

Math 320 Exam Crib Sheet

1. Integration by Parts Formula

$$\int u dv = uv - \int v du$$

Example:

$$\int x \exp(x) dx = x \exp(x) - \int \exp(x) dx + C = x \exp(x) - \exp(x) + C$$

with $u = x$, $dv = \exp(x) dx$, $du = dx$, and $v = \exp(x)$.

2. Example of Partial Fractions

$$\int \frac{5}{(x^2 - 5x + 6)} dx = \int \frac{5}{(x-2)(x-3)} dx$$

Let

$$\begin{aligned} \frac{5}{(x-2)(x-3)} &= \frac{A}{x-2} + \frac{B}{x-3} \\ &= \frac{A(x-3) + B(x-2)}{(x-2)(x-3)} \end{aligned}$$

Therefore

$$(A+B)x = 0 \quad \text{and} \quad -3A - 2B = 5.$$

Solving $A+B=0$ and $-3A-2B=5$ gives $A=-5$ and $B=5$. So finally

$$\int \frac{5}{(x^2 - 5x + 6)} dx = \int \frac{-5}{x-2} dx + \int \frac{+5}{x-3} dx = -5 \ln|x-2| + 5 \ln|x-3| + C.$$

3. Exponentials and the Natural Logarithm: All arguments of \ln are assumed greater than zero.

$$\ln(1) = 0$$

$$\ln(a/b) = \ln(a) - \ln(b)$$

$$\ln(ab) = \ln(a) + \ln(b)$$

$$\ln(a^r) = r \ln(a)$$

$$\int \frac{1}{u} du = \ln |u| + C, \quad u \neq 0$$

$$\exp(\ln(x)) = x$$

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$$\exp(a + b) = \exp(a) \exp(b)$$

$$\exp(a - b) = \frac{\exp(a)}{\exp(b)}$$

$$\exp(ab) = (\exp(a))^b = (\exp(b))^a$$

4. **Taylor Series for $f(x)$ about the point $x = x_o$:**

$$f(x) = \sum_{n=0}^{\infty} \frac{d^n}{dx^n} f(x)|_{x=x_o} \frac{(x - x_o)^n}{n!}$$

6. **Sines and cosines for some angles:**

$$\cos(\pi/6) = \sqrt{3}/2, \quad \sin(\pi/6) = 1/2$$

$$\cos(\pi/3) = 1/2, \quad \sin(\pi/3) = \sqrt{3}/2$$

$$\cos(2\pi/3) = -1/2, \quad \sin(2\pi/3) = \sqrt{3}/2$$

$$\cos(4\pi/3) = -1/2, \quad \sin(4\pi/3) = -\sqrt{3}/2$$

$$\cos(5\pi/3) = 1/2, \quad \sin(5\pi/3) = -\sqrt{3}/2$$

$$\cos(\pi/4) = \sqrt{2}/2, \quad \sin(\pi/4) = \sqrt{2}/2$$

$$\cos(3\pi/4) = -\sqrt{2}/2, \quad \sin(3\pi/4) = \sqrt{2}/2$$

$$\cos(5\pi/4) = -\sqrt{2}/2, \quad \sin(5\pi/4) = -\sqrt{2}/2$$

$$\cos(7\pi/4) = \sqrt{2}/2, \quad \sin(7\pi/4) = -\sqrt{2}/2$$

7. **Definition of $\sinh(x)$ and $\cosh(x)$:** $\sinh(x) = (e^x - e^{-x})/2$, $\cosh(x) = (e^x + e^{-x})/2$