

# CAS integration tests regression report

## Giac 1.9.0-11 via sagemath 9.6 vs. Giac 1.7.0 via sagemath 9.3

Nasser M. Abbasi

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# 1 Summary of regression test table

Table 1: Summary table of regression tests

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1	10	532	0 (not solved)	1 (pass)
2	13	881	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
3	13	882	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
4	13	889	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
5	13	890	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
6	13	914	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
7	13	915	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
8	13	916	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
9	13	920	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
10	13	921	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
11	13	922	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
12	14	1005	0 (not solved)	1 (pass)
13	20	224	0 (not solved)	1 (pass)
14	20	231	0 (not solved)	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
15	25	1735	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
16	25	1738	0 (not solved)	1 (pass)
17	25	1739	0 (not solved)	1 (pass)
18	25	1740	0 (not solved)	1 (pass)
19	25	1741	0 (not solved)	1 (pass)
20	25	1748	0 (not solved)	1 (pass)
21	25	1750	0 (not solved)	1 (pass)
22	25	1751	0 (not solved)	1 (pass)
23	25	2991	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
24	25	2992	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
25	26	245	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
26	26	246	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
27	26	249	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
28	26	252	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
29	26	256	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
30	26	257	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
31	26	258	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
32	26	263	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
33	26	264	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
34	27	1054	0 (not solved)	1 (pass)
35	33	841	0 (not solved)	1 (pass)
36	33	853	0 (not solved)	1 (pass)
37	33	854	0 (not solved)	1 (pass)
38	33	1957	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
39	33	1958	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
40	33	1966	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
41	33	1967	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
42	33	1968	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
43	33	1969	0 (not solved)	1 (pass)
44	34	1179	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
45	34	2225	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
46	34	2227	0 (not solved)	1 (pass)
47	35	18	0 (not solved)	1 (pass)
48	35	19	0 (not solved)	1 (pass)
49	35	20	0 (not solved)	1 (pass)
50	35	21	0 (not solved)	1 (pass)
51	35	22	0 (not solved)	1 (pass)
52	35	23	0 (not solved)	1 (pass)
53	35	24	0 (not solved)	1 (pass)
54	35	25	0 (not solved)	1 (pass)
55	35	26	0 (not solved)	1 (pass)
56	35	27	0 (not solved)	1 (pass)
57	35	28	0 (not solved)	1 (pass)
58	35	29	0 (not solved)	1 (pass)
59	35	30	0 (not solved)	1 (pass)
60	35	31	0 (not solved)	1 (pass)
61	35	44	0 (not solved)	1 (pass)
62	35	45	0 (not solved)	1 (pass)
63	35	46	0 (not solved)	1 (pass)
64	35	47	0 (not solved)	1 (pass)
65	35	48	0 (not solved)	1 (pass)
66	35	49	0 (not solved)	1 (pass)
67	35	50	0 (not solved)	1 (pass)
68	35	51	0 (not solved)	1 (pass)
69	35	52	0 (not solved)	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
70	35	53	0 (not solved)	1 (pass)
71	35	130	0 (not solved)	1 (pass)
72	35	134	0 (not solved)	1 (pass)
73	35	586	-1 (time out)	1 (pass)
74	35	842	-2 (exception)  Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
75	39	3	0 (not solved)	1 (pass)
76	39	4	0 (not solved)	1 (pass)
77	51	163	0 (not solved)	1 (pass)
78	55	744	0 (not solved)	1 (pass)
79	59	201	-1 (time out)	1 (pass)
80	59	263	-1 (time out)	1 (pass)
81	59	295	-1 (time out)	1 (pass)
82	61	27	-1 (time out)	1 (pass)
83	64	111	-2 (exception)  Exception raised: NotImplementedError >> Unable to parse Giac output: 2*(2*sqrt(2)*atan (4*sqrt(sageV	1 (pass)
84	64	185	-2 (exception)  Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
85	65	190	0 (not solved)	1 (pass)
86	65	191	0 (not solved)	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
87	65	192	0 (not solved)	1 (pass)
88	65	193	0 (not solved)	1 (pass)
89	65	206	0 (not solved)	1 (pass)
90	65	207	0 (not solved)	1 (pass)
91	65	208	0 (not solved)	1 (pass)
92	65	209	0 (not solved)	1 (pass)
93	65	210	0 (not solved)	1 (pass)
94	65	228	0 (not solved)	1 (pass)
95	65	229	0 (not solved)	1 (pass)
96	65	230	0 (not solved)	1 (pass)
97	65	231	0 (not solved)	1 (pass)
98	65	249	0 (not solved)	1 (pass)
99	65	250	0 (not solved)	1 (pass)
100	65	251	0 (not solved)	1 (pass)
101	70	164	-2 (exception)  Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
102	70	517	-1 (time out)	1 (pass)
103	70	518	-1 (time out)	1 (pass)
104	70	528	-1 (time out)	1 (pass)
105	70	529	0 (not solved)	1 (pass)
106	74	468	-2 (exception)  Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
107	74	469	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
108	74	470	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
109	74	471	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
110	79	499	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
111	79	558	-1 (time out)	1 (pass)
112	89	128	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
113	89	129	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
114	89	130	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
115	89	144	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
116	92	107	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
117	92	108	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
118	92	109	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
119	92	110	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
120	92	113	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
121	94	107	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
122	94	109	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
123	94	111	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
124	94	112	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
125	94	113	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
126	94	281	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
127	94	410	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
128	94	411	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
129	94	412	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
130	94	413	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
131	95	68	0 (not solved)	1 (pass)
132	98	52	0 (not solved)	1 (pass)
133	103	1137	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
134	103	1138	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
135	103	1143	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
136	103	1144	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
137	103	1149	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
138	103	1150	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
139	103	1155	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
140	103	1156	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
141	103	1161	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
142	103	1162	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
143	103	1167	-1 (time out)	1 (pass)
144	103	1168	-1 (time out)	1 (pass)
145	141	421	0 (not solved)	1 (pass)
146	141	422	-1 (time out)	1 (pass)
147	141	428	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
148	141	429	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
149	141	669	0 (not solved)	1 (pass)
150	141	734	0 (not solved)	1 (pass)
151	141	740	0 (not solved)	1 (pass)
152	149	24	0 (not solved)	1 (pass)
153	153	20	0 (not solved)	1 (pass)
154	153	26	0 (not solved)	1 (pass)
155	164	437	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
156	164	438	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
157	164	439	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
158	164	442	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
159	164	443	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
160	164	447	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
161	164	448	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
162	164	449	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
163	164	452	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
164	164	455	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
165	164	456	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
166	164	486	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage3:=type(sage2) :;OU	1 (pass)
167	164	491	-2 (exception) Exception raised: RuntimeError >> An error occurred running a Giac command:INPUT:sage2OUTPUT : Evaluat	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
168	169	47	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
169	173	26	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
170	173	28	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
171	173	29	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
172	173	31	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
173	173	34	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
174	173	36	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
175	173	37	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
176	173	39	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
177	173	42	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
178	173	44	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
179	173	45	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
180	173	47	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
181	173	106	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
182	173	108	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
183	173	109	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
184	173	111	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
185	173	113	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
186	173	115	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
187	173	117	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
188	173	118	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
189	173	120	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
190	173	122	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
191	173	124	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
192	173	126	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
193	173	127	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
194	173	129	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
195	173	131	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
196	173	133	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
197	176	27	0 (not solved)	1 (pass)
198	180	144	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
199	180	148	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
200	180	150	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
201	180	152	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
202	180	154	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
203	180	156	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
204	180	159	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
205	180	161	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
206	180	163	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
207	180	165	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
208	180	167	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
209	196	549	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
210	196	550	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
211	196	551	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
212	196	552	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
213	196	553	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
214	196	728	0 (not solved)	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
215	196	729	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
216	196	730	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
217	196	731	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
218	197	5	-1 (time out)	1 (pass)
219	197	6	-1 (time out)	1 (pass)
220	197	7	-1 (time out)	1 (pass)
221	197	8	-1 (time out)	1 (pass)
222	197	24	0 (not solved)	1 (pass)
223	199	2	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
224	199	18	0 (not solved)	1 (pass)
225	199	19	0 (not solved)	1 (pass)
226	199	20	0 (not solved)	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
227	199	21	0 (not solved)	1 (pass)
228	199	22	0 (not solved)	1 (pass)
229	199	23	0 (not solved)	1 (pass)
230	199	24	0 (not solved)	1 (pass)
231	199	128	0 (not solved)	1 (pass)
232	199	162	0 (not solved)	1 (pass)
233	199	227	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
234	199	229	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
235	199	244	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
236	199	246	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
237	199	247	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
238	199	296	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
239	199	310	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
240	199	312	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
241	199	313	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
242	199	314	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
243	199	315	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
244	199	316	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
245	199	317	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
246	199	318	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
247	199	383	0 (not solved)	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
248	199	384	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
249	199	386	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
250	199	402	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
251	199	475	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
252	199	476	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
253	199	477	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
254	199	478	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
255	199	479	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0, sageV	1 (pass)
256	199	560	0 (not solved)	1 (pass)
257	199	561	0 (not solved)	1 (pass)
258	199	562	0 (not solved)	1 (pass)
259	199	563	0 (not solved)	1 (pass)
260	199	579	0 (not solved)	1 (pass)
261	199	580	0 (not solved)	1 (pass)
262	199	776	0 (not solved)	1 (pass)
263	199	777	0 (not solved)	1 (pass)
264	199	778	0 (not solved)	1 (pass)
265	199	779	0 (not solved)	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
266	199	793	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
267	199	795	0 (not solved)	1 (pass)
268	199	796	0 (not solved)	1 (pass)
269	201	71	0 (not solved)	1 (pass)
270	201	82	0 (not solved)	1 (pass)
271	204	46	0 (not solved)	1 (pass)
272	204	149	0 (not solved)	1 (pass)
273	206	70	0 (not solved)	1 (pass)
274	206	71	0 (not solved)	1 (pass)
275	206	72	0 (not solved)	1 (pass)
276	206	73	0 (not solved)	1 (pass)
277	206	75	0 (not solved)	1 (pass)
278	206	76	0 (not solved)	1 (pass)
279	206	78	0 (not solved)	1 (pass)
280	206	79	0 (not solved)	1 (pass)
281	206	80	0 (not solved)	1 (pass)
282	206	81	0 (not solved)	1 (pass)
283	206	86	0 (not solved)	1 (pass)
284	206	87	0 (not solved)	1 (pass)
285	206	88	0 (not solved)	1 (pass)
286	206	89	0 (not solved)	1 (pass)
287	206	91	0 (not solved)	1 (pass)
288	206	92	0 (not solved)	1 (pass)
289	206	103	0 (not solved)	1 (pass)
290	206	110	0 (not solved)	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
291	206	111	0 (not solved)	1 (pass)
292	206	112	0 (not solved)	1 (pass)
293	206	113	0 (not solved)	1 (pass)
294	206	117	0 (not solved)	1 (pass)
295	206	118	0 (not solved)	1 (pass)
296	206	119	0 (not solved)	1 (pass)
297	206	120	0 (not solved)	1 (pass)
298	206	121	0 (not solved)	1 (pass)
299	206	122	0 (not solved)	1 (pass)
300	206	123	0 (not solved)	1 (pass)
301	206	124	0 (not solved)	1 (pass)
302	206	125	0 (not solved)	1 (pass)
303	206	127	0 (not solved)	1 (pass)
304	206	128	0 (not solved)	1 (pass)
305	206	129	0 (not solved)	1 (pass)
306	206	132	0 (not solved)	1 (pass)
307	206	135	0 (not solved)	1 (pass)
308	209	403	0 (not solved)	1 (pass)
309	209	1081	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-\operatorname{atan}(i)+\ln(4*\sqrt{2}))*\operatorname{sign}(\dots)$	1 (pass)
310	209	1123	0 (not solved)	1 (pass)
311	209	1648	0 (not solved)	1 (pass)
312	210	42	0 (not solved)	1 (pass)
313	210	214	0 (not solved)	1 (pass)
314	210	529	0 (not solved)	1 (pass)
315	210	756	0 (not solved)	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
316	210	819	-1 (time out)	1 (pass)
317	210	856	0 (not solved)	1 (pass)
318	210	1147	0 (not solved)	1 (pass)
319	210	1197	0 (not solved)	1 (pass)
320	210	1569	0 (not solved)	1 (pass)
321	210	1596	0 (not solved)	1 (pass)
322	210	1763	-1 (time out)	1 (pass)
323	210	1809	0 (not solved)	1 (pass)
324	210	1860	0 (not solved)	1 (pass)
325	210	1905	0 (not solved)	1 (pass)
326	210	1996	-1 (time out)	1 (pass)
327	210	2050	-2 (exception)  Exception raised: NotImplementedError >> Unable to parse Giac output: Undef/Unsigned Inf encountered	1 (pass)
328	210	2306	-1 (time out)	1 (pass)
329	210	2370	0 (not solved)	1 (pass)
330	210	2446	0 (not solved)	1 (pass)
331	210	2680	-1 (time out)	1 (pass)
332	210	2802	0 (not solved)	1 (pass)
333	210	2888	0 (not solved)	1 (pass)
334	210	2912	-2 (exception)  Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
335	210	2955	0 (not solved)	1 (pass)
336	210	3083	0 (not solved)	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
337	210	3165	0 (not solved)	1 (pass)
338	210	3683	0 (not solved)	1 (pass)
339	210	3722	0 (not solved)	1 (pass)
340	210	3788	0 (not solved)	1 (pass)
341	210	3922	0 (not solved)	1 (pass)
342	210	4171	0 (not solved)	1 (pass)
343	210	4209	-2 (exception)  Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
344	210	4323	0 (not solved)	1 (pass)
345	210	4476	0 (not solved)	1 (pass)
346	210	5416	0 (not solved)	1 (pass)
347	210	5600	0 (not solved)	1 (pass)
348	210	5649	0 (not solved)	1 (pass)
349	210	5811	-2 (exception)  Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
350	210	5911	0 (not solved)	1 (pass)
351	210	5947	0 (not solved)	1 (pass)
352	210	6013	0 (not solved)	1 (pass)
353	210	6096	0 (not solved)	1 (pass)
354	210	6201	0 (not solved)	1 (pass)
355	210	6696	0 (not solved)	1 (pass)
356	210	6926	0 (not solved)	1 (pass)
357	210	7043	0 (not solved)	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
358	210	7194	-1 (time out)	1 (pass)
359	210	7444	0 (not solved)	1 (pass)
360	210	7495	0 (not solved)	1 (pass)
361	210	7627	0 (not solved)	1 (pass)
362	210	7683	0 (not solved)	1 (pass)
363	210	7755	-1 (time out)	1 (pass)
364	210	7789	0 (not solved)	1 (pass)
365	210	8043	0 (not solved)	1 (pass)
366	210	8143	-1 (time out)	1 (pass)
367	210	8742	0 (not solved)	1 (pass)
368	210	9011	0 (not solved)	1 (pass)
369	210	9031	0 (not solved)	1 (pass)
370	210	9135	0 (not solved)	1 (pass)
371	210	9215	0 (not solved)	1 (pass)
372	210	9254	0 (not solved)	1 (pass)
373	210	9330	0 (not solved)	1 (pass)
374	210	9649	0 (not solved)	1 (pass)
375	210	9667	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV	1 (pass)
376	210	9708	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-(-2*\ln(3)*\exp(5)$ $*\exp(1)^{21+6*}$	1 (pass)
377	210	9784	0 (not solved)	1 (pass)
378	210	10096	-1 (time out)	1 (pass)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
379	210	10193	0 (not solved)	1 (pass)

## 2 Test file number 10

Test folder name:

test\_cases/0\_Independent\_test\_suites/10\_Timofeev\_Problems

### 2.1 Problem number 532

$$\int \frac{e^{3x/4}}{(-2 + e^{3x/4}) \sqrt{-2 + e^{3x/4} + e^{3x/2}}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh}\left(\frac{2 - 5e^{\frac{3x}{4}}}{4\sqrt{-2 + e^{\frac{3x}{4}} + e^{\frac{3x}{2}}}}\right)}{3}$$

command

```
integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{2}{3} \log\left(\left|\sqrt{e^{\left(\frac{3}{2}x\right)} + e^{\left(\frac{3}{4}x\right)} - 2} - e^{\left(\frac{3}{4}x\right)} + 4\right|\right) + \frac{2}{3} \log\left(\left|\sqrt{e^{\left(\frac{3}{2}x\right)} + e^{\left(\frac{3}{4}x\right)} - 2} - e^{\left(\frac{3}{4}x\right)}\right|\right)$$

## 3 Test file number 13

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.1\_Linear/13\_1.1.1.2-a+b\_x-  
^m-c+d\_x-^n

### 3.1 Problem number 881

$$\int \frac{1}{\sqrt{cx^2} (a + bx)} dx$$

Optimal antiderivative

$$\frac{x \ln(x)}{a\sqrt{cx^2}} - \frac{x \ln(bx + a)}{a\sqrt{cx^2}}$$

command

```
integrate(1/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\log\left(\left|-\left(\sqrt{c}x - \sqrt{cx^2}\right)b - 2a\sqrt{c}\right|\right)}{a\sqrt{c}} - \frac{\log\left(\left|-\sqrt{c}x + \sqrt{cx^2}\right|\right)}{a\sqrt{c}}$$

### 3.2 Problem number 882

$$\int \frac{1}{x\sqrt{cx^2}(a+bx)} dx$$

Optimal antiderivative

$$-\frac{1}{a\sqrt{cx^2}} - \frac{bx \ln(x)}{a^2\sqrt{cx^2}} + \frac{bx \ln(bx+a)}{a^2\sqrt{cx^2}}$$

command

```
integrate(1/x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$-\sqrt{c} \left( \frac{b \log\left(\left|-\left(\sqrt{c}x - \sqrt{cx^2}\right)b - 2a\sqrt{c}\right|\right)}{a^2c} - \frac{b \log\left(\left|-\sqrt{c}x + \sqrt{cx^2}\right|\right)}{a^2c} - \frac{2}{\left(\sqrt{c}x - \sqrt{cx^2}\right)a\sqrt{c}} \right)$$

### 3.3 Problem number 889

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)} dx$$

Optimal antiderivative

$$\frac{x \ln(x)}{ac\sqrt{cx^2}} - \frac{x \ln(bx+a)}{ac\sqrt{cx^2}}$$

command

```
integrate(x^2/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\frac{\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b-2a\sqrt{c}\right|\right)}{a\sqrt{c}} - \frac{\log\left(\left|-\sqrt{c}x+\sqrt{cx^2}\right|\right)}{a\sqrt{c}}}{c}$$

### 3.4 Problem number 890

$$\int \frac{x}{(cx^2)^{3/2}(a+bx)} dx$$

Optimal antiderivative

$$-\frac{1}{ac\sqrt{cx^2}} - \frac{bx \ln(x)}{a^2c\sqrt{cx^2}} + \frac{bx \ln(bx+a)}{a^2c\sqrt{cx^2}}$$

command

```
integrate(x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\frac{b \log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b-2a\sqrt{c}\right|\right)}{a^2c} - \frac{b \log\left(\left|-\sqrt{c}x+\sqrt{cx^2}\right|\right)}{a^2c} - \frac{2}{\left(\sqrt{c}x-\sqrt{cx^2}\right)a\sqrt{c}}}{\sqrt{c}}$$

### 3.5 Problem number 914

$$\int \frac{1}{\sqrt{cx^2} (a + bx)^2} dx$$

Optimal antiderivative

$$\frac{x}{a(bx+a)\sqrt{cx^2}} + \frac{x \ln(x)}{a^2 \sqrt{cx^2}} - \frac{x \ln(bx+a)}{a^2 \sqrt{cx^2}}$$

command

```
integrate(1/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^2\sqrt{c}\operatorname{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)} - \frac{1}{(bx+a)a\sqrt{c}\operatorname{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}$$

### 3.6 Problem number 915

$$\int \frac{1}{x\sqrt{cx^2} (a + bx)^2} dx$$

Optimal antiderivative

$$-\frac{1}{a^2\sqrt{cx^2}} - \frac{bx}{a^2(bx+a)\sqrt{cx^2}} - \frac{2bx \ln(x)}{a^3\sqrt{cx^2}} + \frac{2bx \ln(bx+a)}{a^3\sqrt{cx^2}}$$

command

```
integrate(1/x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{b\left(\frac{2\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^3\operatorname{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)} + \frac{1}{(bx+a)a^2\operatorname{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)} - \frac{1}{a^3\left(\frac{a}{bx+a}-1\right)\operatorname{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}\right)}{\sqrt{c}}$$

### 3.7 Problem number 916

$$\int \frac{1}{x^2 \sqrt{cx^2} (a + bx)^2} dx$$

Optimal antiderivative

$$\frac{2b}{a^3 \sqrt{cx^2}} - \frac{1}{2a^2 x \sqrt{cx^2}} + \frac{b^2 x}{a^3 (bx + a) \sqrt{cx^2}} + \frac{3b^2 x \ln(x)}{a^4 \sqrt{cx^2}} - \frac{3b^2 x \ln(bx + a)}{a^4 \sqrt{cx^2}}$$

command

```
integrate(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{6b^2 \log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^4 \operatorname{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)} + \frac{2b^2}{(bx+a)a^3 \operatorname{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)} - \frac{\frac{6ab^2}{bx+a}-5b^2}{a^4\left(\frac{a}{bx+a}-1\right)^2 \operatorname{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}$$


---


$$2\sqrt{c}$$

### 3.8 Problem number 920

$$\int \frac{x^2}{(cx^2)^{3/2} (a + bx)^2} dx$$

Optimal antiderivative

$$\frac{x}{ac(bx + a) \sqrt{cx^2}} + \frac{x \ln(x)}{a^2 c \sqrt{cx^2}} - \frac{x \ln(bx + a)}{a^2 c \sqrt{cx^2}}$$

command

```
integrate(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^2 \operatorname{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)} + \frac{1}{(bx+a)a \operatorname{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}$$


---


$$c^{\frac{3}{2}}$$

### 3.9 Problem number 921

$$\int \frac{x}{(cx^2)^{3/2} (a+bx)^2} dx$$

Optimal antiderivative

$$-\frac{1}{a^2 c \sqrt{c x^2}} - \frac{b x}{a^2 c (b x + a) \sqrt{c x^2}} - \frac{2 b x \ln(x)}{a^3 c \sqrt{c x^2}} + \frac{2 b x \ln(b x + a)}{a^3 c \sqrt{c x^2}}$$

command

```
integrate(x/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2 b^2 \log\left(\left|-\frac{a}{b x+a}+1\right|\right)}{a^3 \operatorname{sgn}\left(-\frac{b}{b x+a}+\frac{a b}{(b x+a)^2}\right)} + \frac{b^2}{(b x+a) a^2 \operatorname{sgn}\left(-\frac{b}{b x+a}+\frac{a b}{(b x+a)^2}\right)} - \frac{b^2}{a^3\left(\frac{a}{b x+a}-1\right) \operatorname{sgn}\left(-\frac{b}{b x+a}+\frac{a b}{(b x+a)^2}\right)}$$


---


$$b c^{\frac{3}{2}}$$

### 3.10 Problem number 922

$$\int \frac{1}{(cx^2)^{3/2} (a+bx)^2} dx$$

Optimal antiderivative

$$\frac{2 b}{a^3 c \sqrt{c x^2}} - \frac{1}{2 a^2 c x \sqrt{c x^2}} + \frac{b^2 x}{a^3 c (b x + a) \sqrt{c x^2}} + \frac{3 b^2 x \ln(x)}{a^4 c \sqrt{c x^2}} - \frac{3 b^2 x \ln(b x + a)}{a^4 c \sqrt{c x^2}}$$

command

```
integrate(1/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{6 b^2 \log\left(\left|-\frac{a}{b x+a}+1\right|\right)}{a^4 \operatorname{sgn}\left(-\frac{b}{b x+a}+\frac{a b}{(b x+a)^2}\right)} + \frac{2 b^2}{(b x+a) a^3 \operatorname{sgn}\left(-\frac{b}{b x+a}+\frac{a b}{(b x+a)^2}\right)} - \frac{\frac{6 a b^2}{b x+a}-5 b^2}{a^4\left(\frac{a}{b x+a}-1\right)^2 \operatorname{sgn}\left(-\frac{b}{b x+a}+\frac{a b}{(b x+a)^2}\right)}$$


---


$$2 c^{\frac{3}{2}}$$



## 4 Test file number 14

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.1\_Linear/14\_1.1.1.3-a+b\_x-  
^m-c+d\_x-^n-e+f\_x-^p

### 4.1 Problem number 1005

$$\int x^2(1-ax)^{-1-\frac{1}{2}n(1+n)}(1+ax)^{-1-\frac{1}{2}(-1+n)n} dx$$

Optimal antiderivative

$$\frac{(ax+1)^{\frac{(1-n)n}{2}}(-anx+1)(-ax+1)^{-\frac{n(1+n)}{2}}}{a^3n(-n^2+1)}$$

command

```
integrate(x^2*(-a*x+1)^(-1-1/2*n*(1+n))*(a*x+1)^(-1-1/2*(-1+n)*n),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

---


$$a^3nx^3e^{(-\frac{1}{2}n^2\log(ax+1)-\frac{1}{2}n^2\log(-ax+1)+\frac{1}{2}n\log(ax+1)-\frac{1}{2}n\log(-ax+1)-\log(ax+1)-\log(-ax+1))} - a^2x^2e^{(-\frac{1}{2}n^2\log(ax+1)-\frac{1}{2}n^2\log(-ax+1))}$$

## 5 Test file number 20

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.2\_Quadratic/20\_1.1.2.3-  
a+b\_x^2-^p-c+d\_x^2-^q

### 5.1 Problem number 224

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}(x)\sqrt{2}}{2}$$

command

```
integrate(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{4} \sqrt{2} \log(x+1) - \frac{1}{4} \sqrt{2} \log(x-1)$$

## 5.2 Problem number 231

$$\int \frac{1}{\sqrt{1+x^2} \sqrt{2+2x^2}} dx$$

Optimal antiderivative

$$\frac{\arctan(x) \sqrt{2}}{2}$$

command

```
integrate(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{4} \sqrt{2} i \log(ix-1) - \frac{1}{4} \sqrt{2} i \log(-ix-1)$$

## 6 Test file number 25

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.3\_General/25\_1.1.3.2-c\_x-  
^m-a+b\_x^n-p

## 6.1 Problem number 1735

$$\int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2} x} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2}{a\sqrt{a + \frac{b}{x}}}$$

command

```
integrate(1/(a+b/x)^(3/2)/x,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{2 \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a} a} - \frac{2}{a\sqrt{\frac{ax+b}{x}}}$$

## 6.2 Problem number 1738

$$\int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2} x^4} dx$$

Optimal antiderivative

$$-\frac{2\left(a + \frac{b}{x}\right)^{3/2}}{3b^3} + \frac{2a^2}{b^3\sqrt{a + \frac{b}{x}}} + \frac{4a\sqrt{a + \frac{b}{x}}}{b^3}$$

command

```
integrate(1/(a+b/x)^(3/2)/x^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2 \left( \frac{3a^2}{\sqrt{\frac{ax+b}{x}}} + 6a\sqrt{\frac{ax+b}{x}} - \frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x} \right)}{3b^3}$$

### 6.3 Problem number 1739

$$\int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2} x^5} dx$$

Optimal antiderivative

$$\frac{2a\left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{b^4} - \frac{2\left(a + \frac{b}{x}\right)^{\frac{5}{2}}}{5b^4} - \frac{2a^3}{b^4\sqrt{a + \frac{b}{x}}} - \frac{6a^2\sqrt{a + \frac{b}{x}}}{b^4}$$

command

`integrate(1/(a+b/x)^(3/2)/x^5,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2 \left( \frac{5a^3}{\sqrt{\frac{ax+b}{x}}} + 15a^2\sqrt{\frac{ax+b}{x}} - \frac{5(ax+b)a\sqrt{\frac{ax+b}{x}}}{x} + \frac{(ax+b)^2\sqrt{\frac{ax+b}{x}}}{x^2} \right)}{5b^4}$$

## 6.4 Problem number 1740

$$\int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2} x^6} dx$$

Optimal antiderivative

$$-\frac{4a^2\left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{b^5} + \frac{8a\left(a + \frac{b}{x}\right)^{\frac{5}{2}}}{5b^5} - \frac{2\left(a + \frac{b}{x}\right)^{\frac{7}{2}}}{7b^5} + \frac{2a^4}{b^5\sqrt{a + \frac{b}{x}}} + \frac{8a^3\sqrt{a + \frac{b}{x}}}{b^5}$$

command

```
integrate(1/(a+b/x)^(3/2)/x^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$2 \left( \frac{\frac{35a^4}{\sqrt{ax+b}} + 140a^3\sqrt{\frac{ax+b}{x}} - \frac{70(ax+b)a^2\sqrt{\frac{ax+b}{x}}}{x} + \frac{28(ax+b)^2a\sqrt{\frac{ax+b}{x}}}{x^2} - \frac{5(ax+b)^3\sqrt{\frac{ax+b}{x}}}{x^3}}{35b^5} \right)$$

## 6.5 Problem number 1741

$$\int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2} x^7} dx$$

Optimal antiderivative

$$\frac{20a^3\left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{3b^6} - \frac{4a^2\left(a + \frac{b}{x}\right)^{\frac{5}{2}}}{b^6} + \frac{10a\left(a + \frac{b}{x}\right)^{\frac{7}{2}}}{7b^6} - \frac{2\left(a + \frac{b}{x}\right)^{\frac{9}{2}}}{9b^6} - \frac{2a^5}{b^6\sqrt{a + \frac{b}{x}}} - \frac{10a^4\sqrt{a + \frac{b}{x}}}{b^6}$$

command

```
integrate(1/(a+b/x)^(3/2)/x^7,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$2 \left( \frac{63a^5}{\sqrt{\frac{ax+b}{x}}} + 315a^4 \sqrt{\frac{ax+b}{x}} - \frac{210(ax+b)a^3 \sqrt{\frac{ax+b}{x}}}{x} + \frac{126(ax+b)^2 a^2 \sqrt{\frac{ax+b}{x}}}{x^2} - \frac{45(ax+b)^3 a \sqrt{\frac{ax+b}{x}}}{x^3} + \frac{7(a^2 - \frac{6(ax+b)a}{x}) \sqrt{\frac{ax+b}{x}}}{x} \right) \frac{1}{63b^6}$$

## 6.6 Problem number 1748

$$\int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2} x^4} dx$$

Optimal antiderivative

$$\frac{2a^2}{3b^3 \left(a + \frac{b}{x}\right)^{3/2}} - \frac{4a}{b^3 \sqrt{a + \frac{b}{x}}} - \frac{2\sqrt{a + \frac{b}{x}}}{b^3}$$

command

`integrate(1/(a+b/x)^(5/2)/x^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$2 \left( \frac{\left(a^2 - \frac{6(ax+b)a}{x}\right)x}{(ax+b)\sqrt{\frac{ax+b}{x}}} - 3\sqrt{\frac{ax+b}{x}} \right) \frac{1}{3b^3}$$

## 6.7 Problem number 1750

$$\int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2} x^6} dx$$

Optimal antiderivative

$$\frac{2a^4}{3b^5 \left(a + \frac{b}{x}\right)^{\frac{3}{2}}} + \frac{8a \left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{3b^5} - \frac{2 \left(a + \frac{b}{x}\right)^{\frac{5}{2}}}{5b^5} - \frac{8a^3}{b^5 \sqrt{a + \frac{b}{x}}} - \frac{12a^2 \sqrt{a + \frac{b}{x}}}{b^5}$$

command

`integrate(1/(a+b/x)^(5/2)/x^6,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2 \left( 90 a^2 \sqrt{\frac{ax+b}{x}} - \frac{20(ax+b)a \sqrt{\frac{ax+b}{x}}}{x} - \frac{5 \left( a^4 - \frac{12(ax+b)a^3}{x} \right) x}{(ax+b) \sqrt{\frac{ax+b}{x}}} + \frac{3(ax+b)^2 \sqrt{\frac{ax+b}{x}}}{x^2} \right)}{15 b^5}$$

## 6.8 Problem number 1751

$$\int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2} x^7} dx$$

Optimal antiderivative

$$-\frac{2a^5}{3b^6 \left(a + \frac{b}{x}\right)^{\frac{3}{2}}} - \frac{20a^2 \left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{3b^6} + \frac{2a \left(a + \frac{b}{x}\right)^{\frac{5}{2}}}{b^6} - \frac{2 \left(a + \frac{b}{x}\right)^{\frac{7}{2}}}{7b^6} + \frac{10a^4}{b^6 \sqrt{a + \frac{b}{x}}} + \frac{20a^3 \sqrt{a + \frac{b}{x}}}{b^6}$$

command

`integrate(1/(a+b/x)^(5/2)/x^7,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2 \left( 210 a^3 \sqrt{\frac{ax+b}{x}} - \frac{70(ax+b)a^2 \sqrt{\frac{ax+b}{x}}}{x} + \frac{21(ax+b)^2 a \sqrt{\frac{ax+b}{x}}}{x^2} - \frac{7 \left( a^5 - \frac{15(ax+b)a^4}{x} \right) x}{(ax+b) \sqrt{\frac{ax+b}{x}}} - \frac{3(ax+b)^3 \sqrt{\frac{ax+b}{x}}}{x^3} \right)}{21 b^6}$$

### 6.9 Problem number 2991

$$\int \frac{1}{\sqrt{a + b\sqrt{\frac{c}{x}}} x^2} dx$$

Optimal antiderivative

$$-\frac{4\left(a + b\sqrt{\frac{c}{x}}\right)^{\frac{3}{2}}}{3b^2c} + \frac{4a\sqrt{a + b\sqrt{\frac{c}{x}}}}{b^2c}$$

command

```
integrate(1/x^2/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{4\left(\left(b\sqrt{\frac{c}{x}} + a\right)^{\frac{3}{2}}b - 3\sqrt{b\sqrt{\frac{c}{x}} + a}ab\right)\operatorname{sgn}\left(\left(b\sqrt{\frac{c}{x}} + a\right)b - ab\right)}{3b^3c}$$

### 6.10 Problem number 2992

$$\int \frac{1}{\sqrt{a + b\sqrt{\frac{c}{x}}} x^3} dx$$

Optimal antiderivative

$$-\frac{4a^2\left(a + b\sqrt{\frac{c}{x}}\right)^{\frac{3}{2}}}{b^4c^2} + \frac{12a\left(a + b\sqrt{\frac{c}{x}}\right)^{\frac{5}{2}}}{5b^4c^2} - \frac{4\left(a + b\sqrt{\frac{c}{x}}\right)^{\frac{7}{2}}}{7b^4c^2} + \frac{4a^3\sqrt{a + b\sqrt{\frac{c}{x}}}}{b^4c^2}$$

command

```
integrate(1/x^3/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{4\left(5\left(b\sqrt{\frac{c}{x}} + a\right)^{\frac{7}{2}}\operatorname{sgn}\left(\left(b\sqrt{\frac{c}{x}} + a\right)b - ab\right) - 21\left(b\sqrt{\frac{c}{x}} + a\right)^{\frac{5}{2}}\operatorname{asgn}\left(\left(b\sqrt{\frac{c}{x}} + a\right)b - ab\right) + 35\left(b\sqrt{\frac{c}{x}} + a\right)\right)}{35b^4c^2}$$



## 7 Test file number 26

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.3\_General/26\_1.1.3.3-a+b\_x^n-  
^p-c+d\_x^n^q

### 7.1 Problem number 245

$$\int \frac{\left(c + \frac{d}{x}\right)^3}{\sqrt{a + \frac{b}{x}}} dx$$

Optimal antiderivative

$$\frac{c^2(-6ad + bc) \operatorname{arctanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{d\left(-4a^2d^2 + 18abcd + 6b^2c^2 + \frac{bd(2ad+3bc)}{x}\right) \sqrt{a + \frac{b}{x}}}{3ab^2} + \frac{c\left(c + \frac{d}{x}\right)^2 x \sqrt{a + \frac{b}{x}}}{a}$$

command

```
integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{3b^2c^3 \sqrt{\frac{ax+b}{x}}}{\left(a - \frac{ax+b}{x}\right)a} - \frac{3(b^2c^3 - 6abc^2d) \operatorname{arctan}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{2\left(9b^3cd^2 \sqrt{\frac{ax+b}{x}} - 3ab^2d^3 \sqrt{\frac{ax+b}{x}} + \frac{(ax+b)b^2d^3}{x} \sqrt{\frac{ax+b}{x}}\right)}{b^3}$$


---

3b

## 7.2 Problem number 246

$$\int \frac{(c + \frac{d}{x})^2}{\sqrt{a + \frac{b}{x}}} dx$$

Optimal antiderivative

$$\frac{c(-4ad + bc) \operatorname{arctanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{2d^2 \sqrt{a + \frac{b}{x}}}{b} + \frac{c^2 x \sqrt{a + \frac{b}{x}}}{a}$$

command

```
integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\frac{b^2 c^2 \sqrt{ax+b}}{(a - \frac{ax+b}{x})a} + 2d^2 \sqrt{\frac{ax+b}{x}} - \frac{(b^2 c^2 - 4abcd) \operatorname{arctan}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a}}{b}$$

## 7.3 Problem number 249

$$\int \frac{1}{\sqrt{a + \frac{b}{x}} (c + \frac{d}{x})} dx$$

Optimal antiderivative

$$\frac{(2ad + bc) \operatorname{arctanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}} c^2} - \frac{2d^{\frac{3}{2}} \operatorname{arctan}\left(\frac{\sqrt{d} \sqrt{a + \frac{b}{x}}}{\sqrt{-ad + bc}}\right)}{c^2 \sqrt{-ad + bc}} + \frac{x \sqrt{a + \frac{b}{x}}}{ac}$$

command

```
integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$-b^2 \left( \frac{2d^2 \arctan\left(\frac{d\sqrt{ax+b}}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2} b^2 c^2} + \frac{\sqrt{\frac{ax+b}{x}}}{\left(a - \frac{ax+b}{x}\right) abc} - \frac{(bc+2ad) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a} ab^2 c^2} \right)$$

#### 7.4 Problem number 252

$$\int \frac{\left(c + \frac{d}{x}\right)^3}{\left(a + \frac{b}{x}\right)^{3/2}} dx$$

Optimal antiderivative

$$\frac{3c^2(-2ad+bc) \operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{(-2ad+bc)(2a^2d^2-2abcd+3b^2c^2) - \frac{abd^2(2ad+bc)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}} + \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}}$$

command

```
integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2d^3 \sqrt{\frac{ax+b}{x}}}{b} - \frac{3(b^2c^3 - 2abc^2d) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a} a^2} - \frac{2ab^3c^3 - 6a^2b^2c^2d + 6a^3bcd^2 - 2a^4d^3 - \frac{3(ax+b)b^3c^3}{x} + \frac{6(ax+b)ab^2c^2d}{x} - \frac{6(ax+b)d^3}{x}}{b} \left( a\sqrt{\frac{ax+b}{x}} - \frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x} \right) a^2b$$

## 7.5 Problem number 256

$$\int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)} dx$$

Optimal antiderivative

$$\frac{2d^{5/2} \arctan\left(\frac{\sqrt{d} \sqrt{a + \frac{b}{x}}}{\sqrt{-ad + bc}}\right)}{c^2 (-ad + bc)^{3/2}} - \frac{(2ad + 3bc) \operatorname{arctanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2} c^2} + \frac{b(-ad + 3bc)}{a^2 c (-ad + bc) \sqrt{a + \frac{b}{x}}} + \frac{x}{ac \sqrt{a + \frac{b}{x}}}$$

command

```
integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\left( \frac{2d^3 \arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd - ad^2}}\right)}{(b^3c^3 - ab^2c^2d)\sqrt{bcd - ad^2}} + \frac{2abc - \frac{3(ax+b)bc}{x} + \frac{(ax+b)ad}{x}}{(a^2b^2c^2 - a^3bcd) \left( a\sqrt{\frac{ax+b}{x}} - \frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x} \right)} + \frac{(3bc + 2ad) \arctan\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{-a} a^2 b^2 c^2} \right)$$

## 7.6 Problem number 257

$$\int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^2} dx$$

Optimal antiderivative

$$\frac{d^{\frac{5}{2}}(-4ad + 7bc) \arctan\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{-ad + bc}}\right) - (4ad + 3bc) \operatorname{arctanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{c^3(-ad + bc)^{\frac{5}{2}} - a^{\frac{5}{2}}c^3} + \frac{b(2a^2d^2 - 2abcd + 3b^2c^2)}{a^2c^2(-ad + bc)^2\sqrt{a + \frac{b}{x}}} + \frac{d(-2ad + bc)}{ac^2(-ad + bc)\left(c + \frac{d}{x}\right)\sqrt{a + \frac{b}{x}}} + \frac{x}{ac\left(c + \frac{d}{x}\right)\sqrt{a + \frac{b}{x}}}$$

command

```
integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$b^3 \left( \frac{(7bcd^3 - 4ad^4) \arctan\left(\frac{d\sqrt{ax+b}}{x\sqrt{bcd-ad^2}}\right)}{(b^5c^5 - 2ab^4c^4d + a^2b^3c^3d^2)\sqrt{bcd-ad^2}} + \frac{2ab^3c^3 - 2a^2b^2c^2d - \frac{3(ax+b)b^3c^3}{x} + \frac{7(ax+b)ab^2c^2d}{x} - \frac{3(ax+b)a^2bcd}{x}}{(a^2b^4c^4 - 2a^3b^3c^3d + a^4b^2c^2d^2)} \left( abc\sqrt{\frac{ax+b}{x}} - a^2d\sqrt{\frac{ax+b}{x}} \right) \right)$$

## 7.7 Problem number 258

$$\int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{3d^{\frac{5}{2}}(8a^2d^2 - 24abcd + 21b^2c^2) \arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{-ad+bc}}\right)}{4c^4(-ad+bc)^{\frac{7}{2}}} \\
& - \frac{3(2ad+bc) \operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}c^4} + \frac{3b(-ad+2bc)(4a^2d^2 - abcd + 2b^2c^2)}{4a^2c^3(-ad+bc)^3\sqrt{a+\frac{b}{x}}} \\
& + \frac{d(-3ad+2bc)}{2ac^2(-ad+bc)\left(c+\frac{d}{x}\right)^2\sqrt{a+\frac{b}{x}}} \\
& + \frac{d(12a^2d^2 - 21abcd + 4b^2c^2)}{4ac^3(-ad+bc)^2\left(c+\frac{d}{x}\right)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(c+\frac{d}{x}\right)^2\sqrt{a+\frac{b}{x}}}
\end{aligned}$$

command

```
integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{4}b^4 \left( \frac{3(21b^2c^2d^3 - 24abcd^4 + 8a^2d^5) \arctan\left(\frac{d\sqrt{ax+b}}{\sqrt{bcd-ad^2}}\right)}{(b^7c^7 - 3ab^6c^6d + 3a^2b^5c^5d^2 - a^3b^4c^4d^3)\sqrt{bcd-ad^2}} + \frac{4\left(2ab^3c^3 - \frac{3(ax+b)b^3c^3}{x} + \frac{3(ax+b)ab^2c^2d}{x}\right)}{(a^2b^6c^6 - 3a^3b^5c^5d + 3a^4b^4c^4d^2 - a^5b^3c^3d^3)} \right)$$

## 7.8 Problem number 263

$$\int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b(-3ad + 5bc)}{3a^2c(-ad + bc)\left(a + \frac{b}{x}\right)^{\frac{3}{2}}} + \frac{x}{ac\left(a + \frac{b}{x}\right)^{\frac{3}{2}}} - \frac{2d^{\frac{7}{2}} \arctan\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{-ad + bc}}\right)}{c^2(-ad + bc)^{\frac{5}{2}}} \\ & - \frac{(2ad + 5bc) \operatorname{arctanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}c^2} + \frac{b(a^2d^2 - 8abcd + 5b^2c^2)}{a^3c(-ad + bc)^2\sqrt{a + \frac{b}{x}}} \end{aligned}$$

command

```
integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{1}{3} \left( \frac{6d^4 \arctan\left(\frac{d\sqrt{ax+b}}{x\sqrt{bcd-ad^2}}\right)}{(b^4c^4 - 2ab^3c^3d + a^2b^2c^2d^2)\sqrt{bcd-ad^2}} - \frac{2\left(abc - a^2d + \frac{6(ax+b)bc}{x} - \frac{9(ax+b)ad}{x}\right)x}{(a^3b^2c^2 - 2a^4bcd + a^5d^2)(ax+b)\sqrt{\frac{ax+b}{x}}} + \frac{3\sqrt{\frac{ax+b}{x}}}{\left(a - \frac{ax+b}{x}\right)a^3bc} \right)$$

## 7.9 Problem number 264

$$\int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b(6a^2d^2 - 6abcd + 5b^2c^2)}{3a^2c^2(-ad + bc)^2 \left(a + \frac{b}{x}\right)^{3/2}} + \frac{d(-2ad + bc)}{ac^2(-ad + bc) \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)} \\ & + \frac{x}{ac \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)} - \frac{d^{7/2}(-4ad + 9bc) \arctan\left(\frac{\sqrt{d} \sqrt{a + \frac{b}{x}}}{\sqrt{-ad + bc}}\right)}{c^3(-ad + bc)^{7/2}} \\ & - \frac{(4ad + 5bc) \operatorname{arctanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^3} + \frac{b(-2ad + bc)(a^2d^2 - abcd + 5b^2c^2)}{a^3c^2(-ad + bc)^3 \sqrt{a + \frac{b}{x}}} \end{aligned}$$

command

```
integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{1}{3}b^3 \left( \frac{3(9bcd^4 - 4ad^5) \arctan\left(\frac{d\sqrt{ax+b}}{\sqrt{bcd-ad^2}}\right)}{(b^6c^6 - 3ab^5c^5d + 3a^2b^4c^4d^2 - a^3b^3c^3d^3)\sqrt{bcd-ad^2}} - \frac{2\left(abc - a^2d + \frac{6(ax+b)bc}{x} - \frac{12(ax+b)ad}{x}\right)x}{(a^3b^3c^3 - 3a^4b^2c^2d + 3a^5bcd^2 - a^6d^3)(ax+b)\sqrt{bcd-ad^2}} \right)$$



## 8 Test file number 27

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.3\_General/27\_1.1.3.4-e\_x-  
^m-a+b\_x^n-p-c+d\_x^n-q

### 8.1 Problem number 1054

$$\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n) dx$$

Optimal antiderivative

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

command

`integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{c^{14}x^{28n} + 14bc^{13}x^{27n} + 91b^2c^{12}x^{26n} + 364b^3c^{11}x^{25n} + 1001b^4c^{10}x^{24n} + 2002b^5c^9x^{23n} + 3003b^6c^8x^{22n} + 3432b^7c^7x^{21n} + 3432b^8c^6x^{20n} + 2520b^9c^5x^{19n} + 1470b^{10}c^4x^{18n} + 735b^{11}c^3x^{17n} + 210b^{12}c^2x^{16n} + 35b^{13}cx^{15n} + b^{14}x^{14n}}{14n}$$

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## 9 Test file number 33

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.1\_Quadratic/33\_1.2.1.2-  
d+e\_x^m-a+b\_x+c\_x^2-p

### 9.1 Problem number 841

$$\int \frac{d+ex}{(d^2-e^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{ex+d}{3de(-e^2x^2+d^2)^{3/2}} + \frac{2x}{3d^3\sqrt{-e^2x^2+d^2}}$$

command

`integrate((e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{\sqrt{-x^2e^2 + d^2} \left( x \left( \frac{2x^2e^2}{d^3} - \frac{3}{d} \right) - e^{(-1)} \right)}{3(x^2e^2 - d^2)^2}$$

## 9.2 Problem number 853

$$\int \frac{(d + ex)^2}{(d^2 - e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2ex}{5} + \frac{2d}{5}}{e(-e^2x^2 + d^2)^{\frac{5}{2}}} + \frac{x}{5d^2(-e^2x^2 + d^2)^{\frac{3}{2}}} + \frac{2x}{5d^4\sqrt{-e^2x^2 + d^2}}$$

command

`integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{\sqrt{-x^2e^2 + d^2} \left( \left( x^2 \left( \frac{2x^2e^4}{d^4} - \frac{5e^2}{d^2} \right) + 5 \right) x + 2de^{(-1)} \right)}{5(x^2e^2 - d^2)^3}$$

## 9.3 Problem number 854

$$\int \frac{d + ex}{(d^2 - e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{ex + d}{5de(-e^2x^2 + d^2)^{\frac{5}{2}}} + \frac{4x}{15d^3(-e^2x^2 + d^2)^{\frac{3}{2}}} + \frac{8x}{15d^5\sqrt{-e^2x^2 + d^2}}$$

command

```
integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(4x^2\left(\frac{2x^2e^4}{d^5}-\frac{5e^2}{d^3}\right)+\frac{15}{d}\right)x+3e^{(-1)}\right)}{15(x^2e^2-d^2)^3}$$

#### 9.4 Problem number 1957

$$\int \frac{(d+ex)^2}{(ade+(cd^2+ae^2)x+cde x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{2cde x+a e^2+c d^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(ae^2+cd^2)x+cde x^2}}\right)\sqrt{e}}{c^{\frac{3}{2}}d^{\frac{3}{2}}}-\frac{2(ex+d)}{cd\sqrt{ade+(ae^2+cd^2)x+cde x^2}}$$

command

```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{2\left(\frac{(c^2d^4e-2acd^2e^3+a^2e^5)x}{c^3d^5-2ac^2d^3e^2+a^2cde^4}+\frac{c^2d^5-2acd^3e^2+a^2de^4}{c^3d^5-2ac^2d^3e^2+a^2cde^4}\right)}{\sqrt{cdx^2e+ade+(cd^2+ae^2)x}}-\frac{\sqrt{cd}e^{\frac{1}{2}}\log\left(\left|-\sqrt{cd}cd^2e^{\frac{1}{2}}-2\left(\sqrt{cd}xe^{\frac{1}{2}}-\sqrt{cdx^2e+ade+(cd^2+ae^2)x}\right)cde-\sqrt{cd}ae^{\frac{5}{2}}\right|\right)}{c^2d^2}$$

## 9.5 Problem number 1958

$$\int \frac{d + ex}{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2(ex + d)}{(-ae^2 + cd^2) \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}$$

command

```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{2 \left( \frac{(cd^2e - ae^3)x}{c^2d^4 - 2acd^2e^2 + a^2e^4} + \frac{cd^3 - ade^2}{c^2d^4 - 2acd^2e^2 + a^2e^4} \right)}{\sqrt{cdx^2e + ade + (cd^2 + ae^2)x}}$$

## 9.6 Problem number 1966

$$\int \frac{(d + ex)^4}{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(ex + d)^3}{3cd(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}} \\ & + \frac{e^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{2cdex + ae^2 + cd^2}{2\sqrt{c} \sqrt{d} \sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right)}{c^{\frac{5}{2}} d^{\frac{5}{2}}} \\ & - \frac{2e(ex + d)}{c^2 d^2 \sqrt{ade + (ae^2 + cd^2)x + cde x^2}} \end{aligned}$$

command

```
integrate((e*x+d)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2 \left( \left( \frac{4(c^5 d^9 e^3 - 4ac^4 d^7 e^5 + 6a^2 c^3 d^5 e^7 - 4a^3 c^2 d^3 e^9 + a^4 c d e^{11})x}{c^6 d^{10} - 4ac^5 d^8 e^2 + 6a^2 c^4 d^6 e^4 - 4a^3 c^3 d^4 e^6 + a^4 c^2 d^2 e^8} + \frac{3(3c^5 d^{10} e^2 - 11ac^4 d^8 e^4 + 14a^2 c^3 d^6 e^6 - 6a^3 c^2 d^4 e^8 - a^4 c d^2 e^{10} + a^5 e^{12})}{c^6 d^{10} - 4ac^5 d^8 e^2 + 6a^2 c^4 d^6 e^4 - 4a^3 c^3 d^4 e^6 + a^4 c^2 d^2 e^8} \right) x + \sqrt{cd} e^{\frac{3}{2}} \log \left( \left| -\sqrt{cd} cd^2 e^{\frac{1}{2}} - 2 \left( \sqrt{cd} x e^{\frac{1}{2}} - \sqrt{cdx^2 e + ade + (cd^2 + ae^2)x} \right) cde - \sqrt{cd} ae^{\frac{5}{2}} \right| \right)}{c^3 d^3}$$

## 9.7 Problem number 1967

$$\int \frac{(d + ex)^3}{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{2(ex + d)^3}{3(-ae^2 + cd^2)(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}$$

command

```
integrate((e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2 \left( \left( \frac{(c^3 d^6 e^3 - 3ac^2 d^4 e^5 + 3a^2 cd^2 e^7 - a^3 e^9)x}{c^4 d^8 - 4ac^3 d^6 e^2 + 6a^2 c^2 d^4 e^4 - 4a^3 cd^2 e^6 + a^4 e^8} + \frac{3(c^3 d^7 e^2 - 3ac^2 d^5 e^4 + 3a^2 cd^3 e^6 - a^3 de^8)}{c^4 d^8 - 4ac^3 d^6 e^2 + 6a^2 c^2 d^4 e^4 - 4a^3 cd^2 e^6 + a^4 e^8} \right) x + \frac{3(c^3 d^8 e - 3ac^2 d^6 e^3 + 3a^2 cd^4 e^5 - a^3 e^7)}{c^4 d^8 - 4ac^3 d^6 e^2 + 6a^2 c^2 d^4 e^4 - 4a^3 cd^2 e^6 + a^4 e^8} \right)}{3(cdx^2 e + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}$$

## 9.8 Problem number 1968

$$\int \frac{(d + ex)^2}{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{2(ex + d)}{3cd(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}} + \frac{2e(2cdex + ae^2 + cd^2)}{3cd(-ae^2 + cd^2)^2 \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}$$

command

```
integrate((e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$2 \left( \left( \frac{2(c^3 d^5 e^3 - 2ac^2 d^3 e^5 + a^2 cde^7)x}{c^4 d^8 - 4ac^3 d^6 e^2 + 6a^2 c^2 d^4 e^4 - 4a^3 cd^2 e^6 + a^4 e^8} + \frac{3(c^3 d^6 e^2 - ac^2 d^4 e^4 - a^2 cd^2 e^6 + a^3 e^8)}{c^4 d^8 - 4ac^3 d^6 e^2 + 6a^2 c^2 d^4 e^4 - 4a^3 cd^2 e^6 + a^4 e^8} \right) x + \frac{6(ac^2 d^5 e^3 - 2a^2 cd^3 e^5 + a^3 e^8)}{c^4 d^8 - 4ac^3 d^6 e^2 + 6a^2 c^2 d^4 e^4 - 4a^3 cd^2 e^6 + a^4 e^8} \right) \frac{1}{3(cd^2 x^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}$$

## 9.9 Problem number 1969

$$\int \frac{d + ex}{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{2(ex + d)}{3(-ae^2 + cd^2)(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}} + \frac{8e(2cdex + ae^2 + cd^2)}{3(-ae^2 + cd^2)^3 \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}$$

command

```
integrate((e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$2 \left( \left( 4 \left( \frac{2(c^3 d^4 e^3 - ac^2 d^2 e^5)x}{c^4 d^8 - 4ac^3 d^6 e^2 + 6a^2 c^2 d^4 e^4 - 4a^3 cd^2 e^6 + a^4 e^8} + \frac{3(c^3 d^5 e^2 - a^2 cde^6)}{c^4 d^8 - 4ac^3 d^6 e^2 + 6a^2 c^2 d^4 e^4 - 4a^3 cd^2 e^6 + a^4 e^8} \right) x + \frac{3(c^3 d^6 e + 5ac^2 d^4 e^3 - 5a^2 cd^2 e^5 + a^3 e^8)}{c^4 d^8 - 4ac^3 d^6 e^2 + 6a^2 c^2 d^4 e^4 - 4a^3 cd^2 e^6 + a^4 e^8} \right) \frac{1}{3(cd^2 x^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}$$

## 10 Test file number 34

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.1\_Quadratic/34\_1.2.1.3-d+e\_x-^m-f+g\_x-a+b\_x+c\_x^2-^p

## 10.1 Problem number 1179

$$\int \frac{(A + Bx)(bx + cx^2)^{3/2}}{(d + ex)^5} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(d(3Ab e^2 - Bd(-5be + 8cd)) - e(Bd(-11be + 14cd) - 3Ae(-be + 2cd)) x)(cx^2 + bx)^{\frac{3}{2}}}{24d e^2 (-be + cd)(ex + d)^4} \\ & + \frac{2B c^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{x\sqrt{c}}{\sqrt{cx^2 + bx}}\right)}{e^5} \\ & + \frac{(3Ab^4 e^5 - Bd(-5b^4 e^4 - 40b^3 cd e^3 + 240b^2 c^2 d^2 e^2 - 320b c^3 d^3 e + 128c^4 d^4)) \operatorname{arctanh}\left(\frac{bd + (-be + 2cd)x}{2\sqrt{d} \sqrt{-be + cd} \sqrt{cx^2 + bx}}\right)}{128d^{\frac{5}{2}} e^5 (-be + cd)^{\frac{5}{2}}} \\ & - \frac{(d(3Ab^3 e^4 + Bd(5b^3 e^3 + 40b^2 cd e^2 - 112b c^2 d^2 e + 64c^3 d^3)) + e(3Ab^2 e^3(-be + 2cd) + Bd(-5b^3 e^3 + 98b^2 cd e^2 - 64d^2 e^4 (-be + cd)^2 (ex + d)^2))}{64d^2 e^4 (-be + cd)^2 (ex + d)^2} \end{aligned}$$

command

```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/(e*x+d)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

## 10.2 Problem number 2225

$$\int \frac{(d + ex)^3(f + gx)}{(cd^2 - bde - be^2x - ce^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(-beg + cdg + cef)(ex + d)^3}{3c e^2 (-be + 2cd)(d(-be + cd) - b e^2 x - c e^2 x^2)^{\frac{3}{2}}} \\ & + \frac{g \operatorname{arctan}\left(\frac{e(2cx+b)}{2\sqrt{c} \sqrt{d(-be + cd) - b e^2 x - c e^2 x^2}}\right)}{c^{\frac{5}{2}} e^2} - \frac{2g(ex + d)}{c^2 e^2 \sqrt{d(-be + cd) - b e^2 x - c e^2 x^2}} \end{aligned}$$

command

```
integrate((e*x+d)^3*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\sqrt{-ce^2} ge^{(-3)} \log \left( \left| -2 \left( \sqrt{-ce^2} x - \sqrt{-cx^2e^2 + cd^2 - bxe^2 - bde} \right) c - \sqrt{-ce^2} b \right| \right)}{2 \sqrt{-cx^2e^2 + cd^2 - bxe^2 - bde} \left( \left( \frac{c^3 (56c^5d^4ge^4 + 8c^5d^3fe^5 - 116bc^4d^3ge^5 - 12bc^4d^2fe^6 + 90b^2c^3d^2ge^6 + 6b^2c^3dfe^7 - 31b^3c^2dge^7 - b^3c^2}{16c^6d^4e^3 - 32bc^5d^3e^4 + 24b^2c^4d^2e^5 - 8b^3c^3de^6 + b^4c^2e^7} \right) \right)}$$

### 10.3 Problem number 2227

$$\int \frac{(d+ex)(f+gx)}{(cd^2 - bde - be^2x - ce^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(-beg + cdg + cef) (d(-be + 2cd) + e(-be + 2cd) x)}{3ce^2 (-be + 2cd)^2 (d(-be + cd) - be^2x - ce^2x^2)^{3/2}} + \frac{2(-beg - 2cdg + 4cef) (2cx + b)}{3ce (-be + 2cd)^3 \sqrt{d(-be + cd) - be^2x - ce^2x^2}}$$

command

```
integrate((e*x+d)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$2 \sqrt{-cx^2e^2 + cd^2 - bxe^2 - bde} \left( \left( \frac{2(4c^3d^2ge^3 - 8c^3dfe^4 + 4bc^2fe^5 - b^2cge^5)x}{16c^4d^4e^2 - 32bc^3d^3e^3 + 24b^2c^2d^2e^4 - 8b^3cde^5 + b^4e^6} + \frac{3(4bc^2d^2ge^3 - 8bc^2dfe^4 + 4b^2cfe^5 - b^3ge^6)}{16c^4d^4e^2 - 32bc^3d^3e^3 + 24b^2c^2d^2e^4 - 8b^3cde^5} \right) \right)$$



## 11 Test file number 35

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.1\_Quadratic/35\_1.2.1.4-d+e\_x-^m-f+g\_x-^n-a+b\_x+c\_x^2-^p

### 11.1 Problem number 18

$$\int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{x^2(ex+d)}{3de(-e^2x^2+d^2)^{\frac{3}{2}}} - \frac{2}{3e^3\sqrt{-e^2x^2+d^2}}$$

command

```
integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{(x^2(\frac{x}{d} + 3e^{(-1)}) - 2d^2e^{(-3)})\sqrt{-x^2e^2 + d^2}}{3(x^2e^2 - d^2)^2}$$

### 11.2 Problem number 19

$$\int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x^6(ex+d)}{5e^2(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{x^4(7ex+6d)}{15e^4(-e^2x^2+d^2)^{\frac{3}{2}}} - \frac{7d^2 \arctan\left(\frac{ex}{\sqrt{-e^2x^2+d^2}}\right)}{2e^8} \\ & + \frac{x^2(35ex+24d)}{15e^6\sqrt{-e^2x^2+d^2}} + \frac{(35ex+32d)\sqrt{-e^2x^2+d^2}}{10e^8} \end{aligned}$$

command

`integrate(x^7*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{7}{2}d^2 \arcsin\left(\frac{xe}{d}\right)e^{(-8)}\operatorname{sgn}(d) \\ \frac{(96d^7e^{(-8)} + (105d^6e^{(-7)} - (240d^5e^{(-6)} + (245d^4e^{(-5)} - (180d^3e^{(-4)} + (161d^2e^{(-3)} - 15(xe^{(-1)} + 2de^{(-2)})x) \\ 30(x^2e^2 - d^2)^3$$

### 11.3 Problem number 20

$$\int \frac{x^6(d+ex)}{(d^2 - e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{x^5(ex+d)}{5e^2(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{x^3(6ex+5d)}{15e^4(-e^2x^2+d^2)^{\frac{3}{2}}} - \frac{d \arctan\left(\frac{ex}{\sqrt{-e^2x^2+d^2}}\right)}{e^7} \\ + \frac{x(8ex+5d)}{5e^6\sqrt{-e^2x^2+d^2}} + \frac{16\sqrt{-e^2x^2+d^2}}{5e^7}$$

command

`integrate(x^6*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-d \arcsin\left(\frac{xe}{d}\right)e^{(-7)}\operatorname{sgn}(d) \\ \frac{(48d^6e^{(-7)} + (15d^5e^{(-6)} - (120d^4e^{(-5)} + (35d^3e^{(-4)} - (90d^2e^{(-3)} - (15xe^{(-1)} - 23de^{(-2)})x)x)x)x)\sqrt{-x^2}}{15(x^2e^2 - d^2)^3}$$

### 11.4 Problem number 21

$$\int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{x^4(ex+d)}{5e^2(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{x^2(5ex+4d)}{15e^4(-e^2x^2+d^2)^{\frac{3}{2}}} - \frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2+d^2}}\right)}{e^6} + \frac{15ex+8d}{15e^6\sqrt{-e^2x^2+d^2}}$$

command

`integrate(x^5*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\arcsin\left(\frac{xe}{d}\right)e^{(-6)}\operatorname{sgn}(d) - \frac{(8d^5e^{(-6)} + (15d^4e^{(-5)} - (20d^3e^{(-4)} + (35d^2e^{(-3)} - (23xe^{(-1)} + 15de^{(-2)})x)x)x)\sqrt{-x^2e^2+d^2}}{15(x^2e^2-d^2)^3}$$

### 11.5 Problem number 22

$$\int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{x^4(ex+d)}{5de(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{4d^2}{15e^5(-e^2x^2+d^2)^{\frac{3}{2}}} + \frac{4}{5e^5\sqrt{-e^2x^2+d^2}}$$

command

`integrate(x^4*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{(8d^4e^{(-5)} + (3x^2(\frac{x}{d} + 5e^{(-1)}) - 20d^2e^{(-3)})x^2)\sqrt{-x^2e^2+d^2}}{15(x^2e^2-d^2)^3}$$

### 11.6 Problem number 23

$$\int \frac{x^3(d+ex)}{(d^2 - e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{x^2(ex+d)}{5e^2(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{-3ex-2d}{15e^4(-e^2x^2+d^2)^{\frac{3}{2}}} + \frac{x}{5d^2e^3\sqrt{-e^2x^2+d^2}}$$

command

`integrate(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\left(2d^3e^{(-4)} - \left(\frac{3x^3e}{d^2} + 5de^{(-2)}\right)x^2\right)\sqrt{-x^2e^2+d^2}}{15(x^2e^2-d^2)^3}$$

### 11.7 Problem number 24

$$\int \frac{x^2(d+ex)}{(d^2 - e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{x^2(ex+d)}{5de(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{2(-ex+d)}{15de^3(-e^2x^2+d^2)^{\frac{3}{2}}} - \frac{2x}{15d^3e^2\sqrt{-e^2x^2+d^2}}$$

command

`integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\left(\left(x\left(\frac{2x^2e^2}{d^3} - \frac{5}{d}\right) - 5e^{(-1)}\right)x^2 + 2d^2e^{(-3)}\right)\sqrt{-x^2e^2+d^2}}{15(x^2e^2-d^2)^3}$$

### 11.8 Problem number 25

$$\int \frac{x(d+ex)}{(d^2-e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{ex+d}{5e^2(-e^2x^2+d^2)^{5/2}} - \frac{x}{15d^2e(-e^2x^2+d^2)^{3/2}} - \frac{2x}{15d^4e\sqrt{-e^2x^2+d^2}}$$

command

```
integrate(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\left(x^3\left(\frac{2x^2e^3}{d^4} - \frac{5e}{d^2}\right) - 3de^{(-2)}\right)\sqrt{-x^2e^2+d^2}}{15(x^2e^2-d^2)^3}$$

### 11.9 Problem number 26

$$\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{ex+d}{5de(-e^2x^2+d^2)^{5/2}} + \frac{4x}{15d^3(-e^2x^2+d^2)^{3/2}} + \frac{8x}{15d^5\sqrt{-e^2x^2+d^2}}$$

command

```
integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{\sqrt{-x^2e^2+d^2}\left(\left(4x^2\left(\frac{2x^2e^4}{d^5} - \frac{5e^2}{d^3}\right) + \frac{15}{d}\right)x + 3e^{(-1)}\right)}{15(x^2e^2-d^2)^3}$$

### 11.10 Problem number 27

$$\int \frac{d + ex}{x (d^2 - e^2 x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{ex + d}{5d^2 (-e^2 x^2 + d^2)^{\frac{5}{2}}} + \frac{4ex + 5d}{15d^4 (-e^2 x^2 + d^2)^{\frac{3}{2}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{-e^2 x^2 + d^2}}{d}\right)}{d^6} + \frac{8ex + 15d}{15d^6 \sqrt{-e^2 x^2 + d^2}}$$

command

```
integrate((e*x+d)/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{\sqrt{-x^2 e^2 + d^2} \left( \left( \left( x \left( \frac{8xe^5}{d^6} + \frac{15e^4}{d^5} \right) - \frac{20e^3}{d^4} \right) x - \frac{35e^2}{d^3} \right) x + \frac{15e}{d^2} \right) x + \frac{23}{d}}{15 (x^2 e^2 - d^2)^3}$$

$$-\frac{\log\left(\frac{|-2de^{-2}\sqrt{-x^2 e^2 + d^2} e^{(-2)}|}{2|x|}\right)}{d^6}$$

### 11.11 Problem number 28

$$\int \frac{d + ex}{x^2 (d^2 - e^2 x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{ex + d}{5d^2 x (-e^2 x^2 + d^2)^{\frac{5}{2}}} + \frac{5ex + 6d}{15d^4 x (-e^2 x^2 + d^2)^{\frac{3}{2}}}$$

$$- \frac{e \operatorname{arctanh}\left(\frac{\sqrt{-e^2 x^2 + d^2}}{d}\right)}{d^7} + \frac{5ex + 8d}{5d^6 x \sqrt{-e^2 x^2 + d^2}} - \frac{16 \sqrt{-e^2 x^2 + d^2}}{5d^7 x}$$

command

```
integrate((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\sqrt{-x^2e^2 + d^2} \left( \left( \left( 3 \left( x \left( \frac{11xe^6}{d^7} + \frac{5e^5}{d^6} \right) - \frac{25e^4}{d^5} \right) x - \frac{35e^3}{d^4} \right) x + \frac{45e^2}{d^3} \right) x + \frac{23e}{d^2} \right)}{15(x^2e^2 - d^2)^3}$$

$$- \frac{e \log \left( \frac{\left| -2de^{-2} \sqrt{-x^2e^2 + d^2} e \right| e^{(-2)}}{2|x|} \right)}{d^7}$$

$$+ \frac{xe^3}{2 \left( de + \sqrt{-x^2e^2 + d^2} e \right) d^7} - \frac{\left( de + \sqrt{-x^2e^2 + d^2} e \right) e^{(-1)}}{2 d^7 x}$$

## 11.12 Problem number 29

$$\int \frac{d + ex}{x^3 (d^2 - e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{ex + d}{5d^2x^2 (-e^2x^2 + d^2)^{\frac{5}{2}}} + \frac{6ex + 7d}{15d^4x^2 (-e^2x^2 + d^2)^{\frac{3}{2}}} - \frac{7e^2 \operatorname{arctanh} \left( \frac{\sqrt{-e^2x^2 + d^2}}{d} \right)}{2d^8}$$

$$+ \frac{24ex + 35d}{15d^6x^2 \sqrt{-e^2x^2 + d^2}} - \frac{7\sqrt{-e^2x^2 + d^2}}{2d^7x^2} - \frac{16e\sqrt{-e^2x^2 + d^2}}{5d^8x}$$

command

`integrate((e*x+d)/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\begin{aligned}
& - \frac{\sqrt{-x^2 e^2 + d^2} \left( \left( \left( 3 \left( x \left( \frac{11 x e^7}{d^8} + \frac{15 e^6}{d^7} \right) - \frac{25 e^5}{d^6} \right) x - \frac{100 e^4}{d^5} \right) x + \frac{45 e^3}{d^4} \right) x + \frac{58 e^2}{d^3} \right)}{15 (x^2 e^2 - d^2)^3} \\
& - \frac{7 e^2 \log \left( \frac{|-2 d e - 2 \sqrt{-x^2 e^2 + d^2} e| e^{(-2)}}{2 |x|} \right)}{2 d^8} + \frac{x^2 \left( \frac{4 (d e + \sqrt{-x^2 e^2 + d^2} e) e^4}{x} + e^6 \right)}{8 (d e + \sqrt{-x^2 e^2 + d^2} e)^2 d^8} \\
& - \frac{\left( \frac{4 (d e + \sqrt{-x^2 e^2 + d^2} e) d^8 e^8}{x} + \frac{(d e + \sqrt{-x^2 e^2 + d^2} e)^2 d^8 e^6}{x^2} \right) e^{(-8)}}{8 d^{16}}
\end{aligned}$$

### 11.13 Problem number 30

$$\int \frac{x^2 (d + e x)}{(d^2 - e^2 x^2)^{9/2}} dx$$

Optimal antiderivative

$$\frac{x^2 (e x + d)}{7 d e (-e^2 x^2 + d^2)^{7/2}} - \frac{2 (-2 e x + d)}{35 d e^3 (-e^2 x^2 + d^2)^{5/2}} - \frac{4 x}{105 d^3 e^2 (-e^2 x^2 + d^2)^{3/2}} - \frac{8 x}{105 d^5 e^2 \sqrt{-e^2 x^2 + d^2}}$$

command

`integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\left( \left( \left( 4 x^2 \left( \frac{2 x^2 e^4}{d^5} - \frac{7 e^2}{d^3} \right) + \frac{35}{d} \right) x + 21 e^{(-1)} \right) x^2 - 6 d^2 e^{(-3)} \right) \sqrt{-x^2 e^2 + d^2}}{105 (x^2 e^2 - d^2)^4}$$

### 11.14 Problem number 31

$$\int \frac{x^2 (d + e x)}{(d^2 - e^2 x^2)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{x^2 (e x + d)}{9 d e (-e^2 x^2 + d^2)^{9/2}} - \frac{2 (-3 e x + d)}{63 d e^3 (-e^2 x^2 + d^2)^{7/2}} - \frac{2 x}{105 d^3 e^2 (-e^2 x^2 + d^2)^{5/2}} \\
& - \frac{8 x}{315 d^5 e^2 (-e^2 x^2 + d^2)^{3/2}} - \frac{16 x}{315 d^7 e^2 \sqrt{-e^2 x^2 + d^2}}
\end{aligned}$$



command

```
integrate(x^2*(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\left(\left(\left(2\left(4x^2\left(\frac{2x^2e^6}{d^7}-\frac{9e^4}{d^5}\right)+\frac{63e^2}{d^3}\right)x^2-\frac{105}{d}\right)x-45e^{(-1)}\right)x^2+10d^2e^{(-3)}\right)\sqrt{-x^2e^2+d^2}}{315(x^2e^2-d^2)^5}$$

### 11.15 Problem number 44

$$\int \frac{x^5(d+ex)^2}{(d^2-e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{d^4(ex+d)^2}{5e^6(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{22d^3(ex+d)}{15e^6(-e^2x^2+d^2)^{\frac{3}{2}}} - \frac{2d \arctan\left(\frac{ex}{\sqrt{-e^2x^2+d^2}}\right)}{e^6} + \frac{2d(23ex+30d)}{15e^6\sqrt{-e^2x^2+d^2}} + \frac{\sqrt{-e^2x^2+d^2}}{e^6}$$

command

```
integrate(x^5*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-2d \arcsin\left(\frac{xe}{d}\right) e^{(-6)} \operatorname{sgn}(d)$$

$$\frac{(56d^6e^{(-6)} + (30d^5e^{(-5)} - (140d^4e^{(-4)} + (70d^3e^{(-3)} - (105d^2e^{(-2)} + (46de^{(-1)} - 15x)x)x)x)x)\sqrt{-x^2e^2+d^2}}{15(x^2e^2-d^2)^3}$$

### 11.16 Problem number 45

$$\int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{d^3(ex+d)^2}{5e^5(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{17d^2(ex+d)}{15e^5(-e^2x^2+d^2)^{\frac{3}{2}}} - \frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2+d^2}}\right)}{e^5} + \frac{\frac{26ex}{15} + 2d}{e^5\sqrt{-e^2x^2+d^2}}$$

command

`integrate(x^4*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\operatorname{sgn}(d) - \frac{(16d^5e^{(-5)} + (15d^4e^{(-4)} - (40d^3e^{(-3)} + (35d^2e^{(-2)} - 2(15de^{(-1)} + 13x)x)x)x)\sqrt{-x^2e^2+d^2}}{15(x^2e^2-d^2)^3}$$

### 11.17 Problem number 46

$$\int \frac{x^3(d+ex)^2}{(d^2-e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{d^2(ex+d)^2}{5e^4(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{4d(ex+d)}{5e^4(-e^2x^2+d^2)^{\frac{3}{2}}} + \frac{2ex+5d}{5de^4\sqrt{-e^2x^2+d^2}}$$

command

`integrate(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{(2d^4e^{(-4)} + (x^2(\frac{2xe}{d} + 5) - 5d^2e^{(-2)})x^2)\sqrt{-x^2e^2+d^2}}{5(x^2e^2-d^2)^3}$$

### 11.18 Problem number 47

$$\int \frac{x^2(d+ex)^2}{(d^2-e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{d(ex+d)^2}{5e^3(-e^2x^2+d^2)^{5/2}} - \frac{7(ex+d)}{15e^3(-e^2x^2+d^2)^{3/2}} + \frac{x}{15d^2e^2\sqrt{-e^2x^2+d^2}}$$

command

```
integrate(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\left(4d^3e^{(-3)} - \left(x\left(\frac{x^2e^2}{d^2} + 5\right) + 10de^{(-1)}\right)x^2\right)\sqrt{-x^2e^2+d^2}}{15(x^2e^2-d^2)^3}$$

### 11.19 Problem number 48

$$\int \frac{x(d+ex)^2}{(d^2-e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{(ex+d)^2}{5e^2(-e^2x^2+d^2)^{5/2}} - \frac{2(ex+d)}{15de^2(-e^2x^2+d^2)^{3/2}} - \frac{4x}{15d^3e\sqrt{-e^2x^2+d^2}}$$

command

```
integrate(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\left(\left(2x\left(\frac{2x^2e^3}{d^3} - \frac{5e}{d}\right) - 5\right)x^2 - d^2e^{(-2)}\right)\sqrt{-x^2e^2+d^2}}{15(x^2e^2-d^2)^3}$$

### 11.20 Problem number 49

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2ex}{5} + \frac{2d}{5}}{e(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{x}{5d^2(-e^2x^2+d^2)^{\frac{3}{2}}} + \frac{2x}{5d^4\sqrt{-e^2x^2+d^2}}$$

command

```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{\sqrt{-x^2e^2+d^2} \left( \left( x^2 \left( \frac{2x^2e^4}{d^4} - \frac{5e^2}{d^2} \right) + 5 \right) x + 2de^{(-1)} \right)}{5(x^2e^2-d^2)^3}$$

### 11.21 Problem number 50

$$\int \frac{(d+ex)^2}{x(d^2-e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2ex}{5} + \frac{2d}{5}}{d(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{8ex+5d}{15d^3(-e^2x^2+d^2)^{\frac{3}{2}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{-e^2x^2+d^2}}{d}\right)}{d^5} + \frac{16ex+15d}{15d^5\sqrt{-e^2x^2+d^2}}$$

command

```
integrate((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{\sqrt{-x^2e^2+d^2} \left( \left( \left( \left( x \left( \frac{16xe^5}{d^5} + \frac{15e^4}{d^4} \right) - \frac{40e^3}{d^3} \right) x - \frac{35e^2}{d^2} \right) x + \frac{30e}{d} \right) x + 26 \right)}{15(x^2e^2-d^2)^3}$$

$$-\frac{\log\left(\frac{\left| -2de^{-2}\sqrt{-x^2e^2+d^2}e^{\left|e^{(-2)}\right|} \right|}{2|x|}\right)}{d^5}$$

### 11.22 Problem number 51

$$\int \frac{(d+ex)^2}{x^2(d^2-e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e(ex+d)}{5d^2(-e^2x^2+d^2)^{5/2}} + \frac{e(13ex+10d)}{15d^4(-e^2x^2+d^2)^{3/2}} \\ & - \frac{2e \operatorname{arctanh}\left(\frac{\sqrt{-e^2x^2+d^2}}{d}\right)}{d^6} + \frac{e(41ex+30d)}{15d^6\sqrt{-e^2x^2+d^2}} - \frac{\sqrt{-e^2x^2+d^2}}{d^6x} \end{aligned}$$

command

```
integrate((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\begin{aligned} & - \frac{\sqrt{-x^2e^2+d^2} \left( \left( \left( \left( x \left( \frac{41xe^6}{d^6} + \frac{30e^5}{d^5} \right) - \frac{95e^4}{d^4} \right) x - \frac{70e^3}{d^3} \right) x + \frac{60e^2}{d^2} \right) x + \frac{46e}{d} \right)}{15(x^2e^2-d^2)^3} \\ & - \frac{2e \log\left(\frac{|-2de-2\sqrt{-x^2e^2+d^2}e|e^{(-2)}}{2|x|}\right)}{d^6} \\ & + \frac{xe^3}{2\left(de+\sqrt{-x^2e^2+d^2}e\right)d^6} - \frac{\left(de+\sqrt{-x^2e^2+d^2}e\right)e^{(-1)}}{2d^6x} \end{aligned}$$

### 11.23 Problem number 52

$$\int \frac{(d+ex)^2}{x^3(d^2-e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e^2(ex+d)}{5d^3(-e^2x^2+d^2)^{5/2}} + \frac{e^2(6ex+5d)}{5d^5(-e^2x^2+d^2)^{3/2}} - \frac{9e^2 \operatorname{arctanh}\left(\frac{\sqrt{-e^2x^2+d^2}}{d}\right)}{2d^7} \\ & + \frac{2e^2(11ex+10d)}{5d^7\sqrt{-e^2x^2+d^2}} - \frac{\sqrt{-e^2x^2+d^2}}{2d^6x^2} - \frac{2e\sqrt{-e^2x^2+d^2}}{d^7x} \end{aligned}$$

command

```
integrate((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\sqrt{-x^2e^2 + d^2} \left( \left( \left( 2 \left( x \left( \frac{11xe^7}{d^7} + \frac{10e^6}{d^6} \right) - \frac{25e^5}{d^5} \right) x - \frac{45e^4}{d^4} \right) x + \frac{30e^3}{d^3} \right) x + \frac{27e^2}{d^2} \right)}{5(x^2e^2 - d^2)^3} - \frac{9e^2 \log \left( \frac{-2de - 2\sqrt{-x^2e^2 + d^2} e^{e^{(-2)}}}{2|x|} \right)}{2d^7} + \frac{x^2 \left( \frac{8(de + \sqrt{-x^2e^2 + d^2} e)^{e^4}}{x} + e^6 \right)}{8(de + \sqrt{-x^2e^2 + d^2} e)^2 d^7} - \frac{\left( \frac{8(de + \sqrt{-x^2e^2 + d^2} e)^{d^7 e^8}}{x} + \frac{(de + \sqrt{-x^2e^2 + d^2} e)^2 d^7 e^6}{x^2} \right) e^{(-8)}}{8d^{14}}$$

## 11.24 Problem number 53

$$\int \frac{(d + ex)^2}{x^4 (d^2 - e^2 x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{2e^3(ex + d)}{5d^4(-e^2x^2 + d^2)^{\frac{5}{2}}} + \frac{e^3(23ex + 20d)}{15d^6(-e^2x^2 + d^2)^{\frac{3}{2}}} - \frac{7e^3 \operatorname{arctanh}\left(\frac{\sqrt{-e^2x^2 + d^2}}{d}\right)}{d^8} + \frac{2e^3(53ex + 45d)}{15d^8 \sqrt{-e^2x^2 + d^2}} - \frac{\sqrt{-e^2x^2 + d^2}}{3d^6 x^3} - \frac{e \sqrt{-e^2x^2 + d^2}}{d^7 x^2} - \frac{14e^2 \sqrt{-e^2x^2 + d^2}}{3d^8 x}$$

command

```
integrate((e*x+d)^2/x^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\begin{aligned}
& - \frac{\sqrt{-x^2e^2 + d^2} \left( \left( \left( \left( 2x \left( \frac{53xe^8}{d^8} + \frac{45e^7}{d^7} \right) - \frac{235e^6}{d^6} \right) x - \frac{200e^5}{d^5} \right) x + \frac{135e^4}{d^4} \right) x + \frac{116e^3}{d^3} \right)}{15(x^2e^2 - d^2)^3} \\
& + \frac{x^3 \left( \frac{6 \left( de + \sqrt{-x^2e^2 + d^2} e \right) e^6}{x} + \frac{57 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^2 e^4}{x^2} + e^8 \right) e}{24 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^3 d^8} \\
& - \frac{7e^3 \log \left( \frac{\left| -2de - 2\sqrt{-x^2e^2 + d^2} e \right| e^{(-2)}}{2|x|} \right)}{d^8} \\
& - \frac{\left( \frac{57 \left( de + \sqrt{-x^2e^2 + d^2} e \right) d^{16} e^{16}}{x} + \frac{6 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^2 d^{16} e^{14}}{x^2} + \frac{\left( de + \sqrt{-x^2e^2 + d^2} e \right)^3 d^{16} e^{12}}{x^3} \right) e^{(-15)}}{24d^{24}}
\end{aligned}$$

### 11.25 Problem number 130

$$\int \frac{x^2}{(d+ex)(d^2 - e^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2}{3e^3 \sqrt{-e^2x^2 + d^2}} - \frac{x^2}{3de(ex+d) \sqrt{-e^2x^2 + d^2}}$$

command

`integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$+\infty$

### 11.26 Problem number 134

$$\int \frac{1}{x^2(d+ex)(d^2-e^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{e \operatorname{arctanh}\left(\frac{\sqrt{-e^2x^2+d^2}}{d}\right)}{d^5} + \frac{-3ex+4d}{3d^4x\sqrt{-e^2x^2+d^2}} + \frac{1}{3d^2x(ex+d)\sqrt{-e^2x^2+d^2}} - \frac{8\sqrt{-e^2x^2+d^2}}{3d^5x}$$

command

`integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$+\infty$

### 11.27 Problem number 586

$$\int \frac{(d+ex)^3}{(f+gx)^2(d^2-e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4de(ex+d)}{5(dg+ef)^2(-e^2x^2+d^2)^{5/2}} - \frac{e(5d(-3dg+ef) - e(21dg+ef)x)}{15d(dg+ef)^3(-e^2x^2+d^2)^{3/2}} \\ & + \frac{eg^3(-3dg+4ef) \operatorname{arctan}\left(\frac{e^2fx+d^2g}{\sqrt{-d^2g^2+e^2f^2}\sqrt{-e^2x^2+d^2}}\right)}{(-dg+ef)(dg+ef)^4\sqrt{-d^2g^2+e^2f^2}} \\ & + \frac{e(45d^3g^2+e(57d^2g^2+14defg+2e^2f^2)x)}{15d^3(dg+ef)^4\sqrt{-e^2x^2+d^2}} + \frac{g^4\sqrt{-e^2x^2+d^2}}{(-dg+ef)(dg+ef)^4(gx+f)} \end{aligned}$$

command

`integrate((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

output too large to display



## 11.28 Problem number 842

$$\int \frac{\sqrt{d+ex} (a+bx+cx^2)}{(e+fx)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(4ef(-2aef - bdf + 3be^2) - c(-d^2f^2 - 6de^2f + 15e^4)) \operatorname{arctanh}\left(\frac{\sqrt{f} \sqrt{ex+d}}{\sqrt{e} \sqrt{fx+e}}\right)}{4e^{\frac{3}{2}} f^{\frac{7}{2}}} + \frac{2\left(a + \frac{e(-bf+ce)}{f^2}\right) (ex+d)^{\frac{3}{2}}}{(-df+e^2) \sqrt{fx+e}} + \frac{c(ex+d)^{\frac{3}{2}} \sqrt{fx+e}}{2ef^2} + \frac{(4ef(-2aef - bdf + 3be^2) - c(-d^2f^2 - 6de^2f + 15e^4)) \sqrt{ex+d} \sqrt{fx+e}}{4ef^3(-df+e^2)}$$

command

```
integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\left((xe+d)\left(\frac{2(xe+d)ce^{(-1)}}{f} - \frac{(3cdf^4e^2 - 4bf^4e^3 + 5cf^3e^4)e^{(-3)}}{f^5}\right) + \frac{(cd^2f^4e^2 - 4bdf^4e^3 + 6cdf^3e^4 - 8af^4e^4 + 12bf^3e^5 - 15cf^2e^6)e^{(-3)}}{f^5}\right) \sqrt{xe+d}}{4 \sqrt{(xe+d)fe - dfe + e^3}} + \frac{(cd^2f^2 - 4bdf^2e + 6cdf^2e^2 - 8af^2e^2 + 12bfe^3 - 15ce^4)e^{(-\frac{3}{2})} \log\left(\left| -\sqrt{xe+d} \sqrt{f} e^{\frac{1}{2}} + \sqrt{(xe+d)fe - dfe + e^3} \right|\right)}{4 f^{\frac{7}{2}}}$$

## 12 Test file number 39

Test folder name:

```
test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.2_Quartic/39_1.2.2.2-d_x-  
^m-a+b_x^2+c_x^4-^p
```

### 12.1 Problem number 3

$$\int \frac{1}{\sqrt[4]{a^2 + 2abx^2 + b^2x^4}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arcsinh}\left(\frac{x\sqrt{b}}{\sqrt{a}}\right) \sqrt{a} \sqrt{1 + \frac{bx^2}{a}}}{(b^2x^4 + 2abx^2 + a^2)^{\frac{1}{4}} \sqrt{b}}$$

command

```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\arctan\left(\frac{\sqrt{-\frac{bx^2+a}{x^2}}}{\sqrt{b}}\right)}{\sqrt{b}}$$

### 12.2 Problem number 4

$$\int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{3/4}} dx$$

Optimal antiderivative

$$\frac{x(bx^2 + a)}{a(b^2x^4 + 2abx^2 + a^2)^{\frac{3}{4}}}$$

command

```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{1}{a\sqrt{-\frac{bx^2+a}{x^2}}}$$

## 13 Test file number 51

Test folder name:

test\_cases/1\_Algebraic\_functions/1.3\_Miscellaneous/51\_1.3.1\_Rational\_functions

### 13.1 Problem number 163

$$\int x^{14(-1+n)}(b+2cx^n)(bx+cx^{1+n})^{13} dx$$

Optimal antiderivative

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

command

```
integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{c^{14}x^{28n} + 14bc^{13}x^{27n} + 91b^2c^{12}x^{26n} + 364b^3c^{11}x^{25n} + 1001b^4c^{10}x^{24n} + 2002b^5c^9x^{23n} + 3003b^6c^8x^{22n} + 3432b^7c^7x^{21n} + 3432b^8c^6x^{20n} + 252b^9c^5x^{19n} + 14b^{10}c^4x^{18n} + 7b^{11}c^3x^{17n} + b^{12}c^2x^{16n}}{14n}$$

14

## 14 Test file number 55

Test folder name:

test\_cases/2\_Exponentials/55\_2.3\_Exponential\_functions

### 14.1 Problem number 744

$$\int \left( \frac{1}{\sqrt{e^x+x}} + \frac{e^x}{\sqrt{e^x+x}} \right) dx$$

Optimal antiderivative

$$2\sqrt{x+e^x}$$

command

```
integrate(exp(x)/(exp(x)+x)^(1/2)+1/(exp(x)+x)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$2\sqrt{x+e^x}$$

## 15 Test file number 59

Test folder name:

test\_cases/3\_Logarithms/59\_3.2.1-f+g\_x-^m-A+B\_log-e-a+b\_x-over-c+d\_x-^n-^p

### 15.1 Problem number 201

$$\int (ag + bgx)^4 \left( A + B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2B(-ad + bc)^4 g^4 x}{5d^4} + \frac{B(-ad + bc)^3 g^4 (bx + a)^2}{5b d^3} \\ & -\frac{2B(-ad + bc)^2 g^4 (bx + a)^3}{15b d^2} + \frac{B(-ad + bc) g^4 (bx + a)^4}{10bd} \\ & + \frac{2B(-ad + bc)^5 g^4 \ln(dx + c)}{5b d^5} + \frac{g^4 (bx + a)^5 \left( A + B \ln \left( \frac{e(dx+c)^2}{(bx+a)^2} \right) \right)}{5b} \end{aligned}$$

command

```
integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$\begin{aligned} & -\frac{2Ba^5g^4 \log(bx + a)}{5b} + \frac{1}{5} (Ab^4g^4 + Bb^4g^4)x^5 + \frac{(Bb^4cg^4 + 10Aab^3dg^4 + 9Bab^3dg^4)x^4}{10d} \\ & -\frac{2(Bb^4c^2g^4 - 5Bab^3cdg^4 - 15Aa^2b^2d^2g^4 - 11Ba^2b^2d^2g^4)x^3}{15d^2} \\ & + \frac{1}{5} (Bb^4g^4x^5 + 5Bab^3g^4x^4 + 10Ba^2b^2g^4x^3 + 10Ba^3bg^4x^2 + 5Ba^4g^4x) \log \left( \frac{d^2x^2 + 2cdx + c^2}{b^2x^2 + 2abx + a^2} \right) \\ & + \frac{(Bb^4c^3g^4 - 5Bab^3c^2dg^4 + 10Ba^2b^2cd^2g^4 + 10Aa^3bd^3g^4 + 4Ba^3bd^3g^4)x^2}{5d^3} \\ & -\frac{(2Bb^4c^4g^4 - 10Bab^3c^3dg^4 + 20Ba^2b^2c^2d^2g^4 - 20Ba^3bcd^3g^4 - 5Aa^4d^4g^4 + 3Ba^4d^4g^4)x}{5d^4} \\ & + \frac{2(Bb^4c^5g^4 - 5Bab^3c^4dg^4 + 10Ba^2b^2c^3d^2g^4 - 10Ba^3bc^2d^3g^4 + 5Ba^4cd^4g^4) \log(dx + c)}{5d^5} \end{aligned}$$

## 15.2 Problem number 263

$$\int (f + gx)^3 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{B(-ad + bc)g(a^2d^2g^2 - abdg(-cg + 4df) + b^2(c^2g^2 - 4cdfg + 6d^2f^2))x}{2b^3d^3} \\ & - \frac{B(-ad + bc)g^2(-adg - bcg + 4bdf)x^2}{4b^2d^2} - \frac{B(-ad + bc)g^3x^3}{6bd} \\ & - \frac{B(-ag + bf)^4 \ln(bx + a)}{2b^4g} + \frac{(gx + f)^4 \left( A + B \ln \left( \frac{e(bx+a)^2}{(dx+c)^2} \right) \right)}{4g} + \frac{B(-cg + df)^4 \ln(dx + c)}{2d^4g} \end{aligned}$$

command

```
integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$\begin{aligned} & \frac{1}{4} (Ag^3 + Bg^3)x^4 + \frac{(6Abdfg^2 + 6Bbdfg^2 - Bbcg^3 + Badg^3)x^3}{6bd} \\ & + \frac{1}{4} (Bg^3x^4 + 4Bfg^2x^3 + 6Bf^2gx^2 + 4Bf^3x) \log \left( \frac{b^2x^2 + 2abx + a^2}{d^2x^2 + 2cdx + c^2} \right) \\ & + \frac{(6Ab^2d^2f^2g + 6Bb^2d^2f^2g - 4Bb^2cdfg^2 + 4Babd^2fg^2 + Bb^2c^2g^3 - Ba^2d^2g^3)x^2}{4b^2d^2} \\ & + \frac{(4Bab^3f^3 - 6Ba^2b^2f^2g + 4Ba^3bfg^2 - Ba^4g^3) \log(bx + a)}{2b^4} \\ & - \frac{(4Bcd^3f^3 - 6Bc^2d^2f^2g + 4Bc^3dfg^2 - Bc^4g^3) \log(-dx - c)}{2d^4} \\ & + \frac{(2Ab^3d^3f^3 + 2Bb^3d^3f^3 - 6Bb^3cd^2f^2g + 6Bab^2d^3f^2g + 4Bb^3c^2dfg^2 - 4Ba^2bd^3fg^2 - Bb^3c^3g^3 + Ba^3d^3g^3)x}{2b^3d^3} \end{aligned}$$

### 15.3 Problem number 295

$$\int (g + hx)^2 (A + B \log(e(a + bx)^n (c + dx)^{-n})) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{B(-ad + bc)h(-adh - bch + 3bdg)nx}{3b^2d^2} - \frac{B(-ad + bc)h^2nx^2}{6bd} - \frac{B(-ah + bg)^3n \ln(bx + a)}{3b^3h} \\ & + \frac{B(-ch + dg)^3n \ln(dx + c)}{3d^3h} + \frac{(hx + g)^3(A + B \ln(e(bx + a)^n (dx + c)^{-n}))}{3h} \end{aligned}$$

command

```
integrate((h*x+g)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$\begin{aligned} & \frac{1}{3}(Ah^2 + Bh^2)x^3 + \frac{1}{3}(Bh^2nx^3 + 3Bghnx^2 + 3Bg^2nx) \log(bx + a) \\ & - \frac{1}{3}(Bh^2nx^3 + 3Bghnx^2 + 3Bg^2nx) \log(dx + c) \\ & - \frac{(Bbch^2n - Badh^2n - 6Abdgh - 6Bbdgh)x^2}{6bd} \\ & + \frac{(3Bab^2g^2n - 3Ba^2bghn + Ba^3h^2n) \log(bx + a)}{3b^3} \\ & - \frac{(3Bcd^2g^2n - 3Bc^2dghn + Bc^3h^2n) \log(-dx - c)}{3d^3} \\ & - \frac{(3Bb^2cdghn - 3Babd^2ghn - Bb^2c^2h^2n + Ba^2d^2h^2n - 3Ab^2d^2g^2 - 3Bb^2d^2g^2)x}{3b^2d^2} \end{aligned}$$

## 16 Test file number 61

Test folder name:

```
test_cases/3_Logarithms/61_3.2.3_u_log-e-f-a+b_x-^p-c+d_x-^q-r-^s
```

## 16.1 Problem number 27

$$\int (g + hx)^2 \log(e(f(a + bx)^p(c + dx)^q)^r) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-ah + bg)^2 prx}{3b^2} - \frac{(-ch + dg)^2 qrx}{3d^2} - \frac{(-ah + bg) pr(hx + g)^2}{6bh} \\ & - \frac{(-ch + dg) qr(hx + g)^2}{6dh} - \frac{pr(hx + g)^3}{9h} - \frac{qr(hx + g)^3}{9h} - \frac{(-ah + bg)^3 pr \ln(bx + a)}{3b^3h} \\ & - \frac{(-ch + dg)^3 qr \ln(dx + c)}{3d^3h} + \frac{(hx + g)^3 \ln(e(f(bx + a)^p(dx + c)^q)^r)}{3h} \end{aligned}$$

command

```
integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$\begin{aligned} & -\frac{1}{9} (h^2 pr + h^2 qr - 3 h^2 r \log(f) - 3 h^2) x^3 + \frac{1}{3} (h^2 pr x^3 + 3 gh pr x^2 + 3 g^2 pr x) \log(bx + a) \\ & + \frac{1}{3} (h^2 qr x^3 + 3 gh qr x^2 + 3 g^2 qr x) \log(dx + c) \\ & - \frac{(3 bdghpr - adh^2 pr + 3 bdghqr - bch^2 qr - 6 bdghr \log(f) - 6 bdgh) x^2}{6 bd} \\ & + \frac{(3 ab^2 g^2 pr - 3 a^2 bghpr + a^3 h^2 pr) \log(bx + a)}{3 b^3} + \frac{(3 cd^2 g^2 qr - 3 c^2 dghqr + c^3 h^2 qr) \log(-dx - c)}{3 d^3} \\ & - \frac{(3 b^2 d^2 g^2 pr - 3 abd^2 ghpr + a^2 d^2 h^2 pr + 3 b^2 d^2 g^2 qr - 3 b^2 cdghqr + b^2 c^2 h^2 qr - 3 b^2 d^2 g^2 r \log(f) - 3 b^2 d^2 g^2) x}{3 b^2 d^2} \end{aligned}$$

## 17 Test file number 64

Test folder name:

test\_cases/3\_Logarithms/64\_3.5\_Logarithm\_functions

### 17.1 Problem number 111

$$\int \frac{\log(-1 + 4x + 4\sqrt{(-1+x)x})}{x^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -8 \arctan\left(\frac{\sqrt{x}}{\sqrt{x^2-x}}\right) + 4 \arctan(2\sqrt{2}\sqrt{x}) \sqrt{2} \\ & - \frac{2 \ln(-1 + 4x + 4\sqrt{x^2-x})}{\sqrt{x}} - \frac{4 \arctan\left(\frac{2\sqrt{-1+x}\sqrt{2}}{3}\right) \sqrt{x^2-x} \sqrt{2}}{\sqrt{-1+x} \sqrt{x}} \end{aligned}$$

command

```
integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: NotImplementedError

Giac 1.7.0 via sagemath 9.3 output

$$\begin{aligned} & 2\sqrt{2}\pi i - 4\pi i + 4\pi \operatorname{sgn}(\sqrt{x-1} - \sqrt{x}) \\ & - 2\sqrt{2} \left( \pi \operatorname{sgn}(\sqrt{x-1} - \sqrt{x}) + 2 \arctan\left(\frac{\sqrt{2}((\sqrt{x-1} - \sqrt{x})^2 - 1)}{3(\sqrt{x-1} - \sqrt{x})}\right) \right) \\ & + 4\sqrt{2} \arctan\left(\frac{2}{3}\sqrt{2}i\right) + 4\sqrt{2} \arctan(2\sqrt{2}\sqrt{x}) \\ & - \frac{2 \log(4x + 4\sqrt{(x-1)x} - 1)}{\sqrt{x}} - 8 \arctan(i) + 8 \arctan\left(\frac{(\sqrt{x-1} - \sqrt{x})^2 - 1}{2(\sqrt{x-1} - \sqrt{x})}\right) \end{aligned}$$

### 17.2 Problem number 185

$$\int \cos(ax + bx) \log\left(\cos\left(\frac{a}{2} + \frac{bx}{2}\right) \sin\left(\frac{a}{2} + \frac{bx}{2}\right)\right) dx$$

Optimal antiderivative

$$-\frac{\sin(bx+a)}{b} + \frac{\ln\left(\cos\left(\frac{a}{2} + \frac{bx}{2}\right) \sin\left(\frac{a}{2} + \frac{bx}{2}\right)\right) \sin(bx+a)}{b}$$

command



```
integrate(cos(b*x+a)*log(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\log\left(\cos\left(\frac{1}{2}bx + \frac{1}{2}a\right)\sin\left(\frac{1}{2}bx + \frac{1}{2}a\right)\right)\sin(bx + a)}{b} - \frac{\sin(bx + a)}{b}$$

## 18 Test file number 65

Test folder name:

test\_cases/4\_Trig\_functions/4.1\_Sine/65\_4.1.0-a\_sin-<sup>m</sup>-b\_trg-<sup>n</sup>

### 18.1 Problem number 190

$$\int \frac{\sin(a + bx)}{(d \cos(a + bx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2}{bd\sqrt{d \cos(bx + a)}}$$

command

```
integrate(sin(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2}{\sqrt{d \cos(bx + a)} bd}$$

## 18.2 Problem number 191

$$\int \frac{\sin(a + bx)}{(d \cos(a + bx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2}{3bd (d \cos(bx + a))^{\frac{3}{2}}}$$

command

```
integrate(sin(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2}{3 \sqrt{d \cos(bx + a)} bd^2 \cos(bx + a)}$$

## 18.3 Problem number 192

$$\int \frac{\sin(a + bx)}{(d \cos(a + bx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2}{5bd (d \cos(bx + a))^{\frac{5}{2}}}$$

command

```
integrate(sin(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2}{5 \sqrt{d \cos(bx + a)} bd^3 \cos(bx + a)^2}$$

### 18.4 Problem number 193

$$\int \frac{\sin(a + bx)}{(d \cos(a + bx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{2}{7bd (d \cos(bx + a))^{7/2}}$$

command

```
integrate(sin(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2}{7 \sqrt{d \cos(bx + a)} bd^4 \cos(bx + a)^3}$$

### 18.5 Problem number 206

$$\int \frac{\sin^3(a + bx)}{(d \cos(a + bx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(d \cos(bx + a))^{3/2}}{3bd^3} + \frac{2}{bd \sqrt{d \cos(bx + a)}}$$

command

```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2 \left( \frac{\sqrt{d \cos(bx + a)} \cos(bx + a)}{d} + \frac{3}{\sqrt{d \cos(bx + a)}} \right)}{3bd}$$

## 18.6 Problem number 207

$$\int \frac{\sin^3(a + bx)}{(d \cos(a + bx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2}{3bd (d \cos(bx + a))^{3/2}} + \frac{2\sqrt{d \cos(bx + a)}}{bd^3}$$

command

```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2 \left( 3 \sqrt{d \cos(bx + a)} b^2 + \frac{b^2 d}{\sqrt{d \cos(bx + a)} \cos(bx + a)} \right)}{3 b^3 d^3}$$

## 18.7 Problem number 208

$$\int \frac{\sin^3(a + bx)}{(d \cos(a + bx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2}{5bd (d \cos(bx + a))^{5/2}} - \frac{2}{bd^3 \sqrt{d \cos(bx + a)}}$$

command

```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2 \left( 5 b^3 d^3 \cos(bx + a)^2 - b^3 d^3 \right)}{5 \sqrt{d \cos(bx + a)} b^4 d^6 \cos(bx + a)^2}$$

### 18.8 Problem number 209

$$\int \frac{\sin^3(a + bx)}{(d \cos(a + bx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{2}{7bd (d \cos(bx + a))^{7/2}} - \frac{2}{3b d^3 (d \cos(bx + a))^{3/2}}$$

command

```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2 \left( 7 b^4 d^4 \cos(bx + a)^2 - 3 b^4 d^4 \right)}{21 \sqrt{d \cos(bx + a)} b^5 d^8 \cos(bx + a)^3}$$

### 18.9 Problem number 210

$$\int \frac{\sin^3(a + bx)}{(d \cos(a + bx))^{11/2}} dx$$

Optimal antiderivative

$$\frac{2}{9bd (d \cos(bx + a))^{9/2}} - \frac{2}{5b d^3 (d \cos(bx + a))^{5/2}}$$

command

```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(11/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2 \left( 9 b^5 d^5 \cos(bx + a)^2 - 5 b^5 d^5 \right)}{45 \sqrt{d \cos(bx + a)} b^6 d^{10} \cos(bx + a)^4}$$

### 18.10 Problem number 228

$$\int \frac{\csc(a + bx)}{(d \cos(a + bx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sqrt{d \cos(bx + a)}}{\sqrt{d}}\right)}{b d^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d \cos(bx + a)}}{\sqrt{d}}\right)}{b d^{3/2}} + \frac{2}{bd \sqrt{d \cos(bx + a)}}$$

command

```
integrate(csc(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2 \arctan\left(\frac{\sqrt{-d} \tan\left(\frac{1}{2}bx + \frac{1}{2}a\right)^2 - \sqrt{-d \tan\left(\frac{1}{2}bx + \frac{1}{2}a\right)^4 + d}}{\sqrt{-d}}\right)}{\sqrt{-d}} + \frac{\log\left(-\sqrt{-d} \tan\left(\frac{1}{2}bx + \frac{1}{2}a\right)^2 + \sqrt{-d \tan\left(\frac{1}{2}bx + \frac{1}{2}a\right)^4 + d}\right)}{\sqrt{-d}}$$


---

$2bd$

### 18.11 Problem number 229

$$\int \frac{\csc(a + bx)}{(d \cos(a + bx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{d \cos(bx + a)}}{\sqrt{d}}\right)}{b d^{5/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d \cos(bx + a)}}{\sqrt{d}}\right)}{b d^{5/2}} + \frac{2}{3bd (d \cos(bx + a))^{3/2}}$$

command

```
integrate(csc(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{6 \arctan \left( \frac{\sqrt{-d} \tan\left(\frac{1}{2} bx + \frac{1}{2} a\right)^2 - \sqrt{-d \tan\left(\frac{1}{2} bx + \frac{1}{2} a\right)^4 + d}}{\sqrt{-d}} \right) - 3 \log \left( \frac{-\sqrt{-d} \tan\left(\frac{1}{2} bx + \frac{1}{2} a\right)^2 + \sqrt{-d \tan\left(\frac{1}{2} bx + \frac{1}{2} a\right)^4 + d}}{\sqrt{-d}} \right)}{\sqrt{-d}}$$


---


$$6 b d^2$$

### 18.12 Problem number 230

$$\int \frac{\csc(a + bx)}{(d \cos(a + bx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sqrt{d \cos(bx + a)}}{\sqrt{d}}\right)}{b d^{\frac{7}{2}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d \cos(bx + a)}}{\sqrt{d}}\right)}{b d^{\frac{7}{2}}} + \frac{2}{5 b d (d \cos(bx + a))^{\frac{5}{2}}} + \frac{2}{b d^3 \sqrt{d \cos(bx + a)}}$$

command

`integrate(csc(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{10 \arctan \left( \frac{\sqrt{-d} \tan\left(\frac{1}{2} bx + \frac{1}{2} a\right)^2 - \sqrt{-d \tan\left(\frac{1}{2} bx + \frac{1}{2} a\right)^4 + d}}{\sqrt{-d}} \right) + 5 \log \left( \frac{-\sqrt{-d} \tan\left(\frac{1}{2} bx + \frac{1}{2} a\right)^2 + \sqrt{-d \tan\left(\frac{1}{2} bx + \frac{1}{2} a\right)^4 + d}}{\sqrt{-d}} \right)}{\sqrt{-d}}$$


---

### 18.13 Problem number 231

$$\int \frac{\csc(a + bx)}{(d \cos(a + bx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sqrt{d \cos(bx + a)}}{\sqrt{d}}\right)}{\frac{b d^{9/2}}{2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d \cos(bx + a)}}{\sqrt{d}}\right)}{b d^{9/2}} + \frac{2}{7bd(d \cos(bx + a))^{7/2}} + \frac{2}{3bd^3(d \cos(bx + a))^{3/2}}$$

command

```
integrate(csc(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{42 \arctan\left(\frac{\sqrt{-d} \tan\left(\frac{1}{2}bx + \frac{1}{2}a\right)^2 - \sqrt{-d \tan\left(\frac{1}{2}bx + \frac{1}{2}a\right)^4 + d}}{\sqrt{-d}}\right)}{\sqrt{-d}} - \frac{21 \log\left(\left(-\sqrt{-d} \tan\left(\frac{1}{2}bx + \frac{1}{2}a\right)^2 + \sqrt{-d \tan\left(\frac{1}{2}bx + \frac{1}{2}a\right)^4 + d}\right)\right)}{\sqrt{-d}}$$

### 18.14 Problem number 249

$$\int \frac{\csc^3(a + bx)}{(d \cos(a + bx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{5 \arctan\left(\frac{\sqrt{d \cos(bx + a)}}{\sqrt{d}}\right)}{4b d^{3/2}} - \frac{5 \operatorname{arctanh}\left(\frac{\sqrt{d \cos(bx + a)}}{\sqrt{d}}\right)}{4b d^{3/2}} + \frac{5}{2bd \sqrt{d \cos(bx + a)}} - \frac{\csc^2(bx + a)}{2bd \sqrt{d \cos(bx + a)}}$$



command

`integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$10 \arctan \left( \frac{\sqrt{-d} \tan\left(\frac{1}{2}bx + \frac{1}{2}a\right)^2 - \sqrt{-d \tan\left(\frac{1}{2}bx + \frac{1}{2}a\right)^4 + d}}{\sqrt{-d}} \right) + \frac{5 \log \left( \left| -\sqrt{-d} \tan\left(\frac{1}{2}bx + \frac{1}{2}a\right)^2 + \sqrt{-d \tan\left(\frac{1}{2}bx + \frac{1}{2}a\right)^4 + d} \right| \right)}{\sqrt{-d}}$$


---

### 18.15 Problem number 250

$$\int \frac{\csc^3(a + bx)}{(d \cos(a + bx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{7 \arctan\left(\frac{\sqrt{d \cos(bx + a)}}{\sqrt{d}}\right)}{4b d^{5/2}} - \frac{7 \operatorname{arctanh}\left(\frac{\sqrt{d \cos(bx + a)}}{\sqrt{d}}\right)}{4b d^{5/2}} \\ & + \frac{7}{6bd (d \cos(bx + a))^{3/2}} - \frac{\csc^2(bx + a)}{2bd (d \cos(bx + a))^{3/2}} \end{aligned}$$

command

`integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$42 \arctan \left( \frac{\sqrt{-d} \tan\left(\frac{1}{2}bx + \frac{1}{2}a\right)^2 - \sqrt{-d \tan\left(\frac{1}{2}bx + \frac{1}{2}a\right)^4 + d}}{\sqrt{-d}} \right) - \frac{21 \log \left( \left| -\sqrt{-d} \tan\left(\frac{1}{2}bx + \frac{1}{2}a\right)^2 + \sqrt{-d \tan\left(\frac{1}{2}bx + \frac{1}{2}a\right)^4 + d} \right| \right)}{\sqrt{-d}}$$


---

### 18.16 Problem number 251

$$\int \frac{\csc^3(a + bx)}{(d \cos(a + bx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{9 \arctan\left(\frac{\sqrt{d \cos(bx + a)}}{\sqrt{d}}\right)}{4b d^{7/2}} - \frac{9 \operatorname{arctanh}\left(\frac{\sqrt{d \cos(bx + a)}}{\sqrt{d}}\right)}{4b d^{7/2}} + \frac{9}{10bd (d \cos(bx + a))^{5/2}} - \frac{\csc^2(bx + a)}{2bd (d \cos(bx + a))^{5/2}} + \frac{9}{2b d^3 \sqrt{d \cos(bx + a)}}$$

command

```
integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{90 \arctan\left(\frac{\sqrt{-d} \tan\left(\frac{1}{2}bx + \frac{1}{2}a\right)^2 - \sqrt{-d \tan\left(\frac{1}{2}bx + \frac{1}{2}a\right)^4 + d}}{\sqrt{-d}}\right)}{\sqrt{-d}} + \frac{45 \log\left(\frac{-\sqrt{-d} \tan\left(\frac{1}{2}bx + \frac{1}{2}a\right)^2 + \sqrt{-d \tan\left(\frac{1}{2}bx + \frac{1}{2}a\right)^4 + d}}{\sqrt{-d}}\right)}{\sqrt{-d}}$$

## 19 Test file number 70

Test folder name:

test\_cases/4\_Trig\_functions/4.1\_Sine/70\_4.1.1.2-g\_cos-^p-a+b\_sin-^m

### 19.1 Problem number 164

$$\int \frac{\sec^2(c + dx)}{\sqrt{a + a \sin(c + dx)}} dx$$

Optimal antiderivative

$$-\frac{3a \cos(dx + c)}{4d(a + a \sin(dx + c))^{\frac{3}{2}}} - \frac{3 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a} \sqrt{2}}{2\sqrt{a + a \sin(dx + c)}}\right) \sqrt{2}}{8d\sqrt{a}} + \frac{\sec(dx + c)}{d\sqrt{a + a \sin(dx + c)}}$$

command

```
integrate(sec(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$3\sqrt{2} \operatorname{arctan}\left(\frac{\sqrt{2}\left(\sqrt{a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{a \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a + \sqrt{a}}\right)}{2\sqrt{-a}}\right) + \frac{\left(\sqrt{a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{a \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a + \sqrt{a}}\right)}{\sqrt{-a} \operatorname{sgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)}$$

### 19.2 Problem number 517

$$\int \frac{\cos^3(c + dx)}{(a + b \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2(a + b \sin(dx + c))^{\frac{3}{2}}}{3b^3d} + \frac{2a^2 - 2b^2}{b^3d\sqrt{a + b \sin(dx + c)}} + \frac{4a\sqrt{a + b \sin(dx + c)}}{b^3d}$$

command

```
integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2 \left( \frac{3(a^2 - b^2)}{\sqrt{b \sin(dx + c) + a} b^3} - \frac{(b \sin(dx + c) + a)^{\frac{3}{2}} b^6 - 6 \sqrt{b \sin(dx + c) + a} ab^6}{b^9} \right)}{3d}$$

### 19.3 Problem number 518

$$\int \frac{\cos(c + dx)}{(a + b \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2}{bd \sqrt{a + b \sin(dx + c)}}$$

command

`integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{2}{\sqrt{b \sin(dx + c) + a} bd}$$

### 19.4 Problem number 528

$$\int \frac{\cos^3(c + dx)}{(a + b \sin(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2a^2}{3} - \frac{2b^2}{3}}{b^3 d (a + b \sin(dx + c))^{\frac{3}{2}}} - \frac{4a}{b^3 d \sqrt{a + b \sin(dx + c)}} - \frac{2 \sqrt{a + b \sin(dx + c)}}{b^3 d}$$

command

```
integrate(cos(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{2 \left( \frac{3 \sqrt{b \sin(dx+c)+a}}{b^3} + \frac{6(b \sin(dx+c)+a)a-a^2+b^2}{(b \sin(dx+c)+a)^{\frac{3}{2}} b^3} \right)}{3d}$$

## 19.5 Problem number 529

$$\int \frac{\cos(c+dx)}{(a+b \sin(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{2}{3bd(a+b \sin(dx+c))^{\frac{3}{2}}}$$

command

```
integrate(cos(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{2}{3(b \sin(dx+c)+a)^{\frac{3}{2}} bd}$$

## 20 Test file number 74

Test folder name:

test\_cases/4\_Trig\_functions/4.1\_Sine/74\_4.1.2.2-g\_cos-^p-a+b\_sin-^m-c+d\_sin-^n

## 20.1 Problem number 468

$$\int \frac{\cos(c+dx) \cot^3(c+dx)}{\sqrt{a+a \sin(c+dx)}} dx$$

Optimal antiderivative

$$\frac{9 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a \sin(dx+c)}}\right)}{4d\sqrt{a}} - \frac{2 \cos(dx+c)}{d\sqrt{a+a \sin(dx+c)}} + \frac{\cot(dx+c)}{4d\sqrt{a+a \sin(dx+c)}} - \frac{\cot(dx+c) \csc(dx+c)}{2d\sqrt{a+a \sin(dx+c)}}$$

command

```
integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\left(36 \sqrt{2} \sqrt{a} \arctan\left(\frac{\sqrt{2} \sqrt{a} + \sqrt{a}}{\sqrt{-a}}\right) - 18 \sqrt{2} \sqrt{-a} \log\left(\sqrt{2} \sqrt{a} + \sqrt{a}\right) + 54 \sqrt{a} \arctan\left(\frac{\sqrt{2} \sqrt{a} + \sqrt{a}}{\sqrt{-a}}\right) - 27 \sqrt{-a} \log\left(\sqrt{2} \sqrt{a} + \sqrt{a}\right)\right)}{2 \sqrt{2} \sqrt{-a} \sqrt{a} + 3 \sqrt{-a} \sqrt{a}}$$

## 20.2 Problem number 469

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+a \sin(c+dx)}} dx$$

Optimal antiderivative

$$-\frac{7 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a \sin(dx+c)}}\right)}{8d\sqrt{a}} + \frac{9 \cot(dx+c)}{8d\sqrt{a+a \sin(dx+c)}} + \frac{\cot(dx+c) \csc(dx+c)}{12d\sqrt{a+a \sin(dx+c)}} - \frac{\cot(dx+c) (\csc^2(dx+c))}{3d\sqrt{a+a \sin(dx+c)}}$$

command

```
integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\sqrt{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a} \left( \left( \frac{2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\operatorname{asgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)} - \frac{3}{\operatorname{asgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)} \right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \frac{22}{\operatorname{asgn}\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)} \right)$$


---

### 20.3 Problem number 470

$$\int \frac{\cot^4(c + dx) \csc(c + dx)}{\sqrt{a + a \sin(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{11 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{64d\sqrt{a}} - \frac{11 \cot(dx+c)}{64d\sqrt{a+a\sin(dx+c)}} \\ & + \frac{53 \cot(dx+c) \csc(dx+c)}{96d\sqrt{a+a\sin(dx+c)}} + \frac{\cot(dx+c) (\csc^2(dx+c))}{24d\sqrt{a+a\sin(dx+c)}} - \frac{\cot(dx+c) (\csc^3(dx+c))}{4d\sqrt{a+a\sin(dx+c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

## 20.4 Problem number 471

$$\int \frac{\cot^4(c + dx) \csc^2(c + dx)}{\sqrt{a + a \sin(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{9 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{128d\sqrt{a}} - \frac{9 \cot(dx+c)}{128d\sqrt{a+a\sin(dx+c)}} - \frac{3 \cot(dx+c) \csc(dx+c)}{64d\sqrt{a+a\sin(dx+c)}} \\ & + \frac{29 \cot(dx+c) (\csc^2(dx+c))}{80d\sqrt{a+a\sin(dx+c)}} + \frac{\cot(dx+c) (\csc^3(dx+c))}{40d\sqrt{a+a\sin(dx+c)}} - \frac{\cot(dx+c) (\csc^4(dx+c))}{5d\sqrt{a+a\sin(dx+c)}} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

## 21 Test file number 79

Test folder name:

```
test_cases/4_Trig_functions/4.1_Sine/79_4.1.7-d_trig-^m-a+b-c_sin-^n-^p
```

### 21.1 Problem number 499

$$\int (a + b \sin^2(e + fx))^{3/2} \tan^5(e + fx) dx$$



Optimal antiderivative

$$\begin{aligned}
& - \frac{(8a^2 + 40ab + 35b^2) (a + b(\sin^2(fx + e)))^{\frac{3}{2}}}{24(a + b)^2 f} \\
& - \frac{(8a + 9b) (\sec^2(fx + e)) (a + b(\sin^2(fx + e)))^{\frac{5}{2}}}{8(a + b)^2 f} \\
& + \frac{(\sec^4(fx + e)) (a + b(\sin^2(fx + e)))^{\frac{5}{2}}}{4(a + b) f} \\
& + \frac{(8a^2 + 40ab + 35b^2) \operatorname{arctanh}\left(\frac{\sqrt{a + b(\sin^2(fx + e))}}{\sqrt{a + b}}\right)}{8f\sqrt{a + b}} \\
& - \frac{(8a^2 + 40ab + 35b^2) \sqrt{a + b(\sin^2(fx + e))}}{8(a + b) f}
\end{aligned}$$

command

```
integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

**21.2 Problem number 558**

$$\int \frac{\cot(c + dx)}{\sqrt{a + b \sin^4(c + dx)}} dx$$

Optimal antiderivative

$$- \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\sin^4(dx + c))}}{\sqrt{a}}\right)}{2d\sqrt{a}}$$

command

```
integrate(cot(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\arctan\left(\frac{\sqrt{b \sin(dx+c)^4 + a}}{\sqrt{-a}}\right)}{2\sqrt{-a}d}$$

**22 Test file number 89**

Test folder name:

test\_cases/4\_Trig\_functions/4.2\_Cosine/89\_4.2.2.1-a+b\_cos-^m-c+d\_cos-^n

**22.1 Problem number 128**

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{d\sqrt{a}} + \frac{\operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\cos(dx+c)}}\right)\sqrt{2}}{d\sqrt{a}} + \frac{\tan(dx+c)}{d\sqrt{a+a\cos(dx+c)}}$$

command

integrate(sec(d\*x+c)^2/(a+a\*cos(d\*x+c))^(1/2),x, algorithm="giac")

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\sqrt{2} \left( \frac{\sqrt{2}\sqrt{a} \log \left( \frac{\left( \sqrt{a} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - \sqrt{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a} \right)^2 - 4\sqrt{2}|a|-6a}{\left( \sqrt{a} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - \sqrt{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a} \right)^2 + 4\sqrt{2}|a|-6a} \right)}{|a|} + 2 \log \left( \left( \sqrt{a} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - \sqrt{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a} \right) \right) \right)$$

## 22.2 Problem number 129

$$\int \frac{\sec^3(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx$$

Optimal antiderivative

$$\frac{7 \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right) - \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\cos(dx+c)}}\right) \sqrt{2}}{4d\sqrt{a}} - \frac{\tan(dx+c)}{4d\sqrt{a+a\cos(dx+c)}} + \frac{\sec(dx+c)\tan(dx+c)}{2d\sqrt{a+a\cos(dx+c)}}$$

command

```
integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\sqrt{2} \left( \frac{7 \sqrt{2} \sqrt{a} \log \left( \frac{\left( \sqrt{a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{a \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a} \right)^2 - 4 \sqrt{2} |a|^{-6 a}}{\left( \sqrt{a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{a \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a} \right)^2 + 4 \sqrt{2} |a|^{-6 a}} \right)}{|a|} + 8 \log \left( \left( \sqrt{a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{a \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a} \right) \right) \right)$$

## 22.3 Problem number 130

$$\int \frac{\sec^4(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx$$

Optimal antiderivative

$$-\frac{9 \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right) + \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\cos(dx+c)}}\right) \sqrt{2}}{8d\sqrt{a}} + \frac{7 \tan(dx+c)}{8d\sqrt{a+a\cos(dx+c)}} - \frac{\sec(dx+c)\tan(dx+c)}{12d\sqrt{a+a\cos(dx+c)}} + \frac{(\sec^2(dx+c))\tan(dx+c)}{3d\sqrt{a+a\cos(dx+c)}}$$

command

```
integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\sqrt{2} \left( \frac{27 \sqrt{2} \sqrt{a} \log \left( \frac{\left| 2 \left( \sqrt{a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a} \right)^2 - 4 \sqrt{2} |a|^{-6} a}{\left| 2 \left( \sqrt{a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a} \right)^2 + 4 \sqrt{2} |a|^{-6} a} \right|}{|a|} \right)}{48 \log \left( \left( \sqrt{a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a} \right)^2 \right)} \right) + \frac{48 \log \left( \left( \sqrt{a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a} \right)^2 \right)}{48 \log \left( \left( \sqrt{a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a} \right)^2 \right)}$$

## 22.4 Problem number 144

$$\int \frac{\sec(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh} \left( \frac{\sin(dx+c) \sqrt{a}}{\sqrt{a + a \cos(dx+c)}} \right)}{a^{\frac{5}{2}} d} - \frac{\sin(dx+c)}{4d (a + a \cos(dx+c))^{\frac{5}{2}}} - \frac{11 \sin(dx+c)}{16ad (a + a \cos(dx+c))^{\frac{3}{2}}} - \frac{43 \operatorname{arctanh} \left( \frac{\sin(dx+c) \sqrt{a} \sqrt{2}}{2 \sqrt{a + a \cos(dx+c)}} \right) \sqrt{2}}{32a^{\frac{5}{2}} d}$$

command

```
integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$2 \sqrt{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a} \left( \frac{2 \sqrt{2} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2}{a^3} + \frac{13 \sqrt{2}}{a^3} \right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \frac{43 \sqrt{2} \log\left(\left(\sqrt{a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{\frac{1}{a^2}}\right)\right)}{a^{\frac{5}{2}}}$$

## 23 Test file number 92

Test folder name:

test\_cases/4\_Trig\_functions/4.2\_Cosine/92\_4.2.3.1-a+b\_cos-^m-c+d\_cos-^n-A+B\_cos-

### 23.1 Problem number 107

$$\int \frac{\cos^4(c + dx)(A + B \cos(c + dx))}{(a + a \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - B) (\cos^4(dx + c)) \sin(dx + c)}{2d (a + a \cos(dx + c))^{\frac{3}{2}}} - \frac{(15A - 19B) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a} \sqrt{2}}{2\sqrt{a + a \cos(dx + c)}}\right) \sqrt{2}}{4a^{\frac{3}{2}}d} \\ & + \frac{(651A - 799B) \sin(dx + c)}{105ad \sqrt{a + a \cos(dx + c)}} + \frac{(63A - 67B) (\cos^2(dx + c)) \sin(dx + c)}{70ad \sqrt{a + a \cos(dx + c)}} \\ & - \frac{(7A - 11B) (\cos^3(dx + c)) \sin(dx + c)}{14ad \sqrt{a + a \cos(dx + c)}} - \frac{(273A - 397B) \sin(dx + c) \sqrt{a + a \cos(dx + c)}}{210a^2d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$105 \left( 15 \sqrt{2} A - 19 \sqrt{2} B \right) \log\left(\left| -\sqrt{a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + \sqrt{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a} \right|\right) + \frac{\left(\left(\left(\left(\frac{105 \left(\sqrt{2} A a^5 - \sqrt{2} B a^5\right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^3}\right)\right)\right)\right)}{a^{\frac{3}{2}}}$$

### 23.2 Problem number 108

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A-B)(\cos^3(dx+c))\sin(dx+c)}{2d(a+a\cos(dx+c))^{\frac{3}{2}}} + \frac{(11A-15B)\operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\cos(dx+c)}}\right)\sqrt{2}}{4a^{\frac{3}{2}}d} \\ & - \frac{(65A-93B)\sin(dx+c)}{15ad\sqrt{a+a\cos(dx+c)}} - \frac{(5A-9B)(\cos^2(dx+c))\sin(dx+c)}{10ad\sqrt{a+a\cos(dx+c)}} \\ & + \frac{(35A-39B)\sin(dx+c)\sqrt{a+a\cos(dx+c)}}{30a^2d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{15\sqrt{2}(11A-15B)\log\left(-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)}{a^{\frac{3}{2}}} + \left(\left(\left(\frac{15\sqrt{2}(Aa^3-Ba^3)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sqrt{2}\right)}{a^2}\right)\right)$$

60 d

### 23.3 Problem number 109

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A-B)(\cos^2(dx+c))\sin(dx+c)}{2d(a+a\cos(dx+c))^{\frac{3}{2}}} - \frac{(7A-11B)\operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\cos(dx+c)}}\right)\sqrt{2}}{4a^{\frac{3}{2}}d} \\ & + \frac{(9A-13B)\sin(dx+c)}{3ad\sqrt{a+a\cos(dx+c)}} - \frac{(3A-7B)\sin(dx+c)\sqrt{a+a\cos(dx+c)}}{6a^2d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{3 \left( 7 \sqrt{2} A - 11 \sqrt{2} B \right) \log \left( \left| -\sqrt{a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + \sqrt{a \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a} \right| \right)}{a^{\frac{3}{2}}} + \left( \frac{3 \left( \sqrt{2} A a - \sqrt{2} B a \right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 2 \left( 1 \right)}{a} \right)$$


---

12 d

### 23.4 Problem number 110

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(a + a \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{(A - B) \sin(dx + c)}{2d(a + a \cos(dx + c))^{\frac{3}{2}}} + \frac{(3A - 7B) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a + a \cos(dx + c)}}\right) \sqrt{2}}{4a^{\frac{3}{2}}d} + \frac{2B \sin(dx + c)}{ad\sqrt{a + a \cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\left( \frac{\sqrt{2} (Aa^2 - Ba^2) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + \sqrt{2} (Aa^2 - 9Ba^2)}{a^3} \right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + \sqrt{2} (3A - 7B) \log \left( \left| -\sqrt{a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + \sqrt{a \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a} \right| \right)}{\sqrt{a \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a}} + \frac{1}{a^{\frac{3}{2}}}$$


---

4 d

### 23.5 Problem number 113

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3A - 2B) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx+c)}}\right)}{a^{\frac{3}{2}}d} \\ & + \frac{(9A - 5B) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a} \sqrt{2}}{2\sqrt{a + a \cos(dx+c)}}\right) \sqrt{2}}{4a^{\frac{3}{2}}d} \\ & - \frac{(A - B) \tan(dx+c)}{2d(a + a \cos(dx+c))^{\frac{3}{2}}} + \frac{(3A - B) \tan(dx+c)}{2ad\sqrt{a + a \cos(dx+c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\sqrt{2} (9A\sqrt{a} - 5B\sqrt{a}) \log\left(\left(\sqrt{a} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - \sqrt{a \tan^2\left(\frac{1}{2}dx + \frac{1}{2}c\right) + a}\right)^2\right)}{a^2} + \frac{4(3A\sqrt{a} - 2B\sqrt{a}) \log\left(\left(\sqrt{a} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - \sqrt{a \tan^2\left(\frac{1}{2}dx + \frac{1}{2}c\right) + a}\right)^2\right)}{2ad\sqrt{a + a \cos(dx+c)}}$$

## 24 Test file number 94

Test folder name:

test\_cases/4\_Trig\_functions/4.2\_Cosine/94\_4.2.4.2-a+b\*cos-^m-c+d\*cos-^n-A+B\*cos+C\*cos^2-



### 24.1 Problem number 107

$$\int \frac{(A + C \cos^2(c + dx)) \sec^2(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{A \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{d\sqrt{a}} \\ & + \frac{(A+C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\cos(dx+c)}}\right)\sqrt{2}}{d\sqrt{a}} + \frac{A \tan(dx+c)}{d\sqrt{a+a\cos(dx+c)}} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\sqrt{2}(A+C) \log\left(\left(\sqrt{a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{a \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a}\right)^2\right)}{\sqrt{a}} + \frac{A \log\left(\left(\sqrt{a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \sqrt{a \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a}\right)\right)}{\sqrt{a}}$$

### 24.2 Problem number 109

$$\int \frac{(A + C \cos^2(c + dx)) \sec^4(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(9A + 8C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{8d\sqrt{a}} \\ & + \frac{(A+C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\cos(dx+c)}}\right)\sqrt{2}}{d\sqrt{a}} + \frac{(7A+8C) \tan(dx+c)}{8d\sqrt{a+a\cos(dx+c)}} \\ & - \frac{A \sec(dx+c) \tan(dx+c)}{12d\sqrt{a+a\cos(dx+c)}} + \frac{A(\sec^2(dx+c)) \tan(dx+c)}{3d\sqrt{a+a\cos(dx+c)}} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 24.3 Problem number 111

$$\int \frac{\cos^3(c+dx) (A+C \cos^2(c+dx))}{(a+a \cos(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(A+C) (\cos^4(dx+c)) \sin(dx+c)}{2d (a+a \cos(dx+c))^{\frac{3}{2}}} + \frac{(11A+19C) \operatorname{arctanh}\left(\frac{\sin(dx+c) \sqrt{a} \sqrt{2}}{2 \sqrt{a+a \cos(dx+c)}}\right) \sqrt{2}}{4a^{\frac{3}{2}}d} \\ & -\frac{(455A+799C) \sin(dx+c)}{105ad \sqrt{a+a \cos(dx+c)}} - \frac{(35A+67C) (\cos^2(dx+c)) \sin(dx+c)}{70ad \sqrt{a+a \cos(dx+c)}} \\ & + \frac{(7A+11C) (\cos^3(dx+c)) \sin(dx+c)}{14ad \sqrt{a+a \cos(dx+c)}} + \frac{(245A+397C) \sin(dx+c) \sqrt{a+a \cos(dx+c)}}{210a^2d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{105 \left( 11 \sqrt{2} A + 19 \sqrt{2} C \right) \log \left( \left| -\sqrt{a} \tan \left( \frac{1}{2} dx + \frac{1}{2} c \right) + \sqrt{a \tan \left( \frac{1}{2} dx + \frac{1}{2} c \right)^2 + a} \right| \right)}{a^{\frac{3}{2}}} + \left( \left( \left( \left( \frac{105 \left( \sqrt{2} A a^5 + \sqrt{2} C a^5 \right) \tan \left( \frac{1}{2} dx + \frac{1}{2} c \right)}{a^3} \right) \right) \right) \right)$$

#### 24.4 Problem number 112

$$\int \frac{\cos^2(c+dx) (A+C \cos^2(c+dx))}{(a+a \cos(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A+C) (\cos^3(dx+c)) \sin(dx+c)}{2d (a+a \cos(dx+c))^{\frac{3}{2}}} - \frac{(7A+15C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a} \sqrt{2}}{2\sqrt{a+a \cos(dx+c)}}\right) \sqrt{2}}{4a^{\frac{3}{2}}d} \\ & + \frac{(15A+31C) \sin(dx+c)}{5ad\sqrt{a+a \cos(dx+c)}} + \frac{(5A+9C) (\cos^2(dx+c)) \sin(dx+c)}{10ad\sqrt{a+a \cos(dx+c)}} \\ & - \frac{(5A+13C) \sin(dx+c) \sqrt{a+a \cos(dx+c)}}{10a^2d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{5\sqrt{2}(7A+15C) \log\left(\frac{-\sqrt{a} \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + \sqrt{a \tan^2\left(\frac{1}{2}dx+\frac{1}{2}c\right) + a}}{a^{\frac{3}{2}}}\right) + \left(\left(\left(\frac{5\sqrt{2}(Aa^3+Ca^3)}{a^2} \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^2 + \frac{\sqrt{2}(55Aa^3)}{a^2}\right)\right)}{20d}$$

#### 24.5 Problem number 113

$$\int \frac{\cos(c+dx) (A+C \cos^2(c+dx))}{(a+a \cos(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A+C) (\cos^2(dx+c)) \sin(dx+c)}{2d (a+a \cos(dx+c))^{\frac{3}{2}}} + \frac{(3A+11C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a} \sqrt{2}}{2\sqrt{a+a \cos(dx+c)}}\right) \sqrt{2}}{4a^{\frac{3}{2}}d} \\ & - \frac{(3A+13C) \sin(dx+c)}{3ad\sqrt{a+a \cos(dx+c)}} + \frac{(3A+7C) \sin(dx+c) \sqrt{a+a \cos(dx+c)}}{6a^2d} \end{aligned}$$

command

```
integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{3 \left( 3 \sqrt{2} A + 11 \sqrt{2} C \right) \log \left( \left| -\sqrt{a} \tan \left( \frac{1}{2} dx + \frac{1}{2} c \right) + \sqrt{a \tan \left( \frac{1}{2} dx + \frac{1}{2} c \right)^2 + a} \right| \right)}{a^{\frac{3}{2}}} + \left( \frac{3 \left( \sqrt{2} A a + \sqrt{2} C a \right) \tan \left( \frac{1}{2} dx + \frac{1}{2} c \right)^2}{a} + \dots \right)$$


---

12 d

## 24.6 Problem number 281

$$\int \frac{B \cos(c + dx) + C \cos^2(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(B - C) \sin(dx + c)}{2d(a + a \cos(dx + c))^{\frac{3}{2}}} + \frac{(3B - 7C) \operatorname{arctanh} \left( \frac{\sin(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\cos(dx+c)}} \right) \sqrt{2}}{4a^{\frac{3}{2}}d} \\ & + \frac{2C \sin(dx + c)}{ad\sqrt{a + a \cos(dx + c)}} \end{aligned}$$

command

```
integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\left( \frac{\sqrt{2} (Ba^2 - Ca^2) \tan \left( \frac{1}{2} dx + \frac{1}{2} c \right)^2}{a^3} + \frac{\sqrt{2} (Ba^2 - 9Ca^2)}{a^3} \right) \tan \left( \frac{1}{2} dx + \frac{1}{2} c \right)}{\sqrt{a \tan \left( \frac{1}{2} dx + \frac{1}{2} c \right)^2 + a}} + \frac{\sqrt{2} (3B - 7C) \log \left( \left| -\sqrt{a} \tan \left( \frac{1}{2} dx + \frac{1}{2} c \right) + \sqrt{a \tan \left( \frac{1}{2} dx + \frac{1}{2} c \right)^2 + a} \right| \right)}{a^{\frac{3}{2}}}$$


---

4 d

## 24.7 Problem number 410

$$\int \frac{\cos^3(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(a + a \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B + C) (\cos^4(dx + c)) \sin(dx + c)}{2d(a + a \cos(dx + c))^{\frac{3}{2}}} \\ & + \frac{(11A - 15B + 19C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\cos(dx+c)}}\right) \sqrt{2}}{4a^{\frac{3}{2}}d} \\ & - \frac{(455A - 651B + 799C) \sin(dx + c)}{105ad\sqrt{a + a \cos(dx + c)}} - \frac{(35A - 63B + 67C) (\cos^2(dx + c)) \sin(dx + c)}{70ad\sqrt{a + a \cos(dx + c)}} \\ & + \frac{(7A - 7B + 11C) (\cos^3(dx + c)) \sin(dx + c)}{14ad\sqrt{a + a \cos(dx + c)}} \\ & + \frac{(245A - 273B + 397C) \sin(dx + c) \sqrt{a + a \cos(dx + c)}}{210a^2d} \end{aligned}$$

command

`integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="g`

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{105 \left( 11 \sqrt{2} A - 15 \sqrt{2} B + 19 \sqrt{2} C \right) \log \left( \left| -\sqrt{a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + \sqrt{a \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a} \right| \right)}{a^{\frac{3}{2}}} + \frac{\left( \left( \left( \left( \frac{105 \left( \sqrt{2} A a^5 - \sqrt{2} B \right)}{\right)} \right) \right) \right)}{\dots}$$

## 24.8 Problem number 411

$$\int \frac{\cos^2(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(a + a \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B + C) (\cos^3(dx + c)) \sin(dx + c)}{2d (a + a \cos(dx + c))^{\frac{3}{2}}} \\ & - \frac{(7A - 11B + 15C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a} \sqrt{2}}{2\sqrt{a + a \cos(dx + c)}}\right) \sqrt{2}}{4a^{\frac{3}{2}}d} \\ & + \frac{(45A - 65B + 93C) \sin(dx + c)}{15ad \sqrt{a + a \cos(dx + c)}} + \frac{(5A - 5B + 9C) (\cos^2(dx + c)) \sin(dx + c)}{10ad \sqrt{a + a \cos(dx + c)}} \\ & - \frac{(15A - 35B + 39C) \sin(dx + c) \sqrt{a + a \cos(dx + c)}}{30a^2d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="g
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{15 \sqrt{2} (7A - 11B + 15C) \log\left(\left| -\sqrt{a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + \sqrt{a \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a} \right|\right)}{a^{\frac{3}{2}}} + \left(\left(\left(\frac{15 \sqrt{2} (Aa^3 - Ba^3 + Ca^3) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2}{a^2}\right)\right)\right)$$

## 24.9 Problem number 412

$$\int \frac{\cos(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(a + a \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B + C) (\cos^2(dx + c)) \sin(dx + c)}{2d (a + a \cos(dx + c))^{\frac{3}{2}}} \\ & + \frac{(3A - 7B + 11C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a} \sqrt{2}}{2\sqrt{a + a \cos(dx + c)}}\right) \sqrt{2}}{4a^{\frac{3}{2}}d} \\ & - \frac{(3A - 9B + 13C) \sin(dx + c)}{3ad \sqrt{a + a \cos(dx + c)}} + \frac{(3A - 3B + 7C) \sin(dx + c) \sqrt{a + a \cos(dx + c)}}{6a^2d} \end{aligned}$$

command

`integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")`  
 Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{3 \left( 3 \sqrt{2} A - 7 \sqrt{2} B + 11 \sqrt{2} C \right) \log \left( \left| -\sqrt{a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + \sqrt{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a} \right| \right)}{a^{\frac{3}{2}}} + \left( \frac{3 \left( \sqrt{2} A a - \sqrt{2} B a + \sqrt{2} C a \right)}{a} \right)$$

12 d

## 24.10 Problem number 413

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{(A - B + C) \sin(dx + c)}{2d (a + a \cos(dx + c))^{\frac{3}{2}}} + \frac{(A + 3B - 7C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a} \sqrt{2}}{2\sqrt{a + a \cos(dx + c)}}\right) \sqrt{2}}{4a^{\frac{3}{2}}d} + \frac{2C \sin(dx + c)}{ad \sqrt{a + a \cos(dx + c)}}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")`  
 Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\left( \frac{\sqrt{2} (Aa^2 - Ba^2 + Ca^2) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2}{a^3} + \frac{\sqrt{2} (Aa^2 - Ba^2 + 9Ca^2)}{a^3} \right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \sqrt{2} (A + 3B - 7C) \log \left( \left| -\sqrt{a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + \sqrt{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a} \right| \right)}{\sqrt{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a} a^{\frac{3}{2}}}$$

4 d

## 25 Test file number 95

Test folder name:

test\_cases/4\_Trig\_functions/4.2\_Cosine/95\_4.2.7-d\_trig-<sup>m</sup>-a+b-c\_cos-<sup>n</sup>-<sup>p</sup>

### 25.1 Problem number 68

$$\int \frac{\cos(5 + 3x)}{\sqrt{3 + \cos^2(5 + 3x)}} dx$$

Optimal antiderivative

$$\frac{\arcsin\left(\frac{\sin(5+3x)}{2}\right)}{3}$$

command

```
integrate(cos(5+3*x)/(3+cos(5+3*x)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{3} \arcsin\left(\frac{1}{2} \sin(3x + 5)\right)$$

## 26 Test file number 98

Test folder name:

test\_cases/4\_Trig\_functions/4.3\_Tangent/98\_4.3.0-a\_trg-<sup>m</sup>-b\_tan-<sup>n</sup>

### 26.1 Problem number 52

$$\int (b \tan^p(c + dx))^{\frac{1}{p}} dx$$

Optimal antiderivative

$$-\frac{\cot(dx + c) \ln(\cos(dx + c)) (b(\tan^p(dx + c)))^{\frac{1}{p}}}{d}$$

command



`integrate((b*tan(d*x+c)^p)^(1/p),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{4\pi|b|^{\left(\frac{1}{p}\right)} \left[ \frac{dx+c}{\pi} + \frac{1}{2} \right] \tan\left(\frac{\pi \operatorname{sgn}(b)}{4p} - \frac{\pi}{4p}\right) + |b|^{\left(\frac{1}{p}\right)} \log\left(\frac{4}{\tan(dx+c)^2+1}\right) \tan\left(\frac{\pi \operatorname{sgn}(b)}{4p} - \frac{\pi}{4p}\right)^2 - 4c|b|^{\left(\frac{1}{p}\right)} \tan\left(\frac{\pi \operatorname{sgn}(b)}{4p} - \frac{\pi}{4p}\right)}{2\left(d \tan\left(\frac{\pi \operatorname{sgn}(b)}{4p} - \frac{\pi}{4p}\right)^2 + d\right)}$$

## 27 Test file number 103

Test folder name:

`test_cases/4_Trig_functions/4.3_Tangent/103_4.3.2.1-a+b_tan~m-c+d_tan~n`

### 27.1 Problem number 1137

$$\int (a + ia \tan(e + fx))^{5/2} \sqrt{c + d \tan(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-1)^{\frac{1}{4}} a^{\frac{5}{2}} (c^2 + 10Icd + 23d^2) \operatorname{arctanh}\left(\frac{(-1)^{\frac{3}{4}} \sqrt{d} \sqrt{a + Ia \tan(fx + e)}}{\sqrt{a} \sqrt{c + d \tan(fx + e)}}\right)}{4d^{\frac{3}{2}} f} \\ & - \frac{4Ia^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{a} \sqrt{c + d \tan(fx + e)}}{\sqrt{c - Id} \sqrt{a + Ia \tan(fx + e)}}\right) \sqrt{2} \sqrt{c - Id}}{f} \\ & + \frac{a^2(c + 9Id) \sqrt{a + Ia \tan(fx + e)} \sqrt{c + d \tan(fx + e)}}{4df} \\ & - \frac{a^2 \sqrt{a + Ia \tan(fx + e)} (c + d \tan(fx + e))^{\frac{3}{2}}}{2df} \end{aligned}$$

command

`integrate((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\left(2(d \tan(fx + e) + c)^2 a^2 - 2(d \tan(fx + e) + c) a^2 c - 2i(d \tan(fx + e) + c) a^2 d\right) \sqrt{2ad^2 + 2} \sqrt{(d \tan(fx + e) + c)}$$

## 27.2 Problem number 1138

$$\int (a + ia \tan(e + fx))^{3/2} \sqrt{c + d \tan(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ia^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c+d\tan(fx+e)}}{\sqrt{c-Id}\sqrt{a+Ia\tan(fx+e)}}\right)\sqrt{2}\sqrt{c-Id}}{f} \\ & - \frac{(-1)^{\frac{1}{4}}a^{\frac{3}{2}}(Ic+3d)\operatorname{arctanh}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{d}\sqrt{a+Ia\tan(fx+e)}}{\sqrt{a}\sqrt{c+d\tan(fx+e)}}\right)}{f\sqrt{d}} \\ & + \frac{a^2(c+Id)\sqrt{c+d\tan(fx+e)}}{df\sqrt{a+Ia\tan(fx+e)}} - \frac{a^2(c+d\tan(fx+e))^{\frac{3}{2}}}{df\sqrt{a+Ia\tan(fx+e)}} \end{aligned}$$

command

```
integrate((c+d*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\sqrt{2ad^2 + 2\sqrt{(d\tan(fx+e)+c)^2 - 2(d\tan(fx+e)+c)c + c^2 + d^2}} ad(d\tan(fx+e)+c)a \left( \frac{ad^2 + \sqrt{(d\tan(fx+e)+c)^2 - 2(d\tan(fx+e)+c)c + c^2 + d^2}}{2((-id\tan(fx+e) - ic)d + \dots)} \right)}{2((-id\tan(fx+e) - ic)d + \dots)}$$

## 27.3 Problem number 1143

$$\int (a + ia \tan(e + fx))^{5/2} (c + d \tan(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{(-1)^{\frac{1}{4}} a^{\frac{5}{2}} (c - 3Id) (c^2 + 18Icd + 15d^2) \operatorname{arctanh}\left(\frac{(-1)^{\frac{3}{4}} \sqrt{d} \sqrt{a + Ia \tan(fx + e)}}{\sqrt{a} \sqrt{c + d \tan(fx + e)}}\right)}{8d^{\frac{3}{2}} f} \\
 & - \frac{4Ia^{\frac{5}{2}} (c - Id)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{a} \sqrt{c + d \tan(fx + e)}}{\sqrt{c - Id} \sqrt{a + Ia \tan(fx + e)}}\right) \sqrt{2}}{f} \\
 & + \frac{a^2 (c^2 + 14Icd + 19d^2) \sqrt{a + Ia \tan(fx + e)} \sqrt{c + d \tan(fx + e)}}{8df} \\
 & + \frac{a^2 (c + 13Id) \sqrt{a + Ia \tan(fx + e)} (c + d \tan(fx + e))^{\frac{3}{2}}}{12df} \\
 & - \frac{a^2 \sqrt{a + Ia \tan(fx + e)} (c + d \tan(fx + e))^{\frac{5}{2}}}{3df}
 \end{aligned}$$

command

```
integrate((a+I*a*tan(f*x+e))^(5/2)*(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\left(2(d \tan(fx + e) + c)^3 a^2 - 2(d \tan(fx + e) + c)^2 a^2 c - 2i(d \tan(fx + e) + c)^2 a^2 d\right) \sqrt{2ad^2 + 2} \sqrt{(d \tan(fx + e) + c)^2 a^2}$$

## 27.4 Problem number 1144

$$\int (a + ia \tan(e + fx))^{3/2} (c + d \tan(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{2Ia^{\frac{3}{2}} (c - Id)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{a} \sqrt{c + d \tan(fx + e)}}{\sqrt{c - Id} \sqrt{a + Ia \tan(fx + e)}}\right) \sqrt{2}}{f} \\
 & - \frac{(-1)^{\frac{1}{4}} a^{\frac{3}{2}} (3Ic^2 + 18cd - 11Id^2) \operatorname{arctanh}\left(\frac{(-1)^{\frac{3}{4}} \sqrt{d} \sqrt{a + Ia \tan(fx + e)}}{\sqrt{a} \sqrt{c + d \tan(fx + e)}}\right)}{4f \sqrt{d}} \\
 & + \frac{a(3Ic + 5d) \sqrt{a + Ia \tan(fx + e)} \sqrt{c + d \tan(fx + e)}}{4f} \\
 & + \frac{a^2 (c + Id) (c + d \tan(fx + e))^{\frac{3}{2}}}{2df \sqrt{a + Ia \tan(fx + e)}} - \frac{a^2 (c + d \tan(fx + e))^{\frac{5}{2}}}{2df \sqrt{a + Ia \tan(fx + e)}}
 \end{aligned}$$

command

```
integrate((a+I*a*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\sqrt{2ad^2 + 2\sqrt{(d\tan(fx+e)+c)^2 - 2(d\tan(fx+e)+c)c + c^2 + d^2}} ad (d\tan(fx+e)+c)^2 a \left( \frac{\sqrt{ad^2 + \sqrt{(d\tan(fx+e)+c)^2 - 2(d\tan(fx+e)+c)c + c^2 + d^2}}}{2((-id\tan(fx+e) - ic)d + \dots)} \right)}{2((-id\tan(fx+e) - ic)d + \dots)}$$

## 27.5 Problem number 1149

$$\int (a + ia \tan(e + fx))^{5/2} (c + d \tan(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-1)^{\frac{1}{4}} a^{\frac{5}{2}} (5c^4 + 100Ic^3d + 690c^2d^2 - 900Icd^3 - 363d^4) \operatorname{arctanh}\left(\frac{(-1)^{\frac{3}{4}} \sqrt{d} \sqrt{a + Ia \tan(fx + e)}}{\sqrt{a} \sqrt{c + d \tan(fx + e)}}\right)}{64d^{\frac{3}{2}} f} \\ & - \frac{4Ia^{\frac{5}{2}} (c - Id)^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{a} \sqrt{c + d \tan(fx + e)}}{\sqrt{c - Id} \sqrt{a + Ia \tan(fx + e)}}\right) \sqrt{2}}{f} \\ & + \frac{a^2 (5c^3 + 95Ic^2d + 273cd^2 - 149Id^3) \sqrt{a + Ia \tan(fx + e)} \sqrt{c + d \tan(fx + e)}}{64df} \\ & + \frac{a^2 (5c^2 + 90Icd + 107d^2) \sqrt{a + Ia \tan(fx + e)} (c + d \tan(fx + e))^{\frac{3}{2}}}{96df} \\ & + \frac{a^2 (c + 17Id) \sqrt{a + Ia \tan(fx + e)} (c + d \tan(fx + e))^{\frac{5}{2}}}{24df} \\ & - \frac{a^2 \sqrt{a + Ia \tan(fx + e)} (c + d \tan(fx + e))^{\frac{7}{2}}}{4df} \end{aligned}$$

command

```
integrate((a+I*a*tan(f*x+e))^(5/2)*(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{(2(d\tan(fx+e)+c)^4 a^2 - 2(d\tan(fx+e)+c)^3 a^2 c - 2i(d\tan(fx+e)+c)^3 a^2 d) \sqrt{2ad^2 + 2\sqrt{(d\tan(fx+e)+c)^2 - 2(d\tan(fx+e)+c)c + c^2 + d^2}}}{\dots}$$

## 27.6 Problem number 1150

$$\int (a + ia \tan(e + fx))^{3/2} (c + d \tan(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Ia^{\frac{3}{2}}(c - Id)^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{c + d \tan(fx + e)}}{\sqrt{c - Id}\sqrt{a + Ia \tan(fx + e)}}\right) \sqrt{2}}{f} \\ & - \frac{(-1)^{\frac{1}{4}} a^{\frac{3}{2}} (5Ic^3 + 45c^2d - 55Icd^2 - 23d^3) \operatorname{arctanh}\left(\frac{(-1)^{\frac{3}{4}} \sqrt{d} \sqrt{a + Ia \tan(fx + e)}}{\sqrt{a} \sqrt{c + d \tan(fx + e)}}\right)}{8f\sqrt{d}} \\ & + \frac{a(c - 3Id)(5Ic + 3d) \sqrt{a + Ia \tan(fx + e)} \sqrt{c + d \tan(fx + e)}}{8f} \\ & + \frac{a(5Ic + 7d) \sqrt{a + Ia \tan(fx + e)} (c + d \tan(fx + e))^{\frac{3}{2}}}{12f} \\ & + \frac{a^2(c + Id)(c + d \tan(fx + e))^{\frac{5}{2}}}{3df \sqrt{a + Ia \tan(fx + e)}} - \frac{a^2(c + d \tan(fx + e))^{\frac{7}{2}}}{3df \sqrt{a + Ia \tan(fx + e)}} \end{aligned}$$

command

`integrate((a+I*a*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\sqrt{2ad^2 + 2\sqrt{(d \tan(fx + e) + c)^2 - 2(d \tan(fx + e) + c)c + c^2 + d^2}} ad (d \tan(fx + e) + c)^3 a \left( \frac{\sqrt{ad^2 + \sqrt{(d \tan(fx + e) + c)^2 - 2(d \tan(fx + e) + c)c + c^2 + d^2}}}{2((-i d \tan(fx + e) - ic)d + \dots)} \right)}{2((-i d \tan(fx + e) - ic)d + \dots)}$$

## 27.7 Problem number 1155

$$\int \frac{(a + ia \tan(e + fx))^{5/2}}{\sqrt{c + d \tan(e + fx)}} dx$$

Optimal antiderivative

$$\frac{(-1)^{\frac{1}{4}} a^{\frac{5}{2}} (c + 5 Id) \operatorname{arctanh}\left(\frac{(-1)^{\frac{3}{4}} \sqrt{d} \sqrt{a + Ia \tan(fx + e)}}{\sqrt{a} \sqrt{c + d \tan(fx + e)}}\right)}{d^{\frac{3}{2}} f} - \frac{4 Ia^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{a} \sqrt{c + d \tan(fx + e)}}{\sqrt{c - Id} \sqrt{a + Ia \tan(fx + e)}}\right) \sqrt{2}}{f \sqrt{c - Id}} - \frac{a^2 \sqrt{a + Ia \tan(fx + e)} \sqrt{c + d \tan(fx + e)}}{df}$$

command

```
integrate((a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{(2(d \tan(fx + e) + c)a^2 - 2a^2c - 2ia^2d) \sqrt{2ad^2 + 2} \sqrt{(d \tan(fx + e) + c)^2 - 2(d \tan(fx + e) + c)c + c^2 + d^2}}{4((d \tan(fx + e) + c))}$$

## 27.8 Problem number 1156

$$\int \frac{(a + ia \tan(e + fx))^{3/2}}{\sqrt{c + d \tan(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2 Ia^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{a} \sqrt{c + d \tan(fx + e)}}{\sqrt{c - Id} \sqrt{a + Ia \tan(fx + e)}}\right) \sqrt{2}}{f \sqrt{c - Id}} - \frac{2(-1)^{\frac{3}{4}} a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{(-1)^{\frac{3}{4}} \sqrt{d} \sqrt{a + Ia \tan(fx + e)}}{\sqrt{a} \sqrt{c + d \tan(fx + e)}}\right)}{f \sqrt{d}}$$

command

```
integrate((a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\sqrt{2ad^2 + 2\sqrt{(d \tan(fx + e) + c)^2 - 2(d \tan(fx + e) + c)c + c^2 + d^2}} ad a \left( \frac{i(d \tan(fx + e) + c)^2 a^2 d^2 - 2}{ad^2 + \sqrt{(d \tan(fx + e) + c)^2 a^2 d^2 - 2}} \right)}{2((-i d \tan(fx + e) - i c)d + i cd + d^2)}$$

## 27.9 Problem number 1161

$$\int \frac{(a + ia \tan(e + fx))^{5/2}}{(c + d \tan(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(-1)^{\frac{1}{4}} a^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{(-1)^{\frac{3}{4}} \sqrt{d} \sqrt{a + Ia \tan(fx + e)}}{\sqrt{a} \sqrt{c + d \tan(fx + e)}}\right)}{d^{\frac{3}{2}} f} - \frac{4Ia^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{a} \sqrt{c + d \tan(fx + e)}}{\sqrt{c - Id} \sqrt{a + Ia \tan(fx + e)}}\right) \sqrt{2}}{(c - Id)^{\frac{3}{2}} f} + \frac{2a^2(c + Id) \sqrt{a + Ia \tan(fx + e)}}{(c - Id) df \sqrt{c + d \tan(fx + e)}}$$

command

`integrate((a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{(2(d \tan(fx + e) + c)a^2 - 2a^2c - 2i a^2d) \sqrt{2ad^2 + 2\sqrt{(d \tan(fx + e) + c)^2 - 2(d \tan(fx + e) + c)c + c^2 + d^2}}}{4\left((d \tan(fx + e) + c)^2 d^2 - (d \tan(fx + e) + c)c + c^2 + d^2\right)}$$

## 27.10 Problem number 1162

$$\int \frac{(a + ia \tan(e + fx))^{3/2}}{(c + d \tan(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2Ia^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{a} \sqrt{c + d \tan(fx + e)}}{\sqrt{c - Id} \sqrt{a + Ia \tan(fx + e)}}\right) \sqrt{2}}{(c - Id)^{\frac{3}{2}} f} - \frac{2a \sqrt{a + Ia \tan(fx + e)}}{(Ic + d) f \sqrt{c + d \tan(fx + e)}}$$

command

```
integrate((a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\sqrt{2ad^2 + 2\sqrt{(d \tan(fx + e) + c)^2 - 2(d \tan(fx + e) + c)c + c^2 + d^2}} ad a \left( \frac{i(d \tan(fx + e) + c)}{ad^2 + \sqrt{(d \tan(fx + e) + c)^2 a^2 d^2 - 2}} \right)}{2 \left( -i(d \tan(fx + e) + c)^2 d + (i d \tan(fx + e) + ic)cd + (d \tan(fx + e) + c)^2 \right)}$$

## 27.11 Problem number 1167

$$\int \frac{(a + ia \tan(e + fx))^{5/2}}{(c + d \tan(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{4Ia^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{a} \sqrt{c + d \tan(fx + e)}}{\sqrt{c - Id} \sqrt{a + Ia \tan(fx + e)}}\right) \sqrt{2}}{(c - Id)^{\frac{5}{2}} f} + \frac{4Ia^2 \sqrt{a + Ia \tan(fx + e)}}{(c - Id)^2 f \sqrt{c + d \tan(fx + e)}} - \frac{2a(a + Ia \tan(fx + e))^{\frac{3}{2}}}{3(Ic + d) f (c + d \tan(fx + e))^{\frac{3}{2}}}$$

command

```
integrate((a+I*a*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output



Timed out

Giac 1.7.0 via sagemath 9.3 output

$$\frac{(2(d \tan(fx + e) + c)a^2 - 2a^2c - 2i a^2d) \sqrt{2ad^2 + 2 \sqrt{(d \tan(fx + e) + c)^2 - 2(d \tan(fx + e) + c)c + c^2 + d^2}}}{4 \left( (d \tan(fx + e) + c)^3 d^2 - (d \tan(fx + e) + c)c + c^2 + d^2 \right)}$$

**27.12 Problem number 1168**

$$\int \frac{(a + ia \tan(e + fx))^{3/2}}{(c + d \tan(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2Ia^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{a} \sqrt{c + d \tan(fx + e)}}{\sqrt{c - Id} \sqrt{a + Ia \tan(fx + e)}}\right) \sqrt{2}}{(c - Id)^{\frac{5}{2}} f} + \frac{2Ia \sqrt{a + Ia \tan(fx + e)}}{(c - Id)^2 f \sqrt{c + d \tan(fx + e)}} - \frac{2d(a + Ia \tan(fx + e))^{\frac{3}{2}}}{3(c^2 + d^2) f (c + d \tan(fx + e))^{\frac{3}{2}}}$$

command

```
integrate((a+I*a*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\sqrt{2ad^2 + 2 \sqrt{(d \tan(fx + e) + c)^2 - 2(d \tan(fx + e) + c)c + c^2 + d^2}} ad a \left( \frac{i(d \tan(fx + e) + c)^3 d^2 - (d \tan(fx + e) + c)c + c^2 + d^2}{ad^2 + \sqrt{(d \tan(fx + e) + c)^2 - 2(d \tan(fx + e) + c)c + c^2 + d^2}} \right)}{2 \left( -i(d \tan(fx + e) + c)^3 d + i(d \tan(fx + e) + c)^2 cd + (d \tan(fx + e) + c)c + c^2 + d^2 \right)}$$

## 28 Test file number 141

Test folder name:

test\_cases/4\_Trig\_functions/4.7\_Miscellaneous/141\_4.7.7\_Trig\_functions

### 28.1 Problem number 421

$$\int \frac{1}{(5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2}} dx$$

Optimal antiderivative

$$\frac{-3 \cos(ex + d) + 4 \sin(ex + d)}{10e (5 + 4 \cos(ex + d) + 3 \sin(ex + d))^{3/2}} + \frac{\operatorname{arctanh} \left( \frac{\sin(d+ex - \arctan(\frac{3}{4})) \sqrt{2}}{2 \sqrt{1 + \cos \left( d + ex - \arctan \left( \frac{3}{4} \right) \right)}} \right) \sqrt{10}}{100e}$$

command

```
integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{100} \left( \frac{\sqrt{10} \log \left( \frac{-2 \sqrt{10} + 2 \sqrt{\tan \left( \frac{1}{2} x e + \frac{1}{2} d \right)^2 + 1} - 2 \tan \left( \frac{1}{2} x e + \frac{1}{2} d \right) - 6}{2 \sqrt{10} + 2 \sqrt{\tan \left( \frac{1}{2} x e + \frac{1}{2} d \right)^2 + 1} - 2 \tan \left( \frac{1}{2} x e + \frac{1}{2} d \right) - 6} \right)}{\operatorname{sgn} \left( \tan \left( \frac{1}{2} x e + \frac{1}{2} d \right) + 3 \right)} - \frac{20 \left( 19 \left( \sqrt{\tan \left( \frac{1}{2} x e + \frac{1}{2} d \right)^2 + 1} \right) \right)}{\left( \left( \sqrt{\tan \left( \frac{1}{2} x e + \frac{1}{2} d \right)^2 + 1} \right) \right)}$$

## 28.2 Problem number 422

$$\int \frac{1}{(5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}} dx$$

Optimal antiderivative

$$\frac{-3 \cos(ex + d) + 4 \sin(ex + d)}{20e (5 + 4 \cos(ex + d) + 3 \sin(ex + d))^{5/2}} - \frac{3(3 \cos(ex + d) - 4 \sin(ex + d))}{400e (5 + 4 \cos(ex + d) + 3 \sin(ex + d))^{3/2}} + \frac{3 \operatorname{arctanh} \left( \frac{\sin(d+ex - \arctan(\frac{3}{4})) \sqrt{2}}{2 \sqrt{1 + \cos \left( d + ex - \arctan \left( \frac{3}{4} \right) \right)}} \right) \sqrt{10}}{4000e}$$

command

`integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{4000} \left( \frac{3 \sqrt{10} \log \left( \frac{\left| -2 \sqrt{10} + 2 \sqrt{\tan \left( \frac{1}{2} x e + \frac{1}{2} d \right)^2 + 1} - 2 \tan \left( \frac{1}{2} x e + \frac{1}{2} d \right) - 6 \right|}{\left| 2 \sqrt{10} + 2 \sqrt{\tan \left( \frac{1}{2} x e + \frac{1}{2} d \right)^2 + 1} - 2 \tan \left( \frac{1}{2} x e + \frac{1}{2} d \right) - 6 \right|} \right)}{\operatorname{sgn} \left( \tan \left( \frac{1}{2} x e + \frac{1}{2} d \right) + 3 \right)} - 20 \left( 797 \left( \sqrt{\tan \left( \frac{1}{2} x e + \frac{1}{2} d \right)^2} \right) \right) \right)$$

### 28.3 Problem number 428

$$\int \frac{1}{(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{3/2}} dx$$

Optimal antiderivative

$$\frac{3 \cos(ex + d) - 4 \sin(ex + d)}{10e(-5 + 4 \cos(ex + d) + 3 \sin(ex + d))^{3/2}} + \frac{\arctan\left(\frac{\sin(d+ex-\arctan(\frac{3}{4}))\sqrt{2}}{2\sqrt{-1 + \cos\left(d + ex - \arctan\left(\frac{3}{4}\right)\right)}}\right)\sqrt{10}}{100e}$$

command

```
integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{1}{450} \left( \frac{9\sqrt{10} \arctan\left(\frac{1}{10}\sqrt{10} \left(-3i\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)^2 + 1} + 3i \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) - i\right)}{\operatorname{sgn}\left(-3 \tan\left(\frac{1}{2}xe + \frac{1}{2}d\right) + 1\right)} \right) + \frac{10 \left(33i \left(\sqrt{\tan\left(\frac{1}{2}xe + \frac{1}{2}d\right)}\right)\right)}{\dots}$$

### 28.4 Problem number 429

$$\int \frac{1}{(-5 + 4 \cos(d + ex) + 3 \sin(d + ex))^{5/2}} dx$$

Optimal antiderivative

$$\frac{3 \cos(ex + d) - 4 \sin(ex + d)}{20e(-5 + 4 \cos(ex + d) + 3 \sin(ex + d))^{5/2}} - \frac{3(3 \cos(ex + d) - 4 \sin(ex + d))}{400e(-5 + 4 \cos(ex + d) + 3 \sin(ex + d))^{3/2}} - \frac{3 \arctan\left(\frac{\sin(d+ex-\arctan(\frac{3}{4}))\sqrt{2}}{2\sqrt{-1 + \cos\left(d + ex - \arctan\left(\frac{3}{4}\right)\right)}}\right)\sqrt{10}}{4000e}$$

command

```
integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{1}{162000} \left( \frac{243 \sqrt{10} \arctan \left( \frac{1}{10} \sqrt{10} \left( 3i \sqrt{\tan \left( \frac{1}{2} x e + \frac{1}{2} d \right)^2 + 1} - 3i \tan \left( \frac{1}{2} x e + \frac{1}{2} d \right) + i \right)}{\operatorname{sgn} \left( -3 \tan \left( \frac{1}{2} x e + \frac{1}{2} d \right) + 1 \right)} \right) \right) + \frac{10 \left( 15039i \right)}{162000}$$

## 28.5 Problem number 669

$$\int \frac{\cos(3x)}{\sqrt{4 - \sin^2(3x)}} dx$$

Optimal antiderivative

$$\frac{\arcsin \left( \frac{\sin(3x)}{2} \right)}{3}$$

command

```
integrate(cos(3*x)/(4-sin(3*x)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{3} \arcsin \left( \frac{1}{2} \sin(3x) \right)$$

## 28.6 Problem number 734

$$\int \frac{\sec(3x) \tan(3x)}{\sqrt{1 + 5 \cos^2(3x)}} dx$$

Optimal antiderivative

$$\frac{\sec(3x) \sqrt{1 + 5 (\cos^2(3x))}}{3}$$

command

```
integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2\sqrt{5}}{3 \left( \left( \sqrt{5} \cos(3x) - \sqrt{5 \cos^2(3x) + 1} \right)^2 - 1 \right)}$$

## 28.7 Problem number 740

$$\int \frac{\cot(5x) \csc^3(5x)}{\sqrt{1 + \sin^2(5x)}} dx$$

Optimal antiderivative

$$\frac{2 \csc(5x) \sqrt{1 + \sin^2(5x)}}{15} - \frac{(\csc^3(5x)) \sqrt{1 + \sin^2(5x)}}{15}$$

command

```
integrate(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{4 \left( 3 \left( \sqrt{\sin^2(5x) + 1} - \sin(5x) \right)^2 - 1 \right)}{15 \left( \left( \sqrt{\sin^2(5x) + 1} - \sin(5x) \right)^2 - 1 \right)^3}$$

## 29 Test file number 149

Test folder name:

test\_cases/5\_Inverse\_trig\_functions/5.3\_Inverse\_tangent/149\_5.3.3-d+e\_x^m-a+b\_arctan-c\_x^n-p

### 29.1 Problem number 24

$$\int \frac{a + b \operatorname{ArcTan}(cx^2)}{(d + ex)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{bc^2d^3 \arctan(cx^2)}{e(c^2d^4 + e^4)} + \frac{-a - b \arctan(cx^2)}{e(ex + d)} - \frac{2bcde \ln(ex + d)}{c^2d^4 + e^4} \\ & + \frac{bcde \ln(c^2x^4 + 1)}{2c^2d^4 + 2e^4} - \frac{b(c d^2 - e^2) \arctan\left(-1 + x\sqrt{2} \sqrt{c}\right) \sqrt{c} \sqrt{2}}{2(c^2d^4 + e^4)} \\ & - \frac{b(c d^2 - e^2) \arctan\left(1 + x\sqrt{2} \sqrt{c}\right) \sqrt{c} \sqrt{2}}{2(c^2d^4 + e^4)} \\ & - \frac{b(c d^2 + e^2) \ln\left(1 + cx^2 - x\sqrt{2} \sqrt{c}\right) \sqrt{c} \sqrt{2}}{4(c^2d^4 + e^4)} \\ & + \frac{b(c d^2 + e^2) \ln\left(1 + cx^2 + x\sqrt{2} \sqrt{c}\right) \sqrt{c} \sqrt{2}}{4c^2d^4 + 4e^4} \end{aligned}$$

command

```
integrate((a+b*arctan(c*x^2))/(e*x+d)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 30 Test file number 153

Test folder name:

test\_cases/5\_Inverse\_trig\_functions/5.3\_Inverse\_tangent/153\_5.3.7\_Inverse\_tangent\_functions

#### 30.1 Problem number 20

$$\int \sqrt{x} \operatorname{ArcTan}\left(\frac{\sqrt{-e} x}{\sqrt{d + ex^2}}\right) dx$$

Optimal antiderivative

$$\frac{2x^{\frac{3}{2}} \arctan\left(\frac{x\sqrt{-e}}{\sqrt{ex^2+d}}\right)}{3} + \frac{4\sqrt{x} \sqrt{ex^2+d}}{9\sqrt{-e}}$$

$$+ \frac{2d^{\frac{3}{4}} \sqrt{\frac{\cos\left(4 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{-e} (\sqrt{d} + x\sqrt{e}) \sqrt{\frac{e}{(\sqrt{d}})}}}{9 \cos\left(2 \arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right) e^{\frac{5}{4}} \sqrt{ex^2+d}}$$

command

```
integrate(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$+\infty$

#### 30.2 Problem number 26

$$\int x^{3/2} \operatorname{ArcTan}\left(\frac{\sqrt{-e} x}{\sqrt{d + ex^2}}\right) dx$$



Optimal antiderivative

$$\frac{2x^{\frac{5}{2}} \arctan\left(\frac{x\sqrt{-e}}{\sqrt{ex^2+d}}\right)}{5} + \frac{4x^{\frac{3}{2}}\sqrt{ex^2+d}}{25\sqrt{-e}} + \frac{12d\sqrt{-e}\sqrt{x}\sqrt{ex^2+d}}{25e^{\frac{3}{2}}(\sqrt{d}+x\sqrt{e})}$$


---


$$12d^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{-e}(\sqrt{d}+x\sqrt{e}) \sqrt{\frac{e}{(\sqrt{d}+x\sqrt{e})^2}}$$


---


$$25\cos\left(2\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right) e^{\frac{7}{4}}\sqrt{ex^2+d}$$


---


$$6d^{\frac{5}{4}}\sqrt{\frac{\cos\left(4\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) \sqrt{-e}(\sqrt{d}+x\sqrt{e}) \sqrt{\frac{e}{(\sqrt{d}+x\sqrt{e})^2}}$$


---


$$25\cos\left(2\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right) e^{\frac{7}{4}}\sqrt{ex^2+d}$$

command

```
integrate(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$+\infty$

## 31 Test file number 164

Test folder name:

test\_cases/6\_Hyperbolic\_functions/6.1\_Hyperbolic\_sine/164\_6.1.7\_hyper<sup>m</sup>-a+b\_sinh<sup>n</sup>-<sup>p</sup>

### 31.1 Problem number 437

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}} dx$$

Optimal antiderivative

$$-\frac{a^2}{5f(a(\cosh^2(fx+e)))^{\frac{5}{2}}} + \frac{2a}{3f(a(\cosh^2(fx+e)))^{\frac{3}{2}}} - \frac{1}{f\sqrt{a(\cosh^2(fx+e))}}$$

command

```
integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2(15\sqrt{a}e^{(9fx+9e)} + 20\sqrt{a}e^{(7fx+7e)} + 58\sqrt{a}e^{(5fx+5e)} + 20\sqrt{a}e^{(3fx+3e)} + 15\sqrt{a}e^{(fx+e)})}{15af(e^{(2fx+2e)} + 1)^5}$$

### 31.2 Problem number 438

$$\int \frac{\tanh^3(e + fx)}{\sqrt{a + a \sinh^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{a}{3f(a(\cosh^2(fx + e)))^{\frac{3}{2}}} - \frac{1}{f\sqrt{a(\cosh^2(fx + e))}}$$

command

```
integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2(3\sqrt{a}e^{(5fx+5e)} + 2\sqrt{a}e^{(3fx+3e)} + 3\sqrt{a}e^{(fx+e)})}{3af(e^{(2fx+2e)} + 1)^3}$$

### 31.3 Problem number 439

$$\int \frac{\tanh(e + fx)}{\sqrt{a + a \sinh^2(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{1}{f\sqrt{a(\cosh^2(fx + e))}}$$

command

```
integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{2e^{(fx+e)}}{\sqrt{a} f (e^{(2fx+2e)} + 1)}$$

### 31.4 Problem number 442

$$\int \frac{\tanh^4(e + fx)}{\sqrt{a + a \sinh^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{3 \arctan(\sinh(fx + e)) \cosh(fx + e)}{8f \sqrt{a (\cosh^2(fx + e))}} - \frac{3 \tanh(fx + e)}{8f \sqrt{a (\cosh^2(fx + e))}} - \frac{\tanh^3(fx + e)}{4f \sqrt{a (\cosh^2(fx + e))}}$$

command

```
integrate(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\frac{3 \arctan(e^{(fx+e)})}{\sqrt{a}} - \frac{5 \sqrt{a} e^{(7fx+7e)} - 3 \sqrt{a} e^{(5fx+5e)} + 3 \sqrt{a} e^{(3fx+3e)} - 5 \sqrt{a} e^{(fx+e)}}{a(e^{(2fx+2e)}+1)^4}}{4f}$$

### 31.5 Problem number 443

$$\int \frac{\tanh^2(e + fx)}{\sqrt{a + a \sinh^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\arctan(\sinh(fx + e)) \cosh(fx + e)}{2f \sqrt{a (\cosh^2(fx + e))}} - \frac{\tanh(fx + e)}{2f \sqrt{a (\cosh^2(fx + e))}}$$

command

```
integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\arctan(e^{(fx+e)})}{\sqrt{a}} - \frac{\sqrt{a} e^{(3fx+3e)} - \sqrt{a} e^{(fx+e)}}{a(e^{(2fx+2e)}+1)^2}$$

$f$

### 31.6 Problem number 447

$$\int \frac{\tanh^5(e + fx)}{(a + a \sinh^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{a^2}{7f (a (\cosh^2(fx + e)))^{7/2}} + \frac{2a}{5f (a (\cosh^2(fx + e)))^{5/2}} - \frac{1}{3f (a (\cosh^2(fx + e)))^{3/2}}$$

command

```
integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{8 (35 \sqrt{a} e^{(11fx+11e)} - 28 \sqrt{a} e^{(9fx+9e)} + 114 \sqrt{a} e^{(7fx+7e)} - 28 \sqrt{a} e^{(5fx+5e)} + 35 \sqrt{a} e^{(3fx+3e)})}{105 a^2 f (e^{(2fx+2e)} + 1)^7}$$

### 31.7 Problem number 448

$$\int \frac{\tanh^3(e + fx)}{(a + a \sinh^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{a}{5f (a (\cosh^2 (fx + e)))^{\frac{5}{2}}} - \frac{1}{3f (a (\cosh^2 (fx + e)))^{\frac{3}{2}}}$$

command

```
integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{8(5\sqrt{a}e^{(7fx+7e)} - 2\sqrt{a}e^{(5fx+5e)} + 5\sqrt{a}e^{(3fx+3e)})}{15a^2f(e^{(2fx+2e)} + 1)^5}$$

### 31.8 Problem number 449

$$\int \frac{\tanh(e + fx)}{(a + a \sinh^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{1}{3f (a (\cosh^2 (fx + e)))^{\frac{3}{2}}}$$

command

```
integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{8e^{(3fx+3e)}}{3a^{\frac{3}{2}}f(e^{(2fx+2e)} + 1)^3}$$

### 31.9 Problem number 452

$$\int \frac{\tanh^2(e + fx)}{(a + a \sinh^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\arctan(\sinh(fx + e)) \cosh(fx + e)}{8af \sqrt{a (\cosh^2(fx + e))}} + \frac{\tanh(fx + e)}{8af \sqrt{a (\cosh^2(fx + e))}} - \frac{\operatorname{sech}(fx + e)^2 \tanh(fx + e)}{4af \sqrt{a (\cosh^2(fx + e))}}$$

command

```
integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\frac{\arctan(e^{(fx+e)})}{a^{\frac{3}{2}}} + \frac{\sqrt{a} e^{(7fx+7e)} - 7\sqrt{a} e^{(5fx+5e)} + 7\sqrt{a} e^{(3fx+3e)} - \sqrt{a} e^{(fx+e)}}{a^2 (e^{(2fx+2e)} + 1)^4}}{4f}$$

### 31.10 Problem number 455

$$\int \frac{\coth^6(e + fx)}{(a + a \sinh^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\coth(fx + e) \operatorname{csch}(fx + e)^2}{3af \sqrt{a (\cosh^2(fx + e))}} - \frac{\coth(fx + e) \operatorname{csch}(fx + e)^4}{5af \sqrt{a (\cosh^2(fx + e))}}$$

command

```
integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{8(5\sqrt{a} e^{(7fx+7e)} + 2\sqrt{a} e^{(5fx+5e)} + 5\sqrt{a} e^{(3fx+3e)})}{15a^2 f (e^{(2fx+2e)} - 1)^5}$$

## 31.11 Problem number 456

$$\int \frac{\coth^8(e + fx)}{(a + a \sinh^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\coth(fx + e) \operatorname{csch}(fx + e)^2}{3af \sqrt{a (\cosh^2(fx + e))}} - \frac{2 \coth(fx + e) \operatorname{csch}(fx + e)^4}{5af \sqrt{a (\cosh^2(fx + e))}} - \frac{\coth(fx + e) \operatorname{csch}(fx + e)^6}{7af \sqrt{a (\cosh^2(fx + e))}}$$

command

`integrate(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{8 (35 \sqrt{a} e^{(11fx+11e)} + 28 \sqrt{a} e^{(9fx+9e)} + 114 \sqrt{a} e^{(7fx+7e)} + 28 \sqrt{a} e^{(5fx+5e)} + 35 \sqrt{a} e^{(3fx+3e)})}{105 a^2 f (e^{(2fx+2e)} - 1)^7}$$

## 31.12 Problem number 486

$$\int \frac{\tanh^2(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\sqrt{2} \sqrt{\frac{1}{1 + \cosh(2fx + 2e)}} \sqrt{1 + \sinh^2(fx + e)} \operatorname{EllipticE}\left(\frac{\sinh(fx+e)}{\sqrt{1 + \sinh^2(fx + e)}}, \sqrt{1 - \frac{b}{a}}\right) \operatorname{sech}(fx + e) \sqrt{a}}{(a - b) f \sqrt{\frac{\operatorname{sech}(fx + e)^2 (a + b (\sinh^2(fx + e)))}{a}}}$$

$$+ \frac{\sqrt{2} \sqrt{\frac{1}{1 + \cosh(2fx + 2e)}} \sqrt{1 + \sinh^2(fx + e)} \operatorname{EllipticF}\left(\frac{\sinh(fx+e)}{\sqrt{1 + \sinh^2(fx + e)}}, \sqrt{1 - \frac{b}{a}}\right) \operatorname{sech}(fx + e) \sqrt{a}}{(a - b) f \sqrt{\frac{\operatorname{sech}(fx + e)^2 (a + b (\sinh^2(fx + e)))}{a}}}$$

command

`integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$2 \left( \frac{\arctan \left( -\frac{\sqrt{b} e^{(2fx+2e)} - \sqrt{be^{(4fx+4e)} + 4ae^{(2fx+2e)} - 2be^{(2fx+2e)} + b} + \sqrt{b}}{2\sqrt{a-b}} \right) e^e}{\sqrt{a-b}} - \arctan \left( -\frac{\sqrt{b} e^{(2fx+2e)} - \sqrt{be^{(4fx+4e)} + 4ae^{(2fx+2e)} - 2be^{(2fx+2e)} + b}}{2\sqrt{a-b}} \right) \right)$$

### 31.13 Problem number 491

$$\int \frac{\tanh^3(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{(2a+b) \operatorname{arctanh} \left( \frac{\sqrt{a+b(\sinh^2(fx+e))}}{\sqrt{a-b}} \right)}{2(a-b)^{\frac{5}{2}} f} + \frac{2a+b}{2(a-b)^2 f \sqrt{a+b(\sinh^2(fx+e))}} + \frac{\operatorname{sech}(fx+e)^2}{2(a-b) f \sqrt{a+b(\sinh^2(fx+e))}}$$

command

`integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: RuntimeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display



## 32 Test file number 169

Test folder name:

test\_cases/6\_Hyperbolic\_functions/6.2\_Hyperbolic\_cosine/169\_6.2.5\_Hyperbolic\_cosine\_functions

### 32.1 Problem number 47

$$\int \frac{1}{(a + a \cosh(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\sinh(dx + c)}{4d (a + a \cosh(dx + c))^{5/2}} + \frac{3 \sinh(dx + c)}{16ad (a + a \cosh(dx + c))^{3/2}} + \frac{3 \arctan\left(\frac{\sinh(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\cosh(dx+c)}}\right)\sqrt{2}}{32a^{5/2}d}$$

command

```
integrate(1/(a+a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\sqrt{2} \left( \frac{3 \arctan\left(e^{\left(\frac{1}{2} dx + \frac{1}{2} c\right)}\right)}{a^{5/2}} + \frac{3 a^{7/2} e^{\left(\frac{7}{2} dx + \frac{7}{2} c\right)} + 11 a^{7/2} e^{\left(\frac{5}{2} dx + \frac{5}{2} c\right)} - 11 a^{7/2} e^{\left(\frac{3}{2} dx + \frac{3}{2} c\right)} - 3 a^{7/2} e^{\left(\frac{1}{2} dx + \frac{1}{2} c\right)}}{(ae^{(dx+c)}+a)^4 a^2} \right)}{16 d}$$

## 33 Test file number 173

Test folder name:

test\_cases/6\_Hyperbolic\_functions/6.3\_Hyperbolic\_tangent/173\_6.3.7-d\_hyper-^m-a+b-c\_tanh-^n-^p

### 33.1 Problem number 26

$$\int \frac{\sinh^3(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Optimal antiderivative

$$-\frac{a \cosh(dx + c)}{(a + b)^2 d} + \frac{\cosh^3(dx + c)}{3(a + b)d} + \frac{a \operatorname{arctanh}\left(\frac{\operatorname{sech}(dx+c)\sqrt{b}}{\sqrt{a+b}}\right) \sqrt{b}}{(a + b)^{\frac{5}{2}} d}$$

command

```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$24 \sqrt{a^2 - b^2 + 2\sqrt{-ab}(a+b)} (ab - \sqrt{-ab}a) |ae^{(2c)} + be^{(2c)}| \operatorname{arctan} \left( \frac{\sqrt{a^3 e^{(2c)} + a^2 b e^{(2c)} - ab^2 e^{(2c)} - b^3 e^{(2c)} + \sqrt{(a^5 + 3a^4 b + 2a^3 b^2 - 2a^2 b^3 - 3ab^4 - b^5)}}}{\sqrt{a^3 e^{(2c)} + a^2 b e^{(2c)} - ab^2 e^{(2c)} - b^3 e^{(2c)} + \sqrt{(a^5 + 3a^4 b + 2a^3 b^2 - 2a^2 b^3 - 3ab^4 - b^5)}}} \right)$$

### 33.2 Problem number 28

$$\int \frac{\sinh(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Optimal antiderivative

$$\frac{\cosh(dx + c)}{(a + b)d} - \frac{\operatorname{arctanh}\left(\frac{\operatorname{sech}(dx+c)\sqrt{b}}{\sqrt{a+b}}\right) \sqrt{b}}{(a + b)^{\frac{3}{2}} d}$$

command

```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 33.3 Problem number 29

$$\int \frac{\operatorname{csch}(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}(\cosh(dx+c))}{ad} + \frac{\operatorname{arctanh}\left(\frac{\operatorname{sech}(dx+c)\sqrt{b}}{\sqrt{a+b}}\right) \sqrt{b}}{ad\sqrt{a+b}}$$

command

```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 33.4 Problem number 31

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal antiderivative

$$\frac{(a+2b) \operatorname{arctanh}(\cosh(dx+c))}{2a^2d} - \frac{\operatorname{coth}(dx+c) \operatorname{csch}(dx+c)}{2ad} - \frac{\operatorname{arctanh}\left(\frac{\operatorname{sech}(dx+c)\sqrt{b}}{\sqrt{a+b}}\right) \sqrt{b} \sqrt{a+b}}{a^2d}$$

command

```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$2 \left( 3ab - b^2 - \sqrt{-ab} (a-3b) \right) |ae^{(2c)} + be^{(2c)}| \operatorname{arctan} \left( \frac{e^{(dx)}}{\sqrt{\frac{a^3e^{(2c)} - a^2be^{(2c)} + \sqrt{(a^3e^{(2c)} - a^2be^{(2c)})^2 - (a^3e^{(4c)} + a^2be^{(4c)})}}{a^3e^{(4c)} + a^2be^{(4c)}}}} \right)$$


---


$$\left( a^3 - a^2b + 2\sqrt{-ab} a^2 \right) \sqrt{a^2 - b^2 + 2\sqrt{-ab} (a+b)}$$

### 33.5 Problem number 34

$$\int \frac{\sinh^3(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(a-b)\cosh(dx+c)}{(a+b)^3 d} + \frac{\cosh^3(dx+c)}{3(a+b)^2 d} + \frac{ab \operatorname{sech}(dx+c)}{2(a+b)^3 d (a+b - b \operatorname{sech}(dx+c)^2)} \\ & + \frac{(3a-2b) \operatorname{arctanh}\left(\frac{\operatorname{sech}(dx+c)\sqrt{b}}{\sqrt{a+b}}\right) \sqrt{b}}{2(a+b)^{\frac{7}{2}} d} \end{aligned}$$

command

```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 33.6 Problem number 36

$$\int \frac{\sinh(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{3 \cosh(dx+c)}{2(a+b)^2 d} - \frac{\cosh(dx+c)}{2(a+b) d (a+b - b \operatorname{sech}(dx+c)^2)} - \frac{3 \operatorname{arctanh}\left(\frac{\operatorname{sech}(dx+c)\sqrt{b}}{\sqrt{a+b}}\right) \sqrt{b}}{2(a+b)^{\frac{5}{2}} d}$$

command

```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 33.7 Problem number 37

$$\int \frac{\operatorname{csch}(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\operatorname{arctanh}(\cosh(dx + c))}{a^2 d} + \frac{b \operatorname{sech}(dx + c)}{2a(a + b)d(a + b - b \operatorname{sech}(dx + c))^2} \\ & + \frac{(3a + 2b) \operatorname{arctanh}\left(\frac{\operatorname{sech}(dx + c)\sqrt{b}}{\sqrt{a + b}}\right) \sqrt{b}}{2a^2(a + b)^{\frac{3}{2}}d} \end{aligned}$$

command

```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 33.8 Problem number 39

$$\int \frac{\operatorname{csch}^3(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(a + 4b) \operatorname{arctanh}(\cosh(dx + c))}{2a^3 d} - \frac{\operatorname{coth}(dx + c) \operatorname{csch}(dx + c)}{2ad(a + b - b \operatorname{sech}(dx + c))^2} \\ & - \frac{b \operatorname{sech}(dx + c)}{a^2 d(a + b - b \operatorname{sech}(dx + c))^2} - \frac{(3a + 4b) \operatorname{arctanh}\left(\frac{\operatorname{sech}(dx + c)\sqrt{b}}{\sqrt{a + b}}\right) \sqrt{b}}{2a^3 d \sqrt{a + b}} \end{aligned}$$

command

```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 33.9 Problem number 42

$$\int \frac{\sinh^3(c + dx)}{(a + b \tanh^2(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(a - 2b) \cosh(dx + c)}{(a + b)^4 d} + \frac{\cosh^3(dx + c)}{3(a + b)^3 d} + \frac{ab \operatorname{sech}(dx + c)}{4(a + b)^3 d (a + b - b \operatorname{sech}(dx + c))^2} \\ & + \frac{(7a - 4b) b \operatorname{sech}(dx + c)}{8(a + b)^4 d (a + b - b \operatorname{sech}(dx + c))^2} + \frac{5(3a - 4b) \operatorname{arctanh}\left(\frac{\operatorname{sech}(dx + c) \sqrt{b}}{\sqrt{a + b}}\right) \sqrt{b}}{8(a + b)^{\frac{9}{2}} d} \end{aligned}$$

command

```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 33.10 Problem number 44

$$\int \frac{\sinh(c + dx)}{(a + b \tanh^2(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{15 \cosh(dx + c)}{8(a + b)^3 d} - \frac{\cosh(dx + c)}{4(a + b) d (a + b - b \operatorname{sech}(dx + c))^2} \\ & - \frac{5 \cosh(dx + c)}{8(a + b)^2 d (a + b - b \operatorname{sech}(dx + c))^2} - \frac{15 \operatorname{arctanh}\left(\frac{\operatorname{sech}(dx + c) \sqrt{b}}{\sqrt{a + b}}\right) \sqrt{b}}{8(a + b)^{\frac{7}{2}} d} \end{aligned}$$

command

```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 33.11 Problem number 45

$$\int \frac{\operatorname{csch}(c + dx)}{(a + b \tanh^2(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\operatorname{arctanh}(\cosh(dx + c))}{a^3 d} + \frac{b \operatorname{sech}(dx + c)}{4a(a + b) d (a + b - b \operatorname{sech}(dx + c)^2)^2} \\ & + \frac{b(7a + 4b) \operatorname{sech}(dx + c)}{8a^2(a + b)^2 d (a + b - b \operatorname{sech}(dx + c)^2)} + \frac{(15a^2 + 20ab + 8b^2) \operatorname{arctanh}\left(\frac{\operatorname{sech}(dx + c)\sqrt{b}}{\sqrt{a + b}}\right) \sqrt{b}}{8a^3(a + b)^{\frac{5}{2}} d} \end{aligned}$$

command

```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 33.12 Problem number 47

$$\int \frac{\operatorname{csch}^3(c + dx)}{(a + b \tanh^2(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(a + 6b) \operatorname{arctanh}(\cosh(dx + c))}{2a^4 d} - \frac{\operatorname{coth}(dx + c) \operatorname{csch}(dx + c)}{2ad (a + b - b \operatorname{sech}(dx + c)^2)^2} \\ & - \frac{3b \operatorname{sech}(dx + c)}{4a^2 d (a + b - b \operatorname{sech}(dx + c)^2)^2} - \frac{b(11a + 12b) \operatorname{sech}(dx + c)}{8a^3(a + b) d (a + b - b \operatorname{sech}(dx + c)^2)} \\ & - \frac{(15a^2 + 40ab + 24b^2) \operatorname{arctanh}\left(\frac{\operatorname{sech}(dx + c)\sqrt{b}}{\sqrt{a + b}}\right) \sqrt{b}}{8a^4(a + b)^{\frac{3}{2}} d} \end{aligned}$$

command

```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 33.13 Problem number 106

$$\int \frac{\cosh^3(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Optimal antiderivative

$$\frac{(a + 2b) \sinh(dx + c)}{(a + b)^2 d} + \frac{\sinh^3(dx + c)}{3(a + b)d} + \frac{b^2 \arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)}{(a + b)^{\frac{5}{2}} d \sqrt{a}}$$

command

```
integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$24 \left( ab^2 + \sqrt{-ab} b^2 \right) \sqrt{a^2 - b^2 + 2 \sqrt{-ab} (a + b)} |ae^{(2c)} + be^{(2c)}| \arctan \left( \frac{a^3 e^{(2c)} + a^2 b e^{(2c)} - ab^2 e^{(2c)} - b^3 e^{(2c)} + \sqrt{(a^6 + 3a^5 b + 2a^4 b^2 - 2a^3 b^3 - 3a^2 b^4 - ab^5)}}{a^3 e^{(2c)} + a^2 b e^{(2c)} - ab^2 e^{(2c)} - b^3 e^{(2c)} + \sqrt{(a^6 + 3a^5 b + 2a^4 b^2 - 2a^3 b^3 - 3a^2 b^4 - ab^5)}} \right)$$



### 33.14 Problem number 108

$$\int \frac{\cosh(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Optimal antiderivative

$$\frac{\sinh(dx + c)}{(a + b)d} + \frac{b \arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)}{(a+b)^{\frac{3}{2}} d\sqrt{a}}$$

command

```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 33.15 Problem number 109

$$\int \frac{\operatorname{sech}(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)}{d\sqrt{a}\sqrt{a+b}}$$

command

```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\left( a^3 - 10a^2b + 5ab^2 + (5a^2 - 10ab + b^2)\sqrt{-ab} \right) \sqrt{a^2 - b^2 + 2\sqrt{-ab}(a+b)} |ae^{(2c)} + be^{(2c)}| \arctan \left( \frac{ae^{(2c)} - be^{(2c)} + \sqrt{(ae^{(2c)} - be^{(2c)})^2 + (a^2 - b^2)(ae^{(2c)} + be^{(2c)})^2}}{\sqrt{a^6 - 13a^5b - 14a^4b^2 + 14a^3b^3 + 13a^2b^4 - ab^5 + 2(3a^5 - 4a^4b - 14a^3b^2 - 4a^2b^3 + 3ab^4)}\sqrt{-ab}} \right)$$

### 33.16 Problem number 111

$$\int \frac{\operatorname{sech}^3(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Optimal antiderivative

$$-\frac{\arctan(\sinh(dx + c))}{bd} + \frac{\arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)\sqrt{a+b}}{bd\sqrt{a}}$$

command

```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\left(2\sqrt{a^2 - b^2 + 2\sqrt{-ab}(a+b)} ab^2 |ae^{(2c)} + be^{(2c)}| + \sqrt{a^2 - b^2 + 2\sqrt{-ab}(a+b)} \sqrt{-ab}(a+b) |ae^{(2c)} + be^{(2c)}| |b| - (ab^2 - b^3) \sqrt{a}\right)$$

---

(a<sup>3</sup>b+3

### 33.17 Problem number 113

$$\int \frac{\operatorname{sech}^5(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Optimal antiderivative

$$-\frac{(2a + 3b) \arctan(\sinh(dx + c))}{2b^2d} + \frac{(a + b)^{\frac{3}{2}} \arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)}{b^2d\sqrt{a}} - \frac{\operatorname{sech}(dx + c) \tanh(dx + c)}{2bd}$$

command

```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\sqrt{a^2 - b^2 + 2\sqrt{-ab}(a+b)}(a + \sqrt{-ab})|ae^{(2c)} + be^{(2c)}| \arctan \left( \frac{e^{(dx)}}{\sqrt{\frac{ab^2e^{(2c)} - b^3e^{(2c)} + \sqrt{(ab^2e^{(2c)} - b^3e^{(2c)})^2 - (ab^2e^{(2c)} - b^3e^{(2c)})}}{ab^2e^{(4c)} + b^3e^{(4c)}}}} \right)}{a^2b^2 - ab^3 + 2\sqrt{-ab}ab^2}$$

### 33.18 Problem number 115

$$\int \frac{\cosh^3(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{b^2(6a + b) \arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}(a+b)^{\frac{7}{2}}d} + \frac{(a+3b)\sinh(dx+c)}{(a+b)^3d} + \frac{\sinh^3(dx+c)}{3(a+b)^2d} + \frac{b^3\sinh(dx+c)}{2a(a+b)^3d(a+(a+b)(\sinh^2(dx+c)))}$$

command

```
integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 33.19 Problem number 117

$$\int \frac{\cosh(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{b(4a + b) \arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}(a+b)^{\frac{5}{2}}d} + \frac{\sinh(dx+c)}{(a+b)^2 d} + \frac{b^2 \sinh(dx+c)}{2a(a+b)^2 d (a+(a+b)(\sinh^2(dx+c)))}$$

command

```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 33.20 Problem number 118

$$\int \frac{\operatorname{sech}(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{(2a + b) \arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}(a+b)^{\frac{3}{2}}d} + \frac{b \sinh(dx+c)}{2a(a+b)d(a+(a+b)(\sinh^2(dx+c)))}$$

command

```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 33.21 Problem number 120

$$\int \frac{\operatorname{sech}^3(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{\sinh(dx + c)}{2ad(a + (a + b)(\sinh^2(dx + c)))} + \frac{\arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}d\sqrt{a+b}}$$

command

```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 33.22 Problem number 122

$$\int \frac{\operatorname{sech}^5(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{\arctan(\sinh(dx + c))}{b^2d} + \frac{(a + b) \sinh(dx + c)}{2abd(a + (a + b)(\sinh^2(dx + c)))} - \frac{(2a - b) \arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right) \sqrt{a+b}}{2a^{\frac{3}{2}}b^2d}$$

command

```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 33.23 Problem number 124

$$\int \frac{\operatorname{sech}^7(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{(4a + 5b) \arctan(\sinh(dx + c))}{2b^3 d} - \frac{(4a - b)(a + b)^{\frac{3}{2}} \arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}} b^3 d} + \frac{(a + b)(2a + b) \sinh(dx + c)}{2a b^2 d (a + (a + b) (\sinh^2(dx + c)))} - \frac{\operatorname{sech}(dx + c) \tanh(dx + c)}{2bd (a + (a + b) (\sinh^2(dx + c)))}$$

command

```
integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 33.24 Problem number 126

$$\int \frac{\cosh(c + dx)}{(a + b \tanh^2(c + dx))^3} dx$$

Optimal antiderivative

$$\frac{3b(8a^2 + 4ab + b^2) \arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}} (a + b)^{\frac{7}{2}} d} + \frac{\sinh(dx + c)}{(a + b)^3 d} + \frac{b^3 \sinh(dx + c)}{4a (a + b)^3 d (a + (a + b) (\sinh^2(dx + c)))^2} + \frac{3b^2(4a + b) \sinh(dx + c)}{8a^2 (a + b)^3 d (a + (a + b) (\sinh^2(dx + c)))}$$

command

```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 33.25 Problem number 127

$$\int \frac{\operatorname{sech}(c + dx)}{(a + b \tanh^2(c + dx))^3} dx$$

Optimal antiderivative

$$\frac{(8a^2 + 8ab + 3b^2) \arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}(a+b)^{\frac{5}{2}}d} + \frac{b(\cosh^2(dx+c)) \sinh(dx+c)}{4a(a+b)d(a+(a+b)(\sinh^2(dx+c)))^2}$$

$$+ \frac{3b(2a+b) \sinh(dx+c)}{8a^2(a+b)^2d(a+(a+b)(\sinh^2(dx+c)))}$$

command

```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 33.26 Problem number 129

$$\int \frac{\operatorname{sech}^3(c + dx)}{(a + b \tanh^2(c + dx))^3} dx$$

Optimal antiderivative

$$\frac{(4a + 3b) \arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}(a+b)^{\frac{3}{2}}d} + \frac{b \sinh(dx+c)}{4a(a+b)d(a+(a+b)(\sinh^2(dx+c)))^2}$$

$$+ \frac{(4a + 3b) \sinh(dx+c)}{8a^2(a+b)d(a+(a+b)(\sinh^2(dx+c)))}$$

command

```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

## 33.27 Problem number 131

$$\int \frac{\operatorname{sech}^5(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal antiderivative

$$\frac{\sinh(dx+c)}{4ad(a+(a+b)(\sinh^2(dx+c)))^2} + \frac{3\sinh(dx+c)}{8a^2d(a+(a+b)(\sinh^2(dx+c)))}$$

$$+ \frac{3 \arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}d\sqrt{a+b}}$$

command

```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$3 \left( a^3 - 10a^2b + 5ab^2 + (5a^2 - 10ab + b^2)\sqrt{-ab} \right) \sqrt{a^2 - b^2 + 2\sqrt{-ab}(a+b)} |ae^{(2c)} + be^{(2c)}| \arctan \left( \frac{a^3e^{(2c)} - a^2be^{(2c)} + \sqrt{\dots}}{\dots} \right)$$


---


$$a^8 - 13a^7b - 14a^6b^2 + 14a^5b^3 + 13a^4b^4 - a^3b^5 + 2(3a^7 - 4a^6b - 14a^5b^2 - 4a^4b^3 + 3a^3b^4 - 14a^2b^5 + 3ab^6 - b^7)$$

## 33.28 Problem number 133

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal antiderivative

$$-\frac{\arctan(\sinh(dx+c))}{b^3d} + \frac{(a+b)\sinh(dx+c)}{4abd(a+(a+b)(\sinh^2(dx+c)))^2}$$

$$-\frac{(4a-3b)(a+b)\sinh(dx+c)}{8a^2b^2d(a+(a+b)(\sinh^2(dx+c)))}$$

$$+ \frac{(8a^2-4ab+3b^2)\arctan\left(\frac{\sinh(dx+c)\sqrt{a+b}}{\sqrt{a}}\right)\sqrt{a+b}}{8a^{\frac{5}{2}}b^3d}$$



command

```
integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

output too large to display

## 34 Test file number 176

Test folder name:

test\_cases/6\_Hyperbolic\_functions/6.4\_Hyperbolic\_cotangent/176\_6.4.7-d\_hyper-<sup>m</sup>-a+b-<sub>c</sub>-coth-<sup>n</sup>-<sup>p</sup>

### 34.1 Problem number 27

$$\int (a + b \coth^2(x))^{3/2} \tanh^2(x) dx$$

Optimal antiderivative

$$\begin{aligned} & -b^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\coth(x) \sqrt{b}}{\sqrt{a + b (\coth^2(x))}}\right) \\ & + (a + b)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\coth(x) \sqrt{a + b}}{\sqrt{a + b (\coth^2(x))}}\right) - a \sqrt{a + b (\coth^2(x))} \tanh(x) \end{aligned}$$

command

```
integrate((a+b*coth(x)^2)^(3/2)*tanh(x)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\begin{aligned}
& \frac{2b^2 \arctan\left(-\frac{\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} - 2ae^{(2x)} + 2be^{(2x)} + a + b} - \sqrt{a+b}}{2\sqrt{-b}}\right) \operatorname{sgn}(e^{(2x)} - 1)}{\sqrt{-b}} \\
& - \frac{1}{2}(a+b)^{\frac{3}{2}} \log\left(\left|\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} - 2ae^{(2x)} + 2be^{(2x)} + a + b} + \sqrt{a+b}\right|\right) \operatorname{sgn}(e^{(2x)} - 1) \\
& + \frac{1}{2}(a+b)^{\frac{3}{2}} \log\left(\left|-\sqrt{a+b}e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} - 2ae^{(2x)} + 2be^{(2x)} + a + b} + \sqrt{a+b}\right|\right) \operatorname{sgn}(e^{(2x)} - 1) \\
& - \frac{(a^2 + 2ab + b^2) \log\left(\left|-\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} - 2ae^{(2x)} + 2be^{(2x)} + a + b}\right)(a+b) + \sqrt{a+b}(a - \dots)\right|\right)}{2\sqrt{a+b}} \\
& - \frac{4\left(\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} - 2ae^{(2x)} + 2be^{(2x)} + a + b}\right)a^2 \operatorname{sgn}(e^{(2x)} - 1) - \sqrt{a+b}\dots\right)}{\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} - 2ae^{(2x)} + 2be^{(2x)} + a + b}\right)^2 + 2\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} - 2ae^{(2x)} + 2be^{(2x)} + a + b}\right)}
\end{aligned}$$

### 35 Test file number 180

Test folder name:

test\_cases/6\_Hyperbolic\_functions/6.5\_Hyperbolic\_secant/180\_6.5.7-d\_hyper-~m-a+b-c\_sech-~n-~p

#### 35.1 Problem number 144

$$\int \frac{\coth(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Optimal antiderivative

$$\frac{b \ln(b + a(\cosh^2(dx + c)))}{2a(a+b)d} + \frac{\ln(\sinh(dx + c))}{(a+b)d}$$

command

`integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\frac{2dx}{a} - \frac{b \log(ae^{(4dx+4c)} + 2ae^{(2dx+2c)} + 4be^{(2dx+2c)} + a)}{a^2 + ab}}{2d} - \frac{2e^{(2c)} \log(|e^{(2dx+2c)} - 1|)}{ae^{(2c)} + be^{(2c)}}$$

### 35.2 Problem number 148

$$\int \frac{\tanh^5(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{(a + b)^2}{2a^2bd(b + a(\cosh^2(dx + c)))} + \frac{\ln(\cosh(dx + c))}{b^2d} + \frac{(\frac{1}{a^2} - \frac{1}{b^2}) \ln(b + a(\cosh^2(dx + c)))}{2d}$$

command

```
integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\frac{2dx}{a^2} - \frac{2 \log(e^{(2dx+2c)}+1)}{b^2} + \frac{(a^2-b^2) \log(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4be^{(2dx+2c)}+a)}{a^2b^2}}{2d} - \frac{a^2e^{(4dx+4c)}-b^2e^{(4dx+4c)}+2a^2e^{(2dx+2c)}+8ab}{(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4b)}$$

### 35.3 Problem number 150

$$\int \frac{\tanh^3(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{a + b}{2a^2d(b + a(\cosh^2(dx + c)))} + \frac{\ln(b + a(\cosh^2(dx + c)))}{2a^2d}$$

command

```
integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\frac{2dx}{a^2} + \frac{e^{(4dx+4c)}-2e^{(2dx+2c)}+1}{(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4be^{(2dx+2c)}+a)a} - \frac{\log(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4be^{(2dx+2c)}+a)}{a^2}}{2d}$$

### 35.4 Problem number 152

$$\int \frac{\tanh(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{b}{2a^2d(b + a(\cosh^2(dx + c)))} + \frac{\ln(b + a(\cosh^2(dx + c)))}{2a^2d}$$

command

```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\frac{2dx}{a^2} + \frac{e^{(4dx+4c)} + 2e^{(2dx+2c)} + 1}{(ae^{(4dx+4c)} + 2ae^{(2dx+2c)} + 4be^{(2dx+2c)} + a)a} - \frac{\log(ae^{(4dx+4c)} + 2ae^{(2dx+2c)} + 4be^{(2dx+2c)} + a)}{a^2}}{2d}$$

### 35.5 Problem number 154

$$\int \frac{\coth(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^2} dx$$

Optimal antiderivative

$$\frac{b^2}{2a^2(a + b)d(b + a(\cosh^2(dx + c)))} + \frac{b(2a + b)\ln(b + a(\cosh^2(dx + c)))}{2a^2(a + b)^2d} + \frac{\ln(\sinh(dx + c))}{(a + b)^2d}$$

command

```
integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{(2ab + b^2)\log(ae^{(4dx+4c)} + 2ae^{(2dx+2c)} + 4be^{(2dx+2c)} + a)}{a^4 + 2a^3b + a^2b^2} + \frac{2e^{(2c)}\log(|-e^{(2dx+2c)} + 1|)}{a^2e^{(2c)} + 2abe^{(2c)} + b^2e^{(2c)}} - \frac{2dx}{a^2} - \frac{2abe^{(4dx+4c)} + b^2e^{(4dx+4c)} + 4abe^{(2dx+2c)}}{(a^3 + 2a^2b + ab^2)(ae^{(4dx+4c)} + 2ae^{(2dx+2c)} + 4be^{(2dx+2c)} + a)}}{2d}$$

## 35.6 Problem number 156

$$\int \frac{\coth^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^2} dx$$

Optimal antiderivative

$$\frac{b^3}{2a^2(a+b)^2 d (b+a(\cosh^2(dx+c)))} - \frac{\operatorname{csch}(dx+c)^2}{2(a+b)^2 d} + \frac{b^2(3a+b)\ln(b+a(\cosh^2(dx+c)))}{2a^2(a+b)^3 d} + \frac{(a+3b)\ln(\sinh(dx+c))}{(a+b)^3 d}$$

command

```
integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{(3ab^2+b^3)\log(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4be^{(2dx+2c)}+a)}{a^5+3a^4b+3a^3b^2+a^2b^3} + \frac{2(ae^{(2c)}+3be^{(2c)})\log(|-e^{(2dx+2c)}+1|)}{a^3e^{(2c)}+3a^2be^{(2c)}+3ab^2e^{(2c)}+b^3e^{(2c)}} - \frac{2dx}{a^2} - \frac{3ab^2e^{(4dx+4c)}+b^3e^{(4dx+4c)}}{(a^4+3a^3b+3a^2b^2+ab^3)} \cdot \frac{1}{2d}$$

## 35.7 Problem number 159

$$\int \frac{\tanh^5(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal antiderivative

$$-\frac{(a+b)^2}{4a^3d(b+a(\cosh^2(dx+c)))^2} + \frac{a+b}{a^3d(b+a(\cosh^2(dx+c)))} + \frac{\ln(b+a(\cosh^2(dx+c)))}{2a^3d}$$

command

```
integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{4dx}{a^3} - \frac{2\log(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4be^{(2dx+2c)}+a)}{a^3} + \frac{3ae^{(8dx+8c)}-4ae^{(6dx+6c)}+8be^{(6dx+6c)}+2ae^{(4dx+4c)}-16be^{(4dx+4c)}-4ae^{(2dx+2c)}}{(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4be^{(2dx+2c)}+a)^2 a^2} \cdot \frac{1}{4d}$$

## 35.8 Problem number 161

$$\int \frac{\tanh^3(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx$$

Optimal antiderivative

$$-\frac{b(a+b)}{4a^3d(b+a(\cosh^2(dx+c)))^2} + \frac{a+2b}{2a^3d(b+a(\cosh^2(dx+c)))} + \frac{\ln(b+a(\cosh^2(dx+c)))}{2a^3d}$$

command

```
integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\frac{4dx}{a^3} - \frac{2 \log(ae^{(4dx+4c)} + 2ae^{(2dx+2c)} + 4be^{(2dx+2c)} + a)}{a^3} + \frac{3ae^{(8dx+8c)} + 4ae^{(6dx+6c)} + 8be^{(6dx+6c)} + 2ae^{(4dx+4c)} + 4ae^{(2dx+2c)} + 8be^{(2dx+2c)} + a}{(ae^{(4dx+4c)} + 2ae^{(2dx+2c)} + 4be^{(2dx+2c)} + a)^2 a^2}}{4d}$$

## 35.9 Problem number 163

$$\int \frac{\tanh(c + dx)}{(a + b \operatorname{sech}^2(c + dx))^3} dx$$

Optimal antiderivative

$$-\frac{b^2}{4a^3d(b+a(\cosh^2(dx+c)))^2} + \frac{b}{a^3d(b+a(\cosh^2(dx+c)))} + \frac{\ln(b+a(\cosh^2(dx+c)))}{2a^3d}$$

command

```
integrate(tanh(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\frac{4dx}{a^3} - \frac{2 \log(ae^{(4dx+4c)} + 2ae^{(2dx+2c)} + 4be^{(2dx+2c)} + a)}{a^3} + \frac{3ae^{(8dx+8c)} + 12ae^{(6dx+6c)} + 8be^{(6dx+6c)} + 18ae^{(4dx+4c)} + 16be^{(4dx+4c)} + 12ae^{(2dx+2c)} + 8be^{(2dx+2c)} + a}{(ae^{(4dx+4c)} + 2ae^{(2dx+2c)} + 4be^{(2dx+2c)} + a)^2 a^2}}{4d}$$

## 35.10 Problem number 165

$$\int \frac{\coth(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b^3}{4a^3(a+b)d(b+a(\cosh^2(dx+c)))^2} + \frac{b^2(3a+2b)}{2a^3(a+b)^2d(b+a(\cosh^2(dx+c)))} \\ & + \frac{b(3a^2+3ab+b^2)\ln(b+a(\cosh^2(dx+c)))}{2a^3(a+b)^3d} + \frac{\ln(\sinh(dx+c))}{(a+b)^3d} \end{aligned}$$

command

```
integrate(coth(d*x+c)/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2(3a^2b+3ab^2+b^3)\log(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4be^{(2dx+2c)}+a)}{a^6+3a^5b+3a^4b^2+a^3b^3} + \frac{4e^{(2c)}\log(|-e^{(2dx+2c)}+1|)}{a^3e^{(2c)}+3a^2be^{(2c)}+3ab^2e^{(2c)}+b^3e^{(2c)}} - \frac{4dx}{a^3} - \frac{9a^3be^{(8dx+8c)}+9a^2b^2e^{(8dx+8c)}}{a^3}$$

## 35.11 Problem number 167

$$\int \frac{\coth^3(c+dx)}{(a+b\operatorname{sech}^2(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b^4}{4a^3(a+b)^2d(b+a(\cosh^2(dx+c)))^2} + \frac{b^3(2a+b)}{a^3(a+b)^3d(b+a(\cosh^2(dx+c)))} \\ & - \frac{\operatorname{csch}(dx+c)^2}{2(a+b)^3d} + \frac{b^2(6a^2+4ab+b^2)\ln(b+a(\cosh^2(dx+c)))}{2a^3(a+b)^4d} + \frac{(a+4b)\ln(\sinh(dx+c))}{(a+b)^4d} \end{aligned}$$

command

```
integrate(coth(d*x+c)^3/(a+b*sech(d*x+c)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{(6a^2b^2+4ab^3+b^4)\log(ae^{(4dx+4c)}+2ae^{(2dx+2c)}+4be^{(2dx+2c)}+a)}{a^7+4a^6b+6a^5b^2+4a^4b^3+a^3b^4} + \frac{2(ae^{(2c)}+4be^{(2c)})\log(|e^{(2dx+2c)}-1|)}{a^4e^{(2c)}+4a^3be^{(2c)}+6a^2b^2e^{(2c)}+4ab^3e^{(2c)}+b^4e^{(2c)}} - \frac{2dx}{a^3} - \frac{a^5e^{(12dx+12c)}}{a^3}$$

### 36 Test file number 196

Test folder name:

test\_cases/7\_Inverse\_hyperbolic\_functions/7.3\_Inverse\_hyperbolic\_tangent/196\_7.3.6\_Exponentia

#### 36.1 Problem number 549

$$\int \frac{e^{-2 \tanh^{-1}(ax)}}{\sqrt{c - \frac{c}{ax}}} dx$$

Optimal antiderivative

$$\frac{3 \operatorname{arctanh}\left(\frac{\sqrt{c - \frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}} - \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{c - \frac{c}{ax}} \sqrt{2}}{2\sqrt{c}}\right) \sqrt{2}}{a\sqrt{c}} - \frac{x \sqrt{c - \frac{c}{ax}}}{c}$$

command

```
integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$ac \left( \frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{\frac{acx-c}{ax}}}{2 \sqrt{-c}}\right)}{a^2 \sqrt{-c} c} - \frac{3 \arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2 \sqrt{-c} c} - \frac{\sqrt{\frac{acx-c}{ax}}}{a^2 \left(c - \frac{acx-c}{ax}\right) c} \right)$$



## 36.2 Problem number 550

$$\int \frac{e^{-2 \tanh^{-1}(ax)}}{\left(c - \frac{c}{ax}\right)^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{c - \frac{c}{ax}}}{\sqrt{c}}\right)}{a c^{\frac{3}{2}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c - \frac{c}{ax}} \sqrt{2}}{2\sqrt{c}}\right) \sqrt{2}}{a c^{\frac{3}{2}}} - \frac{x \sqrt{c - \frac{c}{ax}}}{c^2}$$

command

```
integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$ac \left( \frac{\sqrt{2} \operatorname{arctan}\left(\frac{\sqrt{2} \sqrt{\frac{acx-c}{ax}}}{2\sqrt{-c}}\right)}{a^2 \sqrt{-c} c^2} - \frac{\operatorname{arctan}\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2 \sqrt{-c} c^2} - \frac{\sqrt{\frac{acx-c}{ax}}}{a^2 \left(c - \frac{acx-c}{ax}\right) c^2} \right)$$

## 36.3 Problem number 551

$$\int \frac{e^{-2 \tanh^{-1}(ax)}}{\left(c - \frac{c}{ax}\right)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c - \frac{c}{ax}}}{\sqrt{c}}\right)}{a c^{\frac{5}{2}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c - \frac{c}{ax}} \sqrt{2}}{2\sqrt{c}}\right) \sqrt{2}}{2a c^{\frac{5}{2}}} + \frac{2}{a c^2 \sqrt{c - \frac{c}{ax}}} - \frac{x}{c^2 \sqrt{c - \frac{c}{ax}}}$$

command

```
integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{2} ac \left( \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{\frac{acx-c}{ax}}}{2\sqrt{-c}}\right)}{a^2 \sqrt{-c} c^3} + \frac{2 \arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2 \sqrt{-c} c^3} + \frac{2 \left(c - \frac{2(acx-c)}{ax}\right)}{\left(c \sqrt{\frac{acx-c}{ax}} - \frac{(acx-c) \sqrt{\frac{acx-c}{ax}}}{ax}\right) a^2 c^3} \right)$$

### 36.4 Problem number 552

$$\int \frac{e^{-2 \tanh^{-1}(ax)}}{\left(c - \frac{c}{ax}\right)^{7/2}} dx$$

Optimal antiderivative

$$\frac{\frac{4}{3a c^2 \left(c - \frac{c}{ax}\right)^{\frac{3}{2}}} - \frac{x}{c^2 \left(c - \frac{c}{ax}\right)^{\frac{3}{2}}} - \frac{3 \operatorname{arctanh}\left(\frac{\sqrt{c - \frac{c}{ax}}}{\sqrt{c}}\right)}{a c^{\frac{7}{2}}}}{\frac{\operatorname{arctanh}\left(\frac{\sqrt{c - \frac{c}{ax}} \sqrt{2}}{2\sqrt{c}}\right) \sqrt{2}}{4a c^{\frac{7}{2}}}} + \frac{7}{2a c^3 \sqrt{c - \frac{c}{ax}}}$$

command

```
integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{12} ac \left( \frac{2 \left( 2c + \frac{15(ax-c)}{ax} \right) x}{(acx-c)ac^4 \sqrt{\frac{acx-c}{ax}}} + \frac{3\sqrt{2} \arctan \left( \frac{\sqrt{2} \sqrt{\frac{acx-c}{ax}}}{2\sqrt{-c}} \right)}{a^2 \sqrt{-c} c^4} + \frac{36 \arctan \left( \frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}} \right)}{a^2 \sqrt{-c} c^4} - \frac{12 \sqrt{\frac{acx-c}{ax}}}{a^2 \left( c - \frac{acx-c}{ax} \right)} \right)$$

### 36.5 Problem number 553

$$\int \frac{e^{-2 \tanh^{-1}(ax)}}{\left( c - \frac{c}{ax} \right)^{9/2}} dx$$

Optimal antiderivative

$$\frac{6}{5a c^2 \left( c - \frac{c}{ax} \right)^{\frac{5}{2}}} + \frac{11}{6a c^3 \left( c - \frac{c}{ax} \right)^{\frac{3}{2}}} - \frac{x}{c^2 \left( c - \frac{c}{ax} \right)^{\frac{5}{2}}} - \frac{5 \operatorname{arctanh} \left( \frac{\sqrt{c - \frac{c}{ax}}}{\sqrt{c}} \right)}{a c^{\frac{9}{2}}} - \frac{\operatorname{arctanh} \left( \frac{\sqrt{c - \frac{c}{ax}} \sqrt{2}}{2\sqrt{c}} \right) \sqrt{2}}{8a c^{\frac{9}{2}}} + \frac{21}{4a c^4 \sqrt{c - \frac{c}{ax}}}$$

command

`integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a/x)^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{120} ac \left( \frac{2 \left( 12c^2 + \frac{50(ax-c)c}{ax} + \frac{255(ax-c)^2}{a^2 x^2} \right) x^2}{(acx-c)^2 c^5 \sqrt{\frac{acx-c}{ax}}} + \frac{15\sqrt{2} \arctan \left( \frac{\sqrt{2} \sqrt{\frac{acx-c}{ax}}}{2\sqrt{-c}} \right)}{a^2 \sqrt{-c} c^5} + \frac{600 \arctan \left( \frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}} \right)}{a^2 \sqrt{-c} c^5} \right)$$

### 36.6 Problem number 728

$$\int \frac{e^{-2 \tanh^{-1}(ax)}}{\sqrt{c - \frac{c}{a^2 x^2}}} dx$$

Optimal antiderivative

$$\frac{(-ax + 1)^2}{a^2 x \sqrt{c - \frac{c}{a^2 x^2}}} + \frac{2(-ax + 1)(ax + 1)}{a^2 x \sqrt{c - \frac{c}{a^2 x^2}}} + \frac{2 \arcsin(ax) \sqrt{-ax + 1} \sqrt{ax + 1}}{a^2 x \sqrt{c - \frac{c}{a^2 x^2}}}$$

command

```
integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{(ax + 1)a^2 \sqrt{c - \frac{2c}{ax + 1}} + \frac{4a^2 c \arctan\left(\frac{\sqrt{c - \frac{2c}{ax + 1}}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 2a^2 \sqrt{c - \frac{2c}{ax + 1}}}{a^3 \operatorname{csgn}\left(-\frac{1}{ax + 1} + 1\right)}$$

### 36.7 Problem number 729

$$\int \frac{e^{-2 \tanh^{-1}(ax)}}{\left(c - \frac{c}{a^2 x^2}\right)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(-ax + 1)^2}{3a^2 \left(c - \frac{c}{a^2 x^2}\right)^{\frac{3}{2}} x} - \frac{2(-ax + 1)^2 (ax + 1) (2ax + 5)}{3a^4 \left(c - \frac{c}{a^2 x^2}\right)^{\frac{3}{2}} x^3} - \frac{2(-ax + 1)^{\frac{3}{2}} (ax + 1)^{\frac{3}{2}} \arcsin(ax)}{a^4 \left(c - \frac{c}{a^2 x^2}\right)^{\frac{3}{2}} x^3}$$

command

```
integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{6(ax+1)a^2\sqrt{c-\frac{2c}{ax+1}} + \frac{24a^2c\arctan\left(\frac{\sqrt{c-\frac{2c}{ax+1}}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{a^2\left(c-\frac{2c}{ax+1}\right)^{\frac{3}{2}}c^2+15a^2\sqrt{c-\frac{2c}{ax+1}}c^3}{c^3}}{6a^3c^2\operatorname{sgn}\left(-\frac{1}{ax+1}+1\right)}$$

**36.8 Problem number 730**

$$\int \frac{e^{-2\operatorname{tanh}^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-ax+1)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}x} + \frac{2(-ax+1)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}x^2} - \frac{2(-ax+1)^3(ax+1)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}x^3} \\ & + \frac{2(-ax+1)^3(ax+1)^2(13ax+28)}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}x^5} + \frac{2(-ax+1)^{\frac{5}{2}}(ax+1)^{\frac{5}{2}}\arcsin(ax)}{a^6\left(c-\frac{c}{a^2x^2}\right)^{\frac{5}{2}}x^5} \end{aligned}$$

command

```
integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\begin{aligned} & \frac{4\arctan\left(\frac{\sqrt{c-\frac{2c}{ax+1}}}{\sqrt{-c}}\right)}{a\sqrt{-c}c^2\operatorname{sgn}\left(-\frac{1}{ax+1}+1\right)} + \frac{8c-\frac{17c}{ax+1}}{4\left(\left(c-\frac{2c}{ax+1}\right)^{\frac{3}{2}}-\sqrt{c-\frac{2c}{ax+1}}c\right)ac^2\operatorname{sgn}\left(-\frac{1}{ax+1}+1\right)} \\ & - \frac{3a^4\left(c-\frac{2c}{ax+1}\right)^{\frac{5}{2}}c^{20}+35a^4\left(c-\frac{2c}{ax+1}\right)^{\frac{3}{2}}c^{21}+345a^4\sqrt{c-\frac{2c}{ax+1}}c^{22}}{120a^5c^{25}\operatorname{sgn}\left(-\frac{1}{ax+1}+1\right)} \end{aligned}$$

### 36.9 Problem number 731

$$\int \frac{e^{-2 \tanh^{-1}(ax)}}{\left(c - \frac{c}{a^2 x^2}\right)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-ax+1)^2}{3a^2 \left(c - \frac{c}{a^2 x^2}\right)^{7/2} x} - \frac{10(-ax+1)^3}{3a^3 \left(c - \frac{c}{a^2 x^2}\right)^{7/2} x^2} - \frac{12(-ax+1)^4}{7a^4 \left(c - \frac{c}{a^2 x^2}\right)^{7/2} x^3} \\ & - \frac{82(-ax+1)^4 (ax+1)}{105a^5 \left(c - \frac{c}{a^2 x^2}\right)^{7/2} x^4} - \frac{2(-ax+1)^4 (ax+1)^2}{35a^6 \left(c - \frac{c}{a^2 x^2}\right)^{7/2} x^5} \\ & - \frac{2(-ax+1)^4 (ax+1)^3 (37ax+72)}{35a^8 \left(c - \frac{c}{a^2 x^2}\right)^{7/2} x^7} - \frac{2(-ax+1)^{7/2} (ax+1)^{7/2} \arcsin(ax)}{a^8 \left(c - \frac{c}{a^2 x^2}\right)^{7/2} x^7} \end{aligned}$$

command

```
integrate(1/(a*x+1)^2*(-a^2*x^2+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\begin{aligned} & \frac{(ax+1) \sqrt{c - \frac{2c}{ax+1}}}{ac^4 \operatorname{sgn}\left(-\frac{1}{ax+1} + 1\right)} - \frac{4 \arctan\left(\frac{\sqrt{c - \frac{2c}{ax+1}}}{\sqrt{-c}}\right)}{a\sqrt{-c} c^3 \operatorname{sgn}\left(-\frac{1}{ax+1} + 1\right)} + \frac{14c - \frac{27c}{ax+1}}{48a \left(c - \frac{2c}{ax+1}\right)^{3/2} c^3 \operatorname{sgn}\left(-\frac{1}{ax+1} + 1\right)} \\ & \frac{15a^6 \left(c - \frac{2c}{ax+1}\right)^{7/2} c^{42} + 189a^6 \left(c - \frac{2c}{ax+1}\right)^{5/2} c^{43} + 1330a^6 \left(c - \frac{2c}{ax+1}\right)^{3/2} c^{44} + 10710a^6 \sqrt{c - \frac{2c}{ax+1}} c^{45}}{3360a^7 c^{49} \operatorname{sgn}\left(-\frac{1}{ax+1} + 1\right)} \end{aligned}$$

### 37 Test file number 197

Test folder name:

test\_cases/7\_Inverse\_hyperbolic\_functions/7.3\_Inverse\_hyperbolic\_tangent/197\_7.3.7\_Inverse\_hy

### 37.1 Problem number 5

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^3} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}}\right)}{2x^2} - \frac{\sqrt{e}\sqrt{ex^2+d}}{2dx}$$

command

```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$\frac{e}{\left(xe^{\frac{1}{2}} - \sqrt{x^2e+d}\right)^2 - d} - \frac{\log\left(-\frac{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}+1}{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}-1}\right)}{4x^2}$$

### 37.2 Problem number 6

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^5} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}}\right)}{4x^4} + \frac{e^{\frac{3}{2}}\sqrt{ex^2+d}}{6d^2x} - \frac{\sqrt{e}\sqrt{ex^2+d}}{12dx^3}$$

command

```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\left(3 \left(xe^{\frac{1}{2}} - \sqrt{x^2e + d}\right)^2 de - d^2e\right)e}{3 \left(\left(xe^{\frac{1}{2}} - \sqrt{x^2e + d}\right)^2 - d\right)^3 d} - \frac{\log\left(-\frac{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e + d}} + 1}{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e + d}} - 1}\right)}{8x^4}$$

**37.3 Problem number 7**

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d + ex^2}}\right)}{x^7} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2 + d}}\right)}{6x^6} + \frac{2e^{\frac{3}{2}}\sqrt{ex^2 + d}}{45d^2x^3} - \frac{4e^{\frac{5}{2}}\sqrt{ex^2 + d}}{45d^3x} - \frac{\sqrt{e}\sqrt{ex^2 + d}}{30dx^5}$$

command

```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$\frac{8 \left(10 \left(xe^{\frac{1}{2}} - \sqrt{x^2e + d}\right)^4 d^2e^2 - 5 \left(xe^{\frac{1}{2}} - \sqrt{x^2e + d}\right)^2 d^3e^2 + d^4e^2\right)e}{45 \left(\left(xe^{\frac{1}{2}} - \sqrt{x^2e + d}\right)^2 - d\right)^5 d^2} - \frac{\log\left(-\frac{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e + d}} + 1}{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e + d}} - 1}\right)}{12x^6}$$



### 37.4 Problem number 8

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^9} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}}\right)}{8x^8} + \frac{3e^{\frac{3}{2}}\sqrt{ex^2+d}}{140d^2x^5} - \frac{e^{\frac{5}{2}}\sqrt{ex^2+d}}{35d^3x^3} + \frac{2e^{\frac{7}{2}}\sqrt{ex^2+d}}{35d^4x} - \frac{\sqrt{e}\sqrt{ex^2+d}}{56dx^7}$$

command

```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\log\left(\frac{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}+1}{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}-1}\right)}{16x^8} + \frac{4\left(35\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^6d^3e^3-21\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^4d^4e^3+7\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2d^5e^3-d^6e^3\right)e}{35\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2-d\right)^7d^3}$$

### 37.5 Problem number 24

$$\int x^{3/2} \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$$

Optimal antiderivative

$$\frac{2x^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}}\right)}{5} - \frac{4x^{\frac{3}{2}}\sqrt{ex^2+d}}{25\sqrt{e}} + \frac{12d\sqrt{x}\sqrt{ex^2+d}}{25e(\sqrt{d}+x\sqrt{e})}$$

$$- \frac{12d^{\frac{5}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticE}\left(\sin\left(2\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{d}+x\sqrt{e}) \sqrt{\frac{ex^2+d}{(\sqrt{d}+x\sqrt{e})}}}{25 \cos\left(2\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right) e^{\frac{5}{4}}\sqrt{ex^2+d}}$$

$$+ \frac{6d^{\frac{5}{4}} \sqrt{\frac{\cos\left(4\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right)}{2}} + \frac{1}{2} \operatorname{EllipticF}\left(\sin\left(2\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right), \frac{\sqrt{2}}{2}\right) (\sqrt{d}+x\sqrt{e}) \sqrt{\frac{ex^2+d}{(\sqrt{d}+x\sqrt{e})}}}{25 \cos\left(2\arctan\left(\frac{e^{\frac{1}{4}}\sqrt{x}}{d^{\frac{1}{4}}}\right)\right) e^{\frac{5}{4}}\sqrt{ex^2+d}}$$

command

```
integrate(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$+\infty$

## 38 Test file number 199

Test folder name:

test\_cases/7\_Inverse\_hyperbolic\_functions/7.4\_Inverse\_hyperbolic\_cotangent/199\_7.4.2\_Exponent

### 38.1 Problem number 2

$$\int e^{\coth^{-1}(ax)} x^2 dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3} + \frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2} + \frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a} + \frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{6} a \left( \frac{3 \log \left( \sqrt{\frac{ax-1}{ax+1}} + 1 \right)}{a^4} - \frac{3 \log \left( \left| \sqrt{\frac{ax-1}{ax+1}} - 1 \right| \right)}{a^4} + \frac{2 \left( \frac{4(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1} - \frac{3(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2} - 9\sqrt{\frac{ax-1}{ax+1}} \right)}{a^4 \left( \frac{ax-1}{ax+1} - 1 \right)^3} \right)$$

## 38.2 Problem number 18

$$\int e^{3 \coth^{-1}(ax)} x^2 dx$$

Optimal antiderivative

$$\frac{11 \operatorname{arctanh} \left( \sqrt{1 - \frac{1}{a^2 x^2}} \right)}{2a^3} - \frac{4\sqrt{1 - \frac{1}{a^2 x^2}}}{a^2 \left( a - \frac{1}{x} \right)} + \frac{14x\sqrt{1 - \frac{1}{a^2 x^2}}}{3a^2} + \frac{3x^2\sqrt{1 - \frac{1}{a^2 x^2}}}{2a} + \frac{x^3\sqrt{1 - \frac{1}{a^2 x^2}}}{3}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{6} a \left( \frac{33 \log \left( \sqrt{\frac{ax-1}{ax+1}} + 1 \right)}{a^4} - \frac{33 \log \left( \left| \sqrt{\frac{ax-1}{ax+1}} - 1 \right| \right)}{a^4} - \frac{24}{a^4 \sqrt{\frac{ax-1}{ax+1}}} + \frac{2 \left( \frac{52(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1} - \frac{21(ax-1)^2}{(ax+1)^2} \right)}{a^4 \left( \frac{ax-1}{ax+1} - 1 \right)} \right)$$

### 38.3 Problem number 19

$$\int e^{3 \coth^{-1}(ax)} x \, dx$$

Optimal antiderivative

$$\frac{9 \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{a^2 x^2}}\right)}{2a^2} - \frac{4\sqrt{1 - \frac{1}{a^2 x^2}}}{a\left(a - \frac{1}{x}\right)} + \frac{3x\sqrt{1 - \frac{1}{a^2 x^2}}}{a} + \frac{x^2\sqrt{1 - \frac{1}{a^2 x^2}}}{2}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{2} a \left( \frac{9 \log\left(\sqrt{\frac{ax-1}{ax+1}} + 1\right)}{a^3} - \frac{9 \log\left(\left|\sqrt{\frac{ax-1}{ax+1}} - 1\right|\right)}{a^3} - \frac{8}{a^3 \sqrt{\frac{ax-1}{ax+1}}} - \frac{2 \left( \frac{5(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1} - 7\sqrt{\frac{ax-1}{ax+1}} \right)}{a^3 \left(\frac{ax-1}{ax+1} - 1\right)^2} \right)$$

### 38.4 Problem number 20

$$\int e^{3 \coth^{-1}(ax)} \, dx$$

Optimal antiderivative

$$\frac{3 \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{a^2 x^2}}\right)}{a} - \frac{4\sqrt{1 - \frac{1}{a^2 x^2}}}{a - \frac{1}{x}} + x\sqrt{1 - \frac{1}{a^2 x^2}}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$a \left( \frac{3 \log \left( \sqrt{\frac{ax-1}{ax+1}} + 1 \right)}{a^2} - \frac{3 \log \left( \left| \sqrt{\frac{ax-1}{ax+1}} - 1 \right| \right)}{a^2} - \frac{2 \left( \frac{3(ax-1)}{ax+1} - 2 \right)}{a^2 \left( \frac{(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1} - \sqrt{\frac{ax-1}{ax+1}} \right)} \right)$$

### 38.5 Problem number 21

$$\int \frac{e^{3 \coth^{-1}(ax)}}{x} dx$$

Optimal antiderivative

$$\operatorname{arccsc}(ax) + \operatorname{arctanh} \left( \sqrt{1 - \frac{1}{a^2 x^2}} \right) - \frac{4a \sqrt{1 - \frac{1}{a^2 x^2}}}{a - \frac{1}{x}}$$

command

`integrate(1/((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-a \left( \frac{2 \arctan \left( \sqrt{\frac{ax-1}{ax+1}} \right)}{a} - \frac{\log \left( \sqrt{\frac{ax-1}{ax+1}} + 1 \right)}{a} + \frac{\log \left( \left| \sqrt{\frac{ax-1}{ax+1}} - 1 \right| \right)}{a} + \frac{4}{a \sqrt{\frac{ax-1}{ax+1}}} \right)$$

### 38.6 Problem number 22

$$\int \frac{e^{3 \coth^{-1}(ax)}}{x^2} dx$$

Optimal antiderivative

$$3a \operatorname{arccsd}(ax) - \frac{2(a + \frac{1}{x})^2}{a \sqrt{1 - \frac{1}{a^2 x^2}}} - 3a \sqrt{1 - \frac{1}{a^2 x^2}}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-2a \left( \frac{\frac{3(ax-1)}{ax+1} + 2}{\frac{(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1} + \sqrt{\frac{ax-1}{ax+1}}} + 3 \arctan \left( \sqrt{\frac{ax-1}{ax+1}} \right) \right)$$

### 38.7 Problem number 23

$$\int \frac{e^{3 \coth^{-1}(ax)}}{x^3} dx$$

Optimal antiderivative

$$-\frac{a^5 \left(1 - \frac{1}{a^2 x^2}\right)^{\frac{5}{2}}}{\left(a - \frac{1}{x}\right)^3} - \frac{3a^3 \left(1 - \frac{1}{a^2 x^2}\right)^{\frac{3}{2}}}{2 \left(a - \frac{1}{x}\right)} + \frac{9a^2 \operatorname{arccsd}(ax)}{2} - \frac{9a^2 \sqrt{1 - \frac{1}{a^2 x^2}}}{2}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$- \left( 9 a \arctan \left( \sqrt{\frac{ax-1}{ax+1}} \right) + \frac{4 a}{\sqrt{\frac{ax-1}{ax+1}}} + \frac{5 (ax-1)a \sqrt{\frac{ax-1}{ax+1}}}{ax+1} + \frac{7 a \sqrt{\frac{ax-1}{ax+1}}}{\left(\frac{ax-1}{ax+1} + 1\right)^2} \right) a$$

### 38.8 Problem number 24

$$\int \frac{e^{3 \coth^{-1}(ax)}}{x^4} dx$$

Optimal antiderivative

$$\frac{11a^3 \operatorname{arccsc}(ax)}{2} - \frac{\left(a + \frac{1}{x}\right)^3}{\sqrt{1 - \frac{1}{a^2 x^2}}} - \frac{a\left(3a + \frac{1}{x}\right)^2 \sqrt{1 - \frac{1}{a^2 x^2}}}{3} - \frac{a^2\left(28a + \frac{3}{x}\right) \sqrt{1 - \frac{1}{a^2 x^2}}}{6}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{1}{3} \left( 33 a^2 \arctan \left( \sqrt{\frac{ax-1}{ax+1}} \right) + \frac{12 a^2}{\sqrt{\frac{ax-1}{ax+1}}} + \frac{52 (ax-1)a^2 \sqrt{\frac{ax-1}{ax+1}}}{ax+1} + \frac{21 (ax-1)^2 a^2 \sqrt{\frac{ax-1}{ax+1}}}{(ax+1)^2} + 39 a^2 \sqrt{\frac{ax-1}{ax+1}} \right) \frac{1}{\left(\frac{ax-1}{ax+1} + 1\right)^3}$$

## 38.9 Problem number 128

$$\int e^{\frac{1}{4} \coth^{-1}(ax)} dx$$

Optimal antiderivative

$$\begin{aligned} & \left(1 - \frac{1}{ax}\right)^{\frac{7}{8}} \left(1 + \frac{1}{ax}\right)^{\frac{1}{8}} x + \frac{\arctan\left(\frac{\left(1 + \frac{1}{ax}\right)^{\frac{1}{8}}}{\left(1 - \frac{1}{ax}\right)^{\frac{1}{8}}}\right)}{2a} + \frac{\operatorname{arctanh}\left(\frac{\left(1 + \frac{1}{ax}\right)^{\frac{1}{8}}}{\left(1 - \frac{1}{ax}\right)^{\frac{1}{8}}}\right)}{2a} \\ & - \frac{\arctan\left(1 - \frac{\left(1 + \frac{1}{ax}\right)^{\frac{1}{8}} \sqrt{2}}{\left(1 - \frac{1}{ax}\right)^{\frac{1}{8}}}\right) \sqrt{2}}{4a} + \frac{\arctan\left(1 + \frac{\left(1 + \frac{1}{ax}\right)^{\frac{1}{8}} \sqrt{2}}{\left(1 - \frac{1}{ax}\right)^{\frac{1}{8}}}\right) \sqrt{2}}{4a} \\ & - \frac{\ln\left(1 + \frac{\left(1 + \frac{1}{ax}\right)^{\frac{1}{4}}}{\left(1 - \frac{1}{ax}\right)^{\frac{1}{4}}} - \frac{\left(1 + \frac{1}{ax}\right)^{\frac{1}{8}} \sqrt{2}}{\left(1 - \frac{1}{ax}\right)^{\frac{1}{8}}}\right) \sqrt{2}}{8a} + \frac{\ln\left(1 + \frac{\left(1 + \frac{1}{ax}\right)^{\frac{1}{4}}}{\left(1 - \frac{1}{ax}\right)^{\frac{1}{4}}} + \frac{\left(1 + \frac{1}{ax}\right)^{\frac{1}{8}} \sqrt{2}}{\left(1 - \frac{1}{ax}\right)^{\frac{1}{8}}}\right) \sqrt{2}}{8a} \end{aligned}$$

command

`integrate(1/((a*x-1)/(a*x+1))^(1/8),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{1}{8} a \left( \frac{2 \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(\sqrt{2} + 2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^2} + \frac{2 \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2} \left(\sqrt{2} - 2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^2} - \sqrt{2} \log\left(\sqrt{\dots}\right) \right)$$

## 38.10 Problem number 162

$$\int \frac{e^{\coth^{-1}(ax)}}{c - acx} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\sqrt{1 - \frac{1}{a^2 x^2}}\right)}{ac} + \frac{2a + \frac{2}{x}}{a^2 c \sqrt{1 - \frac{1}{a^2 x^2}}}$$



command

```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-a \left( \frac{\log \left( \sqrt{\frac{ax-1}{ax+1}} + 1 \right)}{a^2 c} - \frac{\log \left( \left| \sqrt{\frac{ax-1}{ax+1}} - 1 \right| \right)}{a^2 c} - \frac{2}{a^2 c \sqrt{\frac{ax-1}{ax+1}}} \right)$$

### 38.11 Problem number 227

$$\int e^{\coth^{-1}(ax)} (c - acx)^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{8\left(1 + \frac{1}{ax}\right)^{\frac{3}{2}}(-acx+c)^{\frac{7}{2}}}{21a\left(1 - \frac{1}{ax}\right)^{\frac{7}{2}}} - \frac{568\left(1 + \frac{1}{ax}\right)^{\frac{3}{2}}(-acx+c)^{\frac{7}{2}}}{315a^3\left(1 - \frac{1}{ax}\right)^{\frac{7}{2}}x^2} \\ & + \frac{48\left(1 + \frac{1}{ax}\right)^{\frac{3}{2}}(-acx+c)^{\frac{7}{2}}}{35a^2\left(1 - \frac{1}{ax}\right)^{\frac{7}{2}}x} + \frac{2\left(a - \frac{1}{x}\right)^3\left(1 + \frac{1}{ax}\right)^{\frac{3}{2}}x(-acx+c)^{\frac{7}{2}}}{9a^3\left(1 - \frac{1}{ax}\right)^{\frac{7}{2}}} \end{aligned}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$2 \left( \frac{256 \sqrt{2} \sqrt{-c} c^3}{\operatorname{sgn}(c)} - \frac{35 (acx+c)^4 \sqrt{-acx-c}}{c} - \frac{270 (acx+c)^3 \sqrt{-acx-c}}{c} + \frac{756 (acx+c)^2 \sqrt{-acx-c}}{c} + \frac{840 (-acx-c)^{\frac{3}{2}} c^3}{c} \right) \frac{1}{315 a}$$

### 38.12 Problem number 229

$$\int e^{\coth^{-1}(ax)}(c - acx)^{3/2} dx$$

Optimal antiderivative

$$\frac{8a^2c^3\left(1 - \frac{1}{a^2x^2}\right)^{\frac{3}{2}}x^3}{15(-acx + c)^{\frac{3}{2}}} + \frac{2a^2c^2\left(1 - \frac{1}{a^2x^2}\right)^{\frac{3}{2}}x^3}{5\sqrt{-acx + c}}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2\left(\frac{8\sqrt{2}\sqrt{-c}c}{\text{sgn}(c)} - \frac{3(acx+c)^2\sqrt{-acx-c} + 10(-acx-c)^{\frac{3}{2}}c}{\text{csgn}(-acx-c)}\right)}{15a}$$

### 38.13 Problem number 244

$$\int e^{3\coth^{-1}(ax)}(c - acx)^{7/2} dx$$

Optimal antiderivative

$$-\frac{44\left(1 + \frac{1}{ax}\right)^{\frac{5}{2}}(-acx + c)^{\frac{7}{2}}}{63a\left(1 - \frac{1}{ax}\right)^{\frac{7}{2}}} + \frac{214\left(1 + \frac{1}{ax}\right)^{\frac{5}{2}}(-acx + c)^{\frac{7}{2}}}{315a^2\left(1 - \frac{1}{ax}\right)^{\frac{7}{2}}x} + \frac{2\left(1 + \frac{1}{ax}\right)^{\frac{5}{2}}x(-acx + c)^{\frac{7}{2}}}{9\left(1 - \frac{1}{ax}\right)^{\frac{7}{2}}}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2\left(\frac{128\sqrt{2}\sqrt{-c}c^3}{\text{sgn}(c)} + \frac{35(acx+c)^4\sqrt{-acx-c} - 180(acx+c)^3\sqrt{-acx-c}c + 252(acx+c)^2\sqrt{-acx-c}c^2}{\text{csgn}(-acx-c)}\right)}{315a}$$

## 38.14 Problem number 246

$$\int e^{3 \coth^{-1}(ax)} (c - acx)^{3/2} dx$$

Optimal antiderivative

$$\frac{2(ax+1)(-acx+c)^{\frac{3}{2}}}{5 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} a}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2 \left( \frac{4\sqrt{2}\sqrt{-c}c}{\text{sgn}(c)} + \frac{(acx+c)^2\sqrt{-acx-c}}{\text{csgn}(-acx-c)} \right)}{5a}$$

## 38.15 Problem number 247

$$\int e^{3 \coth^{-1}(ax)} \sqrt{c - acx} dx$$

Optimal antiderivative

$$\frac{2\left(1 + \frac{1}{ax}\right)^{\frac{3}{2}} x \sqrt{-acx+c} + 4\sqrt{1 + \frac{1}{ax}} \sqrt{-acx+c}}{3\sqrt{1 - \frac{1}{ax}} + a\sqrt{1 - \frac{1}{ax}}} + \frac{4 \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1 + \frac{1}{ax}}}\right) \sqrt{2}\sqrt{\frac{1}{x}} \sqrt{-acx+c}}{a^{\frac{3}{2}}\sqrt{1 - \frac{1}{ax}}}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\frac{12i\sqrt{2}\sqrt{-c}\arctan(-i)-16\sqrt{2}\sqrt{-c}}{\operatorname{sgn}(c)} + \frac{2\left(6\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)+(-acx-c)^{\frac{3}{2}}-6\sqrt{-acx-c}c\right)}{\operatorname{csgn}(-acx-c)}}{3a}$$

### 38.16 Problem number 296

$$\int e^{\coth^{-1}(ax)} x^2 \sqrt{c-acx} dx$$

Optimal antiderivative

$$\frac{16\left(1+\frac{1}{ax}\right)^{\frac{3}{2}}x\sqrt{-acx+c}}{105a^2\sqrt{1-\frac{1}{ax}}} - \frac{8\left(1+\frac{1}{ax}\right)^{\frac{3}{2}}x^2\sqrt{-acx+c}}{35a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{\frac{3}{2}}x^3\sqrt{-acx+c}}{7\sqrt{1-\frac{1}{ax}}}$$

command

`integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2\left(\frac{22\sqrt{2}\sqrt{-c}}{a^2\operatorname{sgn}(c)} + \frac{15(acx+c)^3\sqrt{-acx-c}-42(acx+c)^2\sqrt{-acx-c}c-35(-acx-c)^{\frac{3}{2}}c^2}{a^2c^3\operatorname{sgn}(-acx-c)}\right)}{105a}$$

### 38.17 Problem number 310

$$\int e^{3\coth^{-1}(ax)} x^3 \sqrt{c-acx} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1576\sqrt{1+\frac{1}{ax}}\sqrt{-acx+c}}{315a^4\sqrt{1-\frac{1}{ax}}} + \frac{472x\sqrt{1+\frac{1}{ax}}\sqrt{-acx+c}}{315a^3\sqrt{1-\frac{1}{ax}}} \\ & + \frac{92x^2\sqrt{1+\frac{1}{ax}}\sqrt{-acx+c}}{105a^2\sqrt{1-\frac{1}{ax}}} + \frac{38x^3\sqrt{1+\frac{1}{ax}}\sqrt{-acx+c}}{63a\sqrt{1-\frac{1}{ax}}} \\ & + \frac{2x^4\sqrt{1+\frac{1}{ax}}\sqrt{-acx+c}}{9\sqrt{1-\frac{1}{ax}}} - \frac{4\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{-acx+c}}{a^{\frac{9}{2}}\sqrt{1-\frac{1}{ax}}} \end{aligned}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1260i\sqrt{2}\sqrt{-c}\operatorname{arctan}(-i)-2584\sqrt{2}\sqrt{-c}}{a^3\operatorname{sgn}(c)} + \frac{2\left(630\sqrt{2}c^{\frac{9}{2}}\operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)-35(acx+c)^4\sqrt{-acx-c}+45(acx+c)^3\right)}{a^3c^4\operatorname{sgn}(c)}$$

315 a

### 38.18 Problem number 312

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2\left(1+\frac{1}{ax}\right)^{\frac{3}{2}}x\sqrt{-acx+c}}{3a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{\frac{5}{2}}x^2\sqrt{-acx+c}}{5\sqrt{1-\frac{1}{ax}}} + \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{-acx+c}}{a^2\sqrt{1-\frac{1}{ax}}} \\ & - \frac{4\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{-acx+c}}{a^{\frac{5}{2}}\sqrt{1-\frac{1}{ax}}} \end{aligned}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{60i\sqrt{2}\sqrt{-c}\arctan(-i)-104\sqrt{2}\sqrt{-c}}{\operatorname{asgn}(c)} + \frac{2\left(30\sqrt{2}c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)-3(acx+c)^2\sqrt{-acx-c}+5(-acx-c)^{\frac{3}{2}}c-3\right)}{ac^2\operatorname{sgn}(-acx-c)} - \frac{\quad}{15a}$$

### 38.19 Problem number 313

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}dx$$

Optimal antiderivative

$$\frac{2\left(1+\frac{1}{ax}\right)^{\frac{3}{2}}x\sqrt{-acx+c}}{3\sqrt{1-\frac{1}{ax}}} + \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{-acx+c}}{a\sqrt{1-\frac{1}{ax}}} - \frac{4\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{-acx+c}}{a^{\frac{3}{2}}\sqrt{1-\frac{1}{ax}}}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{12i\sqrt{2}\sqrt{-c}\arctan(-i)-16\sqrt{2}\sqrt{-c}}{\operatorname{sgn}(c)} + \frac{2\left(6\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)+(-acx-c)^{\frac{3}{2}}-6\sqrt{-acx-c}c\right)}{\operatorname{csgn}(-acx-c)} - \frac{\quad}{3a}$$

## 38.20 Problem number 314

$$\int \frac{e^{3 \coth^{-1}(ax)} \sqrt{c - acx}}{x} dx$$

Optimal antiderivative

$$\frac{2\sqrt{1 + \frac{1}{ax}} \sqrt{-acx + c}}{\sqrt{1 - \frac{1}{ax}}} + \frac{2 \operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right) \sqrt{\frac{1}{x}} \sqrt{-acx + c}}{\sqrt{a} \sqrt{1 - \frac{1}{ax}}} - \frac{4 \operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{\frac{1}{x}}}{\sqrt{a} \sqrt{1 + \frac{1}{ax}}}\right) \sqrt{2} \sqrt{\frac{1}{x}} \sqrt{-acx + c}}{\sqrt{a} \sqrt{1 - \frac{1}{ax}}}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$-c \left( \frac{4\sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{-acx - c}}{2\sqrt{c}}\right)}{\sqrt{c} \operatorname{sgn}(-acx - c)} - \frac{2 \arctan\left(\frac{\sqrt{-acx - c}}{\sqrt{c}}\right)}{\sqrt{c} \operatorname{sgn}(-acx - c)} + \frac{-4i\sqrt{2} \arctan(-i) + 2\sqrt{2} + 2i \arctan(-i)}{\sqrt{-c} \operatorname{sgn}(c)} \right)$$

## 38.21 Problem number 315

$$\int \frac{e^{3 \coth^{-1}(ax)} \sqrt{c - acx}}{x^2} dx$$

Optimal antiderivative

$$\frac{\sqrt{1 + \frac{1}{ax}} \sqrt{-acx + c}}{x \sqrt{1 - \frac{1}{ax}}} + \frac{5 \operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right) \sqrt{a} \sqrt{\frac{1}{x}} \sqrt{-acx + c}}{\sqrt{1 - \frac{1}{ax}}}$$

$$- \frac{4 \operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{\frac{1}{x}}}{\sqrt{a} \sqrt{1 + \frac{1}{ax}}}\right) \sqrt{2} \sqrt{a} \sqrt{\frac{1}{x}} \sqrt{-acx + c}}{\sqrt{1 - \frac{1}{ax}}}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{4 \sqrt{2} a^2 \sqrt{c} \arctan\left(\frac{\sqrt{2} \sqrt{-acx - c}}{2 \sqrt{c}}\right)}{\operatorname{sgn}(-acx - c)} - \frac{5 a^2 \sqrt{c} \arctan\left(\frac{\sqrt{-acx - c}}{\sqrt{c}}\right)}{\operatorname{sgn}(-acx - c)} - \frac{-4i \sqrt{2} a^2 \sqrt{-c} \arctan(-i) + 5i a^2 \sqrt{-c} \arctan\left(\frac{\sqrt{-c}}{c}\right)}{\operatorname{sgn}(c)}$$

$a$

**38.22 Problem number 316**

$$\int \frac{e^{3 \coth^{-1}(ax)} \sqrt{c - acx}}{x^3} dx$$



Optimal antiderivative

$$\begin{aligned}
& \frac{a\left(1 + \frac{1}{ax}\right)^{\frac{3}{2}} \sqrt{-acx + c}}{2x \sqrt{1 - \frac{1}{ax}}} + \frac{7a \sqrt{1 + \frac{1}{ax}} \sqrt{-acx + c}}{4x \sqrt{1 - \frac{1}{ax}}} \\
& + \frac{23a^{\frac{3}{2}} \operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right) \sqrt{\frac{1}{x}} \sqrt{-acx + c}}{4 \sqrt{1 - \frac{1}{ax}}} \\
& - \frac{4a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{\frac{1}{x}}}{\sqrt{a} \sqrt{1 + \frac{1}{ax}}}\right) \sqrt{2} \sqrt{\frac{1}{x}} \sqrt{-acx + c}}{\sqrt{1 - \frac{1}{ax}}}
\end{aligned}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{16 \sqrt{2} a^3 \sqrt{c} \arctan\left(\frac{\sqrt{2} \sqrt{-acx - c}}{2 \sqrt{c}}\right)}{\operatorname{sgn}(-acx - c)} - \frac{23 a^3 \sqrt{c} \arctan\left(\frac{\sqrt{-acx - c}}{\sqrt{c}}\right)}{\operatorname{sgn}(-acx - c)} + \frac{16i \sqrt{2} a^3 \sqrt{-c} \arctan(-i) - 23i a^3 \sqrt{-c} \arctan(-i)}{4a}$$

**38.23 Problem number 317**

$$\int \frac{e^{3 \coth^{-1}(ax)} \sqrt{c - acx}}{x^4} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{a\left(1 + \frac{1}{ax}\right)^{\frac{3}{2}} \sqrt{-acx + c}}{3x^2 \sqrt{1 - \frac{1}{ax}}} + \frac{3a^2\left(1 + \frac{1}{ax}\right)^{\frac{3}{2}} \sqrt{-acx + c}}{4x \sqrt{1 - \frac{1}{ax}}} \\
& + \frac{13a^2 \sqrt{1 + \frac{1}{ax}} \sqrt{-acx + c}}{8x \sqrt{1 - \frac{1}{ax}}} + \frac{45a^{\frac{5}{2}} \operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right) \sqrt{\frac{1}{x}} \sqrt{-acx + c}}{8 \sqrt{1 - \frac{1}{ax}}} \\
& - \frac{4a^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{\frac{1}{x}}}{\sqrt{a} \sqrt{1 + \frac{1}{ax}}}\right) \sqrt{2} \sqrt{\frac{1}{x}} \sqrt{-acx + c}}{\sqrt{1 - \frac{1}{ax}}}
\end{aligned}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{96 \sqrt{2} a^4 \sqrt{c} \arctan\left(\frac{\sqrt{2} \sqrt{-acx - c}}{2 \sqrt{c}}\right)}{\operatorname{sgn}(-acx - c)} - \frac{135 a^4 \sqrt{c} \arctan\left(\frac{\sqrt{-acx - c}}{\sqrt{c}}\right)}{\operatorname{sgn}(-acx - c)} + \frac{96i \sqrt{2} a^4 \sqrt{-c} \arctan(-i) - 135i a^4 \sqrt{-c} \arctan(-i)}{\operatorname{sgn}(c)}$$

24 a

**38.24 Problem number 318**

$$\int \frac{e^{3 \coth^{-1}(ax)} \sqrt{c - acx}}{x^5} dx$$

Optimal antiderivative

$$\frac{a\left(1 + \frac{1}{ax}\right)^{\frac{3}{2}} \sqrt{-acx + c}}{4x^3 \sqrt{1 - \frac{1}{ax}}} + \frac{11a^2\left(1 + \frac{1}{ax}\right)^{\frac{3}{2}} \sqrt{-acx + c}}{24x^2 \sqrt{1 - \frac{1}{ax}}} + \frac{21a^3\left(1 + \frac{1}{ax}\right)^{\frac{3}{2}} \sqrt{-acx + c}}{32x \sqrt{1 - \frac{1}{ax}}} + \frac{107a^3 \sqrt{1 + \frac{1}{ax}} \sqrt{-acx + c}}{64x \sqrt{1 - \frac{1}{ax}}} + \frac{363a^{\frac{7}{2}} \operatorname{arcsinh}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right) \sqrt{\frac{1}{x}} \sqrt{-acx + c}}{64 \sqrt{1 - \frac{1}{ax}}} - \frac{4a^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{\frac{1}{x}}}{\sqrt{a} \sqrt{1 + \frac{1}{ax}}}\right) \sqrt{2} \sqrt{\frac{1}{x}} \sqrt{-acx + c}}{\sqrt{1 - \frac{1}{ax}}}$$

command

`integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{768 \sqrt{2} a^5 \sqrt{c} \arctan\left(\frac{\sqrt{2} \sqrt{-acx - c}}{2 \sqrt{c}}\right)}{\operatorname{sgn}(-acx - c)} - \frac{1089 a^5 \sqrt{c} \arctan\left(\frac{\sqrt{-acx - c}}{\sqrt{c}}\right)}{\operatorname{sgn}(-acx - c)} + \frac{768i \sqrt{2} a^5 \sqrt{-c} \arctan(-i) - 1089i a^5 \sqrt{-c}}{\operatorname{sgn}(c)}$$

### 38.25 Problem number 383

$$\int \frac{e^{\coth^{-1}(ax)}}{c - \frac{c}{ax}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{a^2 x^2}}\right)}{ac} - \frac{2\left(a + \frac{1}{x}\right)}{a^2 c \sqrt{1 - \frac{1}{a^2 x^2}}} + \frac{x \sqrt{1 - \frac{1}{a^2 x^2}}}{c}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$2a \left( \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}} + 1\right)}{a^2c} - \frac{\log\left(\left|\sqrt{\frac{ax-1}{ax+1}} - 1\right|\right)}{a^2c} - \frac{\frac{2(ax-1)}{ax+1} - 1}{a^2c \left( \frac{(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1} - \sqrt{\frac{ax-1}{ax+1}} \right)} \right)$$

### 38.26 Problem number 384

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c - \frac{c}{ax}\right)^2} dx$$

Optimal antiderivative

$$-\frac{4\left(a + \frac{1}{x}\right)}{3a^2c^2 \left(1 - \frac{1}{a^2x^2}\right)^{\frac{3}{2}}} + \frac{3 \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{a^2x^2}}\right)}{a^2c^2} + \frac{-9a - \frac{11}{x}}{3a^2c^2 \sqrt{1 - \frac{1}{a^2x^2}}} + \frac{x \sqrt{1 - \frac{1}{a^2x^2}}}{c^2}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{3}a \left( \frac{9 \log\left(\sqrt{\frac{ax-1}{ax+1}} + 1\right)}{a^2c^2} - \frac{9 \log\left(\left|\sqrt{\frac{ax-1}{ax+1}} - 1\right|\right)}{a^2c^2} - \frac{(ax+1)\left(\frac{12(ax-1)}{ax+1} + 1\right)}{(ax-1)a^2c^2 \sqrt{\frac{ax-1}{ax+1}}} - \frac{6 \sqrt{\frac{ax-1}{ax+1}}}{a^2c^2 \left(\frac{ax-1}{ax+1} - 1\right)} \right)$$

## 38.27 Problem number 386

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c - \frac{c}{ax}\right)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{16\left(a + \frac{1}{x}\right)}{7a^2c^4\left(1 - \frac{1}{a^2x^2}\right)^{\frac{7}{2}}} - \frac{4\left(7a + \frac{17}{x}\right)}{35a^2c^4\left(1 - \frac{1}{a^2x^2}\right)^{\frac{5}{2}}} + \frac{-175a - \frac{307}{x}}{105a^2c^4\left(1 - \frac{1}{a^2x^2}\right)^{\frac{3}{2}}} \\ & + \frac{5 \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{a^2x^2}}\right)}{ac^4} + \frac{-525a - \frac{719}{x}}{105a^2c^4\sqrt{1 - \frac{1}{a^2x^2}}} + \frac{x\sqrt{1 - \frac{1}{a^2x^2}}}{c^4} \end{aligned}$$

command

`integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{420} a \left( \frac{2100 \log\left(\sqrt{\frac{ax-1}{ax+1}} + 1\right)}{a^2c^4} - \frac{2100 \log\left(\left|\sqrt{\frac{ax-1}{ax+1}} - 1\right|\right)}{a^2c^4} - \frac{(ax+1)^3\left(\frac{126(ax-1)}{ax+1} + \frac{595(ax-1)^2}{(ax+1)^2} + \frac{3360(ax-1)^3}{(ax+1)^3}\right)}{(ax-1)^3a^2c^4\sqrt{\frac{ax-1}{ax+1}}} \right)$$

## 38.28 Problem number 402

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c - \frac{c}{ax}\right)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{32\left(a + \frac{1}{x}\right)}{7a^2c^3\left(1 - \frac{1}{a^2x^2}\right)^{\frac{7}{2}}} - \frac{2\left(7a + \frac{13}{x}\right)}{7a^2c^3\left(1 - \frac{1}{a^2x^2}\right)^{\frac{5}{2}}} - \frac{16}{7a^2c^3\left(1 - \frac{1}{a^2x^2}\right)^{\frac{3}{2}}} x \\ & + \frac{6 \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{a^2x^2}}\right)}{ac^3} + \frac{-42a - \frac{59}{x}}{7a^2c^3\sqrt{1 - \frac{1}{a^2x^2}}} + \frac{x\sqrt{1 - \frac{1}{a^2x^2}}}{c^3} \end{aligned}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{14} a \left( \frac{84 \log \left( \sqrt{\frac{ax-1}{ax+1}} + 1 \right)}{a^2 c^3} - \frac{84 \log \left( \left| \sqrt{\frac{ax-1}{ax+1}} - 1 \right| \right)}{a^2 c^3} - \frac{(ax+1)^3 \left( \frac{7(ax-1)}{ax+1} + \frac{28(ax-1)^2}{(ax+1)^2} + \frac{140(ax-1)^3}{(ax+1)^3} + 1 \right)}{(ax-1)^3 a^2 c^3 \sqrt{\frac{ax-1}{ax+1}}} \right)$$

### 38.29 Problem number 475

$$\int \frac{e^{-2 \coth^{-1}(ax)}}{\sqrt{c - \frac{c}{ax}}} dx$$

Optimal antiderivative

$$-\frac{3 \operatorname{arctanh} \left( \frac{\sqrt{c - \frac{c}{ax}}}{\sqrt{c}} \right)}{a\sqrt{c}} + \frac{2 \operatorname{arctanh} \left( \frac{\sqrt{c - \frac{c}{ax}} \sqrt{2}}{2\sqrt{c}} \right) \sqrt{2}}{a\sqrt{c}} + \frac{x \sqrt{c - \frac{c}{ax}}}{c}$$

command

```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$-ac \left( \frac{2 \sqrt{2} \operatorname{arctan} \left( \frac{\sqrt{2} \sqrt{\frac{acx-c}{ax}}}{2 \sqrt{-c}} \right)}{a^2 \sqrt{-c} c} - \frac{3 \operatorname{arctan} \left( \frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}} \right)}{a^2 \sqrt{-c} c} - \frac{\sqrt{\frac{acx-c}{ax}}}{a^2 \left( c - \frac{acx-c}{ax} \right) c} \right)$$

## 38.30 Problem number 476

$$\int \frac{e^{-2 \coth^{-1}(ax)}}{\left(c - \frac{c}{ax}\right)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c - \frac{c}{ax}}}{\sqrt{c}}\right)}{a c^{\frac{3}{2}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c - \frac{c}{ax}} \sqrt{2}}{2\sqrt{c}}\right) \sqrt{2}}{a c^{\frac{3}{2}}} + \frac{x \sqrt{c - \frac{c}{ax}}}{c^2}$$

command

```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$-ac \left( \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{\frac{acx-c}{ax}}}{2\sqrt{-c}}\right)}{a^2 \sqrt{-c} c^2} - \frac{\arctan\left(\frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}}\right)}{a^2 \sqrt{-c} c^2} - \frac{\sqrt{\frac{acx-c}{ax}}}{a^2 \left(c - \frac{acx-c}{ax}\right) c^2} \right)$$

## 38.31 Problem number 477

$$\int \frac{e^{-2 \coth^{-1}(ax)}}{\left(c - \frac{c}{ax}\right)^{5/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{c - \frac{c}{ax}}}{\sqrt{c}}\right)}{a c^{\frac{5}{2}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c - \frac{c}{ax}} \sqrt{2}}{2\sqrt{c}}\right) \sqrt{2}}{2a c^{\frac{5}{2}}} - \frac{2}{a c^2 \sqrt{c - \frac{c}{ax}}} + \frac{x}{c^2 \sqrt{c - \frac{c}{ax}}}$$

command

```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{1}{2}ac \left( \frac{\sqrt{2} \arctan \left( \frac{\sqrt{2} \sqrt{\frac{acx-c}{ax}}}{2\sqrt{-c}} \right)}{a^2 \sqrt{-c} c^3} + \frac{2 \arctan \left( \frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}} \right)}{a^2 \sqrt{-c} c^3} + \frac{2 \left( c - \frac{2(acx-c)}{ax} \right)}{\left( c \sqrt{\frac{acx-c}{ax}} - \frac{(acx-c) \sqrt{\frac{acx-c}{ax}}}{ax} \right) a^2 c^3} \right)$$

### 38.32 Problem number 478

$$\int \frac{e^{-2 \coth^{-1}(ax)}}{\left(c - \frac{c}{ax}\right)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4}{3a c^2 \left(c - \frac{c}{ax}\right)^{3/2}} + \frac{x}{c^2 \left(c - \frac{c}{ax}\right)^{3/2}} + \frac{3 \operatorname{arctanh} \left( \frac{\sqrt{c - \frac{c}{ax}}}{\sqrt{c}} \right)}{a c^{7/2}} \\ & + \frac{\operatorname{arctanh} \left( \frac{\sqrt{c - \frac{c}{ax}} \sqrt{2}}{2\sqrt{c}} \right) \sqrt{2}}{4a c^{7/2}} - \frac{7}{2a c^3 \sqrt{c - \frac{c}{ax}}} \end{aligned}$$

command

```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError



Giac 1.7.0 via sagemath 9.3 output

$$-\frac{1}{12}ac \left( \frac{2 \left( 2c + \frac{15(ax-c)}{ax} \right) x}{(acx-c)ac^4 \sqrt{\frac{acx-c}{ax}}} + \frac{3\sqrt{2} \arctan \left( \frac{\sqrt{2} \sqrt{\frac{acx-c}{ax}}}{2\sqrt{-c}} \right)}{a^2 \sqrt{-c} c^4} + \frac{36 \arctan \left( \frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}} \right)}{a^2 \sqrt{-c} c^4} - \frac{12 \sqrt{\frac{acx-c}{ax}}}{a^2 \left( c - \frac{acx-c}{ax} \right)} \right)$$

### 38.33 Problem number 479

$$\int \frac{e^{-2 \coth^{-1}(ax)}}{\left( c - \frac{c}{ax} \right)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{6}{5a^2 c^2 \left( c - \frac{c}{ax} \right)^{5/2}} - \frac{11}{6a^3 c^3 \left( c - \frac{c}{ax} \right)^{3/2}} + \frac{x}{c^2 \left( c - \frac{c}{ax} \right)^{5/2}} + \frac{5 \operatorname{arctanh} \left( \frac{\sqrt{c - \frac{c}{ax}}}{\sqrt{c}} \right)}{a c^{9/2}} \\ & + \frac{\operatorname{arctanh} \left( \frac{\sqrt{c - \frac{c}{ax}} \sqrt{2}}{2\sqrt{c}} \right) \sqrt{2}}{8a c^{9/2}} - \frac{21}{4a^4 c^4 \sqrt{c - \frac{c}{ax}}} \end{aligned}$$

command

```
integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{1}{120}ac \left( \frac{2 \left( 12c^2 + \frac{50(ax-c)c}{ax} + \frac{255(ax-c)^2}{a^2 x^2} \right) x^2}{(acx-c)^2 c^5 \sqrt{\frac{acx-c}{ax}}} + \frac{15\sqrt{2} \arctan \left( \frac{\sqrt{2} \sqrt{\frac{acx-c}{ax}}}{2\sqrt{-c}} \right)}{a^2 \sqrt{-c} c^5} + \frac{600 \arctan \left( \frac{\sqrt{\frac{acx-c}{ax}}}{\sqrt{-c}} \right)}{a^2 \sqrt{-c} c^5} \right)$$

### 38.34 Problem number 560

$$\int \frac{e^{\coth^{-1}(ax)}}{c - a^2 cx^2} dx$$

Optimal antiderivative

$$\frac{1}{\sqrt{\frac{ax-1}{ax+1}} ac}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{ac \sqrt{\frac{ax-1}{ax+1}}}$$

### 38.35 Problem number 561

$$\int \frac{e^{\coth^{-1}(ax)}}{(c - a^2 cx^2)^2} dx$$

Optimal antiderivative

$$\frac{2}{3 \sqrt{\frac{ax-1}{ax+1}} a c^2} - \frac{-2ax+1}{3 \sqrt{\frac{ax-1}{ax+1}} a c^2 (-a^2 x^2 + 1)}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{(ax+1) \left( \frac{6(ax-1)}{ax+1} - 1 \right)}{(ax-1) \sqrt{\frac{ax-1}{ax+1}}} + 3 \sqrt{\frac{ax-1}{ax+1}}$$


---


$$12 ac^2$$

## 38.36 Problem number 562

$$\int \frac{e^{\coth^{-1}(ax)}}{(c - a^2cx^2)^3} dx$$

Optimal antiderivative

$$\frac{8}{15\sqrt{\frac{ax-1}{ax+1}} ac^3} - \frac{-4ax+1}{15\sqrt{\frac{ax-1}{ax+1}} ac^3 (-a^2x^2+1)^2} - \frac{4(-2ax+1)}{15\sqrt{\frac{ax-1}{ax+1}} ac^3 (-a^2x^2+1)}$$

command

`integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{(ax+1)^2 \left( \frac{20(ax-1)}{ax+1} - \frac{90(ax-1)^2}{(ax+1)^2} - 3 \right) + \frac{5(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1} - 60\sqrt{\frac{ax-1}{ax+1}}}{(ax-1)^2 \sqrt{\frac{ax-1}{ax+1}}} - \frac{240 ac^3}{240 ac^3}$$

## 38.37 Problem number 563

$$\int \frac{e^{\coth^{-1}(ax)}}{(c - a^2cx^2)^4} dx$$

Optimal antiderivative

$$\frac{16}{35\sqrt{\frac{ax-1}{ax+1}} ac^4} - \frac{-6ax+1}{35\sqrt{\frac{ax-1}{ax+1}} ac^4 (-a^2x^2+1)^3} - \frac{2(-4ax+1)}{35\sqrt{\frac{ax-1}{ax+1}} ac^4 (-a^2x^2+1)^2} - \frac{8(-2ax+1)}{35\sqrt{\frac{ax-1}{ax+1}} ac^4 (-a^2x^2+1)}$$

command

`integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{(ax+1)^3 \left( \frac{42(ax-1)}{ax+1} - \frac{175(ax-1)^2}{(ax+1)^2} + \frac{700(ax-1)^3}{(ax+1)^3} - 5 \right) - \frac{70(ax-1)\sqrt{ax-1}}{ax+1} + \frac{7(ax-1)^2\sqrt{ax-1}}{(ax+1)^2} + 525\sqrt{ax-1}}{(ax-1)^3\sqrt{ax+1}}}{2240ac^4}$$

### 38.38 Problem number 579

$$\int \frac{e^{3 \coth^{-1}(ax)}}{(c - a^2cx^2)^3} dx$$

Optimal antiderivative

$$-\frac{8}{35 \left( \frac{ax-1}{ax+1} \right)^{\frac{3}{2}} ac^3} - \frac{-4ax+3}{7 \left( \frac{ax-1}{ax+1} \right)^{\frac{3}{2}} ac^3 (-a^2x^2+1)^2} + \frac{-\frac{24ax}{35} + \frac{36}{35}}{\left( \frac{ax-1}{ax+1} \right)^{\frac{3}{2}} ac^3 (-a^2x^2+1)}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{(ax+1)^3 \left( \frac{28(ax-1)}{ax+1} - \frac{70(ax-1)^2}{(ax+1)^2} + \frac{140(ax-1)^3}{(ax+1)^3} - 5 \right) + 35\sqrt{ax-1}}{(ax-1)^3\sqrt{ax+1}}}{560ac^3}$$

### 38.39 Problem number 580

$$\int \frac{e^{3 \coth^{-1}(ax)}}{(c - a^2cx^2)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{16}{63 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} a c^4} - \frac{-2ax+1}{9 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} a c^4 (-a^2x^2+1)^3} \\ & - \frac{10(-4ax+3)}{63 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} a c^4 (-a^2x^2+1)^2} + \frac{-\frac{16ax}{21} + \frac{8}{7}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} a c^4 (-a^2x^2+1)} \end{aligned}$$

command

`integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{(ax+1)^4 \left( \frac{54(ax-1)}{ax+1} - \frac{189(ax-1)^2}{(ax+1)^2} + \frac{420(ax-1)^3}{(ax+1)^3} - \frac{945(ax-1)^4}{(ax+1)^4} - 7 \right)}{(ax-1)^4 \sqrt{\frac{ax-1}{ax+1}}} + \frac{21(ax-1) \sqrt{\frac{ax-1}{ax+1}}}{ax+1} - 378 \sqrt{\frac{ax-1}{ax+1}}$$


---


$$4032 ac^4$$

### 38.40 Problem number 776

$$\int \frac{e^{\coth^{-1}(ax)}}{c - \frac{c}{a^2x^2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{ax}} \sqrt{1+\frac{1}{ax}}\right)}{ac} - \frac{2\sqrt{1+\frac{1}{ax}}}{ac\sqrt{1-\frac{1}{ax}}} + \frac{x\sqrt{1+\frac{1}{ax}}}{c\sqrt{1-\frac{1}{ax}}}$$

command

`integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$a \left( \frac{\log \left( \sqrt{\frac{ax-1}{ax+1}} + 1 \right)}{a^2 c} - \frac{\log \left( \left| \sqrt{\frac{ax-1}{ax+1}} - 1 \right| \right)}{a^2 c} - \frac{\frac{3(ax-1)}{ax+1} - 1}{a^2 c \left( \frac{(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax+1} - \sqrt{\frac{ax-1}{ax+1}} \right)} \right)$$

### 38.41 Problem number 777

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c - \frac{c}{a^2 x^2}\right)^2} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh} \left( \sqrt{1 - \frac{1}{ax}} \sqrt{1 + \frac{1}{ax}} \right)}{a c^2} - \frac{4}{3a c^2 \left(1 - \frac{1}{ax}\right)^{\frac{3}{2}} \sqrt{1 + \frac{1}{ax}}} + \frac{x}{c^2 \left(1 - \frac{1}{ax}\right)^{\frac{3}{2}} \sqrt{1 + \frac{1}{ax}}} - \frac{11}{3a c^2 \sqrt{1 - \frac{1}{ax}} \sqrt{1 + \frac{1}{ax}}} + \frac{8\sqrt{1 - \frac{1}{ax}}}{3a c^2 \sqrt{1 + \frac{1}{ax}}}$$

command

`integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{12} a \left( \frac{12 \log \left( \sqrt{\frac{ax-1}{ax+1}} + 1 \right)}{a^2 c^2} - \frac{12 \log \left( \left| \sqrt{\frac{ax-1}{ax+1}} - 1 \right| \right)}{a^2 c^2} - \frac{(ax+1) \left( \frac{18(ax-1)}{ax+1} + 1 \right)}{(ax-1)a^2 c^2 \sqrt{\frac{ax-1}{ax+1}}} + \frac{3\sqrt{\frac{ax-1}{ax+1}}}{a^2 c^2} - \frac{24\sqrt{\frac{ax-1}{ax+1}}}{a^2 c^2 \left( \frac{ax-1}{ax+1} \right)} \right)$$

## 38.42 Problem number 778

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c - \frac{c}{a^2 x^2}\right)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{6}{5a c^3 \left(1 - \frac{1}{ax}\right)^{\frac{5}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{3}{2}}} - \frac{29}{15a c^3 \left(1 - \frac{1}{ax}\right)^{\frac{3}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{3}{2}}} \\ & + \frac{x}{c^3 \left(1 - \frac{1}{ax}\right)^{\frac{5}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{3}{2}}} + \frac{\operatorname{arctanh}\left(\sqrt{1 - \frac{1}{ax}} \sqrt{1 + \frac{1}{ax}}\right)}{a c^3} \\ & - \frac{34}{5a c^3 \left(1 + \frac{1}{ax}\right)^{\frac{3}{2}} \sqrt{1 - \frac{1}{ax}}} + \frac{21 \sqrt{1 - \frac{1}{ax}}}{5a c^3 \left(1 + \frac{1}{ax}\right)^{\frac{3}{2}}} + \frac{16 \sqrt{1 - \frac{1}{ax}}}{5a c^3 \sqrt{1 + \frac{1}{ax}}} \end{aligned}$$

command

`integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{240} a \left( \frac{240 \log\left(\sqrt{\frac{ax-1}{ax+1}} + 1\right)}{a^2 c^3} - \frac{240 \log\left(\left|\sqrt{\frac{ax-1}{ax+1}} - 1\right|\right)}{a^2 c^3} - \frac{(ax+1)^2 \left(\frac{40(ax-1)}{ax+1} + \frac{450(ax-1)^2}{(ax+1)^2} + 3\right)}{(ax-1)^2 a^2 c^3 \sqrt{\frac{ax-1}{ax+1}}} - \frac{480}{a^2 c^3} \right)$$

## 38.43 Problem number 779

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c - \frac{c}{a^2 x^2}\right)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{8}{7a c^4 \left(1 - \frac{1}{ax}\right)^{\frac{7}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{5}{2}}} - \frac{11}{7a c^4 \left(1 - \frac{1}{ax}\right)^{\frac{5}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{5}{2}}} - \frac{62}{21a c^4 \left(1 - \frac{1}{ax}\right)^{\frac{3}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{5}{2}}} \\ & + \frac{x}{c^4 \left(1 - \frac{1}{ax}\right)^{\frac{7}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{5}{2}}} + \frac{\operatorname{arctanh}\left(\sqrt{1 - \frac{1}{ax}} \sqrt{1 + \frac{1}{ax}}\right)}{a c^4} \\ & - \frac{269}{21a c^4 \left(1 + \frac{1}{ax}\right)^{\frac{5}{2}} \sqrt{1 - \frac{1}{ax}}} + \frac{262\sqrt{1 - \frac{1}{ax}}}{35a c^4 \left(1 + \frac{1}{ax}\right)^{\frac{5}{2}}} + \frac{163\sqrt{1 - \frac{1}{ax}}}{35a c^4 \left(1 + \frac{1}{ax}\right)^{\frac{3}{2}}} + \frac{128\sqrt{1 - \frac{1}{ax}}}{35a c^4 \sqrt{1 + \frac{1}{ax}}} \end{aligned}$$

command

`integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{6720} a \left( \frac{6720 \log\left(\sqrt{\frac{ax-1}{ax+1}} + 1\right)}{a^2 c^4} - \frac{6720 \log\left(\left|\sqrt{\frac{ax-1}{ax+1}} - 1\right|\right)}{a^2 c^4} - \frac{5(ax+1)^3 \left(\frac{42(ax-1)}{ax+1} + \frac{329(ax-1)^2}{(ax+1)^2} + \frac{2940(ax-1)^3}{(ax+1)^3}\right)}{(ax-1)^3 a^2 c^4 \sqrt{\frac{ax-1}{ax+1}}} \right)$$



## 38.44 Problem number 793

$$\int \frac{e^{3 \coth^{-1}(ax)}}{c - \frac{c}{a^2 x^2}} dx$$

Optimal antiderivative

$$\frac{3 \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{ax}} \sqrt{1 + \frac{1}{ax}}\right)}{ac} - \frac{5\sqrt{1 + \frac{1}{ax}}}{3ac\left(1 - \frac{1}{ax}\right)^{\frac{3}{2}}} + \frac{x\sqrt{1 + \frac{1}{ax}}}{c\left(1 - \frac{1}{ax}\right)^{\frac{3}{2}}} - \frac{14\sqrt{1 + \frac{1}{ax}}}{3ac\sqrt{1 - \frac{1}{ax}}}$$

command

`integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{3} a \left( \frac{9 \log\left(\sqrt{\frac{ax-1}{ax+1}} + 1\right)}{a^2 c} - \frac{9 \log\left(\left|\sqrt{\frac{ax-1}{ax+1}} - 1\right|\right)}{a^2 c} - \frac{(ax+1)\left(\frac{12(ax-1)}{ax+1} + 1\right)}{(ax-1)a^2 c \sqrt{\frac{ax-1}{ax+1}}} - \frac{6\sqrt{\frac{ax-1}{ax+1}}}{a^2 c \left(\frac{ax-1}{ax+1} - 1\right)} \right)$$

## 38.45 Problem number 795

$$\int \frac{e^{3 \coth^{-1}(ax)}}{\left(c - \frac{c}{a^2 x^2}\right)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3 \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{ax}} \sqrt{1 + \frac{1}{ax}}\right)}{a c^3} - \frac{8}{7a c^3 \left(1 - \frac{1}{ax}\right)^{\frac{7}{2}} \sqrt{1 + \frac{1}{ax}}} \\ & - \frac{53}{35a c^3 \left(1 - \frac{1}{ax}\right)^{\frac{5}{2}} \sqrt{1 + \frac{1}{ax}}} - \frac{88}{35a c^3 \left(1 - \frac{1}{ax}\right)^{\frac{3}{2}} \sqrt{1 + \frac{1}{ax}}} \\ & + \frac{x}{c^3 \left(1 - \frac{1}{ax}\right)^{\frac{7}{2}} \sqrt{1 + \frac{1}{ax}}} - \frac{281}{35a c^3 \sqrt{1 - \frac{1}{ax}} \sqrt{1 + \frac{1}{ax}}} + \frac{176\sqrt{1 - \frac{1}{ax}}}{35a c^3 \sqrt{1 + \frac{1}{ax}}} \end{aligned}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{560} a \left( \frac{1680 \log \left( \sqrt{\frac{ax-1}{ax+1}} + 1 \right)}{a^2 c^3} - \frac{1680 \log \left( \left| \sqrt{\frac{ax-1}{ax+1}} - 1 \right| \right)}{a^2 c^3} - \frac{(ax+1)^3 \left( \frac{56(ax-1)}{ax+1} + \frac{350(ax-1)^2}{(ax+1)^2} + \frac{2520(ax-1)^3}{(ax+1)^3} \right)}{(ax-1)^3 a^2 c^3 \sqrt{\frac{ax-1}{ax+1}}} \right)$$

### 38.46 Problem number 796

$$\int \frac{e^{3 \coth^{-1}(ax)}}{\left( c - \frac{c}{a^2 x^2} \right)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{10}{9a c^4 \left(1 - \frac{1}{ax}\right)^{\frac{9}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{3}{2}}} - \frac{29}{21a c^4 \left(1 - \frac{1}{ax}\right)^{\frac{7}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{3}{2}}} \\ & - \frac{105a c^4 \left(1 - \frac{1}{ax}\right)^{\frac{5}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{3}{2}}}{208} - \frac{315a c^4 \left(1 - \frac{1}{ax}\right)^{\frac{3}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{3}{2}}}{1147} \\ & + \frac{x}{c^4 \left(1 - \frac{1}{ax}\right)^{\frac{9}{2}} \left(1 + \frac{1}{ax}\right)^{\frac{3}{2}}} + \frac{3 \operatorname{arctanh} \left( \sqrt{1 - \frac{1}{ax}} \sqrt{1 + \frac{1}{ax}} \right)}{a c^4} \\ & - \frac{1462}{105a c^4 \left(1 + \frac{1}{ax}\right)^{\frac{3}{2}} \sqrt{1 - \frac{1}{ax}}} + \frac{2609 \sqrt{1 - \frac{1}{ax}}}{315a c^4 \left(1 + \frac{1}{ax}\right)^{\frac{3}{2}}} + \frac{1664 \sqrt{1 - \frac{1}{ax}}}{315a c^4 \sqrt{1 + \frac{1}{ax}}} \end{aligned}$$

command

```
integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{20160} a \left( \frac{60480 \log \left( \sqrt{\frac{ax-1}{ax+1}} + 1 \right)}{a^2 c^4} - \frac{60480 \log \left( \left| \sqrt{\frac{ax-1}{ax+1}} - 1 \right| \right)}{a^2 c^4} - \frac{(ax+1)^4 \left( \frac{450(ax-1)}{ax+1} + \frac{2961(ax-1)^2}{(ax+1)^2} + \frac{144}{(ax+1)^3} \right)}{(ax-1)^4 a^2 c^4 \sqrt{\dots}} \right)$$

### 39 Test file number 201

Test folder name:

test\_cases/7\_Inverse\_hyperbolic\_functions/7.5\_Inverse\_hyperbolic\_secant/201\_7.5.2\_Inverse\_hyp

#### 39.1 Problem number 71

$$\int \frac{e^{2\operatorname{sech}^{-1}(ax)}}{x^2} dx$$

Optimal antiderivative

$$-\frac{4a}{3 \left( 1 - \sqrt{\frac{-ax+1}{ax+1}} \right)^3} + \frac{2a}{\left( 1 - \sqrt{\frac{-ax+1}{ax+1}} \right)^2}$$

command

`integrate((1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2/x^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{3 \left( a^2 + \frac{a}{x} \right) a^2 - \left( 9 a^2 + \left( a^2 + \frac{a}{x} \right) \left( \frac{2 \left( a^2 + \frac{a}{x} \right)}{a^2} - 7 \right) \right) \sqrt{a^2 + \frac{a}{x}} \sqrt{-a^2 + \frac{a}{x}} + 3 \left( 2 a^2 - \frac{a}{x} \right) \sqrt{a^2 + \frac{a}{x}} \sqrt{-a^2 + \frac{a}{x}} - \frac{2a}{x^3}}{3 a^3}$$

## 39.2 Problem number 82

$$\int \frac{e^{-\operatorname{sech}^{-1}(ax)}}{x^2} dx$$

Optimal antiderivative

$$-a \operatorname{arctanh}\left(\sqrt{\frac{-ax+1}{ax+1}}\right) - \frac{a}{\left(1 + \sqrt{\frac{-ax+1}{ax+1}}\right)^2} + \frac{a}{1 + \sqrt{\frac{-ax+1}{ax+1}}}$$

command

```
integrate(1/(1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))/x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{1}{2} \left( \sqrt{a^2 + \frac{a}{x}} \sqrt{-a^2 + \frac{a}{x}} \left( \frac{1}{a^2} - \frac{a^2 + \frac{a}{x}}{a^4} \right) - \frac{2 \left( a^2 + \frac{a}{x} \right) a^2 - \left( a^2 + \frac{a}{x} \right)^2}{a^4} - 2 \log \left( \sqrt{a^2 + \frac{a}{x}} - \sqrt{-a^2 + \frac{a}{x}} \right) \right) a$$

## 40 Test file number 204

Test folder name:

test\_cases/8\_Special\_functions/204\_8.1\_Error\_functions

### 40.1 Problem number 46

$$\int (ex)^m \operatorname{Erf}(d(a + b \log(cx^n))) dx$$

Optimal antiderivative

$$\frac{(ex)^{1+m} \operatorname{erf}(d(a + b \ln(cx^n)))}{e(1+m)} + \frac{e^{\frac{(1+m)(-4ab d^2 n + m + 1)}{4b^2 d^2 n^2}} x (ex)^m \operatorname{erf}\left(\frac{1+m-2ab d^2 n - 2b^2 d^2 n \ln(cx^n)}{2bdn}\right) (cx^n)^{-\frac{1+m}{n}}}{1+m}$$

command

```
integrate((e*x)^m*erf(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{x^{m+1} \operatorname{erf}(bdn \log(x) + bd \log(c) + ad) e^m}{m+1} + \frac{\pi \operatorname{erf}\left(-bdn \log(x) - bd \log(c) - ad + \frac{m}{2bdn} + \frac{1}{2bdn}\right) e^{\left(m - \frac{am}{bn} - \frac{a}{bn} + \frac{m^2}{4b^2d^2n^2} + \frac{m}{2b^2d^2n^2} + \frac{1}{4b^2d^2n^2}\right)}}{(\pi + \pi m)c^{\frac{m}{n}}c^{\left(\frac{1}{n}\right)}}$$

## 40.2 Problem number 149

$$\int (ex)^m \operatorname{Erfc}(d(a + b \log(cx^n))) dx$$

Optimal antiderivative

$$-\frac{e^{\frac{(1+m)(-4abd^2n+m+1)}{4b^2d^2n^2}} x (ex)^m \operatorname{erf}\left(\frac{1+m-2abd^2n-2b^2d^2n \ln(cx^n)}{2bdn}\right) (cx^n)^{-\frac{1+m}{n}}}{1+m} + \frac{(ex)^{1+m} \operatorname{erfc}(d(a + b \ln(cx^n)))}{e(1+m)}$$

command

`integrate((e*x)^m*erfc(d*(a+b*log(c*x^n))),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{xx^m \operatorname{erf}(bdn \log(x) + bd \log(c) + ad) e^m}{m+1} + \frac{xx^m e^m}{m+1} - \frac{\pi \operatorname{erf}\left(-bdn \log(x) - bd \log(c) - ad + \frac{m}{2bdn} + \frac{1}{2bdn}\right) e^{\left(m - \frac{am}{bn} - \frac{a}{bn} + \frac{m^2}{4b^2d^2n^2} + \frac{m}{2b^2d^2n^2} + \frac{1}{4b^2d^2n^2}\right)}}{(\pi + \pi m)c^{\frac{m}{n}}c^{\left(\frac{1}{n}\right)}}$$

## 41 Test file number 206

Test folder name:

test\_cases/8\_Special\_functions/206\_8.4\_Trig\_integral\_functions

### 41.1 Problem number 70

$$\int x^3 \text{CosIntegral}(bx) dx$$

Optimal antiderivative

$$\frac{x^4 \text{cosineIntegral}(bx)}{4} + \frac{3 \cos(bx)}{2b^4} - \frac{3x^2 \cos(bx)}{4b^2} + \frac{3x \sin(bx)}{2b^3} - \frac{x^3 \sin(bx)}{4b}$$

command

`integrate(x^3*fresnel_cos(b*x),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{4} x^4 \text{Ci}(bx) - \frac{3(b^2 x^2 - 2) \cos(bx)}{4b^4} - \frac{(b^3 x^3 - 6bx) \sin(bx)}{4b^4}$$

### 41.2 Problem number 71

$$\int x^2 \text{CosIntegral}(bx) dx$$

Optimal antiderivative

$$\frac{x^3 \text{cosineIntegral}(bx)}{3} - \frac{2x \cos(bx)}{3b^2} + \frac{2 \sin(bx)}{3b^3} - \frac{x^2 \sin(bx)}{3b}$$

command

`integrate(x^2*fresnel_cos(b*x),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{3} x^3 \text{Ci}(bx) - \frac{2x \cos(bx)}{3b^2} - \frac{(b^2 x^2 - 2) \sin(bx)}{3b^3}$$

### 41.3 Problem number 72

$$\int x \operatorname{CosIntegral}(bx) dx$$

Optimal antiderivative

$$\frac{x^2 \operatorname{cosineIntegral}(bx)}{2} - \frac{\cos(bx)}{2b^2} - \frac{x \sin(bx)}{2b}$$

command

```
integrate(x*fresnel_cos(b*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{2} x^2 \operatorname{Ci}(bx) - \frac{x \sin(bx)}{2b} - \frac{\cos(bx)}{2b^2}$$

### 41.4 Problem number 73

$$\int \operatorname{CosIntegral}(bx) dx$$

Optimal antiderivative

$$x \operatorname{cosineIntegral}(bx) - \frac{\sin(bx)}{b}$$

command

```
integrate(fresnel_cos(b*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$x \operatorname{Ci}(bx) - \frac{\sin(bx)}{b}$$

### 41.5 Problem number 75

$$\int \frac{\text{CosIntegral}(bx)}{x^2} dx$$

Optimal antiderivative

$$-\frac{\text{cosineIntegral}(bx)}{x} - \frac{\cos(bx)}{x} - b \text{sinIntegral}(bx)$$

command

```
integrate(fresnel_cos(b*x)/x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{bx\Im(\text{Ci}(bx)) \tan\left(\frac{1}{2}bx\right)^2 - bx\Im(\text{Ci}(-bx)) \tan\left(\frac{1}{2}bx\right)^2 + 2bx \text{Si}(bx) \tan\left(\frac{1}{2}bx\right)^2 + bx\Im(\text{Ci}(bx)) - bx\Im(\text{Ci}(-bx))}{2\left(x \tan\left(\frac{1}{2}bx\right)^2 + x\right)} - \frac{\text{Ci}(bx)}{x}$$

### 41.6 Problem number 76

$$\int \frac{\text{CosIntegral}(bx)}{x^3} dx$$

Optimal antiderivative

$$-\frac{b^2 \text{cosineIntegral}(bx)}{4} - \frac{\text{cosineIntegral}(bx)}{2x^2} - \frac{\cos(bx)}{4x^2} + \frac{b \sin(bx)}{4x}$$

command

```
integrate(fresnel_cos(b*x)/x^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{b^2x^2 \text{Ci}(bx) + b^2x^2 \text{Ci}(-bx) - 2bx \sin(bx) + 2 \cos(bx)}{8x^2} - \frac{\text{Ci}(bx)}{2x^2}$$



## 41.7 Problem number 78

$$\int x^3 \operatorname{CosIntegral}(bx)^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x^2}{4b^2} + \frac{x^4 \operatorname{cosineIntegral}(bx)^2}{4} - \frac{3 \operatorname{cosineIntegral}(2bx)}{2b^4} + \frac{3 \operatorname{cosineIntegral}(bx) \cos(bx)}{b^4} \\ & - \frac{3x^2 \operatorname{cosineIntegral}(bx) \cos(bx)}{2b^2} + \frac{3(\cos^2(bx))}{8b^4} - \frac{3 \ln(x)}{2b^4} + \frac{3x \operatorname{cosineIntegral}(bx) \sin(bx)}{b^3} \\ & - \frac{x^3 \operatorname{cosineIntegral}(bx) \sin(bx)}{2b} + \frac{x \cos(bx) \sin(bx)}{b^3} - \frac{13(\sin^2(bx))}{8b^4} + \frac{x^2(\sin^2(bx))}{4b^2} \end{aligned}$$

command

```
integrate(x^3*fresnel_cos(b*x)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\begin{aligned} & \frac{1}{4} x^4 \operatorname{Ci}(bx)^2 - \frac{1}{2} \left( \frac{3(b^2 x^2 - 2) \cos(bx)}{b^4} + \frac{(b^3 x^3 - 6bx) \sin(bx)}{b^4} \right) \operatorname{Ci}(bx) \\ & - \frac{b^2 x^2 \cos(2bx) - 3b^2 x^2 - 4bx \sin(2bx) - 8 \cos(2bx) + 6 \operatorname{Ci}(2bx) + 6 \operatorname{Ci}(-2bx) + 12 \log(x)}{8b^4} \end{aligned}$$

## 41.8 Problem number 79

$$\int x^2 \operatorname{CosIntegral}(bx)^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x}{2b^2} + \frac{x^3 \operatorname{cosineIntegral}(bx)^2}{3} - \frac{4x \operatorname{cosineIntegral}(bx) \cos(bx)}{3b^2} \\ & - \frac{2 \operatorname{sinIntegral}(2bx)}{3b^3} + \frac{4 \operatorname{cosineIntegral}(bx) \sin(bx)}{3b^3} \\ & - \frac{2x^2 \operatorname{cosineIntegral}(bx) \sin(bx)}{3b} + \frac{5 \cos(bx) \sin(bx)}{6b^3} + \frac{x(\sin^2(bx))}{3b^2} \end{aligned}$$

command

```
integrate(x^2*fresnel_cos(b*x)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{3} x^3 \operatorname{Ci}(bx)^2 - \frac{2}{3} \left( \frac{2x \cos(bx)}{b^2} + \frac{(b^2 x^2 - 2) \sin(bx)}{b^3} \right) \operatorname{Ci}(bx) + \frac{5bx \tan(bx)^2 - 2\Im(\operatorname{Ci}(2bx)) \tan(bx)^2 + 2\Im(\operatorname{Ci}(-2bx)) \tan(bx)^2 - 4\operatorname{Si}(2bx) \tan(bx)^2 + 3bx - 2\Im(\operatorname{Ci}(2bx))}{6(b^3 \tan(bx)^2 + b^3)}$$

### 41.9 Problem number 80

$$\int x \operatorname{CosIntegral}(bx)^2 dx$$

Optimal antiderivative

$$\frac{x^2 \operatorname{cosineIntegral}(bx)^2}{2} + \frac{\operatorname{cosineIntegral}(2bx)}{2b^2} - \frac{\operatorname{cosineIntegral}(bx) \cos(bx)}{b^2} + \frac{\ln(x)}{2b^2} - \frac{x \operatorname{cosineIntegral}(bx) \sin(bx)}{b} + \frac{\sin^2(bx)}{2b^2}$$

command

`integrate(x*fresnel_cos(b*x)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{2} x^2 \operatorname{Ci}(bx)^2 - \left( \frac{x \sin(bx)}{b} + \frac{\cos(bx)}{b^2} \right) \operatorname{Ci}(bx) - \frac{\cos(2bx) - \operatorname{Ci}(2bx) - \operatorname{Ci}(-2bx) - 2 \log(x)}{4b^2}$$

### 41.10 Problem number 81

$$\int \operatorname{CosIntegral}(bx)^2 dx$$

Optimal antiderivative

$$x \operatorname{cosineIntegral}(bx)^2 + \frac{\operatorname{sinIntegral}(2bx)}{b} - \frac{2 \operatorname{cosineIntegral}(bx) \sin(bx)}{b}$$

command

```
integrate(fresnel_cos(b*x)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$x \operatorname{Ci}(bx)^2 - \frac{2 \operatorname{Ci}(bx) \sin(bx)}{b} + \frac{\Im(\operatorname{Ci}(2bx)) - \Im(\operatorname{Ci}(-2bx)) + 2 \operatorname{Si}(2bx)}{2b}$$

#### 41.11 Problem number 86

$$\int x^3 \operatorname{CosIntegral}(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{a^4 \operatorname{cosineIntegral}(bx+a)}{4b^4} + \frac{x^4 \operatorname{cosineIntegral}(bx+a)}{4} + \frac{3 \cos(bx+a)}{2b^4} \\ & -\frac{a^2 \cos(bx+a)}{4b^4} + \frac{ax \cos(bx+a)}{2b^3} - \frac{3x^2 \cos(bx+a)}{4b^2} - \frac{a \sin(bx+a)}{2b^4} + \frac{a^3 \sin(bx+a)}{4b^4} \\ & + \frac{3x \sin(bx+a)}{2b^3} - \frac{a^2 x \sin(bx+a)}{4b^3} + \frac{ax^2 \sin(bx+a)}{4b^2} - \frac{x^3 \sin(bx+a)}{4b} \end{aligned}$$

command

```
integrate(x^3*fresnel_cos(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{4} x^4 \operatorname{Ci}(bx+a)$$

$$\frac{a^4 \cos(a)^2 \operatorname{Ci}(bx+a) + a^4 \cos(a)^2 \operatorname{Ci}(-bx-a) + 2b^3 x^3 \sin(bx+a) + a^4 \operatorname{Ci}(bx+a) \sin(a)^2 + a^4 \operatorname{Ci}(-bx-a) \sin(a)^2}{4}$$

## 41.12 Problem number 87

$$\int x^2 \operatorname{CosIntegral}(a + bx) dx$$

Optimal antiderivative

$$\frac{a^3 \operatorname{cosineIntegral}(bx + a)}{3b^3} + \frac{x^3 \operatorname{cosineIntegral}(bx + a)}{3} + \frac{a \cos(bx + a)}{3b^3} - \frac{2x \cos(bx + a)}{3b^2} + \frac{2 \sin(bx + a)}{3b^3} - \frac{a^2 \sin(bx + a)}{3b^3} + \frac{ax \sin(bx + a)}{3b^2} - \frac{x^2 \sin(bx + a)}{3b}$$

command

```
integrate(x^2*fresnel_cos(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{3} x^3 \operatorname{Ci}(bx + a) + \frac{a^3 \cos(a)^2 \operatorname{Ci}(bx + a) + a^3 \cos(a)^2 \operatorname{Ci}(-bx - a) + a^3 \operatorname{Ci}(bx + a) \sin(a)^2 + a^3 \operatorname{Ci}(-bx - a) \sin(a)^2 - 2b^2 x^2 \sin(bx + a)}{6b^3}$$

## 41.13 Problem number 88

$$\int x \operatorname{CosIntegral}(a + bx) dx$$

Optimal antiderivative

$$-\frac{a^2 \operatorname{cosineIntegral}(bx + a)}{2b^2} + \frac{x^2 \operatorname{cosineIntegral}(bx + a)}{2} - \frac{\cos(bx + a)}{2b^2} + \frac{a \sin(bx + a)}{2b^2} - \frac{x \sin(bx + a)}{2b}$$

command

```
integrate(x*fresnel_cos(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{2} x^2 \operatorname{Ci}(bx + a) - \frac{a^2 \cos(a)^2 \operatorname{Ci}(bx + a) + a^2 \cos(a)^2 \operatorname{Ci}(-bx - a) + a^2 \operatorname{Ci}(bx + a) \sin(a)^2 + a^2 \operatorname{Ci}(-bx - a) \sin(a)^2 + 2bx \sin(bx + a)}{4b^2}$$

## 41.14 Problem number 89

$$\int \text{CosIntegral}(a + bx) dx$$

Optimal antiderivative

$$\frac{(bx + a) \text{cosineIntegral}(bx + a)}{b} - \frac{\sin(bx + a)}{b}$$

command

```
integrate(fresnel_cos(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$x \text{Ci}(bx + a) + \frac{a \cos(a)^2 \text{Ci}(bx + a) + a \cos(a)^2 \text{Ci}(-bx - a) + a \text{Ci}(bx + a) \sin(a)^2 + a \text{Ci}(-bx - a) \sin(a)^2 - 2 \cos(a) \sin(bx)}{2b}$$

## 41.15 Problem number 91

$$\int \frac{\text{CosIntegral}(a + bx)}{x^2} dx$$

Optimal antiderivative

$$-\frac{b \text{cosineIntegral}(bx + a)}{a} - \frac{\text{cosineIntegral}(bx + a)}{x} + \frac{b \text{cosineIntegral}(bx) \cos(a)}{a} - \frac{b \text{sinIntegral}(bx) \sin(a)}{a}$$

command

```
integrate(fresnel_cos(b*x+a)/x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\left( \Re(\text{Ci}(bx + a)) \tan\left(\frac{1}{2}a\right)^2 + \Re(\text{Ci}(bx)) \tan\left(\frac{1}{2}a\right)^2 + \Re(\text{Ci}(-bx - a)) \tan\left(\frac{1}{2}a\right)^2 + \Re(\text{Ci}(-bx)) \tan\left(\frac{1}{2}a\right)^2 + 2 \Im(\text{Ci}(bx + a)) \tan\left(\frac{1}{2}a\right) \right)}{x} - \frac{\text{Ci}(bx + a)}{x}$$

### 41.16 Problem number 92

$$\int \frac{\text{CosIntegral}(a + bx)}{x^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b^2 \text{cosineIntegral}(bx + a)}{2a^2} - \frac{\text{cosineIntegral}(bx + a)}{2x^2} \\ & - \frac{b^2 \text{cosineIntegral}(bx) \cos(a)}{2a^2} - \frac{b \cos(bx + a)}{2ax} - \frac{b^2 \cos(a) \text{sinIntegral}(bx)}{2a} \\ & - \frac{b^2 \text{cosineIntegral}(bx) \sin(a)}{2a} + \frac{b^2 \text{sinIntegral}(bx) \sin(a)}{2a^2} \end{aligned}$$

command

```
integrate(fresnel_cos(b*x+a)/x^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 41.17 Problem number 103

$$\int \frac{\text{CosIntegral}(d(a + b \log(cx^n)))}{x} dx$$

Optimal antiderivative

$$\frac{\text{cosineIntegral}(d(a + b \ln(cx^n))) (a + b \ln(cx^n))}{bn} - \frac{\sin(d(a + b \ln(cx^n)))}{bdn}$$

command

```
integrate(fresnel_cos(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{(bdn \log(x) + bd \log(c) + ad) \text{Ci}(bdn \log(x) + bd \log(c) + ad) - \sin(bdn \log(x) + bd \log(c) + ad)}{bdn}$$

## 41.18 Problem number 110

$$\int \text{CosIntegral}(bx) \sin(bx) dx$$

Optimal antiderivative

$$\frac{\text{cosineIntegral}(2bx)}{2b} - \frac{\text{cosineIntegral}(bx) \cos(bx)}{b} + \frac{\ln(x)}{2b}$$

command

```
integrate(fresnel_cos(b*x)*sin(b*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{\cos(bx) \text{Ci}(bx)}{b} + \frac{\text{Ci}(2bx) + \text{Ci}(-2bx) + 2 \log(x)}{4b}$$

## 41.19 Problem number 111

$$\int x \text{CosIntegral}(bx) \sin(bx) dx$$

Optimal antiderivative

$$\frac{x}{2b} - \frac{x \text{cosineIntegral}(bx) \cos(bx)}{b} - \frac{\text{sinIntegral}(2bx)}{2b^2} + \frac{\text{cosineIntegral}(bx) \sin(bx)}{b^2} + \frac{\cos(bx) \sin(bx)}{2b^2}$$

command

```
integrate(x*fresnel_cos(b*x)*sin(b*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\left(\frac{x \cos(bx)}{b} - \frac{\sin(bx)}{b^2}\right) \text{Ci}(bx) + \frac{2bx \tan(bx)^2 - \Im(\text{Ci}(2bx)) \tan(bx)^2 + \Im(\text{Ci}(-2bx)) \tan(bx)^2 - 2 \text{Si}(2bx) \tan(bx)^2 + 2bx - \Im(\text{Ci}(2bx)) + \Im(\text{Ci}(-2bx))}{4(b^2 \tan(bx)^2 + b^2)}$$

## 41.20 Problem number 112

$$\int x^2 \operatorname{CosIntegral}(bx) \sin(bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x^2}{4b} - \frac{\operatorname{cosineIntegral}(2bx)}{b^3} + \frac{2 \operatorname{cosineIntegral}(bx) \cos(bx)}{b^3} - \frac{x^2 \operatorname{cosineIntegral}(bx) \cos(bx)}{b} \\ & + \frac{\cos^2(bx)}{4b^3} - \frac{\ln(x)}{b^3} + \frac{2x \operatorname{cosineIntegral}(bx) \sin(bx)}{b^2} + \frac{x \cos(bx) \sin(bx)}{2b^2} - \frac{\sin^2(bx)}{b^3} \end{aligned}$$

command

```
integrate(x^2*fresnel_cos(b*x)*sin(b*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\begin{aligned} & \left( \frac{2x \sin(bx)}{b^2} - \frac{(b^2 x^2 - 2) \cos(bx)}{b^3} \right) \operatorname{Ci}(bx) \\ & + \frac{2b^2 x^2 + 2bx \sin(2bx) + 5 \cos(2bx) - 4 \operatorname{Ci}(2bx) - 4 \operatorname{Ci}(-2bx) - 8 \log(x)}{8b^3} \end{aligned}$$

## 41.21 Problem number 113

$$\int x^3 \operatorname{CosIntegral}(bx) \sin(bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5x}{2b^3} + \frac{x^3}{6b} + \frac{6x \operatorname{cosineIntegral}(bx) \cos(bx)}{b^3} - \frac{x^3 \operatorname{cosineIntegral}(bx) \cos(bx)}{b} \\ & + \frac{x(\cos^2(bx))}{2b^3} + \frac{3 \operatorname{sinIntegral}(2bx)}{b^4} - \frac{6 \operatorname{cosineIntegral}(bx) \sin(bx)}{b^4} \\ & + \frac{3x^2 \operatorname{cosineIntegral}(bx) \sin(bx)}{b^2} - \frac{4 \cos(bx) \sin(bx)}{b^4} + \frac{x^2 \cos(bx) \sin(bx)}{2b^2} - \frac{3x(\sin^2(bx))}{2b^3} \end{aligned}$$

command

```
integrate(x^3*fresnel_cos(b*x)*sin(b*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output



could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\left(\frac{(b^3 x^3 - 6 b x) \cos(bx)}{b^4} - \frac{3(b^2 x^2 - 2) \sin(bx)}{b^4}\right) \text{Ci}(bx) + \frac{b^3 x^3 \tan(bx)^2 + b^3 x^3 + 3 b^2 x^2 \tan(bx) - 24 b x \tan(bx)^2 + 9 \Im(\text{Ci}(2bx)) \tan(bx)^2 - 9 \Im(\text{Ci}(-2bx)) \tan(bx)^2 + 1}{6(b^4 \tan(bx)^2 + b^4)}$$

#### 41.22 Problem number 117

$$\int \cos(bx) \text{CosIntegral}(bx) dx$$

Optimal antiderivative

$$-\frac{\sinIntegral(2bx)}{2b} + \frac{\cosineIntegral(bx) \sin(bx)}{b}$$

command

```
integrate(fresnel_cos(b*x)*cos(b*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\text{Ci}(bx) \sin(bx)}{b} - \frac{\Im(\text{Ci}(2bx)) - \Im(\text{Ci}(-2bx)) + 2 \text{Si}(2bx)}{4b}$$

#### 41.23 Problem number 118

$$\int x \cos(bx) \text{CosIntegral}(bx) dx$$

Optimal antiderivative

$$-\frac{\cosineIntegral(2bx)}{2b^2} + \frac{\cosineIntegral(bx) \cos(bx)}{b^2} - \frac{\ln(x)}{2b^2} + \frac{x \cosineIntegral(bx) \sin(bx)}{b} - \frac{\sin^2(bx)}{2b^2}$$

command

```
integrate(x*fresnel_cos(b*x)*cos(b*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\left(\frac{x \sin(bx)}{b} + \frac{\cos(bx)}{b^2}\right) \text{Ci}(bx) + \frac{\cos(2bx) - \text{Ci}(2bx) - \text{Ci}(-2bx) - 2 \log(x)}{4b^2}$$

## 41.24 Problem number 119

$$\int x^2 \cos(bx) \operatorname{CosIntegral}(bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3x}{4b^2} + \frac{2x \operatorname{cosineIntegral}(bx) \cos(bx)}{b^2} + \frac{\operatorname{sinIntegral}(2bx)}{b^3} - \frac{2 \operatorname{cosineIntegral}(bx) \sin(bx)}{b^3} \\ & + \frac{x^2 \operatorname{cosineIntegral}(bx) \sin(bx)}{b} - \frac{5 \cos(bx) \sin(bx)}{4b^3} - \frac{x(\sin^2(bx))}{2b^2} \end{aligned}$$

command

```
integrate(x^2*fresnel_cos(b*x)*cos(b*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\begin{aligned} & \left( \frac{2x \cos(bx)}{b^2} + \frac{(b^2 x^2 - 2) \sin(bx)}{b^3} \right) \operatorname{Ci}(bx) \\ & - \frac{5bx \tan(bx)^2 - 2 \Im(\operatorname{Ci}(2bx)) \tan(bx)^2 + 2 \Im(\operatorname{Ci}(-2bx)) \tan(bx)^2 - 4 \operatorname{Si}(2bx) \tan(bx)^2 + 3bx - 2 \Im(\operatorname{Ci}(2bx))}{4(b^3 \tan(bx)^2 + b^3)} \end{aligned}$$

## 41.25 Problem number 120

$$\int x^3 \cos(bx) \operatorname{CosIntegral}(bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{x^2}{2b^2} + \frac{3 \operatorname{cosineIntegral}(2bx)}{b^4} - \frac{6 \operatorname{cosineIntegral}(bx) \cos(bx)}{b^4} \\ & + \frac{3x^2 \operatorname{cosineIntegral}(bx) \cos(bx)}{b^2} - \frac{3(\cos^2(bx))}{4b^4} + \frac{3 \ln(x)}{b^4} - \frac{6x \operatorname{cosineIntegral}(bx) \sin(bx)}{b^3} \\ & + \frac{x^3 \operatorname{cosineIntegral}(bx) \sin(bx)}{b} - \frac{2x \cos(bx) \sin(bx)}{b^3} + \frac{13(\sin^2(bx))}{4b^4} - \frac{x^2(\sin^2(bx))}{2b^2} \end{aligned}$$

command

```
integrate(x^3*fresnel_cos(b*x)*cos(b*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\left( \frac{3(b^2x^2 - 2)\cos(bx)}{b^4} + \frac{(b^3x^3 - 6bx)\sin(bx)}{b^4} \right) \text{Ci}(bx) + \frac{b^2x^2 \cos(2bx) - 3b^2x^2 - 4bx \sin(2bx) - 8 \cos(2bx) + 6 \text{Ci}(2bx) + 6 \text{Ci}(-2bx) + 12 \log(x)}{4b^4}$$

#### 41.26 Problem number 121

$$\int \text{CosIntegral}(2x) \sin(5x) dx$$

Optimal antiderivative

$$\frac{\text{cosineIntegral}(3x)}{10} + \frac{\text{cosineIntegral}(7x)}{10} - \frac{\text{cosineIntegral}(2x) \cos(5x)}{5}$$

command

```
integrate(fresnel_cos(2*x)*sin(5*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{1}{5} \cos(5x) \text{Ci}(2x) + \frac{1}{10} \text{Ci}(7x) + \frac{1}{10} \text{Ci}(3x)$$

#### 41.27 Problem number 122

$$\int \cos(5x) \text{CosIntegral}(2x) dx$$

Optimal antiderivative

$$-\frac{\text{sinIntegral}(3x)}{10} - \frac{\text{sinIntegral}(7x)}{10} + \frac{\text{cosineIntegral}(2x) \sin(5x)}{5}$$

command

```
integrate(fresnel_cos(2*x)*cos(5*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{5} \text{Ci}(2x) \sin(5x) - \frac{1}{10} \text{Si}(7x) - \frac{1}{10} \text{Si}(3x)$$

## 41.28 Problem number 123

$$\int x^2 \operatorname{CosIntegral}(a + bx) \sin(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{ax}{2b^2} + \frac{x^2}{4b} - \frac{\operatorname{cosineIntegral}(2bx + 2a)}{b^3} + \frac{a^2 \operatorname{cosineIntegral}(2bx + 2a)}{2b^3} \\ & + \frac{2 \operatorname{cosineIntegral}(bx + a) \cos(bx + a)}{b^3} - \frac{x^2 \operatorname{cosineIntegral}(bx + a) \cos(bx + a)}{b} \\ & + \frac{\cos^2(bx + a)}{4b^3} + \frac{\cos(2bx + 2a)}{2b^3} - \frac{\ln(bx + a)}{b^3} + \frac{a^2 \ln(bx + a)}{2b^3} \\ & + \frac{a \operatorname{sinIntegral}(2bx + 2a)}{b^3} + \frac{2x \operatorname{cosineIntegral}(bx + a) \sin(bx + a)}{b^2} \\ & - \frac{a \cos(bx + a) \sin(bx + a)}{2b^3} + \frac{x \cos(bx + a) \sin(bx + a)}{2b^2} \end{aligned}$$

command

`integrate(x^2*fresnel_cos(b*x+a)*sin(b*x+a),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\begin{aligned} & \left( \frac{2x \sin(bx + a)}{b^2} - \frac{(b^2 x^2 - 2) \cos(bx + a)}{b^3} \right) \operatorname{Ci}(bx + a) \\ & + \frac{2b^2 x^2 \tan(bx + a)^2 - 4abx \tan(bx + a)^2 + 4a^2 \log(|bx + a|) \tan(bx + a)^2 + 2a^2 \Re(\operatorname{Ci}(2bx + 2a)) \tan(bx + a)^2}{2b^3} \end{aligned}$$

## 41.29 Problem number 124

$$\int x \operatorname{CosIntegral}(a + bx) \sin(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x}{2b} - \frac{a \operatorname{cosineIntegral}(2bx + 2a)}{2b^2} - \frac{x \operatorname{cosineIntegral}(bx + a) \cos(bx + a)}{b} - \frac{a \ln(bx + a)}{2b^2} \\ & - \frac{\operatorname{sinIntegral}(2bx + 2a)}{2b^2} + \frac{\operatorname{cosineIntegral}(bx + a) \sin(bx + a)}{b^2} + \frac{\cos(bx + a) \sin(bx + a)}{2b^2} \end{aligned}$$

command

```
integrate(x*fresnel_cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\left(\frac{x \cos (b x+a)}{b}-\frac{\sin (b x+a)}{b^2}\right) \operatorname{Ci}(b x+a) \\ +\frac{2 b x \tan (b x)^2 \tan (a)^2-2 a \log (|b x+a|) \tan (b x)^2 \tan (a)^2-a \Re(\operatorname{Ci}(2 b x+2 a)) \tan (b x)^2 \tan (a)^2-a \Re(\operatorname{Ci}(-2 b x-2 a)) \tan (b x)^2 \tan (a)^2}{4 b}$$

### 41.30 Problem number 125

$$\int \operatorname{CosIntegral}(a+b x) \sin (a+b x) d x$$

Optimal antiderivative

$$\frac{\operatorname{cosineIntegral}(2 b x+2 a)}{2 b}-\frac{\operatorname{cosineIntegral}(b x+a) \cos (b x+a)}{b}+\frac{\ln (b x+a)}{2 b}$$

command

```
integrate(fresnel_cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\cos (b x+a) \operatorname{Ci}(b x+a)}{b} \\ +\frac{\cos (2 a)^2 \operatorname{Ci}(2 b x+2 a)+\cos (2 a)^2 \operatorname{Ci}(-2 b x-2 a)+\operatorname{Ci}(2 b x+2 a) \sin (2 a)^2+\operatorname{Ci}(-2 b x-2 a) \sin (2 a)^2+2 \operatorname{Ci}(2 a) \sin (2 a)}{4 b}$$

## 41.31 Problem number 127

$$\int x^2 \cos(a + bx) \operatorname{CosIntegral}(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{x}{b^2} + \frac{a \operatorname{cosineIntegral}(2bx + 2a)}{b^3} + \frac{2x \operatorname{cosineIntegral}(bx + a) \cos(bx + a)}{b^2} \\ & - \frac{a \cos(2bx + 2a)}{4b^3} + \frac{x \cos(2bx + 2a)}{4b^2} + \frac{a \ln(bx + a)}{b^3} + \frac{\operatorname{sinIntegral}(2bx + 2a)}{b^3} \\ & - \frac{a^2 \operatorname{sinIntegral}(2bx + 2a)}{2b^3} - \frac{2 \operatorname{cosineIntegral}(bx + a) \sin(bx + a)}{b^3} \\ & + \frac{x^2 \operatorname{cosineIntegral}(bx + a) \sin(bx + a)}{b} - \frac{\cos(bx + a) \sin(bx + a)}{b^3} - \frac{\sin(2bx + 2a)}{8b^3} \end{aligned}$$

command

```
integrate(x^2*fresnel_cos(b*x+a)*cos(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\left( \frac{2x \cos(bx + a)}{b^2} + \frac{(b^2 x^2 - 2) \sin(bx + a)}{b^3} \right) \operatorname{Ci}(bx + a) - \frac{a^2 \Im(\operatorname{Ci}(2bx + 2a)) \tan(bx + a)^2 - a^2 \Im(\operatorname{Ci}(-2bx - 2a)) \tan(bx + a)^2 + 2a^2 \operatorname{Si}(2bx + 2a) \tan(bx + a)^2 + 5bx}{b^3}$$

## 41.32 Problem number 128

$$\int x \cos(a + bx) \operatorname{CosIntegral}(a + bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\operatorname{cosineIntegral}(2bx + 2a)}{2b^2} + \frac{\operatorname{cosineIntegral}(bx + a) \cos(bx + a)}{b^2} + \frac{\cos(2bx + 2a)}{4b^2} \\ & - \frac{\ln(bx + a)}{2b^2} + \frac{a \operatorname{sinIntegral}(2bx + 2a)}{2b^2} + \frac{x \operatorname{cosineIntegral}(bx + a) \sin(bx + a)}{b} \end{aligned}$$

command

```
integrate(x*fresnel_cos(b*x+a)*cos(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\left( \frac{x \sin(bx + a)}{b} + \frac{\cos(bx + a)}{b^2} \right) \text{Ci}(bx + a) + a \Im(\text{Ci}(2bx + 2a)) \tan(bx)^2 \tan(a)^2 - a \Im(\text{Ci}(-2bx - 2a)) \tan(bx)^2 \tan(a)^2 + 2a \text{Si}(2bx + 2a) \tan(bx)^2 \tan(a)$$

### 41.33 Problem number 129

$$\int \cos(a + bx) \text{CosIntegral}(a + bx) dx$$

Optimal antiderivative

$$-\frac{\text{sinIntegral}(2bx + 2a)}{2b} + \frac{\text{cosineIntegral}(bx + a) \sin(bx + a)}{b}$$

command

`integrate(fresnel_cos(b*x+a)*cos(b*x+a),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{\text{Ci}(bx + a) \sin(bx + a)}{b} - \frac{\Im(\text{Ci}(2bx + 2a)) - \Im(\text{Ci}(-2bx - 2a)) + 2 \text{Si}(2bx + 2a)}{4b}$$

### 41.34 Problem number 132

$$\int \text{CosIntegral}(c + dx) \sin(a + bx) dx$$

Optimal antiderivative

$$\frac{\text{cosineIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2b} + \frac{\text{cosineIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2b} - \frac{\text{cosineIntegral}(dx + c) \cos(bx + a)}{b} - \frac{\text{sinIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b} - \frac{\text{sinIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b}$$

command

```
integrate(fresnel_cos(d*x+c)*sin(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 41.35 Problem number 135

$$\int \cos(a + bx) \operatorname{CosIntegral}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\cos\left(a - \frac{bc}{d}\right) \operatorname{sinIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right)}{2b} - \frac{\cos\left(a - \frac{bc}{d}\right) \operatorname{sinIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right)}{2b} \\ & - \frac{\operatorname{cosineIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b} \\ & - \frac{\operatorname{cosineIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b} + \frac{\operatorname{cosineIntegral}(dx + c) \sin(bx + a)}{b} \end{aligned}$$

command

```
integrate(fresnel_cos(d*x+c)*cos(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

output too large to display



## 42 Test file number 209

Test folder name:

test\_cases/209\_Blake\_problems

### 42.1 Problem number 403

$$\int \frac{1}{\sqrt[4]{1+x^4}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{x}{(x^4+1)^{\frac{1}{4}}}\right)}{2} + \frac{\operatorname{arctanh}\left(\frac{x}{(x^4+1)^{\frac{1}{4}}}\right)}{2}$$

command

```
integrate(1/(x^4+1)^(1/4),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{1}{2} \arctan\left(\frac{(x^4+1)^{\frac{1}{4}}}{x}\right) + \frac{1}{4} \log\left(\frac{(x^4+1)^{\frac{1}{4}}}{x} + 1\right) - \frac{1}{4} \log\left(\frac{(x^4+1)^{\frac{1}{4}}}{x} - 1\right)$$

### 42.2 Problem number 1081

$$\int \frac{1+x}{\sqrt{-7+4x+14x^2-12x^3+x^4}} dx$$

Optimal antiderivative

$$\arctan\left(\frac{-4+4x}{1-2x+x^2-\sqrt{x^4-12x^3+14x^2+4x-7}}\right) + \ln(-1+x) - \ln\left(-5+6x-x^2+\sqrt{x^4-12x^3+14x^2+4x-7}\right)$$

command

```
integrate((1+x)/(x^4-12*x^3+14*x^2+4*x-7)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: NotImplementedError

Giac 1.7.0 via sagemath 9.3 output

$$\begin{aligned}
 & \arctan \left( \frac{\frac{1}{7} \sqrt{7} \left( \sqrt{2} + \frac{3 \left( \sqrt{7} \sqrt{-\frac{10}{x} - \frac{7}{x^2} + 1} - 4 \sqrt{2} \right)}{\frac{7}{x} + 5} \right)}{\operatorname{sgn} \left( -\frac{1}{x^2} + \frac{1}{x^3} \right)} \right) \\
 - & \log \left( \frac{10 \sqrt{7} + 40 \sqrt{2} + \frac{50 \left( \sqrt{7} \sqrt{-\frac{10}{x} - \frac{7}{x^2} + 1} - 4 \sqrt{2} \right)}{\frac{7}{x} + 5}}{\operatorname{sgn} \left( -\frac{1}{x^2} + \frac{1}{x^3} \right)} \right) \\
 + & \log \left( \frac{-2 \sqrt{7} + 8 \sqrt{2} + \frac{10 \left( \sqrt{7} \sqrt{-\frac{10}{x} - \frac{7}{x^2} + 1} - 4 \sqrt{2} \right)}{\frac{7}{x} + 5}}{\operatorname{sgn} \left( -\frac{1}{x^2} + \frac{1}{x^3} \right)} \right)
 \end{aligned}$$

### 42.3 Problem number 1123

$$\int \frac{\sqrt{1-4x+x^2} + (1-4x+x^2)^{3/2}}{\sqrt{1-4x+x^2} + (1-4x+x^2)^{3/2} - (1-4x+x^2)^{5/2}} dx$$

Optimal antiderivative

*Unintegrable*

command

```
integrate(((x^2-4*x+1)^(1/2)+(x^2-4*x+1)^(3/2))/((x^2-4*x+1)^(1/2)+(x^2-4*x+1)^(3/2)-(x^2-4*x+1)^(5/2)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\begin{aligned}
& \frac{\left( \left( \sqrt{\frac{1}{2}\sqrt{5} + \frac{7}{2}} - 2 \right)^2 + 4\sqrt{\frac{1}{2}\sqrt{5} + \frac{7}{2}} - 6 \right) \log \left( x + \sqrt{\frac{1}{2}\sqrt{5} + \frac{7}{2}} - 2 \right)}{2 \left( 2 \left( \sqrt{\frac{1}{2}\sqrt{5} + \frac{7}{2}} - 2 \right)^3 + 12 \left( \sqrt{\frac{1}{2}\sqrt{5} + \frac{7}{2}} - 2 \right)^2 + 17\sqrt{\frac{1}{2}\sqrt{5} + \frac{7}{2}} - 32 \right)} \\
& \frac{\left( \left( \sqrt{\frac{1}{2}\sqrt{5} + \frac{7}{2}} + 2 \right)^2 - 4\sqrt{\frac{1}{2}\sqrt{5} + \frac{7}{2}} - 6 \right) \log \left( x - \sqrt{\frac{1}{2}\sqrt{5} + \frac{7}{2}} - 2 \right)}{2 \left( 2 \left( \sqrt{\frac{1}{2}\sqrt{5} + \frac{7}{2}} + 2 \right)^3 - 12 \left( \sqrt{\frac{1}{2}\sqrt{5} + \frac{7}{2}} + 2 \right)^2 + 17\sqrt{\frac{1}{2}\sqrt{5} + \frac{7}{2}} + 32 \right)} \\
& + \frac{\left( \left( \sqrt{-\frac{1}{2}\sqrt{5} + \frac{7}{2}} - 2 \right)^2 + 4\sqrt{-\frac{1}{2}\sqrt{5} + \frac{7}{2}} - 6 \right) \log \left( x + \sqrt{-\frac{1}{2}\sqrt{5} + \frac{7}{2}} - 2 \right)}{2 \left( 2 \left( \sqrt{-\frac{1}{2}\sqrt{5} + \frac{7}{2}} - 2 \right)^3 + 12 \left( \sqrt{-\frac{1}{2}\sqrt{5} + \frac{7}{2}} - 2 \right)^2 + 17\sqrt{-\frac{1}{2}\sqrt{5} + \frac{7}{2}} - 32 \right)} \\
& + \frac{\left( \left( \sqrt{-\frac{1}{2}\sqrt{5} + \frac{7}{2}} + 2 \right)^2 - 4\sqrt{-\frac{1}{2}\sqrt{5} + \frac{7}{2}} - 6 \right) \log \left( x - \sqrt{-\frac{1}{2}\sqrt{5} + \frac{7}{2}} - 2 \right)}{2 \left( 2 \left( \sqrt{-\frac{1}{2}\sqrt{5} + \frac{7}{2}} + 2 \right)^3 - 12 \left( \sqrt{-\frac{1}{2}\sqrt{5} + \frac{7}{2}} + 2 \right)^2 + 17\sqrt{-\frac{1}{2}\sqrt{5} + \frac{7}{2}} + 32 \right)}
\end{aligned}$$

#### 42.4 Problem number 1648

$$\int \frac{1}{\sqrt[4]{-1 - 3x^4 - 2x^8 + 2x^{12} + 3x^{16} + x^{20}}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{(x^{20}+3x^{16}+2x^{12}-2x^8-3x^4-1)^{\frac{1}{4}}2^{\frac{3}{4}}}{2x(x^4+1)}\right)2^{\frac{3}{4}}}{4} + \frac{\operatorname{arctanh}\left(\frac{(x^{20}+3x^{16}+2x^{12}-2x^8-3x^4-1)^{\frac{1}{4}}2^{\frac{3}{4}}}{2x(x^4+1)}\right)2^{\frac{3}{4}}}{4}$$

command

```
integrate(1/(x^20+3*x^16+2*x^12-2*x^8-3*x^4-1)^(1/4),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{4} \cdot 2^{\frac{3}{4}} \arctan\left(\frac{2^{\frac{3}{4}}(x^4-1)^{\frac{1}{4}}}{2x}\right) - \frac{1}{8} \cdot 2^{\frac{3}{4}} \log\left(2^{\frac{1}{4}} + \frac{(x^4-1)^{\frac{1}{4}}}{x}\right) + \frac{1}{8} \cdot 2^{\frac{3}{4}} \log\left(2^{\frac{1}{4}} - \frac{(x^4-1)^{\frac{1}{4}}}{x}\right)$$

## 43 Test file number 210

Test folder name:

test\_cases/210\_Hebisch

### 43.1 Problem number 42

$$\int e^{-17+e} \frac{6x+e^{17}(-5+4x+x^2)}{e^{17}} + \frac{6x+e^{17}(-5+4x+x^2)}{e^{17}} (6+e^{17}(4+2x)) dx$$

Optimal antiderivative

$$5 + e^{(4+x+6e^{-17})x-5}$$

command

```
integrate(((2*x+4)*exp(17)+6)*exp(((x^2+4*x-5)*exp(17)+6*x)/exp(17))*exp(exp(((x^2+4*x-5)*exp(17)+6*x)/exp(17)))/exp(17), x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e\left(e\left(\left(x^2e^{17}+4xe^{17}+6x-5e^{17}\right)e^{(-17)}\right)\right)$$

### 43.2 Problem number 214

$$\int \frac{e^{\frac{2}{3}(5e^{9+6x+x^2}-3x-3x^2 \log(e^2+2x))} \left(-6e^2 - 12x - 12x^2 + e^{9+6x+x^2}(120x + 40x^2 + e^2(60 + 20x)) + (-12e^2x - 24x^2)\right)}{3e^2 + 6x} dx$$

Optimal antiderivative

$$e^{\frac{10e^{(3+x)^2}}{3} - 2x - 2x^2 \ln(e^2+2x)}$$

command

```
integrate((((-12*exp(2)*x-24*x^2)*log(exp(2)+2*x)+((20*x+60)*exp(2)+40*x^2+120*x)*exp(x^2+6*x+6*exp(2)-12*x^2-12*x)*exp(-x^2*log(exp(2)+2*x)+5/3*exp(x^2+6*x+9)-x)^2/(3*exp(2)+6*x)), x, algo
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e\left(-2x^2 \log(2x+e^2) - 2x + \frac{10}{3} e^{(x^2+6x+9)}\right)$$

### 43.3 Problem number 529

$$\int \frac{e^{3-x} - \frac{e^{3-x}(1+5e^{-3+x})}{x}}{5x} \left( -1 - x - e^{-3+x}x + 6e^{-3+x} + \frac{e^{3-x}(1+5e^{-3+x})}{x}x \right) dx$$

Optimal antiderivative

$$\frac{6x}{5} - 1 - \frac{x e^{-5 - \frac{e^{3-x}}{x}}}{5} - e^3$$

command

```
integrate(1/5*(6*x*exp(-3+x)*exp((5*x*exp(-3+x)+1)/x/exp(-3+x))-x*exp(-3+x)-x-1)/x/exp(-3+x)/exp((5*x*exp(-3+x)+1)/x/exp(-3+x)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{1}{5} x e^{\left(-\frac{5x+e^{(-x+3)}}{x}\right)} + \frac{6}{5} x$$

### 43.4 Problem number 756

$$\int \frac{e^{1+8x+x^2+x \log\left(\frac{7-3e^4+6x}{3x}\right)} \left( -49 - 62x - 12x^2 + e^4(21+6x) + (-7+3e^4-6x) \log\left(\frac{7-3e^4+6x}{3x}\right) \right)}{-7+3e^4-6x} dx$$

Optimal antiderivative

$$e^{x \left( 8 + \ln\left(\frac{7-e^4}{3x} + 2\right) + x \right) + 1}$$

command

```
integrate(((3*exp(4)-6*x-7)*log(1/3*(-3*exp(4)+6*x+7)/x)+(6*x+21)*exp(4)-12*x^2-62*x-49)*exp(x*log(1/3*(-3*exp(4)+6*x+7)/x)+x^2+8*x+1)/(3*exp(4)-6*x-7),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left(x^2+x \log\left(-\frac{e^4}{x} + \frac{7}{3x} + 2\right) + 8x + 1\right)}$$

### 43.5 Problem number 819

$$\int \frac{10x - 3e^{2x}x - 6e^xx^3 - 3x^5 + (e^{3x}(6 + 12x) + e^{2x}(36x^2 + 12x^3) + e^x(-20 + 30x^4)) \log(10x - 3e^{2x}x - 6e^xx^3 - 3x^5)}{-10x + 3e^{2x}x + 6e^xx^3 + 3x^5}$$

Optimal antiderivative

$$e^x \ln\left(x - 3x\left((e^x + x^2)^2 - 3\right)\right)^2 - x$$

command

```
integrate(((3*x*exp(x)^3+6*exp(x)^2*x^3+(3*x^5-10*x)*exp(x))*log(-3*x*exp(x)^2-6*exp(x)*x^3-3*x^5+10*x)^2+((12*x+6)*exp(x)^3+(12*x^3+36*x^2)*exp(x)^2+(30*x^4-20)*exp(x))*log(-3*x*exp(x)^2-6*exp(x)*x^3-3*x^5+10*x)-3*x*exp(x)^2-6*exp(x)*x^3-3*x^5+10*x)/(3*x*exp(x)^2+6*exp(x)*x,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$e^x \log\left(-3x^5 - 6x^3e^x - 3xe^{2x} + 10x\right)^2 - x$$

### 43.6 Problem number 856

$$\int \frac{e^{\frac{-x^2+x \log(x^2)+\log\left(\frac{16-\log(3+x)}{e}\right)}{x}} \left(95x - 16x^2 - 16x^3 + (-6x + x^2 + x^3) \log(3+x) + (-48 - 16x + (3+x) \log(3+x))\right)}{-48x^2 - 16x^3 + (3x^2 + x^3) \log(3+x)}$$

Optimal antiderivative

$$-e^{\ln(x^2)+\frac{\ln\left(\frac{-\ln(3+x)+16}{e}\right)}{x}} - x$$

command

```
integrate((((3+x)*log(3+x)-16*x-48)*log((-log(3+x)+16)/exp(1))+(x^3+x^2-6*x)*log(3+x)-16*x^3-16*x^2+95*x)*exp((log((-log(3+x)+16)/exp(1))+x*log(x^2)-x^2)/x)/((x^3+3*x^2)*log(3+x)-16*x^3-48*x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-e^{\left(-x+\frac{\log\left(\frac{-e^{(-1)} \log(x+3)+16 e^{(-1)}}{x}\right)+\log(x^2)}{x}\right)}$$

### 43.7 Problem number 1147

$$\int \frac{(-x + 2x \log(x) + (1 + e^x + 2x) \log^2(x)) \sqrt{\frac{x^4 + (2x^2 + 2e^x x^2 + 2x^3 + 2x^4) \log(x) + (1 + e^{2x} + 2x + 3x^2 + 2x^3) \log^2(x)}{\log^2(x)}}}{x^2 \log(x) + (1 + e^x + x + x^2) \log^2(x)} dx$$

Optimal antiderivative

$$\sqrt{\left(-1 - x - \frac{x^2}{\ln(x)} - x^2 - e^x\right)^2}$$

command

```
integrate(((exp(x)+2*x+1)*log(x)^2+2*x*log(x)-x)*((exp(x)^2+(2*x^2+2*x+2)*exp(x)+x^4+2*x^3+3*x^2+2*x+1)*log(x)^2)/(x^2*log(x)+(1+exp(x)+x+x^2)*log(x)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{1}{3} \log(x)^3 + \frac{3}{2} x^2 - \frac{1}{2} \log(x)^2 + 2x + \frac{x^2}{\log(x)} + e^x$$

### 43.8 Problem number 1197

$$\int \frac{e^{\frac{-2x+3x^2+2x^3+8e^8 \log(x^2)}{2+x}} (-8x + 24x^2 + 30x^3 + 8x^4 + e^8(64 + 32x) - 16e^8 x \log(x^2))}{4x + 4x^2 + x^3} dx$$

Optimal antiderivative

$$2e^{\frac{8e^8 \ln(x^2)}{2+x} - x + 2x^2}$$

command

```
integrate((-16*x*exp(8)*log(x^2)+(32*x+64)*exp(8)+8*x^4+30*x^3+24*x^2-8*x)*exp((8*exp(8)*log(2*x)/(2+x))/(x^3+4*x^2+4*x)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$2e^{\left(\frac{2x^3}{x+2} + \frac{3x^2}{x+2} + \frac{8e^8 \log(x^2)}{x+2} - \frac{2x}{x+2}\right)}$$

### 43.9 Problem number 1569

$$\int \frac{e^{9+2x(10+2x^2)} + e^{9+x(100x+20x^3)} + e^9(250x^2+50x^4)}{x^2} \frac{(100e^9x^4 + e^{9+2x}(-20 + 20x + 4x^3) + e^{9+x}(-100x + 100x^2 + 20x^3 + \dots))}{x^3}$$

Optimal antiderivative

$$e^{\frac{10(x^2+5)e^9(x+\frac{e^x}{5})(5x+e^x)}{x^2}}$$

command

```
integrate(((4*x^3+20*x-20)*exp(9)*exp(x)^2+(20*x^4+20*x^3+100*x^2-100*x)*exp(9)*exp(x)+100*x^2
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left(50x^2e^9+20xe^{(x+9)}+\frac{100e^{(x+9)}}{x}+\frac{10e^{(2x+9)}}{x^2}+250e^9+2e^{(2x+9)}\right)}$$

### 43.10 Problem number 1596

$$\int \frac{e^{\frac{2}{5} \left( 5e^{\frac{-2e^2+x \log^2\left(\frac{2+3x}{x}\right)}{x^2}} - x \right)} \left( -4x^3 - 6x^4 + e^{\frac{-2e^2+x \log^2\left(\frac{2+3x}{x}\right)}{x^2}} \left( e^2(80 + 120x) - 40x \log\left(\frac{2+3x}{x}\right) + (-20x - 30x^2) \log \dots \right) \right)}{10x^3 + 15x^4}$$

Optimal antiderivative

$$e^{2e^{\frac{\ln\left(4-\frac{-2+x}{x}\right)^2-\frac{2e^2}{x}}{x}}-\frac{2x}{5}}$$

command

```
integrate(((((-30*x^2-20*x)*log((2+3*x)/x)^2-40*x*log((2+3*x)/x)+(120*x+80)*exp(2))*exp((x*log(2*exp(2))/x^2)-6*x^4-4*x^3)*exp(exp((x*log((2+3*x)/x)^2-2*exp(2))/x^2)-1/5*x)^2/(15*x^4+10*x^3
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left(-\frac{2}{5}x+2e^{\left(\frac{\log\left(\frac{2}{x}+3\right)^2-\frac{2e^2}{x^2}}{x}\right)}\right)}$$



### 43.11 Problem number 1763

$$\int \frac{(-2x^2 - 8e^x x^2) \log^2(x) + ((-2x^2 + 8e^x x^3) \log(x) + (-2x^2 + 8e^x x^3) \log^2(x)) \log(e^{-x}(-2 + 8e^x x)) + ((2x + 8e^x x^2) \log(x) + (-2x^2 + 8e^x x^3) \log^2(x)) \log(e^{-x}(-2 + 8e^x x))}{(2x + 8e^x x^2) \log(x) + (-2x^2 + 8e^x x^3) \log^2(x)}$$

Optimal antiderivative

$$(x - \ln(\ln(8x - 2e^{-x})))^2 \ln(x)^2$$

command

```
integrate(((8*exp(x)*x-2)*log(x)*log((8*exp(x)*x-2)/exp(x))*log(log((8*exp(x)*x-2)/exp(x)))^2+
8*exp(x)*x^2+2*x)*log(x)^2+(-16*exp(x)*x^2+4*x)*log(x))*log((8*exp(x)*x-2)/exp(x))+
(8*exp(x)*x^2)/exp(x))+((8*exp(x)*x^3-2*x^2)*log(x)^2+(8*exp(x)*x^3-2*x^2)*log(x))*log((8*exp(x)*x-
2)/exp(x))+(-8*exp(x)*x^2-2*x^2)*log(x)^2)/(4*exp(x)*x^2-x)/log((8*exp(x)*x-2)/exp(x)),x, alg
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$x^2 \log(x)^2 - 2x \log(x)^2 \log(-x + \log(2) + \log(4xe^x - 1)) + \log(x)^2 \log(-x + \log(2) + \log(4xe^x - 1))^2$$

### 43.12 Problem number 1809

$$\int \frac{e^{-e^{-5+e^5-x}} (20x - 4x^2 - x^3) \left( -20 + 8x + 3x^2 + e^{-5+e^5-x} (-20x + 4x^2 + x^3) \right)}{-20x + 4x^2 + x^3} dx$$

Optimal antiderivative

$$e^{\ln((-x^2-4x+20)x) - e^{5-x-5}}$$

command

```
integrate(((x^3+4*x^2-20*x)*exp(exp(5)-x-5)+3*x^2+8*x-20)*exp(log(-x^3-4*x^2+20*x)-
exp(exp(5)-x-5)))/(x^3+4*x^2-20*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\left( x^3 e^{\left( -x+e^5-e^{(-x+e^5-5)} \right) -5} + 4x^2 e^{\left( -x+e^5-e^{(-x+e^5-5)} \right) -5} - 20xe^{\left( -x+e^5-e^{(-x+e^5-5)} \right) -5} \right) e^{(x-e^5+5)}$$

### 43.13 Problem number 1860

$$\int \frac{e^{9-x} \log\left(\frac{e^{e^2} \log^2(-4+x)}{x^2}\right)}{\log\left(\frac{e^{e^2} \log^2(-4+x)}{x^2}\right)} \left( -2e^9 x + e^9(-8+2x) \log(-4+x) + (4x-x^2) \log(-4+x) \log^2\left(\frac{e^{e^2} \log^2(-4+x)}{x^2}\right) \right) \frac{dx}{(-4x+x^2) \log(-4+x) \log^2\left(\frac{e^{e^2} \log^2(-4+x)}{x^2}\right)}$$

Optimal antiderivative

$$e^{\frac{e^9}{\ln\left(\frac{e^{e^2} \ln(-4+x)^2}{x^2}\right)} - x}$$

command

```
integrate((-x^2+4*x)*log(x-4)*log(exp(exp(2))*log(x-4)^2/x^2)^2+(2*x-8)*exp(9)*log(x-4)-2*x*exp(9))*exp((-x*log(exp(exp(2))*log(x-4)^2/x^2)+exp(9))/log(exp(exp(2))*log(x-4)^2/x^2))/(x^2-4*x)/log(x-4)/log(exp(exp(2))*log(x-4)^2/x^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left( -x + \frac{e^9}{\log\left(\frac{e^{e^2} \log(x-4)^2}{x^2}\right)} \right)}$$

### 43.14 Problem number 1905

$$\int -\frac{e^{25+\frac{1}{5}} \left( -1 + \log^{-\frac{e^{25}x}{5+2x}}(5) \right) \log^{-\frac{e^{25}x}{5+2x}}(5) \log(\log(5))}{25+20x+4x^2} dx$$

Optimal antiderivative

$$e^{\frac{\ln(\ln(5))x e^{25}}{-2x-5} - \frac{1}{5}}$$

command

```
integrate(-exp(25)*log(log(5))*exp(-x*exp(25)*log(log(5))/(5+2*x))*exp(1/5*exp(-x*exp(25)*log(1/5)/(4*x^2+20*x+25)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left(\frac{1}{5} \log(5) - \frac{x e^{25}}{2x+5} - \frac{1}{5}\right)}$$

### 43.15 Problem number 1996

$$\int \frac{-1500e^{15-3x}x^5 + e^{20-4x}(-2500x^5 + 2500x^6) + e^x(e^{20-4x}(2500x^3 - 2500x^4) + e^{15-3x}(-1500x^3 + 1500x^4)) + \dots}{\dots}$$

Optimal antiderivative

$$\left(x \ln\left(\frac{x}{2} - \frac{e^x}{2x}\right) + \frac{5x e^{5-x}}{3}\right)^4$$

command

```
integrate(((324*exp(x)*x^3-324*x^5)*log(1/2*(-exp(x)+x^2)/x)^4+((( -540*x^4+2160*x^3)*exp(5-x)+324*x^4-324*x^3)*exp(x)+(540*x^6-2160*x^5)*exp(5-x)-324*x^5)*log(1/2*(-exp(x)+x^2)/x)^3+((2700*x^4+5400*x^3)*exp(5-x)^2+(1620*x^4-1620*x^3)*exp(5-x))*exp(x)+(2700*x^6-5400*x^5)*exp(5-x)^2-1620*x^5*exp(5-x))*log(1/2*(-exp(x)+x^2)/x)^2+((( -4500*x^4+6000*x^3)*exp(5-x)^3+(2700*x^4-2700*x^3)*exp(5-x)^2)*exp(x)+(4500*x^6-6000*x^5)*exp(5-x)^3-2700*x^5*exp(5-x)^2)*log(1/2*(-exp(x)+x^2)/x)+((-2500*x^4+2500*x^3)*exp(5-x)^4+(1500*x^4-1500*x^3)*exp(5-x)^3)*exp(x)+(2500*x^5*exp(5-x)^4-1500*x^5*exp(5-x)^3)/(81*exp(x)-81*x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{81} \left(81 x^4 e^{(4x)} \log(2)^4 - 324 x^4 e^{(4x)} \log(2)^3 \log(x^2 - e^x) + 486 x^4 e^{(4x)} \log(2)^2 \log(x^2 - e^x)^2 - 324 x^4 e^{(4x)} \log(2) \log(x^2 - e^x)^3 + \dots\right)$$

### 43.16 Problem number 2050

$$\int \frac{e^2(2+e)}{-e^2 + 6e^{2x+ex}} dx$$

Optimal antiderivative

$$\frac{1}{3} + \ln\left(\frac{e^2 e^{-x(e+2)}}{2} - 3\right)$$

command

```
integrate((exp(1)+2)*exp(2)/(6*exp(x*exp(1)+2*x)-exp(2)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: NotImplementedError

Giac 1.7.0 via sagemath 9.3 output

$$-\left(x(e+2)e^{(-2)} - e^{(-2)} \log\left(\left|-e^2 + 6e^{(x(e+2))}\right|\right)\right)e^2$$

### 43.17 Problem number 2306

$$\int \frac{e^{16x^8}(-839808 - 209952x) + e^{16x^8}(10917504x^7 + 6718464x^8 + 839808x^9) \log(13 + 8x + x^2) + e^{12x^8}(279936 +$$

Optimal antiderivative

$$\left(\frac{9e^{4x^8}}{\ln((4+x)^2 - 3)^4} - 1\right)^4$$

command

```
integrate((( -1152*x^9-9216*x^8-14976*x^7)*exp(x^8)^4*log(x^2+8*x+13)^13+(288*x+1152)*exp(x^8)
7776*x-31104)*exp(x^8)^8*log(x^2+8*x+13)^8+(-279936*x^9-2239488*x^8-3639168*x^7)*exp(x^8)^12*
209952*x-839808)*exp(x^8)^16)/(x^2+8*x+13)/log(x^2+8*x+13)^17,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$\frac{9\left(4e^{(4x^8)} \log(x^2 + 8x + 13)^{12} - 54e^{(8x^8)} \log(x^2 + 8x + 13)^8 + 324e^{(12x^8)} \log(x^2 + 8x + 13)^4 - 729e^{(16x^8)}\right)}{\log(x^2 + 8x + 13)^{16}}$$

### 43.18 Problem number 2370

$$\int \frac{\frac{1}{27}e^{3x}x^2 + \frac{1}{3}e^{2x}x^3 + e^xx^4 + x^5 + e^{x + \frac{100}{9 - \frac{2e^{2x}}{3} + 2e^x x + x^2}} \left( \frac{1}{27}e^{3x}(1-x) + 200x + x^3 - x^4 + \frac{1}{9}e^{2x}(3x - 3x^2) + \frac{1}{3}e^x(200x \right)}{\frac{1}{27}e^{3x}x^2 + \frac{1}{3}e^{2x}x^3 + e^xx^4 + x^5}$$

Optimal antiderivative

$$x - \frac{e^x e^{\left(\frac{e^{-\ln(3)+x}}{2} + \frac{x}{2}\right)^2}}{x}$$

command

```
integrate((((1-x)*exp(-log(3)+x)^3+(-3*x^2+3*x)*exp(-log(3)+x)^2+(-3*x^3+3*x^2+200*x)*exp(-log(3)+x)-x^4+x^3+200*x)*exp(x)*exp(100/(exp(-log(3)+x)^2+2*x*exp(-log(3)+x)+x^2))+x^2*exp(-log(3)+x)^3+3*x^3*exp(-log(3)+x)^2+3*x^4*exp(-log(3)+x)+x^5)/(x^2*exp(-log(3)+x)^3+3*x^3*exp(-log(3)+x)^2+3*x^4*exp(-log(3)+x)+x^5),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{x^2 - e^{\left(\frac{9x^3+6x^2e^x+xe^{(2x)}+900}{9x^2+6xe^x+e^{(2x)}}\right)}}{x}$$

### 43.19 Problem number 2446

$$\int \frac{e^{\frac{1}{3}(-e^{-4+\sqrt[4]{x}}-3x\log(3))} \left(-e^{-4+\sqrt[4]{x}}\sqrt[4]{x} - 12x\log(3)\right)}{48x} dx$$

Optimal antiderivative

$$\frac{e^{-\frac{e^{x^{\frac{1}{4}}-4}}{3}-x\ln(3)}}{4}$$

command

```
integrate(1/48*(-x^(1/4)*exp(x^(1/4)-4)-12*x*log(3))*exp(-1/3*exp(x^(1/4)-4)-x*log(3))/x,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{1}{4}e^{\left(-\left(x^{\frac{1}{4}}-4\right)^4\log(3)-16\left(x^{\frac{1}{4}}-4\right)^3\log(3)-96\left(x^{\frac{1}{4}}-4\right)^2\log(3)-256\left(x^{\frac{1}{4}}-4\right)\log(3)-\frac{1}{3}e^{\left(x^{\frac{1}{4}}-4\right)}-256\log(3)\right)}$$

### 43.20 Problem number 2680

$$\int \frac{12x - 12x^2 - 12x^3 + 24x^4 - 12x^5 + (-12 + 12x + 24x^2 - 48x^3 + 24x^4) \log(2) + (-12x + 24x^2 - 12x^3) \log^2(2)}{(-4x^2 + 11x^3 - 10x^4 + 3x^5 + (8x - 22x^2))}$$

Optimal antiderivative

$$\frac{4x - \frac{4x}{(\ln(2)-x)(x^2-x)}}{\ln\left(2 - \frac{3x}{2}\right)}$$

command

```
integrate((((12*x^3-40*x^2+44*x-16)*log(2)^2+(-24*x^4+80*x^3-88*x^2+44*x-16)*log(2)+12*x^5-40*x^4+44*x^3-40*x^2+44*x-16)*log(2-3/2*x)+(-12*x^3+24*x^2-12*x)*log(2)^2+(24*x^4-48*x^3+24*x^2+12*x-12)*log(2)-12*x^5+24*x^4-12*x^3-12*x^2+12*x)/((3*x^3-10*x^2+11*x-4)*log(2)^2+(-6*x^4+20*x^3-22*x^2+8*x)*log(2)+3*x^5-10*x^4+11*x^3-4*x^2)/log(2-3/2*x)^2,x,al
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$\frac{4(x^3 - x^2 \log(2) - x^2 + x \log(2) + 1)}{x^2 \log(2) - x \log(2)^2 - x^2 \log(-3x + 4) + x \log(2) \log(-3x + 4) - x \log(2) + \log(2)^2 + x \log(-3x + 4) - \log^2(2)}$$

### 43.21 Problem number 2802

$$\int \frac{e^{\frac{9x}{\log\left(\frac{e^{10}+2e^5x+x^2}{e^8}\right)}} \left(36x + (-18e^5 - 18x) \log\left(\frac{e^{10}+2e^5x+x^2}{e^8}\right)\right)}{e^{\frac{18x}{\log\left(\frac{e^{10}+2e^5x+x^2}{e^8}\right)}} (e^5 + x) \log^2\left(\frac{e^{10}+2e^5x+x^2}{e^8}\right) + e^{\frac{9x}{\log\left(\frac{e^{10}+2e^5x+x^2}{e^8}\right)}} (2e^7 + 2e^2x) \log^2\left(\frac{e^{10}+2e^5x+x^2}{e^8}\right) + (e^9 + e^4x) \log\left(\frac{e^{10}+2e^5x+x^2}{e^8}\right)}$$

Optimal antiderivative

$$\frac{2}{e^{\frac{9x}{\ln\left(\left(e^5+x\right)^2e^{-8}\right)}} + e^2}$$

command

```
integrate((-18*exp(5)-18*x)*log((exp(5)^2+2*x*exp(5)+x^2)/exp(4)^2)+36*x)*exp(9*x/log((exp(5)
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$x^2 e^2 \log(x^2 + 2xe^5 + e^{10})^3 + x^2 e^{\left(\frac{9x}{\log(x^2 + 2xe^5 + e^{10}) - 8}\right)} \log(x^2 + 2xe^5 + e^{10})^3 - 26x^2 e^2 \log(x^2 + 2xe^5 + e^{10})^2 - 26$$

### 43.22 Problem number 2888

$$\int \frac{1}{3} \left( -5 + e^{-9-x+e^5x-e^xx} (-3 + 3e^5 + e^x(-3-3x)) \right) dx$$

Optimal antiderivative

$$1 - \frac{5x}{3} + e^{x(e^5-e^x)-x-9}$$

command

```
integrate(1/3*((-3*x-3)*exp(x)+3*exp(5)-3)*exp(-exp(x)*x*x*exp(5)-x-9)-5/3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\frac{5}{3}x + e^{(xe^5-xe^x-x-9)}$$

### 43.23 Problem number 2912

$$\int \frac{e^{\frac{4-4x-3x^2+2x^3+x^4}{x-2e^8x+e^2e^8x}} \left( 4 + 3x^2 - 4x^3 - 3x^4 + e^{e^8x} (-4 - 3x^2 + 4x^3 + 3x^4 + e^8(-8x + 8x^2 + 6x^3 - 4x^4 - 2x^5)) \right)}{-x^2 + 3e^{e^8x}x^2 - 3e^{2e^8x}x^2 + e^{3e^8x}x^2} dx$$

Optimal antiderivative

$$\frac{(x^2+x-2)^2}{e^{x(1-e^xe^8)^2}}$$

command

```
integrate((((-2*x^5-4*x^4+6*x^3+8*x^2-8*x)*exp(4)^2+3*x^4+4*x^3-3*x^2-4)*exp(x*exp(4)^2)-3*x^4-4*x^3+3*x^2+4)*exp((x^4+2*x^3-3*x^2-4*x+4)/(x*exp(x*exp(4)^2)^2-2*x*exp(x*exp(4)^2)+x))3*x^2*exp(x*exp(4)^2)^2+3*x^2*exp(x*exp(4)^2)-x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left( \frac{x^4}{xe^{(2xe^8)}-2xe^{(xe^8)}+x} + \frac{2x^3}{xe^{(2xe^8)}-2xe^{(xe^8)}+x} - \frac{3x^2}{xe^{(2xe^8)}-2xe^{(xe^8)}+x} - \frac{4x}{xe^{(2xe^8)}-2xe^{(xe^8)}+x} + \frac{4}{xe^{(2xe^8)}-2xe^{(xe^8)}+x} \right)}$$

### 43.24 Problem number 2955

$$\int e^{\frac{6561+5832x^2-2916x^3+1944x^4-1944x^5+774x^6-432x^7+232x^8+e^4x^8+e^5x^8-68x^9+24x^{10}-8x^{11}+x^{12}+e^2(162x^4+72x^6-36x^7+8x^8-8x^9+2x^{10})+(-2916x^2)}{}}$$

Optimal antiderivative

$$e^{\left(e^2+\left(\frac{9}{x^2}-\ln(x)+2-x\right)^2\right)+e^5} - 1$$

command

```
integrate(((4*x^9+4*x^8+72*x^6)*log(x)^3+(12*x^10-12*x^9-24*x^8+108*x^7-540*x^6-1944*x^4)*log
36*x^10+48*x^8-648*x^7+1296*x^6-2916*x^5+8748*x^4+17496*x^2)*log(x)+(4*x^10-4*x^9-
8*x^8+36*x^7-180*x^6-648*x^4)*exp(2)+4*x^12-20*x^11+24*x^10-20*x^9-140*x^8+864*x^7-
1980*x^6+6804*x^5-9720*x^4+14580*x^3-37908*x^2-52488)*exp((x^8*log(x)^4+(4*x^9-8*x^8-
36*x^6)*log(x)^3+(2*x^8*exp(2)+6*x^10-24*x^9+24*x^8-108*x^7+216*x^6+486*x^4)*log(x)^2+((4*x^9
8*x^8-36*x^6)*exp(2)+4*x^11-24*x^10+48*x^9-140*x^8+432*x^7-432*x^6+972*x^5-1944*x^4-
2916*x^2)*log(x)+x^8*exp(5)+x^8*exp(2)^2+(2*x^10-8*x^9+8*x^8-36*x^7+72*x^6+162*x^4)*exp(2)+x^
8*x^11+24*x^10-68*x^9+232*x^8-432*x^7+774*x^6-1944*x^5+1944*x^4-2916*x^3+5832*x^2+6561)/x^8)/
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left(x^4+4x^3\log(x)+6x^2\log(x)^2+4x\log(x)^3+\log(x)^4-8x^3+2x^2e^2-24x^2\log(x)+4xe^2\log(x)-24x\log(x)^2+2e^2\log(x)^2-8\log(x)^3+24x^2-8xe^2+\right)}$$

### 43.25 Problem number 3083

$$\int \frac{(80-20x)\log\left(\frac{1}{3}e^{-16+8x-x^2}(-1-15e^{16-8x+x^2})\right)}{3+45e^{16-8x+x^2}} dx$$

Optimal antiderivative

$$\frac{5\ln\left(-\frac{e^{-(4-x)^2}}{3}-5\right)^2}{3}$$

command

```
integrate((-20*x+80)*log(1/3*(-15*exp(x^2-8*x+16)-1)/exp(x^2-8*x+16)))/(45*exp(x^2-
8*x+16)+3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{160}{3}x^2 - \frac{1280}{3}x - \frac{160}{3}\log\left(15e^{(x^2-8x+16)}+1\right)$$



### 43.26 Problem number 3165

$$\int \frac{-1 + 6x - x^2 + 3 \log(x)}{(12x^2 - 4x^3 + 4x \log(x)) \log\left(\frac{2x^3}{3x - x^2 + \log(x)}\right) + (3x^2 - x^3 + x \log(x)) \log\left(\frac{2x^3}{3x - x^2 + \log(x)}\right) \log\left(\log\left(\frac{2x^3}{3x - x^2 + \log(x)}\right)\right)}$$

Optimal antiderivative

$$\ln\left(-12 - 3 \ln\left(\ln\left(\frac{2x^2}{3 + \frac{\ln(x)}{x} - x}\right)\right)\right)$$

command

```
integrate((3*log(x)-x^2+6*x-1)/((x*log(x)-x^3+3*x^2)*log(2*x^3/(log(x)-x^2+3*x))*log(log(2*x^3/(log(x)-x^2+3*x))))+(4*x*log(x)-4*x^3+12*x^2)*log(2*x^3/(log(x)-x^2+3*x))),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\log(\log(i\pi + \log(2) - \log(x^2 - 3x - \log(x)) + 3 \log(x)) + 4)$$

### 43.27 Problem number 3683

$$\int \frac{-4x - 4e^x x^2 - 8x^3 + (4 + 4e^x x + 8x^2) \log(x) + (e^x(2 - 2x) + 2x^2 - 2x^3 + (2 - 2x) \log(x)) \log(e^x + x^2 + \log(x))}{(-e^x x^2 - x^4 + (e^x x - x^2 + x^3) \log(x) + x \log^2(x)) \log(e^x + x^2 + \log(x)) \log(\log(e^x + x^2 + \log(x)))}$$

Optimal antiderivative

$$\ln\left((\ln(x) - x)^2 \ln(\ln(\ln(x) + x^2 + e^x))^4\right)$$

command

```
integrate((((2-2*x)*log(x)+(2-2*x)*exp(x)-2*x^3+2*x^2)*log(log(x)+x^2+exp(x))*log(log(log(x)+4*exp(x)*x^2-8*x^3-4*x)/(x*log(x)^2+(exp(x)*x+x^3-x^2)*log(x)-exp(x)*x^2-x^4)/log(log(x)+x^2+log(x))))))
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$2 \log(-x + \log(x)) + 4 \log(\log(\log(x^2 + e^x + \log(x))))$$

### 43.28 Problem number 3722

$$\int \frac{e^{-\frac{50e^{3-x}}{x}} \left( e^{3-x}(7200 + 7200x) + e^{\frac{50e^{3-x}}{x}} (18x^2 + 18x^3) + e^{\frac{25e^{3-x}}{x}} (-72x^2 + e^{3-x}(-1800 - 3600x - 1800x^2)) \right)}{x^2} dx$$

Optimal antiderivative

$$3 \left( 1 - 4e^{-\frac{25e^{3-x}}{x}} + x \right) \left( 3 - 12e^{-\frac{25e^{3-x}}{x}} + 3x \right)$$

command

```
integrate(((18*x^3+18*x^2)*exp(25*exp(3-x)/x)^2+((-1800*x^2-3600*x-1800)*exp(3-x)-72*x^2)*exp(25*exp(3-x)/x)+(7200*x+7200)*exp(3-x))/x^2/exp(25*exp(3-x)/x)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$9 \left( x^2 e^{(-x+3)} + 2x e^{(-x+3)} - 8x e^{\left( -\frac{x^2-3x+25e^{(-x+3)}}{x} \right)} + 16 e^{\left( -\frac{x^2-3x+50e^{(-x+3)}}{x} \right)} - 8 e^{\left( -\frac{x^2-3x+25e^{(-x+3)}}{x} \right)} \right) e^{(x-3)}$$

### 43.29 Problem number 3788

$$\int e^{\frac{-6+5e^{10}x-5x\log(25x)}{-10+5x}} \frac{(16 - 10e^{10} - 5x + 10\log(25x))}{20 - 20x + 5x^2} dx$$

Optimal antiderivative

$$e^{\frac{x(e^{10} - \ln(25x) - \frac{6}{5})}{-2+x}}$$

command

```
integrate((10*log(25*x)-10*exp(5)^2-5*x+16)*exp((-5*x*log(25*x)+5*x*exp(5)^2-6)/(5*x-10))/(5*x^2-20*x+20),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left( \frac{xe^{10}}{x-2} - \frac{x\log(25x)}{x-2} - \frac{6}{5(x-2)} \right)}$$

### 43.30 Problem number 3922

$$\int \frac{(1 + e^5 - 2x) \log^2(1 + e^5 - 2x) + e^{\frac{x}{\log(1+e^5-2x)}} (-2x^2 \log(\frac{3x}{2}) + (-x - e^5x + 2x^2) \log(1 + e^5 - 2x) \log(\frac{3x}{2}))}{(x + e^5x - 2x^2) \log^2(1 + e^5 - 2x) \log(\frac{3x}{2})} dx$$

Optimal antiderivative

$$\ln\left(\ln\left(\frac{3x}{2}\right) e^{-e^{\frac{x}{\ln(e^5+1-2x)}}}\right)$$

command

```
integrate((((-x*exp(5)+2*x^2-x)*log(3/2*x)*log(exp(5)+1-2*x)-2*x^2*log(3/2*x))*exp(x/log(exp(2*x)))+(exp(5)+1-2*x)*log(exp(5)+1-2*x)^2)/(x*exp(5)-2*x^2+x)/log(3/2*x)/log(exp(5)+1-2*x)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-e^{\left(\frac{x}{\log(-2x+e^5+1)}\right)} + \log(-\log(2) + \log(3x))$$

### 43.31 Problem number 4171

$$\int \frac{e^{4x \log(5) \log(e^{3x}-x)} (-100x \log(5) + 300e^{3x}x \log(5) + (100e^{3x} \log(5) - 100x \log(5)) \log(e^{3x}-x))}{e^{3x}-x} dx$$

Optimal antiderivative

$$25 e^{4x \ln(5) \ln(e^{3x}-x)}$$

command

```
integrate((((100*log(5)*exp(x)^3-100*x*log(5))*log(exp(x)^3-x)+300*x*log(5)*exp(x)^3-100*x*log(5))*exp(2*x*log(5)*log(exp(x)^3-x))^2/(exp(x)^3-x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$25 e^{(4x \log(5) \log(-x+e^{(3x)}))}$$

### 43.32 Problem number 4209

$$\int e^{\frac{2(-e^6+x+4e^3x-4x^2+(2+e^6-x-4e^3x+4x^2+(e^6x-x^2-4e^3x^2+4x^3)\log(4))\log(x))}{-1+(1+x\log(4))\log(x)}} \frac{(-4-2x-8e^3x+16x^2+(4x+16e^3x-32x^2+x+(-2x-2x^2\log(4))))}{x+(-2x-2x^2\log(4))} dx$$

Optimal antiderivative

$$e^{2(e^3-2x)^2-2x+\frac{4}{1-\frac{1}{\ln(x)}+2x\ln(2)}}$$

command

```
integrate(((4*(-8*x^3*exp(3)+16*x^4-2*x^3)*log(2)^2+2*(-16*x^2*exp(3)+32*x^3-4*x^2-4*x)*log(2)-8*x*exp(3)+16*x^2-2*x)*log(x)^2+(2*(16*x^2*exp(3)-32*x^3+4*x^2)*log(2)+16*x*exp(3)-32*x^2+4*x)*log(x)-8*x*exp(3)+16*x^2-2*x-4)*exp(((2*(x*exp(3)^2-4*x^2*exp(3)+4*x^3-x^2)*log(2)+exp(3)^2-4*x*exp(3)+4*x^2-x+2)*log(x)-exp(3)^2+4*x*exp(3)-4*x^2+x)/((2*x*log(2)+11))^2/((4*x^3*log(2)^2+4*x^2*log(2)+x)*log(x)^2+(-4*x^2*log(2)-2*x)*log(x)+x),x, algorithm="g
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left(\frac{16x^3\log(2)\log(x)}{2x\log(2)\log(x)+\log(x)-1} - \frac{16x^2e^3\log(2)\log(x)}{2x\log(2)\log(x)+\log(x)-1} - \frac{4x^2\log(2)\log(x)}{2x\log(2)\log(x)+\log(x)-1} + \frac{4xe^6\log(2)\log(x)}{2x\log(2)\log(x)+\log(x)-1} + \frac{8x^2\log(x)}{2x\log(2)\log(x)+\log(x)-1} - \frac{8xe^3\log(x)}{2x\log(2)\log(x)+\log(x)-1}\right)}$$

### 43.33 Problem number 4323

$$\int e^{-7+e^{e+2x}-x} (e(1-x) + 2e^{1+e+2x}x) dx$$

Optimal antiderivative

$$e e^{e^{e+2x}-x-7} x$$

command

```
integrate((2*x*exp(1)*exp(exp(1)+2*x)+(1-x)*exp(1))/exp(-exp(exp(1)+2*x)+x+7),x, algorithm="g
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$xe^{(-x+e^{(2x+e)}-6)}$$

### 43.34 Problem number 4476

$$\int e^{-4 + \frac{-2x + e^4 \log(x+x^2) \log(\log(x))}{e^4 \log(x+x^2)}} \frac{((12x + 24x^2) \log(x) + (-12x - 12x^2) \log(x) \log(x+x^2) + e^4(6 + 6x) \log^2(x+x^2))}{(x+x^2) \log(x) \log^2(x+x^2)} dx$$

Optimal antiderivative

$$6 e^{\ln(\ln(x)) - \frac{2e^{-4}x}{\ln(x^2+x)}}$$

command

```
integrate(((6+6*x)*exp(4)*log(x^2+x)^2+(-12*x^2-12*x)*log(x)*log(x^2+x)+(24*x^2+12*x)*log(x)^2*x)/exp(4)/log(x^2+x))/(x^2+x)/exp(4)/log(x)/log(x^2+x)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$6 e^{\left(-\frac{2xe^{-4}}{\log(x^2+x)} + \log(\log(x))\right)}$$

### 43.35 Problem number 5416

$$\int \frac{e^{-2x} \left( -e^{2x} x^2 + e^{\frac{2e^{-2x}(-e^4 + e^{2x}(-3+x))}{x}} (24e^{2x} + e^4(8 + 16x)) \right)}{x^2} dx$$

Optimal antiderivative

$$4 e^{\frac{-6+2x-2e^4-2x}{x}} + 20 - x$$

command

```
integrate(((24*exp(2*x)+(16*x+8)*exp(4))*exp((( -3+x)*exp(2*x)-exp(4))/x/exp(2*x))^2-exp(2*x)*x^2)/exp(2*x)/x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-x + 4 e^{\left(\frac{2(x - e^{(-2x+4)} - 3)}{x}\right)}$$

### 43.36 Problem number 5600

$$\int e^{\frac{20x + x \log\left(\frac{1 - x \log\left(\frac{e^{-x} \log(2)}{x}\right)}{x}\right)}{-1 + x \log\left(\frac{e^{-x} \log(2)}{x}\right)}} \left( -19 - x - x^2 + 20x \log\left(\frac{e^{-x} \log(2)}{x}\right) + \left(-1 + x \log\left(\frac{e^{-x} \log(2)}{x}\right)\right) \log\left(\frac{1 - x \log\left(\frac{e^{-x} \log(2)}{x}\right)}{x}\right) \right) dx$$

Optimal antiderivative

$$e^{x \left( 20 + \ln\left(\frac{1}{x} - \ln\left(\frac{\ln(2)e^{-x}}{x}\right)\right) \right)}$$

command

```
integrate(((x*log(log(2)/exp(x)/x)-1)*log((-x*log(log(2)/exp(x)/x)+1)/x)+20*x*log(log(2)/exp(x)^2-x-19)*exp(x*log((-x*log(log(2)/exp(x)/x)+1)/x)+20*x)/(x*log(log(2)/exp(x)/x)-1),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left(x \log\left(\frac{1}{x} - \log\left(\frac{e^{(-x)} \log(2)}{x}\right)\right) + 20x\right)}$$

### 43.37 Problem number 5649

$$\int e^{\frac{-7x + e^{36 + e^4 + 12x^2 + x^4 + e^2(-12 - 2x^2)}}{16 + \log(x)}} \left( -105 + e^{36 + e^4 + 12x^2 + x^4 + e^2(-12 - 2x^2)} (15 + 384x^2 - 64e^2x^2 + 64x^4) + (-7 + e^{36 - 7x}) \right) / (256 + 32 \log(x) + \log^2(x)) dx$$

Optimal antiderivative

$$e^{\frac{x \left( e^{(e^2 - x^2 - 6)^2} - 7 \right)}{16 + \ln(x)}}$$

command

```
integrate(((((-4*x^2*exp(2)+4*x^4+24*x^2+1)*exp(exp(2)^2+(-2*x^2-12)*exp(2)+x^4+12*x^2+36)-7)*log(x)+(-64*x^2*exp(2)+64*x^4+384*x^2+15)*exp(exp(2)^2+(-2*x^2-12)*exp(2)+x^4+12*x^2+36)-105)*exp((x*exp(exp(2)^2+(-2*x^2-12)*exp(2)+x^4+12*x^2+36)-7*x)/(16+log(x)))/(log(x)^2+32*log(x))),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left(\frac{x e^{(x^4 - 2x^2 e^2 + 12x^2 + e^4 - 12e^2 + 36)}}{\log(x) + 16} - \frac{7x}{\log(x) + 16}\right)}$$

### 43.38 Problem number 5811

$$\int \frac{15x - 50e^x x + 10e^{2x} x - 5x^2 + (15x - 10x^2) \log(x) + (-125 + 50e^x - 5e^{2x} + (-15x + 5x^2) \log(x)) \log(25 - 10e^x + e^{2x})}{(-25x + 10e^x x - e^{2x} x + (-3x^2 + x^3) \log(x)) \log(25 - 10e^x + e^{2x} + (3x - x^2) \log(x)) \log(\log(25 - 10e^x + e^{2x}))} dx$$

Optimal antiderivative

$$\frac{5}{\ln\left(\frac{\ln(\ln((5-e^x)^2+(3-x)x \ln(x)))}{x}\right)}$$

command

```
integrate((((5*x^2-15*x)*log(x)-5*exp(x)^2+50*exp(x)-125)*log((-x^2+3*x)*log(x)+exp(x)^2-10*exp(x)+25)*log(log((-x^2+3*x)*log(x)+exp(x)^2-10*exp(x)+25))+(-10*x^2+15*x)*log(x)+10*x*exp(50*exp(x)*x-5*x^2+15*x)/((x^3-3*x^2)*log(x)-x*exp(x)^2+10*exp(x)*x-25*x)/log((-x^2+3*x)*log(x)+10*exp(x)+25)/log(log((-x^2+3*x)*log(x)+exp(x)^2-10*exp(x)+25))/log(log(log((-x^2+3*x)*log(x)+10*exp(x)+25))/x)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{5}{\log(x) - \log(\log(\log(-x^2 \log(x) + 3x \log(x) + e^{(2x)} - 10e^x + 25)))}$$

### 43.39 Problem number 5911

$$\int \frac{e^{3-x \log(\frac{1}{3}(e^5 x - 3 \log(x)))} (-12 + 4e^5 x + (4e^5 x - 12 \log(x)) \log(\frac{1}{3}(e^5 x - 3 \log(x))))}{-e^5 x + 3 \log(x)} dx$$

Optimal antiderivative

$$4e^{-x \ln(-\ln(x) + \frac{x e^5}{3}) + 3} - 16$$

command

```
integrate(((((-12*log(x)+4*x*exp(5))*log(-log(x)+1/3*x*exp(5))+4*x*exp(5)-12)*exp(-x*log(-log(x)+1/3*x*exp(5))+3)/(3*log(x)-x*exp(5))),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$4e^{(-x \log(\frac{1}{3} x e^5 - \log(x)) + 3)}$$

### 43.40 Problem number 5947

$$\int \frac{e^{2x} \left( e^4(48 - 96x) - 168x + 168x^2 + e^{\frac{1}{3}(2-2x)}(-48 + 128x) \right)}{48e^8x^2 + 48e^{\frac{2}{3}(2-2x)}x^2 - 168e^4x^3 + 147x^4 + e^{\frac{1}{3}(2-2x)}(-96e^4x^2 + 168x^3)} dx$$

Optimal antiderivative

$$\frac{e^{2x}}{\left( \frac{7x}{4} + e^{-\frac{2x}{3} + \frac{2}{3}} - e^4 \right) x}$$

command

```
integrate(((128*x-48)*exp(-2/3*x+2/3)+(-96*x+48)*exp(4)+168*x^2-168*x)*exp(x)^2/(48*x^2*exp(-2/3*x+2/3)^2+(-96*x^2*exp(4)+168*x^3)*exp(-2/3*x+2/3)+48*x^2*exp(4)^2-168*x^3*exp(4)+147*x^4)
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

---


$$235298 x^8 e^{\left(\frac{2}{3} x\right)} + 268912 x^7 e^{\frac{2}{3}} + 352947 x^7 e^{\left(\frac{2}{3} x\right)} - 806736 x^7 e^{\left(\frac{2}{3} x+4\right)} - 768320 x^6 e^{\frac{14}{3}} + 403368 x^6 e^{\frac{2}{3}} + 1152480 x^6$$

### 43.41 Problem number 6013

$$\int \frac{e^{-1+e^{\frac{1}{x}}-x} \left( -e^{\frac{1}{x}} - x^2 \right)}{x^2} dx$$

Optimal antiderivative

$$\frac{e^{\ln(x^2)+e^{\frac{1}{x}}-x-1}}{x^2}$$

command

```
integrate((-exp(1/x)-x^2)*exp(log(x^2)+exp(1/x)-x-1)/x^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left(-x+e^{\frac{1}{x}}-1\right)}$$



## 43.42 Problem number 6096

$$\int \frac{e^{\frac{1}{2} \left( 5e^{\frac{1}{2}} x - 5x^2 - 5x^3 \right)} \left( 2x + 10x^3 + 15x^4 + e^{\frac{1}{x^2}} (10 - 5x^2) \right)}{2e^{5e^{\frac{1}{2}} x - 5x^2 - 5x^3} x - 4e^{\frac{1}{2} \left( 5e^{\frac{1}{2}} x - 5x^2 - 5x^3 \right)} x^2 + 2x^3} dx$$

Optimal antiderivative

$$\frac{x}{e^{\frac{5x}{2} e^{\frac{1}{2}} x - \frac{5x^3}{2} - \frac{5x^2}{2}} - x}$$

command

```
integrate((( -5*x^2+10)*exp(1/x^2)+15*x^4+10*x^3+2*x)*exp(5/2*x*exp(1/x^2)-5/2*x^3-5/2*x^2)/(2*x*exp(5/2*x*exp(1/x^2)-5/2*x^3-5/2*x^2)^2-4*x^2*exp(5/2*x*exp(1/x^2)-5/2*x^3-5/2*x^2)+2*x^3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{x}{x - e^{\left( -\frac{5}{2} x^3 - \frac{5}{2} x^2 + \frac{5}{2} x e^{\left( \frac{1}{x^2} \right)} \right)}}$$

## 43.43 Problem number 6201

$$\int \frac{e^{\frac{1}{12} e^{-5+2x}} \left( -2 - \frac{1}{3} e^{-5+2x} x \log(x) \right)}{25x - 10e^{\frac{1}{12} e^{-5+2x}} x \log(x) + e^{\frac{1}{6} e^{-5+2x}} x \log^2(x)} dx$$

Optimal antiderivative

$$\frac{2}{\ln(x) e^{e^{-\ln(12)+2x-5}} - 5}$$

command

```
integrate((-4*x*exp(-log(12)+2*x-5)*log(x)-2)*exp(exp(-log(12)+2*x-5))/(x*log(x)^2*exp(exp(-log(12)+2*x-5))^2-10*x*log(x)*exp(exp(-log(12)+2*x-5))+25*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{2 \left( x e^{(2x)} \log(x) + 6 e^5 \right)}{x e^{(2x + \frac{1}{12} e^{(2x-5)})} \log(x)^2 - 5 x e^{(2x)} \log(x) + 6 e^{\left( \frac{1}{12} e^{(2x-5)} + 5 \right)} \log(x) - 30 e^5}$$

### 43.44 Problem number 6696

$$\int \frac{(-e^5 + x)^{\frac{20x}{-x^2 + \log^4(7)}} (20x^3 - 20x \log^4(7) + (20e^5 x^2 - 20x^3 + (20e^5 - 20x) \log^4(7)) \log(-e^5 + x))}{e^5 x^4 - x^5 + (-2e^5 x^2 + 2x^3) \log^4(7) + (e^5 - x) \log^8(7)} dx$$

Optimal antiderivative

$$e^{\frac{20 \ln(-e^5 + x)}{\frac{\ln(7)^4}{x} - x}}$$

command

```
integrate((((20*exp(5)-20*x)*log(7)^4+20*x^2*exp(5)-20*x^3)*log(-exp(5)+x)-20*x*log(7)^4+20*x*exp(5)+x)/(log(7)^4-x^2))^4/((exp(5)-x)*log(7)^8+(-2*x^2*exp(5)+2*x^3)*log(7)^4+x^4*exp(5)-x^5),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{20(x-e^5)}{(x-e^5)^{\log(7)^4-(x-e^5)^2-2(x-e^5)e^5-e^{10}}} \frac{20e^5}{(x-e^5)^{\log(7)^4-(x-e^5)^2-2(x-e^5)e^5-e^{10}}}$$

### 43.45 Problem number 6926

$$\int \frac{e^{-x+x\frac{e^3+4x}{x}} \left( x^2 + x\frac{e^3+4x}{x} (-e^3 - 4x + e^3 \log(x)) \right)}{x^2} dx$$

Optimal antiderivative

$$-e^{\frac{(e^3+4x)\ln(x)}{x}-x}$$

command

```
integrate(((log(x)*exp(3)-exp(3)-4*x)*exp((exp(3)+4*x)*log(x)/x)+x^2)*exp(exp((exp(3)+4*x)*log(x)/x)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-e^{\left(x\frac{e^3}{x}+4-x\right)}$$

### 43.46 Problem number 7043

$$\int \frac{4ex + e(160x - 40x^2) \log(4) + e(-16 + 4x) \log(4 - x) + e^{\frac{x}{e}} \left( 4e + e(80 - 20x) \log(4) + e^{\frac{x}{e}} (20x - 5x^2) \log(4) \right)}{e(-16 + 4x)}$$

Optimal antiderivative

$$(\ln(4 - x) - 10x \ln(2)) \left( x + e^{\frac{e^{-1}x}{4}} \right) + 25$$

command

```
integrate((((x-4)*exp(x/exp(1))*log(-x+4)+2*(-5*x^2+20*x)*log(2)*exp(x/exp(1))+2*(-20*x+80)*exp(1)*log(2)+4*exp(1))*exp(1/4*exp(x/exp(1)))+(4*x-16)*exp(1)*log(-x+4)+2*(-40*x^2+160*x)*exp(1)*log(2)+4*x*exp(1))/(4*x-16)/exp(1),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$-\left( 10x^2e \log(2) + 10xe \left( \frac{1}{4} e^{(xe^{(-1)})+1} \right) \log(2) - xe \log(-x+4) - e \left( \frac{1}{4} e^{(xe^{(-1)})+1} \right) \log(-x+4) \right) e^{(-1)}$$

### 43.47 Problem number 7194

$$\int \frac{16x^2 + e^x(8x + 8x^2) + (6x + e^{2x}x + 2x^3 + e^x(3 + 3x^2)) \log(3 + e^xx + x^2) + (64x + e^x(32 + 32x) + (24 + 8e^x)) \log(3 + e^xx + x^2)}{(3 + e^xx + x^2) \log(3 + e^xx + x^2)}$$

Optimal antiderivative

$$e^x + (4 \ln(\ln(3 + (x + e^x)x) (2 \ln(3) - 1)) + x)^2$$

command

```
integrate((((8*exp(x)*x+8*x^2+24)*log(exp(x)*x+x^2+3)+(32*x+32)*exp(x)+64*x)*log((2*log(3)-1)*log(exp(x)*x+x^2+3))+(x*exp(x)^2+(3*x^2+3)*exp(x)+2*x^3+6*x)*log(exp(x)*x+x^2+3)+(8*x^2+8*x))
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$x^2 + 8x \log(2 \log(3) - 1) + 8x \log(\log(x^2 + xe^x + 3)) + 32 \log(2 \log(3) - 1) \log(\log(x^2 + xe^x + 3)) + 16 \log(\log(x^2 + xe^x + 3))^2 + e^x$$

### 43.48 Problem number 7444

$$\int e^{-4+e^{16}(1-x)+4x+x^2-x^3+e^8(-2x+2x^2)} (4 - e^{16} + 2x - 3x^2 + e^8(-2 + 4x)) dx$$

Optimal antiderivative

$$e^{(4-(x-e^8)^2)(-1+x)}$$

command

```
integrate((-exp(8)^2+(4*x-2)*exp(8)-3*x^2+2*x+4)*exp((1-x)*exp(8)^2+(2*x^2-2*x)*exp(8)-x^3+x^2+4*x-4),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e^{(-x^3+2x^2e^8+x^2-xe^{16}-2xe^8+4x+e^{16}-4)}$$

### 43.49 Problem number 7495

$$\int \frac{80e^{-2e^5+2x} + 40e^{-e^5+x} \log(5) + 5 \log^2(5) + e^{\frac{5e^{-e^5+x} + 2e^{-e^5+x} \log(\log(4))}{4e^{-e^5+x} + \log(5)}} (5e^{-e^5+x} \log(5) + 2e^{-e^5+x} \log(5) \log(\log(4)))}{16e^{-2e^5+2x} + 8e^{-e^5+x} \log(5) + \log^2(5)} dx$$

Optimal antiderivative

$$5x + e^{\frac{5+\ln(4\ln(2)^2)}{\ln(5)e^{e^5-x}+4}}$$

command

```
integrate(((log(5)*exp(-exp(5)+x)*log(4*log(2)^2)+5*log(5)*exp(-exp(5)+x))*exp((exp(-exp(5)+x)*log(4*log(2)^2)+5*exp(-exp(5)+x))/(4*exp(-exp(5)+x)+log(5)))+80*exp(-exp(5)+x)^2+40*exp(5)+x)+5*log(5)^2)/(16*exp(-exp(5)+x)^2+8*log(5)*exp(-exp(5)+x)+log(5)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\left( 5xe^x + 5e^x \log(4e^{(x-e^5)} + \log(5)) - 5e^x \log(-4e^{(x-e^5)} - \log(5)) + e^{\left( \frac{4xe^{(x-e^5)} + x \log(5) + 2e^{(x-e^5)} \log(2) + 2e^{(x-e^5)} \log(\log(4))}{4e^{(x-e^5)} + \log(5)} \right)} \right)$$

## 43.50 Problem number 7627

$$\int \frac{1}{5} e^{-6 + \frac{4e^{2x}x + e^{3+x}(20x+4x^2) + e^6(25x+10x^2+x^3)}{5e^6}} (e^{2x}(8+16x) + e^6(50+40x+6x^2) + e^{3+x}(40+56x+8x^2)) dx$$

Optimal antiderivative

$$2e^{\frac{x(5+2e^xe^{-3+x})^2}{5}}$$

command

```
integrate(1/5*((16*x+8)*exp(x)^2+(8*x^2+56*x+40)*exp(3)*exp(x)+(6*x^2+40*x+50)*exp(3)^2)*exp(
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$2e^{\left(\frac{1}{5}x^3 + \frac{4}{5}x^2e^{(x-3)} + 2x^2 + \frac{4}{5}xe^{(2x-6)} + 4xe^{(x-3)} + 5x\right)}$$

## 43.51 Problem number 7683

$$\int \frac{e^{-x}(-1 + (1-x)\log(-\frac{x}{2}) - \log^2(-\frac{x}{2}))}{\log^2(-\frac{x}{2})} dx$$

Optimal antiderivative

$$\frac{\left(x + \frac{x^2}{\ln(-\frac{x}{2})}\right) e^{-x}}{x}$$

command

```
integrate((-log(-1/2*x)^2+(1-x)*log(-1/2*x)-1)/exp(x)/log(-1/2*x)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\frac{xe^{(-x)} + e^{(-x)} \log\left(-\frac{1}{2}x\right)}{\log\left(-\frac{1}{2}x\right)}$$

### 43.52 Problem number 7755

$$\int \frac{-6x + 2x^3 + e^{10}(-6x + 2x^3) + e^5(-12x + 4x^3) + e^{2-2x}(-6x + 6x^2 + 2x^3 - 2x^4) + e^{1-x}(-12x + 6x^2 + 4x^3 - 1)}{1}$$

Optimal antiderivative

$$\ln\left(1 + x^2(e^{1-x} + e^5 + 1)^2\right)(x^2 - 3)$$

command

```
integrate(((2*x^3*exp(1-x)^2+(4*x^3*exp(5)+4*x^3)*exp(1-x)+2*x^3*exp(5)^2+4*x^3*exp(5)+2*x^3+x)^2+(2*x^2*exp(5)+2*x^2)*exp(1-x)+x^2*exp(5)^2+2*x^2*exp(5)+x^2+1)+(-2*x^4+2*x^3+6*x^2-6*x)*exp(1-x)^2+((-2*x^4+4*x^3+6*x^2-12*x)*exp(5)-2*x^4+4*x^3+6*x^2-12*x)*exp(1-x)+(2*x^3-6*x)*exp(5)^2+(4*x^3-12*x)*exp(5)+2*x^3-6*x)/(x^2*exp(1-x)^2+(2*x^2*exp(5)+2*x^2)*exp(1-x)+x^2*exp(5)^2+2*x^2*exp(5)+x^2+1), x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$\begin{aligned} & -2x^3 + x^2 \log\left(x^2 e^2 + x^2 e^{(2x)} + x^2 e^{(2x+10)} + 2x^2 e^{(2x+5)} + 2x^2 e^{(x+6)} + 2x^2 e^{(x+1)} + e^{(2x)}\right) \\ & + 6x - 3 \log\left(x^2 e^2 + x^2 e^{(2x)} + x^2 e^{(2x+10)} + 2x^2 e^{(2x+5)} + 2x^2 e^{(x+6)} + 2x^2 e^{(x+1)} + e^{(2x)}\right) \end{aligned}$$

### 43.53 Problem number 7789

$$\int x^{35+609x+2650x^2+625x^3+e^2(4+x)}(72 + 1218x + 5300x^2 + 1250x^3 + e^2(8 + 2x) + (1218x + 2e^2x + 10600x^2 + 3750x^3) \log(x)) dx$$

Optimal antiderivative

$$2e^{((3+25x)^2+e^2)(4+x)\ln(x)}$$

command

```
integrate(((2*exp(2)*x+3750*x^3+10600*x^2+1218*x)*log(x)+(2*x+8)*exp(2)+1250*x^3+5300*x^2+1218*x^2+72*x^2+1218*x+5300*x^2+1250*x^3)/((3+25*x)^2+e^2)*(4+x)*ln(x)), x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$2e^{(625x^3 \log(x) + 2650x^2 \log(x) + xe^2 \log(x) + 609x \log(x) + 4e^2 \log(x) + 36 \log(x))}$$

### 43.54 Problem number 8043

$$\int \frac{e^{-2e^{2-x^2} \log(3)+2x \log(3)} x - 2x \log(3) - 4e^{2-x^2} x^2 \log(3) + \left(1 + e^{-2e^{2-x^2} \log(3)+2x \log(3)} x\right) \log\left(e^{2e^{2-x^2} \log(3)-2x \log(3)}\right)}{\left(x + e^{-2e^{2-x^2} \log(3)+2x \log(3)} x^2\right) \log\left(e^{2e^{2-x^2} \log(3)-2x \log(3)}\right) \left(-1 - e^{-2e^{2-x^2} \log(3)+2x \log(3)} x\right) \log\left(x \log\left(e^{2e^{2-x^2} \log(3)-2x \log(3)}\right)\right)}$$

Optimal antiderivative

$$\ln\left(\ln\left(x \ln\left(-e^{-2 \ln(3)\left(x-e^{-x^2+2}\right)} - x\right)\right)\right)$$

command

```
integrate(((x*exp(-log(3)*exp(-x^2+2)+x*log(3))^2+1)*log((-x*exp(-log(3)*exp(-x^2+2)+x*log(3))
1)/exp(-log(3)*exp(-x^2+2)+x*log(3))^2)+x*exp(-log(3)*exp(-x^2+2)+x*log(3))^2-4*x^2*log(3)*exp
x^2+2)-2*x*log(3))/(x^2*exp(-log(3)*exp(-x^2+2)+x*log(3))^2+x)/log((-x*exp(-log(3)*exp(-
x^2+2)+x*log(3))^2-1)/exp(-log(3)*exp(-x^2+2)+x*log(3))^2)/log(x*log((-x*exp(-log(3)*exp(-
x^2+2)+x*log(3))^2-1)/exp(-log(3)*exp(-x^2+2)+x*log(3))^2)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\log\left(\log\left(x \log\left(-\left(xe^{\left(2x \log(3)-2e^{(-x^2+2)} \log(3)}\right)} + 1\right) e^{\left(-2x \log(3)+2e^{(-x^2+2)} \log(3)}\right)}\right)\right)\right)$$

### 43.55 Problem number 8143

$$\int \frac{-192x^6 - 96x^7 + e^5(-192x^3 - 240x^4 - 72x^5 - 48x^6 - 24x^7) + (-96x^6 + e^5(-192x^3 - 120x^4 - 24x^6)) \log(3)}{256}$$

Optimal antiderivative

$$\frac{\left(e^{\frac{x}{4}} - 3\right) (\ln(3) + 2 + x)^2}{\left(4 + e^5 + \frac{e^5}{x^2}\right)^2}$$

command

```
integrate((((x^6+x^4+16*x^3)*exp(5)+4*x^6)*log(3)^2+((2*x^7+12*x^6+2*x^5+44*x^4+64*x^3)*exp(
48*x^3*exp(5)*log(3)^2+((-24*x^6-120*x^4-192*x^3)*exp(5)-96*x^6)*log(3)+(-24*x^7-48*x^6-
72*x^5-240*x^4-192*x^3)*exp(5)-96*x^7-192*x^6)/((4*x^6+12*x^4+12*x^2+4)*exp(5)^3+(48*x^6+96*x
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

output too large to display

### 43.56 Problem number 8742

$$\int e^{\frac{3x+e^{200+40x+2x^2}x-\log(-2+e^2+e^5)}{x}} \left( e^{200+40x+2x^2} (40x^2 + 4x^3) + \log(-2 + e^2 + e^5) \right) \frac{dx}{x^2}$$

Optimal antiderivative

$$1 + e^{e^{2(x+10)^2} - \frac{\ln(e^5+e^2-2)}{x}} + 3$$

command

```
integrate((log(exp(5)+exp(2)-2)+(4*x^3+40*x^2)*exp(x^2+20*x+100)^2)*exp((-log(exp(5)+exp(2)-2)+x*exp(x^2+20*x+100)^2+3*x)/x)/x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left(-\frac{\log(e^5+e^2-2)}{x} + e^{(2x^2+40x+200)} + 3\right)}$$

### 43.57 Problem number 9011

$$\int e^{\frac{3+e^{10}(-x^2-x^3)}{-e^{10}x^2+e^{10}x^2\log\left(\frac{x}{5\log(x)}\right)}} \left( 3 + e^{10}(-x^2 - x^3) + (3 + e^{10}(x^2 + 2x^3)) \log(x) + (-6 - e^{10}x^3) \log(x) \log\left(\frac{x}{5\log(x)}\right) \right) \frac{dx}{e^{10}x^3 \log(x) - 2e^{10}x^3 \log(x) \log\left(\frac{x}{5\log(x)}\right) + e^{10}x^3 \log(x) \log^2\left(\frac{x}{5\log(x)}\right)}$$

Optimal antiderivative

$$\frac{\frac{3e^{-10}}{x^2} - x - 1}{e^{\ln\left(\frac{x}{5\ln(x)}\right) - 1}}$$

command

```
integrate((-x^3*exp(5)^2-6)*log(x)*log(1/5*x/log(x))+((2*x^3+x^2)*exp(5)^2+3)*log(x)+(-x^3-x^2)*exp(5)^2+3)*exp(((x^3-x^2)*exp(5)^2+3)/(x^2*exp(5)^2*log(1/5*x/log(x))-x^2*exp(5)^2*2*x^3*exp(5)^2*log(x)*log(1/5*x/log(x))+x^3*exp(5)^2*log(x)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left(-\frac{x^3 e^{10}}{x^2 e^{10} \log\left(\frac{x}{5 \log(x)}\right) - x^2 e^{10}} - \frac{x^2 e^{10}}{x^2 e^{10} \log\left(\frac{x}{5 \log(x)}\right) - x^2 e^{10}} + \frac{3}{x^2 e^{10} \log\left(\frac{x}{5 \log(x)}\right) - x^2 e^{10}}\right)}$$



### 43.58 Problem number 9031

$$\int e^{\frac{\log^2\left(\frac{-25+e^{2+2x}+e^{2+x}(-40-8x)-10x-x^2+e^2(525+185x+11x^2-x^3)}{e^2(25+10x+x^2)}\right)}{(e^{2+2x}(16+4x)+e^{2+x}(-320-144x-16x^2)+e^2(-250-125-75x-15x^2-x^3+e^{2+2x}(5+x)+e^{2+x}(-200-80x-8x^2))} dx$$

Optimal antiderivative

$$e^{\ln\left(5+\left(\frac{e^x}{5+x}-4\right)^2-x-e^{-2}\right)^2}$$

command

```
integrate(((4*x+16)*exp(2)*exp(x)^2+(-16*x^2-144*x-320)*exp(2)*exp(x)+(-2*x^3-30*x^2-150*x-250)*exp(2))*log((exp(2)*exp(x)^2+(-8*x-40)*exp(2)*exp(x)+(-x^3+11*x^2+185*x+525)*exp(2)*x^2-10*x-25)/(x^2+10*x+25)/exp(2))*exp(log((exp(2)*exp(x)^2+(-8*x-40)*exp(2)*exp(x)+(-x^3+11*x^2+185*x+525)*exp(2)-x^2-10*x-25)/(x^2+10*x+25)/exp(2))^2)/((5+x)*exp(2)*exp(x)^2+(-8*x^2-80*x-200)*exp(2)*exp(x)+(-x^4+6*x^3+240*x^2+1450*x+2625)*exp(2)-x^3-15*x^2-75*x-125),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left(\log\left(-\frac{x^3 e^2}{x^2 e^2 + 10 x e^2 + 25 e^2} + \frac{11 x^2 e^2}{x^2 e^2 + 10 x e^2 + 25 e^2} - \frac{x^2}{x^2 e^2 + 10 x e^2 + 25 e^2} + \frac{185 x e^2}{x^2 e^2 + 10 x e^2 + 25 e^2} - \frac{8 x e^{(x+2)}}{x^2 e^2 + 10 x e^2 + 25 e^2} - \frac{10 x}{x^2 e^2 + 10 x e^2 + 25 e^2} + \frac{525 e^2}{x^2 e^2 + 10 x e^2 + 25 e^2}\right)} dx$$

### 43.59 Problem number 9135

$$\int e^{\frac{-x-e^{2x}x+\log(e^2x)}{-4+x}} \frac{(-4+5x+e^{2x}(4x+8x^2-2x^3)-x\log(e^2x))}{16x-8x^2+x^3} dx$$

Optimal antiderivative

$$2 + e^{\frac{\ln(e^2x)-xe^{2x}-x}{-4+x}}$$

command

```
integrate((-x*log(exp(2)*x)+(-2*x^3+8*x^2+4*x)*exp(2*x)+5*x-4)*exp((log(exp(2)*x)-x*exp(2*x)-x)/(x-4))/(x^3-8*x^2+16*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left(-\frac{x e^{(2x)}}{x-4} - \frac{x}{x-4} + \frac{\log(xe^2)}{x-4}\right)}$$

### 43.60 Problem number 9215

$$\int \frac{1}{3} e^{\frac{1}{3}(e^{31/5}x^2 + e^{\frac{6}{5}+x}x^2)} \left( 2e^{31/5}x + e^{\frac{6}{5}+x}(2x + x^2) \right) dx$$

Optimal antiderivative

$$e^{\frac{(e^5 + e^x)e^{\frac{6}{5}}x^2}{3}}$$

command

`integrate(1/3*((x^2+2*x)*exp(3/5)^2*exp(x)+2*x*exp(3/5)^2*exp(5/2)^2)*exp(1/3*x^2*exp(3/5)^2*`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left(\frac{1}{3}x^2e^{\frac{31}{5}} + \frac{1}{3}x^2e^{\left(x + \frac{6}{5}\right)}\right)}$$

### 43.61 Problem number 9254

$$\int \frac{e^{-3 + \frac{8ex^2 + e^2(4x^2 + e(-2x + 4x^2)) - 8e^3 \log(\log(x))}{e^{3x}}} (-8e^3 + (8ex^2 + e^2(4x^2 + 4ex^2)) \log(x) + 8e^3 \log(x) \log(\log(x)))}{x^2 \log(x)} dx$$

Optimal antiderivative

$$e^{8xe^{-2} + 4x - \frac{8 \ln(\ln(x))}{x} + 4e^{-1}x - 2}$$

command

`integrate((8*exp(1)*exp(2)*log(x)*log(log(x)))+(4*x^2*exp(1)+4*x^2)*exp(2)+8*x^2*exp(1))*log(8*exp(1)*exp(2))*exp((-8*exp(1)*exp(2)*log(log(x)))+(4*x^2-2*x)*exp(1)+4*x^2)*exp(2)+8*x^2*ex`

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left(4xe^{(-1)} + 8xe^{(-2)} + 4x - \frac{8 \log(\log(x))}{x} - 2\right)}$$

**43.62 Problem number 9330**

$$\int \frac{e^{\frac{1}{16} \left( 1 + 8 \log \left( 2 \log \left( \frac{e^5 + e^x}{x} \right) \right) + 16 \log^2 \left( 2 \log \left( \frac{e^5 + e^x}{x} \right) \right) \right)} \left( -e^5 + e^x(-1 + x) + (-4e^5 + e^x(-4 + 4x)) \log \left( 2 \log \left( \frac{e^5 + e^x}{x} \right) \right) \right)}{(2e^5 x + 2e^x x) \log \left( \frac{e^5 + e^x}{x} \right)} dx$$

Optimal antiderivative

$$e^{\left( -\frac{1}{4} - \ln \left( 2 \ln \left( \frac{e^5 + e^x}{x} \right) \right) \right)^2}$$

command

```
integrate(((((-4+4*x)*exp(x)-4*exp(5))*log(2*log((exp(5)+exp(x))/x))+(-1+x)*exp(x)-exp(5))*exp(log(2*log((exp(5)+exp(x))/x))^2+1/2*log(2*log((exp(5)+exp(x))/x))+1/16)/(2*exp(x)
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left( \log \left( 2 \log \left( \frac{e^5 + e^x}{x} \right) \right)^2 + \frac{1}{2} \log \left( 2 \log \left( \frac{e^5 + e^x}{x} \right) \right) + \frac{1}{16} \right)}$$

**43.63 Problem number 9649**

$$\int \frac{e^{\frac{x^2 + (2+x^2) \log^2(x^2) + x \log(e^{4/5} x) \log^2(x^2)}{x}} \left( x^2 + (8 + 4x^2) \log(x^2) + 4x \log(e^{4/5} x) \log(x^2) + (-2 + x + x^2) \log^2(x^2) \right)}{x^2} dx$$

Optimal antiderivative

$$e^{x + \ln(x^2)^2 \left( \frac{2}{x} + x + \ln \left( x e^{\frac{4}{5}} \right) \right)}$$

command

```
integrate((4*x*log(x^2)*log(x*exp(4/5)))+(x^2+x-2)*log(x^2)^2+(4*x^2+8)*log(x^2)+x^2)*exp((x*1
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left( x \log(x^2)^2 + \log(x^2)^2 \log \left( x e^{\frac{4}{5}} \right) + x + \frac{2 \log(x^2)^2}{x} \right)}$$

### 43.64 Problem number 9667

$$\int e^{\frac{16x-81x^2-24e^{\frac{1}{5}(5x+\log(5))}x^2+9e^{\frac{2}{5}(5x+\log(5))}x^3}{16-24e^{\frac{1}{5}(5x+\log(5))}x+9e^{\frac{2}{5}(5x+\log(5))}x^2}} \left( -64 + 648x - 108e^{\frac{2}{5}(5x+\log(5))}x^2 + 27e^{\frac{3}{5}(5x+\log(5))}x^3 + e^{\frac{1}{5}(5x+\log(5))}(144x^2 - 64 + 144e^{\frac{1}{5}(5x+\log(5))}x - 108e^{\frac{2}{5}(5x+\log(5))}x^2 + 27e^{\frac{3}{5}(5x+\log(5))}x^3) \right) dx$$

Optimal antiderivative

$$e^{x - \frac{9}{\left(\frac{4}{3x} - e^{-\frac{\ln(5)}{5}} + x\right)^2}}$$

command

```
integrate((27*x^3*exp(1/5*log(5)+x)^3-108*x^2*exp(1/5*log(5)+x)^2+(486*x^3+144*x)*exp(1/5*log(5)+x)-81*x^2*exp(1/5*log(5)+x)-81*x^2+16*x)/(9*x^2*exp(1/5*log(5)+x)+16)/(27*x^3*exp(1/5*log(5)+x)^3-108*x^2*exp(1/5*log(5)+x)^2+144*x*exp(1/5*log(5)+x)-81*x^2+16*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: TypeError

Giac 1.7.0 via sagemath 9.3 output

$$e^{\left( \frac{9x^3e^{(2x+\frac{2}{5}\log(5))}}{9x^2e^{(2x+\frac{2}{5}\log(5))}-24xe^{(x+\frac{1}{5}\log(5))}+16} - \frac{24x^2e^{(x+\frac{1}{5}\log(5))}}{9x^2e^{(2x+\frac{2}{5}\log(5))}-24xe^{(x+\frac{1}{5}\log(5))}+16} - \frac{81x^2}{9x^2e^{(2x+\frac{2}{5}\log(5))}-24xe^{(x+\frac{1}{5}\log(5))}+16} + \frac{16}{9x^2e^{(2x+\frac{2}{5}\log(5))}-24xe^{(x+\frac{1}{5}\log(5))}+16} \right)}$$

### 43.65 Problem number 9708

$$\int \frac{e^5(-e^{21} - 3x^2) \log(3)}{(e^{21}x + x^3)^2} dx$$

Optimal antiderivative

$$e^{5+\ln\left(\frac{\ln(3)}{(e^{21}+x^2)x}\right)}$$

command

```
integrate((-exp(21)-3*x^2)*exp(log(log(3)/(x*exp(21)+x^3))+5)/(x*exp(21)+x^3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Exception raised: NotImplementedError

Giac 1.7.0 via sagemath 9.3 output

$$\frac{e^5 \log(3)}{x^3 + xe^{21}}$$

### 43.66 Problem number 9784

$$\int \frac{e^{5x}(-45x + 15x^2 + e^x(15x^2 - 5x^3)) + (36 - 24x + e^x(-12x + 8x^2)) \log(e^{-x}(-3 + e^x x)) + (-36x + 12x^2 + e^x(15x^2 - 5x^3))}{9x - 3x^2 + e^x(-3x^2 + x^3)}$$

Optimal antiderivative

$$4 \ln\left(\frac{1}{3}x^2 - x\right) \ln(x - 3e^{-x}) - e^{5x}$$

command

```
integrate((((8*x^2-12*x)*exp(x)-24*x+36)*log((exp(x)*x-3)/exp(x))+((4*x^2-12*x)*exp(x)+12*x^2-36*x)*log(1/3*x^2-x)+((-5*x^3+15*x^2)*exp(x)+15*x^2-45*x)*exp(5*x))/((x^3-3*x^2)*exp(x)-3*x^2+9*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$4x \log(3) - 4 \log(3) \log(xe^x - 3) - 4x \log(x - 3) + 4 \log(xe^x - 3) \log(x - 3) - 4x \log(x) + 4 \log(xe^x - 3) \log(x) - e^{(5x)}$$

### 43.67 Problem number 10096

$$\int \frac{8000 + 4800x + 960x^2 - 56x^3}{(8000x + 4800x^2 + 960x^3 + 44x^4 - 4x^5 + (6000x + 2400x^2 + 240x^3 - 5x^4) \log(x) + (1500x + 300x^2) \log^2(x) + 1000x^3 \log^3(x))}$$

Optimal antiderivative

$$\ln\left(\ln\left(\frac{\left(x - \frac{x^4}{(4x+20+5\ln(x))^2}\right)^2}{x}\right)\right)$$

command

```
integrate((125*log(x)^3+(300*x+1500)*log(x)^2+(-35*x^3+240*x^2+2400*x+6000)*log(x)-12*x^4-56*x^3+960*x^2+4800*x+8000)/(125*x*log(x)^3+(300*x^2+1500*x)*log(x)^2+(-5*x^4+240*x^3+4*x^5+44*x^4+960*x^3+4800*x^2+8000*x)/log((625*x*log(x)^4+(2000*x^2+10000*x)*log(x)^3+(-50*x^4+2400*x^3+24000*x^2+60000*x)*log(x)^2+(-80*x^5+880*x^4+19200*x^3+96000*x^2+160000*x)*log(x)^3+32*x^6-64*x^5+4320*x^4+38400*x^3+128000*x^2+160000*x)/(625*log(x)^4+(2000*x+10000)*log(x)^3+(10000*x^2+10000*x)*log(x)^2+10000*x*log(x)+10000))),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

Timed out

Giac 1.7.0 via sagemath 9.3 output

$$\begin{aligned} & \log \left( -\log \left( x^6 - 32x^5 - 80x^4 \log(x) - 50x^3 \log(x)^2 - 64x^4 \right. \right. \\ & \quad + 880x^3 \log(x) + 2400x^2 \log(x)^2 + 2000x \log(x)^3 + 625 \log(x)^4 \\ & \quad + 4320x^3 + 19200x^2 \log(x) + 24000x \log(x)^2 + 10000 \log(x)^3 + 38400x^2 \\ & \quad \left. \left. + 96000x \log(x) + 60000 \log(x)^2 + 128000x + 160000 \log(x) + 160000 \right) \right) \\ & + \log \left( 256x^4 + 1280x^3 \log(x) + 2400x^2 \log(x)^2 + 2000x \log(x)^3 + 625 \log(x)^4 \right. \\ & \quad + 5120x^3 + 19200x^2 \log(x) + 24000x \log(x)^2 + 10000 \log(x)^3 + 38400x^2 \\ & \quad \left. + 96000x \log(x) + 60000 \log(x)^2 + 128000x + 160000 \log(x) + 160000 \right) - \log(x) \end{aligned}$$

### 43.68 Problem number 10193

$$\int e^{-e^{1+e^{32-2x+x^2+e^{16}(-2+2x)}}} \left( -3x + e^{1+e^{32-2x+x^2+e^{16}(-2+2x)}} (-4x^2 + 4e^{16}x^2 + 4x^3) \right. \\ \left. + \left( 2x + e^{1+e^{32-2x+x^2+e^{16}(-2+2x)}} (2x^2 - 2e^{16}x^2 - 2x^3) \right) \log(x) \right) dx$$

Optimal antiderivative

$$(\ln(x) - 2) x^2 e^{-e^{(e^{16}+x-1)^2}}$$

command

```
integrate(((((-2*x^2*exp(16)-2*x^3+2*x^2)*exp(exp(16)^2+(-2+2*x)*exp(16)+x^2-2*x+1)+2*x)*log(x)
4*x^2)*exp(exp(16)^2+(-2+2*x)*exp(16)+x^2-2*x+1)-3*x)/exp(exp(exp(16)^2+(-2+2*x)*exp(16)+x^2-
2*x+1))),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

could not integrate

Giac 1.7.0 via sagemath 9.3 output

$$\left( x^2 e^{\left( x^2 + 2xe^{16} - 2x + e^{32} - 2e^{16} - e^{(x^2 + 2xe^{16} - 2x + e^{32} - 2e^{16} + 1)} + 1 \right)} \right) \log(x) - 2x^2 e^{\left( x^2 + 2xe^{16} - 2x + e^{32} - 2e^{16} - e^{(x^2 + 2xe^{16} - 2x + e^{32} - 2e^{16} + 1)} \right)}$$