

1 Find the Laplace transform of  $f(t) = \begin{cases} t & \text{if } 0 \leq t < 1 \\ 1 & \text{if } 1 \leq t < 2 \\ 0 & \text{if } t > 2 \end{cases}$

2 \_\_\_\_\_ " \_\_\_\_\_  $g(t) = e^{-\alpha t} \cos \beta t$ .

3 Find the inverse Laplace transform of  $\frac{4}{(s^2 - 2s - 3)}$

4 \_\_\_\_\_ " \_\_\_\_\_  $\sum_{k=1}^5 \frac{a_k}{s^2 + k^2}$

Using Laplace transform solve the IVPs

5 (IVP)  $\begin{cases} y' + 3y = 10 \sin t \\ y(0) = 0 \end{cases}$  (Note: solution can be obtained by integrating factor)

6 (IVP)  $\begin{cases} y'' + y = 2 \cos t \\ y(0) = 6, y'(0) = 0 \end{cases}$

7 Sketch  $f(t) = (t-1)u(t-1)$  and find its Laplace transform.

8  $f(t) = \begin{cases} e^t & \text{if } t \in (0, 1) \\ 0 & \text{otherwise} \end{cases}$  Use the shifting property to compute  $\mathcal{L}\{f\}(s)$

9 Find the inverse Laplace transform of  $3 \frac{1 - e^{-\pi s}}{s^2 + 9}$

10 Solve the IVP using Laplace transform

(IVP)  $\begin{cases} y'' + 3y' + 2y = g(t) = \begin{cases} 4t & \text{if } t \in (0, 1) \\ 8 & \text{if } t > 1 \end{cases} \\ y(0) = y'(0) = 0 \end{cases}$

11 Find a,  $\mathcal{L}(te^{-t} \cos t)$  b,  $\mathcal{L}^{-1}\left(\frac{s}{(s^2 - 9)^2}\right)$

12 Compute a,  $1 * \sin wt$  b,  $e^t * e^{-t}$

13 Find the solution by applying convolution (over)

$$\begin{cases} y'' + 4y = \begin{cases} 1 & \text{if } t \in (0,1) \\ 0 & \text{if } t > 1 \end{cases} \\ y(0) = 1, y'(0) = 0 \end{cases}$$

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Solve the IVPs  
transform:

by means of Laplace

$$a) \begin{cases} y_1' = 6y_1 + 9y_2 \\ y_2' = y_1 + 6y_2 \end{cases}$$

$$y_1(0) = -3$$

$$y_2(0) = -3$$

$$b) \begin{cases} y_1'' + y_2 = -5 \cos 2t \\ y_2'' + y_1 = 5 \cos 2t \end{cases}$$

$$y_1(0) = y_1'(0) = 1$$

$$y_2(0) = -y_2'(0) = -1$$