

HW 8 (First HW on Laplace), Math 601  
Fall 2018  
University Of Wisconsin, Milwaukee

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December 30, 2018

Compiled on December 30, 2018 at 3:53pm

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# 1 problems description

Homework

MATH 601

HW 8 Nov 16, 2018. First on Laplace

1 Find the Laplace transform of  $f(t) = \begin{cases} t & \text{if } 0 \leq t < 1 \\ 1 & \text{if } 1 \leq t < 2 \\ 0 & \text{if } t > 2 \end{cases}$

2 Find the Laplace transform of  $g(t) = e^{-\alpha t} \cos \beta t$ .

3 Find the inverse Laplace transform of  $\frac{4}{(s^2 - 2s - 3)}$

4 Find the inverse Laplace transform of  $\sum_{k=1}^5 \frac{ak}{s^2 + k^2}$

Using Laplace transform solve the IVPs

5 (IVP)  $\begin{cases} y' + 3y = 10 \sin t \\ y(0) = 0 \end{cases}$

(Note: solution can be obtained by integrating factor)

6 (IVP)  $\begin{cases} y'' + y = 2 \cos t \\ y(0) = 6, y'(0) = 0 \end{cases}$

7 Sketch  $f(t) = (t-1)u(t-1)$  and find its Laplace transform.

8  $f(t) = \begin{cases} e^t & \text{if } t \in (0, 1) \\ 0 & \text{otherwise} \end{cases}$  Use the shifting property to compute  $\mathcal{L}\{f\}(s)$

9 Find the inverse Laplace transform of  $3 \frac{1 - e^{-\pi s}}{s^2 + 9}$

10 Solve the IVP using Laplace transform

(IVP)  $\begin{cases} y'' + 3y' + 2y = g(t) = \begin{cases} 4t & \text{if } t \in (0, 1) \\ 8 & \text{if } t > 1 \end{cases} \\ y(0) = y'(0) = 0 \end{cases}$

11 Find a,  $\mathcal{L}(te^{-t} \cos t)$  b,  $\mathcal{L}^{-1}\left(\frac{s}{(s^2 - 9)^2}\right)$

12 Compute a,  $1 * \sin wt$  b,  $e^t * e^{-t}$

13 Find the solution by applying convolution (over)

$$\begin{cases} y'' + 4y = \begin{cases} 1 & \text{if } t \in (0,1) \\ 0 & \text{if } t > 1 \end{cases} \\ y(0) = 1, y'(0) = 0 \end{cases}$$

**14** Solve the IVPs by means of Laplace transform:

$$a) \begin{cases} y_1' = 6y_1 + 9y_2 \\ y_2' = y_1 + 6y_2 \end{cases} \quad \begin{matrix} y_1(0) = -3 \\ y_2(0) = -3 \end{matrix}$$

$$b) \begin{cases} y_1'' + y_2 = -5 \cos 2t \\ y_2'' + y_1 = 5 \cos 2t \end{cases} \quad \begin{matrix} y_1(0) = y_1'(0) = 1 \\ y_2(0) = -y_2'(0) = -1 \end{matrix}$$

## 2 Key solution

MATH 601 HW H5<sup>2</sup> Solutions

Laplace tr.

①

$$\begin{aligned}
 \boxed{1} \quad \mathcal{L}(f(t)) &= \int_0^{\infty} e^{-st} f(t) dt = \int_0^1 e^{-st} \cdot t dt + \int_1^2 e^{-st} dt = \\
 &= -\frac{1}{s} e^{-st} \cdot t \Big|_{t=0}^1 + \frac{1}{s} \int_0^1 e^{-st} dt - \frac{1}{s} e^{-st} \Big|_1^2 \\
 &= -\frac{1}{s} e^{-s} + 0 + \frac{1}{s} \int_0^1 e^{-st} dt - \frac{1}{s} e^{-2s} + \frac{1}{s} e^{-s} = \\
 &= -\frac{1}{s} e^{-2s} + \frac{1}{s} \left[ -\frac{1}{s} e^{-st} \Big|_0^1 \right] = -\frac{1}{s^2} e^{-s} + \frac{1}{s^2} - \frac{1}{s} e^{-2s} \\
 &= \frac{1}{s^2} (1 - e^{-s}) - \frac{1}{s} e^{-2s}
 \end{aligned}$$

$$\boxed{2} \quad e^{-\alpha t} \cos \beta t = \mathcal{L}(\cos \beta t)(s + \alpha) = \frac{s + \alpha}{(s + \alpha)^2 + \beta^2}$$

$$\boxed{3} \quad \mathcal{L}^{-1}\left(\frac{4}{s^2 + 2s - 3}\right) = \mathcal{L}^{-1}\left(\frac{1}{s-1}\right) - \mathcal{L}^{-1}\left(\frac{1}{s+3}\right) = e^t - e^{-3t}$$

$$\frac{4}{s^2 + 2s - 3} = \frac{4}{(s-1)(s+3)} = \frac{A}{s-1} + \frac{B}{s+3} = \frac{As + 3A + Bs - B}{(s-1)(s+3)}$$

$$\begin{aligned}
 \frac{1}{s-1} - \frac{1}{s+3} &\Rightarrow \begin{cases} A+B=0 & \Rightarrow A=-B \\ 3A-B=4 & \Rightarrow 4A=4 \Rightarrow A=1 \\ & \Rightarrow B=-1 \end{cases}
 \end{aligned}$$

$$\boxed{4} \quad \mathcal{L}^{-1}\left(\sum_{k=1}^5 \frac{a_k}{s+k^2}\right) = \sum_{k=1}^5 \frac{a_k}{k} \mathcal{L}^{-1}\left(\frac{k}{s+k^2}\right) = \sum_{k=1}^5 \frac{a_k}{k} \sin kt =$$

$$= a_1 \sin t + \frac{a_2}{2} \sin 2t + \frac{a_3}{3} \sin 3t + \frac{a_4}{4} \sin 4t + \frac{a_5}{5} \sin 5t$$

(2)

$$\boxed{5} \text{ (VP)} \begin{cases} y' + 3y = 10 \sin t \\ y(0) = 0 \end{cases}$$

$$\mathcal{L}(y') = sY - y(0) = sY$$

Transforming the equation

$$sY + 3Y = 10 \frac{1}{s^2+1} \Rightarrow Y(s) = \frac{10}{(s+3)(s^2+1)} =$$

$$\begin{cases} A+B=0 \Rightarrow A=-B \\ C+3B=0 \mid C=3A \leftarrow \\ A+3C=10 \mid \frac{C}{3} + 3C = 10 \end{cases}$$

$$10C = 30$$

$$\begin{cases} C=3 \\ A=1 \\ B=-1 \end{cases}$$

$$\text{Thus } Y(s) = \frac{1}{s+3} - \frac{s}{s^2+1} + \frac{3}{s^2+1}$$

$$\mathcal{L}^{-1} \Downarrow$$

$$\underline{y(t) = e^{-3t} - \cos t + 3 \sin t}$$

Indeed solution

$$\begin{cases} y' + 3y = [-3e^{-3t} + \sin t + 3 \cos t] + [3y^{-3t} - 3 \cos t + 9 \sin t] = 10 \sin t \\ y(0) = 1 - 1 + 0 = 0 \end{cases}$$

$$\boxed{6} \begin{cases} y'' + y = 2 \cos t \\ y(0) = 6 \\ y'(0) = 0 \end{cases} \Rightarrow \mathcal{L}$$

$$s^2 Y - s y(0) - y'(0) + Y = 2 \frac{s}{s^2+1}$$

$$Y = \frac{2s}{(s^2+1)(s^2+1)} + \frac{6s}{s^2+1} =$$

$$\Rightarrow y(t) = 2 \overbrace{\mathcal{L}^{-1} \left( \frac{s}{s^2+1} \cdot \frac{1}{s^2+1} \right)}^{\cos t * \sin t} + 6 \mathcal{L}^{-1} \left( \frac{s}{s^2+1} \right)$$

$$= 2 \cdot \int_0^t \cos \tau \sin(t-\tau) d\tau + 6 \cos t = \underline{t \sin t + 6 \cos t}$$

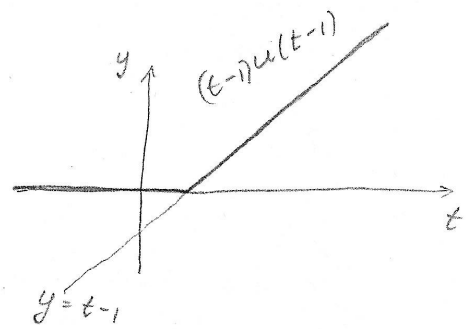
(3)

swice

$$\begin{aligned}
 \cos t * \sin t &= \int_0^t \cos \tau \sin(t-\tau) d\tau = \\
 &= \int_0^t \cos \tau (\sin t \cos \tau - \cos t \sin \tau) d\tau = \\
 &= \sin t \int_0^t \underbrace{\cos^2 \tau}_{\frac{1+\cos 2\tau}{2}} d\tau - \frac{1}{2} \cos t \int_0^t \underbrace{2 \sin \tau \cos \tau}_{\sin^2 \tau} d\tau = \\
 &= \frac{t}{2} + \frac{\sin 2\tau}{4} \Big|_0^t - \frac{1}{2} \cos t \sin^2 \tau \Big|_0^t = \frac{t}{2} \sin t + \frac{\sin^2 \cos t}{2} - \frac{1}{2} \cos t \sin^2 t = \frac{t}{2} \sin t
 \end{aligned}$$

7

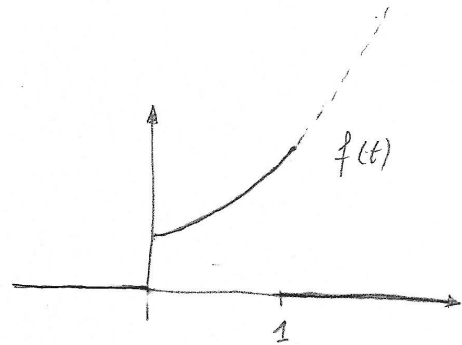
$$\begin{aligned} & \mathcal{L}[(t-1)u(t-1)] \\ &= e^{-s} \mathcal{L}(t-1) = \\ & e^{-s} [\mathcal{L}(t) - \mathcal{L}(1)] = \\ & \underline{e^{-s} \left[ \frac{1}{s^2} + \frac{1}{s} \right]} \end{aligned}$$



(4)

8

$$f(t) = \begin{cases} e^t & \text{if } t \in (0, \infty) \\ 0 & \text{otherwise} \end{cases}$$



$$f(t) = e^t u(t) - e^t u(t-1)$$

$$\mathcal{L}(f(t)) = \mathcal{L}(e^t u(t)) - \mathcal{L}(e^t u(t-1)) =$$

$$\mathcal{L}(e^t) - \mathcal{L}(g(t-1)u(t-1)) = e^{-s} \mathcal{L}(g(t))$$

$$\text{where } g(t) = e^{(t+1)} = e e^t$$

$$\underline{\underline{\mathcal{L}(f(t))}} = \frac{1}{s-1} - e^{-s} \mathcal{L}(e \cdot e^t) = \frac{1}{s-1} - e^{-s} \cdot e \cdot \frac{1}{s-1} = \underline{\underline{\frac{1-e^{-s}}{s-1}}}$$

9

$$\mathcal{L}^{-1} \left( 3 \frac{1-e^{-\pi s}}{s^2+9} \right) = \mathcal{L}^{-1} \left( \frac{3}{s^2+9} \right) - \mathcal{L}^{-1} \left[ e^{-\pi s} \frac{3}{s^2+9} \right] =$$

"  $\sin 3t$ 

"

F(s)  $f(t) = \sin 3t$ 

$$\underline{\sin 3(t-\pi) \cdot u(t-\pi)}$$

$$= \sin 3t - \underbrace{\sin 3(t-\pi)}_{\sin 3t \cos(3\pi) - \cos 3t \sin 3\pi} u(t-\pi) = \sin 3t + \sin 3t u(t-\pi)$$

$$\sin 3t \underbrace{\cos(3\pi)}_{-1} - \cos 3t \underbrace{\sin 3\pi}_{0}$$



Thus

$$\mathcal{L}^{-1} \left( 3 \frac{1 - e^{-\pi s}}{s^2 + 9} \right) = \begin{cases} \sin 3t & \text{if } t < \pi \\ 2 \sin 3t & \text{if } t \geq \pi \end{cases} \quad (5)$$

$$\boxed{10} \quad \begin{cases} y'' + 3y' + 2y = g(t) = \begin{cases} 4t & \text{if } t \in (0, 1) \\ 8 & \text{if } t > 1 \end{cases} \\ y(0) = y'(0) = 0 \end{cases}$$

$$g(t) = 4t(1 - u(t-1)) + 8u(t-1) \quad \text{if } t \geq 0$$

use Laplace transform:

$$\mathcal{L}(y'' + 3y' + 2y) = \mathcal{L}(g(t))$$

$$[s^2 Y - s^2 y(0) - s y'(0)] + 3[sY - y(0)] + 2Y = \mathcal{L}(4t(1 - u(t-1))) + \mathcal{L}[8u(t-1)]$$

$$Y(s^2 + 3s + 2) = \mathcal{L}(4t) - \mathcal{L}(4t u(t-1)) + 8\mathcal{L}(u(t-1))$$

$$= \frac{4}{s^2} - 4 \frac{s+1}{s^2} e^{-s} + \frac{8e^{-s}}{s}$$

$$\mathcal{L}(t u(t-1)) = \mathcal{L}((t+1-1)u(t-1))$$

$$= e^{-s} \mathcal{L}(t+1) = e^{-s} \left( \frac{1}{s^2} + \frac{1}{s} \right)$$

$$\Rightarrow Y = \frac{4}{s^2(s+1)(s+2)} + e^{-s} \frac{4s-4}{s^2(s+1)(s+2)}$$

$$= \frac{4}{s^2(s+1)(s+2)} + e^{-s} \frac{4}{s(s+1)(s+2)}$$

$$- e^{-s} \frac{4}{s^2(s+1)(s+2)}$$

Use Partial fraction decomposition.

$$\frac{4}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} = \frac{As^2 + 3As + 2A + Bs^2 + 2Bs + Cs^2 + Cs}{s(s+1)(s+2)}$$

$$\left. \begin{array}{l} A+B+C=0 \\ 3A+2B+C=0 \end{array} \right\} \begin{array}{l} B+C=-2 \\ 2B+C=-6 \end{array} \quad \left| \quad \begin{array}{l} B=-4 \\ C=2 \end{array} \right.$$

$$2A=4$$

$$A=2$$

$$\frac{4}{s(s+1)(s+2)} = \frac{2}{s} - \frac{4}{s+1} + \frac{2}{s+2}$$

$$\frac{4}{s^2(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{D}{s+2} = \frac{-3}{s} + \frac{2}{s^2} + \frac{4}{s+1} - \frac{1}{s+2}$$

$$= \frac{As^3 + 3As^2 + 2As + Bs^2 + 3Bs + 2B + Cs^3 + 2Cs^2 + Ds^3 + Ds^2}{s^2(s+1)(s+2)}$$

$$A + C + D = 0$$

$$3A + B + 2C + D = 0$$

$$2A + 3B = 0$$

$$2B = 4$$

$$\Rightarrow B=2$$

$$A = -3, B = 2, C = -4, D = 1$$

$$A + C + D = 0$$

$$3A + 2C + D = -2$$

$$2A = -6$$

$$A = -3$$

$$C + D = 3$$

$$2C + D = -2 + 9 = 7$$

↓

$$C = 7 - 3 = 4$$

$$D = -1$$

$$Y = \frac{-3}{s} + \frac{2}{s^2} + \frac{4}{s+1} - \frac{1}{s+2} + e^{-s} \frac{2}{s} - e^{-s} \frac{4}{s+1} + e^{-s} \frac{2}{s+2}$$

$$+ e^{-s} \frac{3}{s} - e^{-s} \frac{2}{s^2} - e^{-s} \frac{4}{s+1} + e^{-s} \frac{1}{s+2}$$

$$= \frac{-3}{s} + \frac{2}{s^2} + \frac{4}{s+1} - \frac{1}{s+2} + e^{-s} \frac{5}{s} - e^{-s} \frac{2}{s^2} - e^{-s} \frac{8}{s+1} + e^{-s} \frac{3}{s+2}$$

$$y(t) = -3 + 2t + 4e^{-t} - e^{-2t} + 5u(t-1) + 2(t-1)u(t-1) - 8e^{(t-1)}u(t-1)$$

$$+ 3e^{2(t-1)}u(t-1)$$

(8)

II

$$a) \mathcal{L}(t e^{-t} \cos t) = -\frac{d}{ds} \mathcal{L}(e^{-t} \cos t) = -\frac{d}{ds} \mathcal{L}(\cos t)(s+1)$$

$$= -\frac{d}{ds} \left[ \frac{s+1}{(s+1)^2+1} \right] =$$

$$-\left[ \frac{(s+1)^2+1 - (s+1) \cdot 2(s+1)}{[(s+1)^2+1]^2} \right] =$$

$$= \frac{(s+1)^2 - 2(s+1)^2 + 1}{[(s+1)^2+1]^2} = \frac{(s+1)^2 - 1}{[(s+1)^2+1]^2}$$

$$b) \mathcal{L}^{-1} \left[ \frac{s}{(s^2-9)^2} \right] = \mathcal{L}^{-1} \left( \frac{d}{ds} \left[ \frac{1}{2(9-s^2)} \right] \right) = -t \mathcal{L}^{-1} \left( -\frac{1}{2} \frac{1}{(s-3)(s+3)} \right)$$

$$\left/ \left[ \frac{1}{2(9-s^2)} \right]' = \frac{1}{2} \left[ -\frac{-2s}{(9-s^2)^2} \right] = \frac{s}{(s^2-9)^2} \right/$$

$$= \frac{t}{2} \mathcal{L}^{-1} \left( \frac{A}{s-3} + \frac{B}{s+3} \right)$$

$$= \frac{t}{2} \left( \frac{1}{6} e^{3t} - \frac{1}{6} e^{-3t} \right) =$$

$$\underline{\underline{\frac{t}{12} (e^{3t} - e^{-3t})}}$$

$$\left/ \frac{1}{(s-3)(s+3)} = \frac{As+3A+Bs-3B}{(s-3)(s+3)} \right/$$

$$A+B=0$$

$$3A-3B=1$$

$$6A=1$$

$$A=1/6$$

$$B=-1/6$$

(10)

12

compute

$$\begin{aligned}
 a) \quad 1 * \sin \omega t &= \int_0^t 1 \sin \omega(\tau-t) d\tau = \int_0^t \sin \omega \tau \cos \omega t \\
 &\quad - \cos \omega \tau \sin \omega t d\tau \\
 &= \sin \omega t \int_0^t \cos \omega \tau d\tau - \cos \omega t \int_0^t \sin \omega \tau d\tau = \\
 &= \sin \omega t \left[ \frac{\sin \omega \tau}{\omega} \Big|_0^t \right] - \cos \omega t \left[ -\frac{\cos \omega \tau}{\omega} \Big|_0^t \right] = \\
 &= \frac{\sin^2 \omega t}{\omega} + \frac{\cos^2 \omega t}{\omega} - \frac{\cos \omega t}{\omega} = \frac{1 - \cos \omega t}{\omega}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad e^t * e^{-t} &= \int_0^t e^t e^{-(t-\tau)} d\tau = e^{2t} \int_0^t e^{-\tau} d\tau = \\
 &= e^{2t} \left[ -e^{-\tau} \Big|_0^t \right] = e^{2t} [1 - e^{-t}] = e^{2t} - e^{3t}
 \end{aligned}$$

13

$$\begin{cases} y'' + 4y = \begin{cases} 1 & \text{if } t \in (0,1) \\ 0 & \text{if } t > 0 \end{cases} = (1 - u(t-1)) & \text{when } t \geq 0 \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

$$\begin{aligned}
 \text{Laplace tr.} \quad s^2 Y - s y(0) - y'(0) - 4Y &= \mathcal{L}(1) - \mathcal{L}(u(t-1)) \\
 &= \frac{1}{s} - \frac{e^{-s}}{s}
 \end{aligned}$$

$$\Rightarrow Y = \frac{1}{s^2 - 4} \left[ s + \frac{1}{s} - \frac{e^{-s}}{s} \right]$$

$$= \frac{s}{(s-2)(s+2)} + \frac{1}{s(s-2)(s+2)} - e^{-s} \frac{1}{s(s-2)(s+2)}$$

$$= \frac{1}{2} \frac{1}{s-2} + \frac{1}{2} \frac{1}{s+2} - \frac{1}{4} \frac{1}{s} + \frac{1}{8} \frac{1}{s-2} + \frac{1}{8} \frac{1}{s+2} + \frac{1}{4} e^{-s} \frac{1}{s} - \frac{1}{8} e^{-s} \frac{1}{s-2} - \frac{1}{8} e^{-s} \frac{1}{s+2} = *$$

$$\frac{s}{(s-2)(s+2)} = \frac{A}{s-2} + \frac{B}{s+2} = \frac{As+2A+Bs-2B}{(s-2)(s+2)} \quad \text{(11)}$$

$$\begin{aligned} A+B &= 1 & 2A-1 & & A=B &= \frac{1}{2} \\ 2(A-B) &= 0 & \Rightarrow & & A &= B \end{aligned}$$

$$\frac{1}{s(s-2)(s+2)} = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s+2} = \frac{As^2-4A+Bs^2+2Bs+Cs^2-2Cs}{s(s-2)(s+2)}$$

$$= -\frac{1}{4} \frac{1}{s} + \frac{1}{8} \frac{1}{s-2} + \frac{1}{8} \frac{1}{s+2} \quad \left| \begin{array}{l} A+B+C=0 \\ 2(B-C)=0 \\ -4A=1 \end{array} \right. \quad \begin{array}{l} 2B=\frac{1}{4} \\ B=C=\frac{1}{8} \\ B=C \\ A=-\frac{1}{4} \end{array}$$

$$Y = -\frac{1}{4} \frac{1}{s} + \frac{1}{8} \frac{1}{s-2} + \frac{1}{8} \frac{1}{s+2} + \frac{1}{4} e^{-s} \frac{1}{s} - \frac{1}{8} e^{-s} \frac{1}{s-2} - \frac{1}{8} e^{-s} \frac{1}{s+2}$$

$$y(t) = -\frac{1}{4} + \frac{1}{8} e^{2t} + \frac{1}{8} e^{-2t} + \frac{1}{4} u(t-1) - \frac{1}{8} e^{2(t-1)} u(t-1)$$

$$- \frac{1}{8} e^{-2(t-1)} u(t-1)$$

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(12)

$$a) \quad \begin{aligned} y_1' &= 6y_1 + 9y_2 & y_1(0) &= -3 \\ y_2' &= y_1 + 6y_2 & y_2(0) &= -3 \end{aligned}$$

$$\begin{aligned} sY_1 - y_1(0) &= 6Y_1 - 9Y_2 \\ sY_2 - y_2(0) &= Y_1 + 6Y_2 \end{aligned} \Rightarrow \begin{bmatrix} 6-s & -9 \\ 1 & 6-s \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$Y_1 = \frac{1}{(6-s)^2 + 9} [3(6-s) + 27]$$

$$Y_2 = \frac{1}{(6-s)^2 + 9} [3(6-s) - 3]$$

$$Y_1 = -3 \frac{s-6}{(s-6)^2 + 9} + 9 \frac{3}{(s-6)^2 + 9}$$

$$\underline{y_1(t)} = -3 e^{6t} \mathcal{L}^{-1}\left(\frac{s}{s^2+9}\right) + 9 e^{6t} \mathcal{L}^{-1}\left(\frac{3}{s^2+9}\right) =$$

$$e^{6t} (-3 \cos 3t + 9 \sin 3t)$$

$$Y_2 = -3 \frac{s-6}{(s-6)^2 + 9} - \frac{3}{(6-s)^2 + 9}$$

$$\underline{y_2(t)} = -3 e^{6t} \mathcal{L}^{-1}\left(\frac{s}{s^2+9}\right) - e^{6t} \mathcal{L}^{-1}\left(\frac{3}{s^2+9}\right) =$$

$$e^{6t} (-3 \cos 3t - \sin 3t)$$

$$b) \quad \begin{aligned} y_1'' + y_2 &= -5 \cos 2t & y_1(0) &= y_1'(0) = 1 \\ y_2'' + y_1 &= 5 \cos 2t & y_2(0) &= -y_2'(0) = -1 \end{aligned}$$

$$\begin{cases} s^2 Y_1 - s y_1(0) - y_1'(0) + Y_2 = -5 \frac{s}{s^2+4} \\ s^2 Y_2 - s y_2(0) - y_2'(0) + Y_1 = 5 \frac{s}{s^2+4} \end{cases}$$

(13)

$$\begin{bmatrix} s^2 & 1 \\ 1 & s^2 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} -5 \frac{s}{s^2+4} + s + 1 \\ 5 \frac{s}{s^2+4} + s - 1 \end{bmatrix}$$

$$Y_1 = \frac{1}{s^4-1} \left[ s^2 \left[ -5 \frac{s}{s^2+4} + s + 1 \right] - \left[ 5 \frac{s}{s^2+4} + s - 1 \right] \right] =$$

$$= -5 \frac{s^2+1}{s^4-1} \cdot \frac{s}{s^2+4} + \frac{s^2-1}{s^4-1} s + \frac{s^2+1}{s^4-1} = \frac{-5s}{(s-1)(s+1)(s^2+4)}$$

$$+ \frac{s}{s^2+1} + \frac{1}{(s-1)(s+1)} =$$

$$Y_1 = \frac{1}{2} \frac{1}{s-1} + \frac{5}{2} \frac{1}{s+1} - 3 \frac{s}{s^2+4} + \frac{2}{s^2+4} + \frac{s}{s^2+1} + \frac{1}{2} \frac{1}{s-1} - \frac{1}{2} \frac{1}{s+1}$$

$$\frac{A}{s-1} + \frac{B}{s+1} + \frac{Cs+D}{s^2+4} = \frac{As^3 + As^2 + 4As + 4A + Bs^3 - Bs^2 + 4Bs - 4B + Cs^3 - Cs + Ds^2 - D}{(s-1)(s+1)(s^2+4)}$$

$$\begin{array}{l} s^3: A+B+C=0 \\ s^2: A-B+D=0 \\ s: 4A-4B-C=-5 \\ 1: 4A-4B-D=0 \end{array} \left\{ \begin{array}{l} A+B+D=5 \\ A-B+D=0 \\ 4A-4B-D=-10 \\ \Rightarrow D-C=-5 \\ C=D-5 \end{array} \right. \left. \begin{array}{l} 2B=5 \quad B=5/2 \\ A+D=5/2 \\ 4A-D=0 \end{array} \right\} \begin{array}{l} A=1/2 \\ D=2 \\ C=-3 \end{array} = \frac{-5s}{(s-1)(s+1)(s^2+4)}$$

$$\frac{1}{(s-1)(s+1)} = \frac{A}{s-1} + \frac{B}{s+1} = \frac{As+A+Bs-B}{(s-1)(s+1)}$$

$$A+B=0$$

$$A-B=1$$

$$\begin{array}{l} A=1/2 \\ B=-1/2 \end{array}$$

$$y_1 = e^t + 2e^{-t} - 3 \cos 2t + \sin 2t + \cos t$$

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$$Y_2 = \frac{1}{s^4-1} \left[ s^2 \left[ 5 \frac{s}{s^2+4} + s-1 \right] + \left[ 5 \frac{s}{s^2+4} - s-1 \right] \right]$$

$$= 5 \frac{s^2+1}{s^4-1} \frac{s}{s^2+4} + \frac{s^2-1}{s^4-1} s - \frac{s^2+1}{s^4-1} = \frac{5s}{(s-1)(s+1)(s^2+4)}$$

$$+ \frac{s}{s^2+1} - \frac{1}{(s-1)(s+1)}$$

$$Y_2 = \underbrace{-\frac{1}{2} \frac{1}{s-1}} - \underbrace{\frac{5}{2} \frac{1}{s+1}} + 3 \frac{s}{s^2+4} - \frac{2}{s^2+4} + \frac{s}{s^2+1} - \underbrace{\frac{1}{2} \frac{1}{s-1}} + \underbrace{\frac{1}{2} \frac{1}{s+1}}$$

$$-\frac{1}{s-1} - 2 \frac{1}{s+1} + 3 \frac{s}{s^2+4} - \frac{2}{s^2+4} + \frac{s}{s^2+1}$$

$$\underline{y_2(t) = -e^t - 2e^{-t} + 3\cos 2t - \sin 2t + \cos t}$$