^5 + (-12 + 12x + 24x^2 - 48x^3 + 24x^4)\log(2) + (-12x + 24x^2 - 12x^3)\log^2(2)}{(-4x^2 + 11x^3 - 10x^4 + 3x^5 + (8x - 22x^2}$$

Optimal antiderivative

$$\frac{4x - \frac{4x}{(\ln(2)-x)(x^2-x)}}{\ln\left(2 - \frac{3x}{2}\right)}$$

command

```
Integrate[(12*x - 12*x^2 - 12*x^3 + 24*x^4 - 12*x^5 + (-12 + 12*x + 24*x^2 - 48*x^3 + 24*x^4)
12*x + 24*x^2 - 12*x^3)*Log[2]^2 + (-16 + 44*x - 40*x^2 + 44*x^3 - 40*x^4 + 12*x^5 + (-
16 + 44*x - 88*x^2 + 80*x^3 - 24*x^4)*Log[2] + (-16 + 44*x - 40*x^2 + 12*x^3)*Log[2]^2)*Log[(
4*x^2 + 11*x^3 - 10*x^4 + 3*x^5 + (8*x - 22*x^2 + 20*x^3 - 6*x^4)*Log[2] + (-4 + 11*x - 10*x^
```

Mathematica 13.1 output

$$\int \frac{12x - 12x^2 - 12x^3 + 24x^4 - 12x^5 + (-12 + 12x + 24x^2 - 48x^3 + 24x^4)\log(2) + (-12x + 24x^2 - 12x^3)\log^2(2)}{(-4x^2 + 11x^3 - 10x^4 + 3x^5 + (8x - 22x^2}$$

Mathematica 12.3 output

$$\frac{4(3x^5 + x(-3 + 3\log^2(2) - \log(8)) + \log(8) + 3x^3(1 + \log^2(2) + \log(16)) - x^4(6 + \log(64)) - x^2(-3 + 6\log^2(2) -}{3(-1+x)^2(x - \log(2))^2\log\left(2 - \frac{3x}{2}\right)}$$

22.17 Problem number 2812

$$\int \frac{108x + 126x^2 + 36x^3 + 50x^5 + (100x^3 + 150x^4 + 50x^5)\log(3) + e^{3x}(-50x^2 + (-100 - 150x - 50x^2)\log(3)) +}{-200x^5 - 300x^6 - 150x^7 - 25x^8 + e^{3x}(200x^2 + 300x^3 + 150x^4 + 25}$$

Optimal antiderivative

$$\frac{\frac{9}{(-5\,\mathrm{e}^{x}+5x)^2} + \frac{x}{2+x} + \ln(3)}{x\,(2+x)} + 3$$

command

```
Integrate[(108*x + 126*x^2 + 36*x^3 + 50*x^5 + (100*x^3 + 150*x^4 + 50*x^5)*Log[3] + E^(3*x)*
50*x^2 + (-100 - 150*x - 50*x^2)*Log[3]) + E^(2*x)*(150*x^3 + (300*x + 450*x^2 + 150*x^3)*Log
36 - 126*x - 90*x^2 - 18*x^3 - 150*x^4 + (-300*x^2 - 450*x^3 - 150*x^4)*Log[3]))/(-
200*x^5 - 300*x^6 - 150*x^7 - 25*x^8 + E^(3*x)*(200*x^2 + 300*x^3 + 150*x^4 + 25*x^5) + E^(2*
600*x^3 - 900*x^4 - 450*x^5 - 75*x^6) + E^x*(600*x^4 + 900*x^5 + 450*x^6 + 75*x^7)),x]
```

Mathematica 13.1 output

$$\int \frac{108x + 126x^2 + 36x^3 + 50x^5 + (100x^3 + 150x^4 + 50x^5)\log(3) + e^{3x}(-50x^2 + (-100 - 150x - 50x^2)\log(3)) +}{-200x^5 - 300x^6 - 150x^7 - 25x^8 + e^{3x}(200x^2 + 300x^3 + 150x^4 + 25}$$

Mathematica 12.3 output

$$\frac{1}{100}\left(-\frac{50(-2 + x\log(3) + \log(9))}{(2+x)^2} + \frac{\frac{36}{(e^x-x)^2(2+x)} + 25\log(9)}{x}\right)$$

22.18 Problem number 2856

$$\int \frac{3x^3 + 4^{25x}(-2x^2 - 25x^3\log(4)) + e^{2\log^2(x)}(x - 25\,4^{25x}x\log(4) + (-4^{1+25x} + 4x)\log(x)) + e^{\log^2(x)}(4x^2 + 4^{25x}(}{x}$$

Optimal antiderivative

$$-\left(x - \mathrm{e}^{50x\ln(2)}\right)\left(x + \mathrm{e}^{\ln(x)^2}\right)\left(-x - \mathrm{e}^{\ln(x)^2}\right)$$

command

```
Integrate[(3*x^3 + 4^(25*x)*(-2*x^2 - 25*x^3*Log[4]) + E^(2*Log[x]^2)*(x - 25*4^(25*x)*x*Log[
4^(1 + 25*x) + 4*x)*Log[x]) + E^Log[x]^2*(4*x^2 + 4^(25*x)*(-2*x - 50*x^2*Log[4]) + (-
(4^(1 + 25*x)*x) + 4*x^2)*Log[x]))/x,x]
```

Mathematica 13.1 output

$$\int \frac{3x^3 + 4^{25x}(-2x^2 - 25x^3\log(4)) + e^{2\log^2(x)}(x - 25\,4^{25x}x\log(4) + (-4^{1+25x} + 4x)\log(x)) + e^{\log^2(x)}(4x^2 + 4^{25x}(}{x}$$

Mathematica 12.3 output

$$-\left((2^{50x} - x)\left(e^{\log^2(x)} + x\right)^2\right)$$

22.19 Problem number 2913

$$\int \frac{(50+(100+100x)\log(2)+(100+100x)\log(x))\log\left(4e^{2x}x^2\log(2)+4e^{2x}x^2\log(x)\right)+(-50\log(2)-50\log(x))}{x^3\log(2)+x^3\log(x)}$$

Optimal antiderivative

$$\frac{25\ln\left(4\,\mathrm{e}^{2x}x^2(\ln(2)+\ln(x))\right)^2}{x^2}$$

command

```
Integrate[((50 + (100 + 100*x)*Log[2] + (100 + 100*x)*Log[x])*Log[4*E^(2*x)*x^2*Log[2] + 4*E^
50*Log[2] - 50*Log[x])*Log[4*E^(2*x)*x^2*Log[2] + 4*E^(2*x)*x^2*Log[x]]^2)/(x^3*Log[2] + x^3*
```

Mathematica 13.1 output

$$\int \frac{(50+(100+100x)\log(2)+(100+100x)\log(x))\log\left(4e^{2x}x^2\log(2)+4e^{2x}x^2\log(x)\right)+(-50\log(2)-50\log(x))}{x^3\log(2)+x^3\log(x)}$$

Mathematica 12.3 output

$$50\left(-2+\frac{\log^2\left(4e^{2x}x^2\log(2x)\right)}{2x^2}\right)$$

22.20 Problem number 3121

$$\int \frac{3^{-1/x}\sqrt[x]{\log\left(x^4\right)}\left(-48+12\log\left(x^4\right)\log\left(\frac{\log(x^4)}{3}\right)\right)}{\left(625x^2+50x^2\log(4)+x^2\log^2(4)\right)\log\left(x^4\right)+3^{-1/x}\left(-50x^2-2x^2\log(4)\right)\log^{1+\frac{1}{x}}\left(x^4\right)+3^{-2/x}x^2\log^{1+\frac{2}{x}}\left(x^4\right)}\,dx$$

Optimal antiderivative

$$\frac{12}{-25-2\ln(2)+\mathrm{e}^{\frac{\ln\left(\frac{\ln\left(x^4\right)}{3}\right)}{x}}}$$

command

```
Integrate[(Log[x^4]^x^(-1)*(-48 + 12*Log[x^4]*Log[Log[x^4]/3]))/(3^x^(-1)*((625*x^2 + 50*x^2*
50*x^2 - 2*x^2*Log[4])*Log[x^4]^(1 + x^(-1)))/3^x^(-1) + (x^2*Log[x^4]^(1 + 2/x))/3^(2/x))),x
```

Mathematica 13.1 output

$$\int \frac{3^{-1/x}\sqrt[x]{\log\left(x^4\right)}\left(-48+12\log\left(x^4\right)\log\left(\frac{\log(x^4)}{3}\right)\right)}{\left(625x^2+50x^2\log(4)+x^2\log^2(4)\right)\log\left(x^4\right)+3^{-1/x}\left(-50x^2-2x^2\log(4)\right)\log^{1+\frac{1}{x}}\left(x^4\right)+3^{-2/x}x^2\log^{1+\frac{2}{x}}\left(x^4\right)}\,dx$$

Mathematica 12.3 output

$$-\frac{4\;3^{1+\frac{1}{x}}}{3^{\frac{1}{x}}(25+\log(4))-\sqrt[x]{\log\left(x^4\right)}}$$

22.21 Problem number 3124

$$\int \frac{-1000-5000x-6200x^2-2900x^3-100}{256x^3+448x^4+288x^5+144x^6+104x^7+48x^8+8x^9+(-192x^3-240x^4-96x^5-60x^6-48x^7-12x^8)\log(5}$$

Optimal antiderivative

$$\left(5-\frac{5}{\left(\ln(5)-\ln\left(\frac{x}{x^3+x+4}\right)-4-2x\right)x}\right)^2$$

command

```
Integrate[(-1000 - 5000*x - 6200*x^2 - 2900*x^3 - 1000*x^4 - 1000*x^5 - 400*x^6 + (200 + 1850
200*x - 50*x^2 - 50*x^4)*Log[5]^2 + (-200 - 1850*x - 1600*x^2 - 350*x^3 - 300*x^4 - 300*x^5 +
200*x - 50*x^2 - 50*x^4)*Log[x/(4 + x + x^3)]^2)/(256*x^3 + 448*x^4 + 288*x^5 + 144*x^6 + 104
192*x^3 - 240*x^4 - 96*x^5 - 60*x^6 - 48*x^7 - 12*x^8)*Log[5] + (48*x^3 + 36*x^4 + 6*x^5 + 12
4*x^3 - x^4 - x^6)*Log[5]^3 + (192*x^3 + 240*x^4 + 96*x^5 + 60*x^6 + 48*x^7 + 12*x^8 + (-
96*x^3 - 72*x^4 - 12*x^5 - 24*x^6 - 12*x^7)*Log[5] + (12*x^3 + 3*x^4 + 3*x^6)*Log[5]^2)*Log[x
12*x^3 - 3*x^4 - 3*x^6)*Log[5])*Log[x/(4 + x + x^3)]^2 + (4*x^3 + x^4 + x^6)*Log[x/(4 + x + x
```

Mathematica 13.1 output

$$\int \frac{-1000-5000x-6200x^2-2900x^3-100}{256x^3+448x^4+288x^5+144x^6+104x^7+48x^8+8x^9+(-192x^3-240x^4-96x^5-60x^6-48x^7-12x^8)\log(5}$$

Mathematica 12.3 output

$$-\frac{25\left(x(32+136x+96x^2-48x^3+66x^4+45x^5-18x^6+20x^7+6x^8-x^9+2x^{10})\log^3\left(\frac{x}{5(4+x+x^3)}\right)+(4+x+x}{}$$

22.22 Problem number 3351

$$\int \frac{6x+2x^2-25x^3-81x^4-63x^5-19x^6-2x^7+\left(6x+81x^2+243x^3+189x^4+57x^5+6x^6\right)\log\left(\frac{e^2}{3x}\right)+\left(-81x\right.}{-27x^3-27x^4-9x^5-x^6+(81x^2+81x^3+27x^4+3x^5)\log\left(\frac{e^2}{3x}\right)+(-81x-81x^2}$$

Optimal antiderivative

$$x^2+\frac{x^2}{(3+x)^2\left(\ln\left(\frac{e^2}{3x}\right)-x\right)^2}+x+10$$

command

```
Integrate[(6*x + 2*x^2 - 25*x^3 - 81*x^4 - 63*x^5 - 19*x^6 - 2*x^7 + (6*x + 81*x^2 + 243*x^3
81*x - 243*x^2 - 189*x^3 - 57*x^4 - 6*x^5)*Log[E^2/(3*x)]^2 + (27 + 81*x + 63*x^2 + 19*x^3 +
27*x^3 - 27*x^4 - 9*x^5 - x^6 + (81*x^2 + 81*x^3 + 27*x^4 + 3*x^5)*Log[E^2/(3*x)] + (-
81*x - 81*x^2 - 27*x^3 - 3*x^4)*Log[E^2/(3*x)]^2 + (27 + 27*x + 9*x^2 + x^3)*Log[E^2/(3*x)]^3
```

Mathematica 13.1 output

$$\int \frac{6x + 2x^2 - 25x^3 - 81x^4 - 63x^5 - 19x^6 - 2x^7 + (6x + 81x^2 + 243x^3 + 189x^4 + 57x^5 + 6x^6)\log\left(\frac{e^2}{3x}\right) + (-81x}{-27x^3 - 27x^4 - 9x^5 - x^6 + (81x^2 + 81x^3 + 27x^4 + 3x^5)\log\left(\frac{e^2}{3x}\right) + (-81x - 81x^2}$$

Mathematica 12.3 output

output too large to display

22.23 Problem number 3670

$$\int \frac{3^{2/x}(x^4)^{-2/x}\left(-2x^2 + (-8x + 2x^2)\log(x) + 2x\log(x)\log\left(\frac{x^4}{3}\right) + 3^{-1/x}(x^4)^{\frac{1}{x}}\left(40x\log(x) + (160 - 40x)\log^2(x\right.\right.}{x\log^3(x)}$$

Optimal antiderivative

$$\left(20 - \frac{x\,\mathrm{e}^{-\frac{\ln\left(\frac{x^4}{3}\right)}{x}}}{\ln(x)}\right)^2$$

command

```
Integrate[(3^(2/x)*(-2*x^2 + (-8*x + 2*x^2)*Log[x] + 2*x*Log[x]*Log[x^4/3] + ((x^4)^x^(-
1)*(40*x*Log[x] + (160 - 40*x)*Log[x]^2 - 40*Log[x]^2*Log[x^4/3]))/3^x^(-1)))/(x*(x^4)^(2/x)*
```

Mathematica 13.1 output

$$\int \frac{3^{2/x}(x^4)^{-2/x}\left(-2x^2 + (-8x + 2x^2)\log(x) + 2x\log(x)\log\left(\frac{x^4}{3}\right) + 3^{-1/x}(x^4)^{\frac{1}{x}}\left(40x\log(x) + (160 - 40x)\log^2(x\right.\right.}{x\log^3(x)}$$

Mathematica 12.3 output

$$\frac{3^{\frac{1}{x}}x(x^4)^{-2/x}\left(3^{\frac{1}{x}}x - 40(x^4)^{\frac{1}{x}}\log(x)\right)}{\log^2(x)}$$

22.24 Problem number 3955

$$\int \frac{-6\log(4)+(6x^2-9x^3)\log^2(x)+(6\log(4)\log(x)+(3x^2-3x^3)\log^2(x))\log\left(\frac{4\log(4)+(2x^2-2x^3)\log(x)}{\log(x)}\right)\log\left(\log\left(\right.\right.}{(-2x^2\log(4)\log(x)+(-x^4+x^5)\log^2(x))\log\left(\frac{4\log(4)+(2x^2-2x^3)\log(x)}{\log(x)}\right)\log^2\left(\log\left(\frac{4\log(4)+(2x^2-2x^3}{\log(x)}\right.\right.}$$

Optimal antiderivative

$$\frac{3}{\ln\left(\ln\left(\ln\left(e^{-2x^2+2x}\right)x+\frac{8\ln(2)}{\ln(x)}\right)\right)x}$$

command

```
Integrate[(-6*Log[4] + (6*x^2 - 9*x^3)*Log[x]^2 + (6*Log[4]*Log[x] + (3*x^2 - 3*x^3)*Log[x]^2
2*x^2*Log[4]*Log[x] + (-x^4 + x^5)*Log[x]^2)*Log[(4*Log[4] + (2*x^2 - 2*x^3)*Log[x])/Log[x]]*
```

Mathematica 13.1 output

$$\int \frac{-6\log(4)+(6x^2-9x^3)\log^2(x)+(6\log(4)\log(x)+(3x^2-3x^3)\log^2(x))\log\left(\frac{4\log(4)+(2x^2-2x^3)\log(x)}{\log(x)}\right)\log\left(\log\left(\right.\right.}{(-2x^2\log(4)\log(x)+(-x^4+x^5)\log^2(x))\log\left(\frac{4\log(4)+(2x^2-2x^3)\log(x)}{\log(x)}\right)\log^2\left(\log\left(\frac{4\log(4)+(2x^2-2x^3}{\log(x)}\right.\right.}$$

Mathematica 12.3 output

$$-\frac{3(\log(256)-2(-1+x)x^2\log(x))\left(\log(16)+x^2(-2+3x)\log^2(x)\right)}{x\left(-2\log(4)+(-1+x)x^2\log(x)\right)\left(\log(256)+2x^2(-2+3x)\log^2(x)\right)\log\left(\log\left(-2(-1+x)x^2+\frac{\log(256)}{\log(x)}\right)\right)}$$

22.25 Problem number 4166

$$\int \frac{-36+e^{x+x^2\log(3x)}(-16+16x+16x^2+32x^2\log(3x))}{81+16e^{2x+2x^2\log(3x)}+e^{x+x^2\log(3x)}(72-32x)-72x+16x^2}\,dx$$

Optimal antiderivative

$$\frac{x}{x-\frac{9}{4}-e^{x^2\ln(3x)+x}}$$

command

```
Integrate[(-36 + E^(x + x^2*Log[3*x])*(-16 + 16*x + 16*x^2 + 32*x^2*Log[3*x]))/(81 + 16*E^(2*
```

Mathematica 13.1 output

$$\int \frac{-36+e^{x+x^2\log(3x)}(-16+16x+16x^2+32x^2\log(3x))}{81+16e^{2x+2x^2\log(3x)}+e^{x+x^2\log(3x)}(72-32x)-72x+16x^2}\,dx$$

Mathematica 12.3 output

$$-\frac{4x}{9-4x+4\,3^{x^2}e^x x^{x^2}}$$

22.26 Problem number 4890

$$\int \frac{2+x-2x^2-x\log(16)+\left(-x+x^2+(-1+x)\log(16)-\log\left(x^2\right)\right)\log\left(x-x^2+(1-x)\log(16)+\log\left(x^2\right)\right)\log \cdots}{(x^3-\cdots}$$

Optimal antiderivative

$$\frac{\ln\left(\ln\left(\ln\left(\ln\left(x^2\right)+(x+4\ln(2))\,(1-x)\right)\right)\right)+1}{x}$$

command

```
Integrate[(2 + x - 2*x^2 - x*Log[16] + (-x + x^2 + (-1 + x)*Log[16] - Log[x^2])*Log[x - x^2 +
x + x^2 + (-1 + x)*Log[16] - Log[x^2])*Log[x - x^2 + (1 - x)*Log[16] + Log[x^2]]*Log[Log[x - 
```

Mathematica 13.1 output

$$\int \frac{2+x-2x^2-x\log(16)+\left(-x+x^2+(-1+x)\log(16)-\log\left(x^2\right)\right)\log\left(x-x^2+(1-x)\log(16)+\log\left(x^2\right)\right)\log \cdots}{(x^3-\cdots}$$

Mathematica 12.3 output

$$\frac{1}{x}+\frac{\log\left(\log\left(\log\left(-((-1+x)(x+\log(16)))+\log\left(x^2\right)\right)\right)\right)}{x}$$

22.27 Problem number 4908

$$\int \frac{\left(625+1500x+1350x^2+540x^3+81x^4\right)\log(2)+\left(625+1500x+1350x^2+540x^3+81x^4\right)\log^2(2)+e^{2x}\left(4x^2+\cdots\right.}{(625+1500x+1350x^2+540\cdots}$$

Optimal antiderivative

$$\frac{x}{\ln(2)-\frac{2\,\mathrm{e}^{x}}{x\left(\left(3+\frac{5}{x}\right)^2-\mathrm{e}^{x}+\ln(x)\right)}}+x$$

command

```
Integrate[((625 + 1500*x + 1350*x^2 + 540*x^3 + 81*x^4)*Log[2] + (625 + 1500*x + 1350*x^2 + 5
10*x^2 + 22*x^3 + 18*x^4 + (-100*x - 170*x^2 - 96*x^3 - 18*x^4)*Log[2] + (-50*x^2 - 60*x^3 - 
4*x^3 + 2*x^4 + (-4*x^3 - 2*x^4)*Log[2] - 2*x^4*Log[2]^2))*Log[x] + (x^4*Log[2] + x^4*Log[2]^
100*x - 120*x^2 - 36*x^3)*Log[2] + (-50*x^2 - 60*x^3 - 18*x^4)*Log[2]^2) + ((50*x^2 + 60*x^3 
4*x^3*Log[2] - 2*x^4*Log[2]^2))*Log[x] + x^4*Log[2]^2*Log[x]^2),x]
```

Mathematica 13.1 output

$$\int \frac{\left(625+1500x+1350x^2+540x^3+81x^4\right)\log(2)+\left(625+1500x+1350x^2+540x^3+81x^4\right)\log^2(2)+e^{2x}\left(4x^2+\cdots\right.}{(625+1500x+1350x^2+540\cdots}$$

Mathematica 12.3 output

$$\frac{x\left(1+\log(2)-\frac{e^x x\left(100\log(2)+60x\log(2)-x^2\left(36\log^2(2)+\log(4)-18\log(2)\log(4)\right)+e^x x\left(-4+x^2\log(4)+x\left(4-\log^2(4)+\log(2)\log(16)\right)\right)\right)}{\left((50+30x-x^2)\log(2)+e^x x(-2+2x+x^2\log(2))\right)\left(-(5+3x)^2\log(2)+e^x x(2+x\log(2))-x^2\log(2)\log(x)\right)}\right)}{\log(2)}$$

22.28 Problem number 5022

$$\int \frac{500-5x^2+125\log(3)+(-625-125\log(3))\log}{25x^2+10x^3+x^4+(-1250x-250x^2+(-250x-50x^2)\log(3))\log(x)+\left(15625+6250\log(3)+625\log^2(3)\right)\log}$$

Optimal antiderivative

$$\frac{x}{x\left(1+\frac{x}{5}\right)+5\left(\ln(\ln(x))-5-\ln(3)\right)\ln(x)}$$

command

```
Integrate[(500 - 5*x^2 + 125*Log[3] + (-625 - 125*Log[3])*Log[x] + (-125 + 125*Log[x])*Log[Lo
1250*x - 250*x^2 + (-250*x - 50*x^2)*Log[3])*Log[x] + (15625 + 6250*Log[3] + 625*Log[3]^2)*Lo
6250 - 1250*Log[3])*Log[x]^2)*Log[Log[x]] + 625*Log[x]^2*Log[Log[x]]^2),x]
```

Mathematica 13.1 output

$$\int \frac{500-5x^2+125\log(3)+(-625-125\log(3))\log}{25x^2+10x^3+x^4+(-1250x-250x^2+(-250x-50x^2)\log(3))\log(x)+\left(15625+6250\log(3)+625\log^2(3)\right)\log}$$

Mathematica 12.3 output

$$\frac{5x}{x(5+x)+25\log(x)\left(-5+\log\left(\frac{\log(x)}{3}\right)\right)}$$

22.29 Problem number 5264

$$\int \frac{-1000x^2-600x^3-620x^4-308x^5-60x^6-4x^7+\left(40x-1192x^2-660x^3-1124x^4-608x^5-120x^6-8x^7+\right.}{}$$

Optimal antiderivative

$$\left(\ln(5)\left(x^2+\frac{\ln(2)+2}{5+x-\ln(x)^2}\right)+x^2+2\right)^2$$

command

```
Integrate[(-1000*x^2 - 600*x^3 - 620*x^4 - 308*x^5 - 60*x^6 - 4*x^7 + (40*x - 1192*x^2 - 660*
80 - 16*x - 40*x^2 - 8*x^3 + (-40 - 8*x - 20*x^2 - 4*x^3)*Log[2])*Log[5] + (-16 - 40*x^2 - 8*
16 - 20*x^2 - 4*x^3)*Log[2] - 4*Log[2]^2)*Log[5]^2)*Log[x] + (600*x^2 + 240*x^3 + 324*x^4 + 1
8*x + 680*x^2 + 252*x^3 + 624*x^4 + 240*x^5 + 24*x^6 + (-4*x + 40*x^2 + 6*x^3)*Log[2])*Log[5]
120*x^2 - 24*x^3 - 60*x^4 - 12*x^5 + (-128*x^2 - 24*x^3 - 120*x^4 - 24*x^5 - 4*x^2*Log[2])*Lo
8*x^2 - 60*x^4 - 12*x^5 - 4*x^2*Log[2])*Log[5]^2)*Log[x]^4 + (8*x^2 + 4*x^4 + (8*x^2 + 8*x^4)
125*x - 75*x^2 - 15*x^3 - x^4 + (75*x + 30*x^2 + 3*x^3)*Log[x]^2 + (-15*x - 3*x^2)*Log[x]^4 +
```

Mathematica 13.1 output

$Aborted

Mathematica 12.3 output

$$4x^2(1+\log(5))+x^4(1+\log(5))^2$$
$$+\frac{x^2\left(\log^2(2)\log^2(5)+5\log(4)\log(5)(9+10\log(5))+\log^2(25)+\log(2)\log(5)(-90-98\log(5)+\log(25))\right)+x^4(1-}{}$$
$$+\frac{-8000\log(5)(8+\log(16))-4800x\log(5)(8+\log(16))-160x^2\log(5)(188+200\log(5)+5\log(4)(9+10\log(5))-}{}$$

22.30 Problem number 5265

$$\int \frac{36-e^{3x}x+12\log(2)+\log^2(2)+e^{2x}(1-12x-2x\log(2))+e^x\left(12-32x+(2-12x)\log(2)-x\log^2(2)\right)+(72}{36x+e^{2x}x+12x\log(2)+x\log^2(2)+e^x(12x+2x\log(2}$$

Optimal antiderivative

$$\ln(2x)-\mathrm{e}^x-\frac{4}{6+\ln(2)+\mathrm{e}^x}+\ln(x)^2$$

command

```
Integrate[(36 - E^(3*x)*x + 12*Log[2] + Log[2]^2 + E^(2*x)*(1 - 12*x - 2*x*Log[2]) + E^x*(12
```

Mathematica 13.1 output

$$\int \frac{36-e^{3x}x+12\log(2)+\log^2(2)+e^{2x}(1-12x-2x\log(2))+e^x\left(12-32x+(2-12x)\log(2)-x\log^2(2)\right)+(72}{36x+e^{2x}x+12x\log(2)+x\log^2(2)+e^x(12x+2x\log(2}$$

Mathematica 12.3 output

$$-e^x-\frac{24-2\log^3(2)+\log^2(2)(-12+\log(4))+\log(16)-\log(2)\log(4096)+\log(4)\log(4096)}{(6+\log(2))\left(6+e^x+\log(2)\right)}$$
$$+\log(x)+\log^2(x)$$

22.31 Problem number 5410

$$\int \frac{-5x^6+ex^6+e^{-\frac{3x}{-5+e}}\left(320-240x+60x^2-5x^3+e\left(-64+48x-12x^2+x^3\right)\right)+e^{-\frac{2x}{-5+e}}\left(-240x^2+120x^3-15\right.}{-5x^6+ex^6+e^{-\frac{3x}{-5+e}}\left(320-240x+60x^2-5x^3+e\left(-64+48x-12x^2+x^3\right)\right)+e^{-\frac{2x}{-5+e}}\left(-240x^2+12\right.}$$

Optimal antiderivative

$$x-\frac{4x^2}{\left(\frac{(-4+x)\mathrm{e}^{\frac{x}{5-\mathrm{e}}}}{x}+x\right)^2}$$

command

```
Integrate[(-5*x^6 + E*x^6 + (320 - 240*x + 60*x^2 - 5*x^3 + E*(-64 + 48*x - 12*x^2 + x^3))/E^
5 + E)) + (-240*x^2 + 120*x^3 - 15*x^4 + E*(48*x^2 - 24*x^3 + 3*x^4))/E^((2*x)/(-5 + E)) + (-
320*x^3 + 132*x^4 - 23*x^5 + E*(64*x^3 - 20*x^4 + 3*x^5))/E^(x/(-5 + E)))/(-5*x^6 + E*x^6 + (
64 + 48*x - 12*x^2 + x^3))/E^((3*x)/(-5 + E)) + (-240*x^2 + 120*x^3 - 15*x^4 + E*(48*x^2 - 24
5 + E)) + (60*x^4 - 15*x^5 + E*(-12*x^4 + 3*x^5))/E^(x/(-5 + E))),x]
```

Mathematica 13.1 output

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Mathematica 12.3 output

$$e^{-\frac{20}{-5+e}}(-4+x)x\left(2e^{\frac{4e}{-5+e}}(-8+x)^3(-4+x)+4e^{\frac{4e+x}{-5+e}}(-8+x)^3x^2+2e^{\frac{2(2e+x)}{-5+e}}(-8+x)^3x^3+4e^{\frac{15+e+x}{-5+e}}x^2(160-24\right.$$

22.32 Problem number 5820

$$\int \frac{5x-x^3+e^2(5-x^2)+e^{2e^{2e^{\log(x)\log(e^2+x)}}}\left(e^2+x+e^{2e^{\log(x)\log(e^2+x)}+\log(x)\log(e^2+x)}(-4x\log(x)+(\right.}{25x-10x^2+11x^3-2x^4+x^5+e^{4e^{2e^{\log(x)\log(e^2+x)}}}(e^2+x)+e^2(25-10x+11x^2-2x^3+x^4)+e^{2e^{2e^{\log(x)\log(e^2+x)}}}}$$

Optimal antiderivative

$$\frac{x}{x^2-x+\mathrm{e}^{2\,\mathrm{e}^{2\,\mathrm{e}^{\ln(x)\ln\left(x+\mathrm{e}^2\right)}}}+5}$$

command

```
Integrate[(5*x - x^3 + E^2*(5 - x^2) + E^(2*E^(2*E^(Log[x]*Log[E^2 + x])))*(E^2 + x + E^(2*E^
4*x*Log[x] + (-4*E^2 - 4*x)*Log[E^2 + x])))/(25*x - 10*x^2 + 11*x^3 - 2*x^4 + x^5 + E^(4*E^(2
```

Mathematica 13.1 output

$$\int \frac{5x-x^3+e^2(5-x^2)+e^{2e^{2e^{\log(x)\log(e^2+x)}}}\left(e^2+x+e^{2e^{\log(x)\log(e^2+x)}+\log(x)\log(e^2+x)}(-4x\log(x)+(\right.}{25x-10x^2+11x^3-2x^4+x^5+e^{4e^{2e^{\log(x)\log(e^2+x)}}}(e^2+x)+e^2(25-10x+11x^2-2x^3+x^4)+e^{2e^{2e^{\log(x)\log(e^2+x)}}}}$$

Mathematica 12.3 output

$$\frac{x\left(4e^{2x^{\log(e^2+x)}+\log(x)\log(e^2+x)}x(5-x+x^2)\log(x)+(e^2+x)\left(x-2x^2+4e^{2x^{\log(e^2+x)}+\log(x)\log(e^2+x)}(5-\right.\right.}{\left(5+e^{2e^{2x^{\log(e^2+x)}}}-x+x^2\right)\left(4e^{2x^{\log(e^2+x)}}x^{1+\log(e^2+x)}(5-x+x^2)\log(x)+(e^2+x)\left(x-2x^2+4e^{2x^{\log(e^2+x)}}x^{\log(\right.\right.}}$$

22.33 Problem number 5856

$$\int \frac{96 + 96x - 36x^2 + 12x^2\log(5) + e^x\left(-96 - 96x + 30x^2 - 6x^3 - 3x^4 - 12x^2\log(5)\right) +}{64 + 48x^2 + 9x^4 + (32x + 12x^3)\log(5) + 4x^2\log^2(5) + e^x\left(-64 - 40x^2 - 6x^4 + (-32x - 10x^3)\log(5) - 4x^2\log^2\right.}$$

Optimal antiderivative

$$\frac{3(2+x)\,x}{x^2 + x\left(\frac{x}{-e^x+2} + \ln(5)\right) + 4} + 9$$

command

```
Integrate[(96 + 96*x - 36*x^2 + 12*x^2*Log[5] + E^x*(-96 - 96*x + 30*x^2 - 6*x^3 - 3*x^4 - 12
64 - 40*x^2 - 6*x^4 + (-32*x - 10*x^3)*Log[5] - 4*x^2*Log[5]^2) + E^(2*x)*(16 + 8*x^2 + x^4 +
```

Mathematica 13.1 output

$$\int \frac{96 + 96x - 36x^2 + 12x^2\log(5) + e^x\left(-96 - 96x + 30x^2 - 6x^3 - 3x^4 - 12x^2\log(5)\right) +}{64 + 48x^2 + 9x^4 + (32x + 12x^3)\log(5) + 4x^2\log^2(5) + e^x\left(-64 - 40x^2 - 6x^4 + (-32x - 10x^3)\log(5) - 4x^2\log^2\right.}$$

Mathematica 12.3 output

$$\frac{3\left(256 + 3x^6 + 96x(-2 + \log(25)) + x^5(-12 + 9\log(5) + \log(25)) + 4x^2\left(56 + 4\log(5)(-3 + \log(25)) - 5\log(25) +\right.\right.}{(-}$$

22.34 Problem number 6066

$$\int \frac{10x - 20x^2 + 30x^3 - 20x^4 - 2x^5 + 10x^6 - 20x^7 + 20x^8 - 10x^9 + 2x^{10} + \left(5x - 16x^3 + 52x^4 - 60x^5 + 28x^6 - 4\right.}{x^5 - 4x^6 + 6x^7 - 4x^8 + x^9 + (8x^3 - 18}$$

Optimal antiderivative

$$x^2 - 2x + \frac{5}{(4-x)\ln(3) + (x - \ln(x) - x^2)^2}$$

command

```
Integrate[(10*x - 20*x^2 + 30*x^3 - 20*x^4 - 2*x^5 + 10*x^6 - 20*x^7 + 20*x^8 - 10*x^9 + 2*x^
32*x + 48*x^2 - 18*x^3 + 2*x^4)*Log[3]^2 + (-10 + 10*x - 20*x^2 + 8*x^4 - 32*x^5 + 48*x^6 - 3
12*x^3 + 36*x^4 - 36*x^5 + 12*x^6 + (-16*x + 20*x^2 - 4*x^3)*Log[3])*Log[x]^2 + (8*x^2 - 16*x
2*x + 2*x^2)*Log[x]^4)/(x^5 - 4*x^6 + 6*x^7 - 4*x^8 + x^9 + (8*x^3 - 18*x^4 + 12*x^5 - 2*x^6)
4*x^4 + 12*x^5 - 12*x^6 + 4*x^7 + (-16*x^2 + 20*x^3 - 4*x^4)*Log[3])*Log[x] + (6*x^3 - 12*x^4
4*x^2 + 4*x^3)*Log[x]^3 + x*Log[x]^4),x]
```

Mathematica 13.1 output

$$\int \frac{10x - 20x^2 + 30x^3 - 20x^4 - 2x^5 + 10x^6 - 20x^7 + 20x^8 - 10x^9 + 2x^{10} + \left(5x - 16x^3 + 52x^4 - 60x^5 + 28x^6 - 4\right.}{x^5 - 4x^6 + 6x^7 - 4x^8 + x^9 + (8x^3 - 18}$$

Mathematica 12.3 output

$$\frac{16x^{11}\log(3) - 20\log(81) + x^3\left(-48\log^3(3) + 88\log(81) + (84 + \log(9))\log^2(81) + 6\log^2(3)(28 + \log(81)) - 24\log\right.}{}$$

22.35 Problem number 6250

$$\int \frac{e^{2x}(4x+16x^2)+e^x(-8x-32x^2)\log(3)+(4x+16x^2)\log^2(3)+(e^{2x}(-8x-16x^2)-8x^3\log(3)+(-8x-16x}{(e^{2x}(1+8x+16x^2)+e^x(-2-16x-32x^2)\log(3)+(1+8x+16x^2)\log^2(3))\log^2(x)+(e^x(2x}$$

Optimal antiderivative

$$\frac{4x}{\ln(x)\left(\frac{x\ln(x)}{\ln(3)-e^x}-\frac{1+4x}{x}\right)}$$

command

```
Integrate[(E^(2*x)*(4*x + 16*x^2) + E^x*(-8*x - 32*x^2)*Log[3] + (4*x + 16*x^2)*Log[3]^2 + (E
8*x - 16*x^2) - 8*x^3*Log[3] + (-8*x - 16*x^2)*Log[3]^2 + E^x*(8*x^3 + (16*x + 32*x^2)*Log[3]
2 - 16*x - 32*x^2)*Log[3] + (1 + 8*x + 16*x^2)*Log[3]^2)*Log[x]^2 + (E^x*(2*x^2 + 8*x^3) + (-
2*x^2 - 8*x^3)*Log[3])*Log[x]^3 + x^4*Log[x]^4),x]
```

Mathematica 13.1 output

$$\int \frac{e^{2x}(4x+16x^2)+e^x(-8x-32x^2)\log(3)+(4x+16x^2)\log^2(3)+(e^{2x}(-8x-16x^2)-8x^3\log(3)+(-8x-16x}{(e^{2x}(1+8x+16x^2)+e^x(-2-16x-32x^2)\log(3)+(1+8x+16x^2)\log^2(3))\log^2(x)+(e^x(2x}$$

Mathematica 12.3 output

$$-\frac{4x^2\left(\frac{e^{2x}(1+4x)+(1+4x)\log^2(3)-e^x(8x\log(3)+\log(9))}{\log(x)}-\frac{x^2(e^{3x}(-2-11x-8x^2+16x^3)+2\log^3(3)+12x\log^3(3)+x^2\log^2(3)(1+16\log(3))+x^3(}{}\right.}{}$$

22.36 Problem number 6633

$$\int \frac{-4x-2e^3x-6x^2+(10+4e^3)\log(3)+(-6x-2e^3x}{(-6x-2e^3x+6x^2+(6+2e^3-6x)\log(3))\log^2(x-\log(3))+(-x+\log(3))\log(2x)\log^2(x-\log(3))+((-12x}$$

Optimal antiderivative

$$\frac{x}{\ln(-\ln(3)+x)+\ln\left(\left(\frac{e^3+\frac{\ln(2x)}{2}+3}{x}-3\right)^2\right)}$$

command

```
Integrate[(-4*x - 2*E^3*x - 6*x^2 + (10 + 4*E^3)*Log[3] + (-6*x - 2*E^3*x + 6*x^2 + (6 + 2*E^
x + 2*Log[3] + (-x + Log[3])*Log[x - Log[3]]) + (-6*x - 2*E^3*x + 6*x^2 + (6 + 2*E^3 - 6*x)*L
x + Log[3])*Log[2*x])*Log[(36 + 4*E^6 + E^3*(24 - 24*x) - 72*x + 36*x^2 + (12 + 4*E^3 - 12*x)
6*x - 2*E^3*x + 6*x^2 + (6 + 2*E^3 - 6*x)*Log[3])*Log[x - Log[3]]^2 + (-x + Log[3])*Log[2*x]*
12*x - 4*E^3*x + 12*x^2 + (12 + 4*E^3 - 12*x)*Log[3])*Log[x - Log[3]] + (-2*x + 2*Log[3])*Log
6*x - 2*E^3*x + 6*x^2 + (6 + 2*E^3 - 6*x)*Log[3] + (-x + Log[3])*Log[2*x])*Log[(36 + 4*E^6 +
```

Mathematica 13.1 output

$$\int \frac{-4x - 2e^3x - 6x^2 + (10 + 4e^3)\log(3) + (-6x - 2e^3x}{(-6x - 2e^3x + 6x^2 + (6 + 2e^3 - 6x)\log(3))\log^2(x - \log(3)) + (-x + \log(3))\log(2x)\log^2(x - \log(3)) + ((-12x}$$

Mathematica 12.3 output

$$\frac{x}{\log\left(\frac{1}{4}(x - \log(3))\right) + \log\left(\frac{(6+2e^3-6x+\log(2x))^2}{x^2}\right)}$$

22.37 Problem number 6675

$$\int \frac{-2 + e^x(1 + (-2 - x)\log(5)) + (-2x + e^x x\log(5))\log(x) + (2 + e^x(-1 + (2 + x)\,]}{(2x^2 + e^x(-x^2 + (2x^2 + x^3)\log(5)))\log(x) + (4x + e^x(-2x + (4x + 2x^2)\log(5)))\log(x)\log\left(\frac{e^x\log(5)\log(x)}{2+e^x(-1+(2+x)\log(5}\right.}$$

Optimal antiderivative

$$\frac{x}{x + \ln\left(\frac{\ln(x)}{x - \frac{1-2\,\mathrm{e}^{-x}}{\ln(5)} + 2}\right)} + 2$$

command

```
Integrate[(-2 + E^x*(1 + (-2 - x)*Log[5]) + (-2*x + E^x*x*Log[5])*Log[x] + (2 + E^x*(-
1 + (2 + x)*Log[5]))*Log[x]*Log[(E^x*Log[5]*Log[x])/(2 + E^x*(-1 + (2 + x)*Log[5]))])/((2*x^2
x^2 + (2*x^2 + x^3)*Log[5]))*Log[x] + (4*x + E^x*(-2*x + (4*x + 2*x^2)*Log[5]))*Log[x]*Log[(E
1 + (2 + x)*Log[5]))] + (2 + E^x*(-1 + (2 + x)*Log[5]))*Log[x]*Log[(E^x*Log[5]*Log[x])/(2 + E
1 + (2 + x)*Log[5]))]^2),x]
```

Mathematica 13.1 output

$$\int \frac{-2 + e^x(1 + (-2 - x)\log(5)) + (-2x + e^x x\log(5))\log(x) + (2 + e^x(-1 + (2 + x)\,]}{(2x^2 + e^x(-x^2 + (2x^2 + x^3)\log(5)))\log(x) + (4x + e^x(-2x + (4x + 2x^2)\log(5)))\log(x)\log\left(\frac{e^x\log(5)\log(x)}{2+e^x(-1+(2+x)\log(5}\right.}$$

Mathematica 12.3 output

$$\frac{x}{x + \log\left(\frac{e^x\log(5)\log(x)}{2+e^x(-1+(2+x)\log(5))}\right)}$$

22.38 Problem number 6680

$$\int e^{-7+x^2-25^{\frac{4x}{e^2}}\left(x^2\right)^{\frac{4x}{e^2}}}\left(e^2\left(1+2x^2\right)+25^{\frac{4x}{e^2}}\left(x^2\right)^{\frac{4x}{e^2}}\left(-8x-4x\log\left(25x^2\right)\right)\right)\,dx$$

Optimal antiderivative

$$x\,\mathrm{e}^{-\mathrm{e}^{4x\ln\left(25x^2\right)\mathrm{e}^{-2}}+x^2-5}$$

command

```
Integrate[E^(-7 + x^2 - 25^((4*x)/E^2)*(x^2)^((4*x)/E^2))*(E^2*(1 + 2*x^2) + 25^((4*x)/E^2)*(
8*x - 4*x*Log[25*x^2])),x]
```

Mathematica 13.1 output

$$\int e^{-7+x^2-25^{\frac{4x}{e^2}}\left(x^2\right)^{\frac{4x}{e^2}}}\left(e^2\left(1+2x^2\right)+25^{\frac{4x}{e^2}}\left(x^2\right)^{\frac{4x}{e^2}}\left(-8x-4x\log\left(25x^2\right)\right)\right)\,dx$$

Mathematica 12.3 output

$$e^{-5+x^2-25^{\frac{4x}{e^2}}\left(x^2\right)^{\frac{4x}{e^2}}}x$$

22.39 Problem number 6838

$$\int \frac{4x-5x^2-8x^3+10x^4+\left(-x+2x^3\right)\log(5)+\left(4x-10x^2+e^{x^2}(40-100x-10\log(5))-x\log(5)\right)\log\left(e^{-x^2}\left(1\right.\right.}{10e^{x^2}+x}$$

Optimal antiderivative

$$(4-5x-\ln(5))\ln\left(10+x\,\mathrm{e}^{-x^2}\right)x$$

command

```
Integrate[(4*x - 5*x^2 - 8*x^3 + 10*x^4 + (-x + 2*x^3)*Log[5] + (4*x - 10*x^2 + E^x^2*(40 - 1
```

Mathematica 13.1 output

$$\int \frac{4x-5x^2-8x^3+10x^4+\left(-x+2x^3\right)\log(5)+\left(4x-10x^2+e^{x^2}(40-100x-10\log(5))-x\log(5)\right)\log\left(e^{-x^2}\left(1\right.\right.}{10e^{x^2}+x}$$

Mathematica 12.3 output

$$-x(-4+5x+\log(5))\log\left(10+e^{-x^2}x\right)$$

22.40 Problem number 7310

$$\int \frac{-108+144x^3+2x^6+e^{-1+x}(-54+72x^3+x^6)+(e^{-1+x}(9x-24x^4+16x^7)+e^{-1+x}x^7\log(x))\log\left(\frac{9-24x^3+16}{}\right.}{9x-24x^4+16x^7+x^7\log(x)}$$

Optimal antiderivative

$$\ln\left(\ln(x)+\left(4-\frac{3}{x^3}\right)^2\right)(2+e^{-1+x})$$

command

```
Integrate[(-108 + 144*x^3 + 2*x^6 + E^(-1 + x)*(-54 + 72*x^3 + x^6) + (E^(-1 + x)*(9*x - 24*x
1 + x)*x^7*Log[x])*Log[(9 - 24*x^3 + 16*x^6 + x^6*Log[x])/x^6])/(9*x - 24*x^4 + 16*x^7 + x^7*
```

Mathematica 13.1 output

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Mathematica 12.3 output

$$\frac{-12e\log(x)+e^x\log\left(\frac{(3-4x^3)^2}{x^6}+\log(x)\right)+2e\log(9-24x^3+16x^6+x^6\log(x))}{e}$$

22.41 Problem number 7387

$$\int \frac{180x^3-120x^4\log(3)+e^{2x}(200x-600x^2+200x^3\log(3))+e^x(360x^2-120x^3-180x^4+(-200x^3+60x^4+60}{9x^2-27x^3+9x^4\log(3)+e^{2x}(25-}$$

Optimal antiderivative

$$\frac{4(e^x-\ln(x(x\ln(3)-3)+1))x}{\frac{e^x}{x}+\frac{3}{5}}$$

command

```
Integrate[(180*x^3 - 120*x^4*Log[3] + E^(2*x)*(200*x - 600*x^2 + 200*x^3*Log[3]) + E^x*(360*x
200*x^3 + 60*x^4 + 60*x^5)*Log[3]) + (-60*x^2 + 180*x^3 - 60*x^4*Log[3] + E^x*(-200*x + 700*x
200*x^3 + 100*x^4)*Log[3]))*Log[1 - 3*x + x^2*Log[3]])/(9*x^2 - 27*x^3 + 9*x^4*Log[3] + E^(2*
```

Mathematica 13.1 output

$$\int \frac{180x^3-120x^4\log(3)+e^{2x}(200x-600x^2+200x^3\log(3))+e^x(360x^2-120x^3-180x^4+(-200x^3+60x^4+60}{9x^2-27x^3+9x^4\log(3)+e^{2x}(25-}$$

Mathematica 12.3 output

$$\frac{4x^2(5-15x+x^2\log(243))(e^x-\log(1-3x+x^2\log(3)))}{(5e^x+3x)(1-3x+x^2\log(3))}$$

22.42 Problem number 7414

$$\int \frac{e^{\frac{2\left(4x^2+x^3+\log\left(\frac{15}{40+3e^x}\right)\right)}{4x^2+x^3}}\left(e^x\left(-24x-6x^2\right)+\left(-640+e^x(-48-18x)-240x\right)\log\left(\frac{15}{40+3e^x}\right)\right)}{640x^3+320x^4+40x^5+e^x\left(48x^3+24x^4+3x^5\right)}\,dx$$

Optimal antiderivative

$$\mathrm{e}^{\frac{2x+\frac{2\ln\left(\frac{5}{\mathrm{e}^x+\frac{40}{3}}\right)}{(4+x)x}}{x}}$$

command

```
Integrate[(E^((2*(4*x^2 + x^3 + Log[15/(40 + 3*E^x)]))/(4*x^2 + x^3))*(E^x*(-24*x - 6*x^2) + 
640 + E^x*(-48 - 18*x) - 240*x)*Log[15/(40 + 3*E^x)]))/(640*x^3 + 320*x^4 + 40*x^5 + E^x*(48*
```

Mathematica 13.1 output

$$\int \frac{e^{\frac{2\left(4x^2+x^3+\log\left(\frac{15}{40+3e^x}\right)\right)}{4x^2+x^3}}\left(e^x\left(-24x-6x^2\right)+\left(-640+e^x(-48-18x)-240x\right)\log\left(\frac{15}{40+3e^x}\right)\right)}{640x^3+320x^4+40x^5+e^x\left(48x^3+24x^4+3x^5\right)}\,dx$$

Mathematica 12.3 output

$$15^{\frac{2}{x^2(4+x)}}e^2\left(\frac{1}{40+3e^x}\right)^{\frac{2}{x^2(4+x)}}$$

22.43 Problem number 7882

$$\int \frac{120x^2-120x\log\left(\frac{x}{4}\right)+120x\log^2\left(\frac{x}{4}\right)}{4x^2-4x^3+x^4+\left(4x-2x^2\right)\log^2\left(\frac{x}{4}\right)+\log^4\left(\frac{x}{4}\right)}\,dx$$

Optimal antiderivative

$$\frac{60x}{\frac{\ln\left(\frac{x}{4}\right)^2}{x}-x+2}$$

command

```
Integrate[(120*x^2 - 120*x*Log[x/4] + 120*x*Log[x/4]^2)/(4*x^2 - 4*x^3 + x^4 + (4*x - 2*x^2)*
```

Mathematica 13.1 output

$Aborted

Mathematica 12.3 output

$$-\frac{120x^2}{2(-2+x)x-2\log^2\left(\frac{x}{4}\right)}$$

22.44 Problem number 8195

$$\int \frac{-12+e^x\left(-3-3x-x^2\right)+\left(e^{3x}\left(6x+2x^2\right)+e^{2x}\left(24x+8x^2\right)\right)\log\left(\frac{16+4e^x}{3+x}\right)+\left(e^{3x}\left(6x+2x^2\right)+e^{2x}\left(24x+8x^2\right)\right.}{\left(12x+4x^2+e^x\left(3x+x^2\right)\right)\log\left(\frac{16+4e^x}{3+x}\right)+\left(12x+4x^2+e^x\left(3x+x^2\right)\right)\log\left(\frac{x}{\log(3)}\right)}$$

Optimal antiderivative

$$e^{2x}-\ln\left(\ln\left(\frac{x}{\ln(3)}\right)+\ln\left(\frac{4\,e^x+16}{3+x}\right)\right)$$

command

```
Integrate[(-12 + E^x*(-3 - 3*x - x^2) + (E^(3*x)*(6*x + 2*x^2) + E^(2*x)*(24*x + 8*x^2))*Log[
```

Mathematica 13.1 output

$$\int \frac{-12+e^x\left(-3-3x-x^2\right)+\left(e^{3x}\left(6x+2x^2\right)+e^{2x}\left(24x+8x^2\right)\right)\log\left(\frac{16+4e^x}{3+x}\right)+\left(e^{3x}\left(6x+2x^2\right)+e^{2x}\left(24x+8x^2\right)\right.}{\left(12x+4x^2+e^x\left(3x+x^2\right)\right)\log\left(\frac{16+4e^x}{3+x}\right)+\left(12x+4x^2+e^x\left(3x+x^2\right)\right)\log\left(\frac{x}{\log(3)}\right)}$$

Mathematica 12.3 output

$$e^{2x}-\log\left(\log\left(\frac{4+e^x}{3+x}\right)+\log\left(\frac{4x}{\log(3)}\right)\right)$$

22.45 Problem number 8361

$$\int \frac{12x^2\log(2)+3e^{2/x}x^2\log(2)+3e^{\frac{2\left(x^3+x^3\log(2)\right)}{\log(2)}}x^2\log(2)+e^{\frac{1}{x}}\left(5+12x^2\right)\log(2)+e^{\frac{x^3+x^3\log(2)}{\log(2)}}\left(15x^4-6e^{\frac{1}{x}}x^2\log(2\right.}{12x^2\log(2)+12e^{\frac{1}{x}}x^2\log(2)+3e^{2/x}x^2\log(2)+3e^{\frac{2\left(x^3+x^3\log(2)\right)}{\log(2)}}x^2\log(2)+e^{\frac{x^3+x^3\log(2)}{\log(2)}}\left(-12x^2\log(2)\right.}$$

Optimal antiderivative

$$\frac{5}{3\left(e^{\frac{1}{x}}-e^{\frac{x^2(x\ln(2)+x)}{\ln(2)}}+2\right)}+x$$

command

```
Integrate[(12*x^2*Log[2] + 3*E^(2/x)*x^2*Log[2] + 3*E^((2*(x^3 + x^3*Log[2]))/Log[2])*x^2*Log
1)*(5 + 12*x^2)*Log[2] + E^((x^3 + x^3*Log[2])/Log[2])*(15*x^4 - 6*E^x^(-1)*x^2*Log[2] + (-
12*x^2 + 15*x^4)*Log[2]))/(12*x^2*Log[2] + 12*E^x^(-1)*x^2*Log[2] + 3*E^(2/x)*x^2*Log[2] + 3*
12*x^2*Log[2] - 6*E^x^(-1)*x^2*Log[2])),x]
```

Mathematica 13.1 output

$$\int \frac{12x^2\log(2)+3e^{2/x}x^2\log(2)+3e^{\frac{2\left(x^3+x^3\log(2)\right)}{\log(2)}}x^2\log(2)+e^{\frac{1}{x}}\left(5+12x^2\right)\log(2)+e^{\frac{x^3+x^3\log(2)}{\log(2)}}\left(15x^4-6e^{\frac{1}{x}}x^2\log(2\right.}{12x^2\log(2)+12e^{\frac{1}{x}}x^2\log(2)+3e^{2/x}x^2\log(2)+3e^{\frac{2\left(x^3+x^3\log(2)\right)}{\log(2)}}x^2\log(2)+e^{\frac{x^3+x^3\log(2)}{\log(2)}}\left(-12x^2\log(2)\right.}$$

Mathematica 12.3 output

$$\frac{1}{3}\left(3x+\frac{15\left(2+e^{\frac{1}{x}}\right)x^4(1+\log(2))+e^{\frac{1}{x}}\log(32)}{\left(2+e^{\frac{1}{x}}-e^{x^3\left(1+\frac{1}{\log(2)}\right)}\right)\left(e^{\frac{1}{x}}\log(2)+x^4\left(6+e^{\frac{1}{x}}(3+\log(8))+\log(64)\right)\right)}\right)$$

22.46 Problem number 8653

$$\int \frac{2x^3\log^2(3)-4x\log^3(3)+\left(-2x^5\log(3)+14x^3\log^2(3)\right)\log(x)-12x^5\log(3)\log^2(x)+2x^7\log^3(x)}{-8\log^3(3)+12x^2\log^2(3)\log(x)-6x^4\log(3)\log^2(x)+x^6\log^3(x)}\,dx$$

Optimal antiderivative

$$\left(x-\frac{x}{-\frac{\ln(x)x^2}{\ln(3)}+2}\right)^2$$

command

```
Integrate[(2*x^3*Log[3]^2 - 4*x*Log[3]^3 + (-2*x^5*Log[3] + 14*x^3*Log[3]^2)*Log[x] - 12*x^5*
8*Log[3]^3 + 12*x^2*Log[3]^2*Log[x] - 6*x^4*Log[3]*Log[x]^2 + x^6*Log[x]^3),x]
```

Mathematica 13.1 output

$$\int \frac{2x^3\log^2(3)-4x\log^3(3)+\left(-2x^5\log(3)+14x^3\log^2(3)\right)\log(x)-12x^5\log(3)\log^2(x)+2x^7\log^3(x)}{-8\log^3(3)+12x^2\log^2(3)\log(x)-6x^4\log(3)\log^2(x)+x^6\log^3(x)}\,dx$$

Mathematica 12.3 output

$$x^2\left(8\log^2(3)\log^3(9)-x^6\left(\log^2(3)+\log(3)\log(9)-\log^2(9)\right)+4x^2\log^2(9)\left(\log^2(3)-\log(3)\log(9)+\log^2(9)\right)+x^4\left(\right.\right.$$

22.47 Problem number 8762

$$\int \frac{}{-5x+6x^2-x^3+(5x-6x^2+x^3)\log(5)+(5x-6x^2+x^3)\log(5-x)+(e^5(-15x+18x^2-3x^3)+e^5(15x-1}$$

Optimal antiderivative

$$\frac{x^2}{\left(x\ln\left(\frac{x^2-x}{\ln(5-x)+\ln(5)-1}\right)+x\,\mathrm{e}^{-5}\right)^2}$$

command

```
Integrate[(E^15*(10 - 24*x + 6*x^2) + E^15*(-10 + 22*x - 4*x^2)*Log[5] + E^15*(-10 + 22*x - 4
5*x + 6*x^2 - x^3 + (5*x - 6*x^2 + x^3)*Log[5] + (5*x - 6*x^2 + x^3)*Log[5 - x] + (E^5*(-
15*x + 18*x^2 - 3*x^3) + E^5*(15*x - 18*x^2 + 3*x^3)*Log[5] + E^5*(15*x - 18*x^2 + 3*x^3)*Log
x + x^2)/(-1 + Log[5] + Log[5 - x])] + (E^10*(-15*x + 18*x^2 - 3*x^3) + E^10*(15*x - 18*x^2 +
x + x^2)/(-1 + Log[5] + Log[5 - x])]^2 + (E^15*(-5*x + 6*x^2 - x^3) + E^15*(5*x - 6*x^2 + x^3
x + x^2)/(-1 + Log[5] + Log[5 - x])]^3),x]
```

Mathematica 13.1 output

$$\int \frac{}{-5x+6x^2-x^3+(5x-6x^2+x^3)\log(5)+(5x-6x^2+x^3)\log(5-x)+(e^5(-15x+18x^2-3x^3)+e^5(15x-1}$$

Mathematica 12.3 output

$$\frac{e^{10}}{\left(1+e^5\log\left(\frac{(-1+x)x}{-1+\log(-5(-5+x))}\right)\right)^2}$$

22.48 Problem number 8818

$$\int \frac{-2+x+((-12+7x)\log(3)+(12-7x)\log(x))\log(-\log(3)+\log(x))}{(-256x^7+256x^8-64x^9)\log(3)+(256x^7-256x^8+64x^9)\log(x)}\,dx$$

Optimal antiderivative

$$\frac{\ln(\ln(x)-\ln(3))}{64x^6(-2+x)}$$

command

```
Integrate[(-2 + x + ((-12 + 7*x)*Log[3] + (12 - 7*x)*Log[x])*Log[-Log[3] + Log[x]])/((-
256*x^7 + 256*x^8 - 64*x^9)*Log[3] + (256*x^7 - 256*x^8 + 64*x^9)*Log[x]),x]
```

Mathematica 13.1 output

$$\int \frac{-2+x+((-12+7x)\log(3)+(12-7x)\log(x))\log(-\log(3)+\log(x))}{(-256x^7+256x^8-64x^9)\log(3)+(256x^7-256x^8+64x^9)\log(x)}\,dx$$

Mathematica 12.3 output

$$\frac{\log\left(\log\left(\frac{x}{3}\right)\right)}{64(-2+x)x^6}$$

22.49 Problem number 9331

$$\int \frac{-32\log(3)+32\log(3)\log\left(\frac{x}{3}\right)+(-8+16x)\log(3)\log^2\left(\frac{x}{3}\right)-8\log(3)\log^2\left(\frac{x}{3}\right)\log(x)}{\left(-4x\log\left(\frac{x}{3}\right)-x^2\log^2\left(\frac{x}{3}\right)+x\log^2\left(\frac{x}{3}\right)\log(x)\right)\log^2\left(\frac{16x^2+8x^3\log\left(\frac{x}{3}\right)+x^4\log^2\left(\frac{x}{3}\right)+\left(-8x^2\log\left(\frac{x}{3}\right)-2x^3\log^2\left(\frac{x}{3}\right)\right)\log(x)+x^2\log}{\log^2\left(\frac{x}{3}\right)}\right.}$$

Optimal antiderivative

$$\frac{4\ln(3)}{\ln\left(x^2\left(\frac{4}{\ln\left(\frac{x}{3}\right)}-\ln(x)+x\right)^2\right)}$$

command

```
Integrate[(-32*Log[3] + 32*Log[3]*Log[x/3] + (-8 + 16*x)*Log[3]*Log[x/3]^2 - 8*Log[3]*Log[x/3
4*x*Log[x/3] - x^2*Log[x/3]^2 + x*Log[x/3]^2*Log[x])*Log[(16*x^2 + 8*x^3*Log[x/3] + x^4*Log[x
8*x^2*Log[x/3] - 2*x^3*Log[x/3]^2)*Log[x] + x^2*Log[x/3]^2*Log[x]^2)/Log[x/3]^2]^2),x]
```

Mathematica 13.1 output

$$\int \frac{-32\log(3)+32\log(3)\log\left(\frac{x}{3}\right)+(-8+16x)\log(3)\log^2\left(\frac{x}{3}\right)-8\log(3)\log^2\left(\frac{x}{3}\right)\log(x)}{\left(-4x\log\left(\frac{x}{3}\right)-x^2\log^2\left(\frac{x}{3}\right)+x\log^2\left(\frac{x}{3}\right)\log(x)\right)\log^2\left(\frac{16x^2+8x^3\log\left(\frac{x}{3}\right)+x^4\log^2\left(\frac{x}{3}\right)+\left(-8x^2\log\left(\frac{x}{3}\right)-2x^3\log^2\left(\frac{x}{3}\right)\right)\log(x)+x^2\log}{\log^2\left(\frac{x}{3}\right)}\right.}$$

Mathematica 12.3 output

$$\frac{4\log(3)\left(-4-\log(81)+\log^2\left(\frac{x}{3}\right)(-1+2x-\log(x))+4\log(x)\right)}{\left(-4+4\log\left(\frac{x}{3}\right)+\log^2\left(\frac{x}{3}\right)(-1+2x-\log(x))\right)\log\left(\frac{x^2\left(4+\log\left(\frac{x}{3}\right)(x-\log(x))\right)^2}{\log^2\left(\frac{x}{3}\right)}\right)}$$

22.50 Problem number 9462

$$\int \frac{8x-9\log(2)+(-2x+2\log}{-49x+28x^2-4x^3+(49-28x+4x^2)\log(2)+(-4x+4\log(2))\log^2(x)+(-14x+4x^2+(14-4x)\log(2))\log}$$

Optimal antiderivative

$$-\frac{x}{\ln(-2\ln(2)+2x)-2\ln(x)-2x+7}$$

command

```
Integrate[(8*x - 9*Log[2] + (-2*x + 2*Log[2])*Log[x] + (x - Log[2])*Log[2*x - 2*Log[2]])/(-
49*x + 28*x^2 - 4*x^3 + (49 - 28*x + 4*x^2)*Log[2] + (-4*x + 4*Log[2])*Log[x]^2 + (-
14*x + 4*x^2 + (14 - 4*x)*Log[2])*Log[2*x - 2*Log[2]] + (-x + Log[2])*Log[2*x - 2*Log[2]]^2 +
28 + 8*x)*Log[2] + (4*x - 4*Log[2])*Log[2*x - 2*Log[2]])),x]
```

Mathematica 13.1 output

$$\int \frac{8x - 9\log(2) + (-2x + 2\log}{-49x + 28x^2 - 4x^3 + (49 - 28x + 4x^2)\log(2) + (-4x + 4\log(2))\log^2(x) + (-14x + 4x^2 + (14 - 4x)\log(2))\log}$$

Mathematica 12.3 output

$$\frac{x(2x - \log(4))}{2(x - \log(2))(-7 + 2x + 2\log(x) - \log(2x - \log(4)))}$$

22.51 Problem number 9582

$$\int \frac{-16x - 16x^2\log(2) + \left(-20 + 20x - 4x^3\right)\log^2(2) + \left(16x\log\right.}{64x - 32x^2 + 4x^3 + \left(-80x + 84x^2 - 32x^3 + 4x^4\right)\log(2) + \left(25x - 40x^2 + 26x^3 - 8x^4 + x^5\right)\log^2(2) + \left(\left(-64x +\right.\right.}$$

Optimal antiderivative

$$\frac{4}{\frac{5}{\frac{2}{\ln(2)} + x - \ln(-x)} + x - 4}$$

command

```
Integrate[(-16*x - 16*x^2*Log[2] + (-20 + 20*x - 4*x^3)*Log[2]^2 + (16*x*Log[2] + 8*x^2*Log[2
x] - 4*x*Log[2]^2*Log[-x]^2)/(64*x - 32*x^2 + 4*x^3 + (-80*x + 84*x^2 - 32*x^3 + 4*x^4)*Log[2
64*x + 32*x^2 - 4*x^3)*Log[2] + (40*x - 42*x^2 + 16*x^3 - 2*x^4)*Log[2]^2)*Log[-x] + (16*x -
x]^2),x]
```

Mathematica 13.1 output

$$\int \frac{-16x - 16x^2\log(2) + \left(-20 + 20x - 4x^3\right)\log^2(2) + \left(16x\log\right.}{64x - 32x^2 + 4x^3 + \left(-80x + 84x^2 - 32x^3 + 4x^4\right)\log(2) + \left(25x - 40x^2 + 26x^3 - 8x^4 + x^5\right)\log^2(2) + \left(\left(-64x +\right.\right.}$$

Mathematica 12.3 output

$$-\frac{4\left(-x^4\log^3(2) + x^3\log(2)\left(5\log^2(2) - \log(4) + \log(2)\log(16)\right) + x^2\left(-15\log^3(2) - \log^2(2)(-2 + \log(16)) + \log^2(\right.\right.}{\left(x^3\log^2(2) - \log^2(1\right.}$$

22.52 Problem number 9617

$$\int \frac{\left(10+e^x(2-2x)+e^{x^2}(2-4x^2)+8\log(2)\right)\log\left(\frac{4x}{5+e^x+e^{x^2}-3x+4\log(2)}\right)}{5x+e^xx+e^{x^2}x-3x^2+4x\log(2)}\,dx$$

Optimal antiderivative

$$\ln\left(\frac{x}{\ln(2)-\frac{3x}{4}+\frac{5}{4}+\frac{e^x}{4}+\frac{e^{x^2}}{4}}\right)^2$$

command

```
Integrate[((10 + E^x*(2 - 2*x) + E^x^2*(2 - 4*x^2) + 8*Log[2])*Log[(4*x)/(5 + E^x + E^x^2 - 3
```

Mathematica 13.1 output

$$\int \frac{\left(10+e^x(2-2x)+e^{x^2}(2-4x^2)+8\log(2)\right)\log\left(\frac{4x}{5+e^x+e^{x^2}-3x+4\log(2)}\right)}{5x+e^xx+e^{x^2}x-3x^2+4x\log(2)}\,dx$$

Mathematica 12.3 output

$$\log^2\left(\frac{4x}{e^x+e^{x^2}-3x+5\left(1+\frac{4\log(2)}{5}\right)}\right)$$

22.53 Problem number 9713

$$\int \frac{42+144x+108x^2+3\,2^{2+\frac{4x}{3}}\left(\frac{1}{x^2}\right)^{2x/3}x^4+2^{2x/3}\left(\frac{1}{x^2}\right)^{x/3}\left(51x^2+70x^3+x^3\log\left(\frac{4}{x^2}\right)\right)}{48+144x+108x^2+3\,2^{2+\frac{4x}{3}}\left(\frac{1}{x^2}\right)^{2x/3}x^4+2^{2x/3}\left(\frac{1}{x^2}\right)^{x/3}(48x^2+72x^3)}\,dx$$

Optimal antiderivative

$$x-\frac{x}{4\left(3x+e^{\frac{x\ln\left(\frac{4}{x^2}\right)}{3}}x^2+2\right)}$$

command

```
Integrate[(42 + 144*x + 108*x^2 + 3*2^(2 + (4*x)/3)*(x^(-2))^((2*x)/3)*x^4 + 2^((2*x)/3)*(x^(
2))^(x/3)*(51*x^2 + 70*x^3 + x^3*Log[4/x^2]))/(48 + 144*x + 108*x^2 + 3*2^(2 + (4*x)/3)*(x^(-
2))^((2*x)/3)*x^4 + 2^((2*x)/3)*(x^(-2))^(x/3)*(48*x^2 + 72*x^3)),x]
```

Mathematica 13.1 output

$$\int \frac{42+144x+108x^2+3\ 2^{2+\frac{4x}{3}}\left(\frac{1}{x^2}\right)^{2x/3}x^4+2^{2x/3}\left(\frac{1}{x^2}\right)^{x/3}\left(51x^2+70x^3+x^3\log\left(\frac{4}{x^2}\right)\right)}{48+144x+108x^2+3\ 2^{2+\frac{4x}{3}}\left(\frac{1}{x^2}\right)^{2x/3}x^4+2^{2x/3}\left(\frac{1}{x^2}\right)^{x/3}(48x^2+72x^3)}\,dx$$

Mathematica 12.3 output

$$\frac{1}{4}x\left(4-\frac{1}{2+2^{2x/3}\left(\frac{1}{x^2}\right)^{-1+\frac{x}{3}}+3x}\right)$$

22.54 Problem number 10072

$$\int \frac{-700+e^{\frac{1}{25}(-125+145x-16x^2+25x\log(3x))}\left(25-170x+32x^2-25x\log(3x)\right)}{19600-1400e^{\frac{1}{25}(-125+145x-16x^2+25x\log(3x))}+25e^{\frac{2}{25}(-125+145x-16x^2+25x\log(3x))}}\,dx$$

Optimal antiderivative

$$\frac{x}{\mathrm{e}^{x+4+x\ln(3x)-\left(\frac{4x}{5}-3\right)^2}-28}$$

command

```
Integrate[(-700 + E^((-125 + 145*x - 16*x^2 + 25*x*Log[3*x])/25)*(25 - 170*x + 32*x^2 - 25*x*
125 + 145*x - 16*x^2 + 25*x*Log[3*x])/25) + 25*E^((2*(-125 + 145*x - 16*x^2 + 25*x*Log[3*x]))
```

Mathematica 13.1 output

$$\int \frac{-700+e^{\frac{1}{25}(-125+145x-16x^2+25x\log(3x))}\left(25-170x+32x^2-25x\log(3x)\right)}{19600-1400e^{\frac{1}{25}(-125+145x-16x^2+25x\log(3x))}+25e^{\frac{2}{25}(-125+145x-16x^2+25x\log(3x))}}\,dx$$

Mathematica 12.3 output

$$-\frac{e^{5+\frac{16x^2}{25}}x}{28e^{5+\frac{16x^2}{25}}-3^xe^{29x/5}x^x}$$

22.55 Problem number 10272

$$\int \frac{6x^3-4x^2\log(2)+e^x\left(-16x^5+\left(-16x^3+32x^4\right)\log(2)+\left(20x^2-24x^3\right)\log^2(2)+\left(-8x+8x^2\right)\log^3(2)+(1-x}{x^2+e^x\left(8x^3-8x^2\log(2)+2x\log^2(2)\right)+e^{2x}\left(16x^4-32x^3\log(2)+\right.}$$

Optimal antiderivative

$$\frac{4x}{4\,\mathrm{e}^x+\frac{4x}{(2x-\ln(2))^2}}-x^2$$

command

```
Integrate[(6*x^3 - 4*x^2*Log[2] + E^x*(-16*x^5 + (-16*x^3 + 32*x^4)*Log[2] + (20*x^2 - 24*x^3
8*x + 8*x^2)*Log[2]^3 + (1 - x)*Log[2]^4) + E^(2*x)*(-32*x^5 + 64*x^4*Log[2] - 48*x^3*Log[2]^
```

Mathematica 13.1 output

$$\int \frac{6x^3 - 4x^2\log(2) + e^x\left(-16x^5 + \left(-16x^3 + 32x^4\right)\log(2) + \left(20x^2 - 24x^3\right)\log^2(2) + \left(-8x + 8x^2\right)\log^3(2) + (1 - x}{x^2 + e^x\left(8x^3 - 8x^2\log(2) + 2x\log^2(2)\right) + e^{2x}\left(16x^4 - 32x^3\log(2) +\right.}$$

Mathematica 12.3 output

$$x\left(-x\right.$$

$$+\frac{64x^7 - \log^6(2) + x\log^5(2)(8 + \log(2)) - 64x^6(-1 + \log(8)) - x^2\log^3(2)\left(4\log^2(2) + \log(4) + 2\log(2)(9 + \log(16\right.}{(2x - \log(2))^3\left(2x^2 - x(-2 + \log(2)) + \log(2)\right)\left(x + 4\right.}$$