HW 9, Math 320, Spring 2017

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Contents

0.1 Section 4.3 problem 6 (page 252)

problem Determine whether the given vectors are L.I. or L.D. Do this by inspection without solving linear system of equations

$$\bar{v}_1 = (1,0,0)$$
$$\bar{v}_2 = (1,1,0)$$
$$\bar{v}_3 = (1,1,1)$$

solution

The equation $c_1\bar{v}_1 + c_2\bar{v}_2 + c_3\bar{v}_3 = \bar{0}$ gives

$$c_1(1,0,0) + c_2(1,1,0) + c_3(1,1,1) = (0,0,0)$$
$$(c_1 + c_2 + c_3, c_2 + c_3, c_3) = (0,0,0)$$

Hence $c_3 = 0$ and $c_2 = 0$ and $c_1 = 0$ is the only solution. Therefore definition of linear independence (page 248), the vectors are linearly independent.

0.2 Section 4.3 problem 7

problem Determine whether the given vectors are L.I. or L.D. Do this by inspection without solving linear system of equations

$$v_1 = (2,1,0,0)$$
$$v_2 = (3,0,1,0)$$
$$v_3 = (4,0,0,1)$$

solution

The equation $c_1\bar{v}_1 + c_2\bar{v}_2 + c_3\bar{v}_3 = \bar{0}$ gives

$$c_1(2,1,0,0) + c_2(3,0,1,0) + c_3(4,0,0,1) = (0,0,0,0)$$
$$(2c_1 + 3c_2 + 4c_3, c_1, c_2, c_3) = (0,0,0,0)$$

Therefore, we see by inspection (comparing terms) that $c_3 = 0, c_2 = 0, c_1 = 0$. Therefore definition of linear independence (page 248), the vectors are linearly independent.

0.3 Section 4.3 problem 15

problem Express the indicated vector w as linear combination of the given vectors v_i if this is possible. If not, show it is impossible

$$\bar{w} = (4,5,6)$$
$$\bar{v}_1 = (2,-1,4)$$
$$\bar{v}_2 = (3,0,1)$$
$$\bar{v}_3 = (1,2,-1)$$

solution

The equation $c_1\bar{v}_1 + c_2\bar{v}_2 + c_3\bar{v}_3 = \bar{w}$ gives (in matrix form)

$$\begin{pmatrix} 2 & 3 & 1 \\ -1 & 0 & 2 \\ 4 & 1 & -1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$$

We now solve for c_1, c_2, c_3. Let $R_2 = R_2 + \frac{1}{2}R_1$ therefore

$$\begin{pmatrix} 2 & 3 & 1 \\ 0 & \frac{3}{2} & \frac{5}{2} \\ 4 & 1 & -1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ 6 \end{pmatrix}$$

$R_3 = R_3 - 2R_1$ gives

$$\begin{pmatrix} 2 & 3 & 1 \\ 0 & \frac{3}{2} & \frac{5}{2} \\ 0 & -5 & -3 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix}$$

$R_3 = R_3 - \frac{10}{3}R_2$ gives

$$\begin{pmatrix} 2 & -1 & 4 \\ 0 & \frac{3}{2} & \frac{5}{2} \\ 0 & 0 & \frac{16}{3} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ \frac{64}{3} \end{pmatrix}$$

Therefore, since there are no zero pivots at end of forward Gaussian elimination, the solution is unique and not zero. (by backward substitution,

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$$

Hence

$$\begin{aligned} \bar{w} &= c_1\bar{v}_1 + c_2\bar{v}_2 + c_3\bar{v}_3 \\ &= 3\bar{v}_1 - 2\bar{v}_2 + 4\bar{v}_3 \end{aligned}$$

0.4 Section 4.3 problem 20

<u>problem</u> Three vectors v_1, v_2, v_3 are given. If they are L.I., show this. Otherwise, find a nontrivial linear combination of them that is equal to the zero vector.

$$\begin{aligned} \bar{v}_1 &= (1, 1, -1, 1) \\ \bar{v}_2 &= (2, 1, 1, 1) \\ \bar{v}_3 &= (3, 1, 4, 1) \end{aligned}$$

<u>solution</u>

Here the space is $\mathbb{R}^4$, but only 3 vectors are given. Therefore theorem 3 at page 252 is used. This theorem says that, if we set the A matrix, with its columns as the given vectors above, then the vectors are L.I. iff there is a 3×3 submatrix inside A which has nonzero determinant. To show this, Gaussian eliminating is used.

$$A=\begin{pmatrix}1&2&3\\1&1&1\\-1&1&4\\1&1&1\end{pmatrix}\xrightarrow{R_2=R_2-R_1}\begin{pmatrix}1&2&3\\0&-1&-2\\-1&1&4\\1&1&1\end{pmatrix}\xrightarrow{R_3=R_3+R_1}\begin{pmatrix}1&2&3\\0&-1&-2\\0&3&7\\1&1&1\end{pmatrix}\xrightarrow{R_4=R_4-R_1}$$

$$\begin{pmatrix}1&2&3\\0&-1&-2\\0&3&7\\0&-1&-2\end{pmatrix}\xrightarrow{R_3=R_3+3R_2}\begin{pmatrix}1&2&3\\0&-1&-2\\0&0&1\\0&-1&-2\end{pmatrix}\xrightarrow{R_4=R_4-R_2}\begin{pmatrix}1&2&3\\0&-1&-2\\0&0&1\\0&0&0\end{pmatrix}$$

The above shows that there is a submatrix of size 3×3 which has nonzero determinant. It is the matrix of the first 3 rows

$$\begin{pmatrix}1&2&3\\0&-1&-2\\0&0&1\end{pmatrix}$$

This has nonzero determinant. Since it is diagonal, its determinant is the product of diagonal elements. Since no diagonal element is zero, the determinant is not zero. This implies vectors are linearly independent.

0.5 Section 4.3 problem 24

problem The vectors $\bar{v}_i$ are known to be L.I., apply the definition of L.I. to show that the vectors u_i are also L.I.

$$\begin{aligned}\bar{u}_1&=\bar{v}_1+\bar{v}_2\\\bar{u}_2&=2\bar{v}_1+3\bar{v}_2\end{aligned}$$

solution

We will examine

$$a\bar{u}_1+b\bar{u}_2=\bar{0}$$

To see if this is satisfied only for $a=0,b=0$.

$$\begin{aligned}a\bar{u}_1+b\bar{u}_2&=\bar{0}\\a\left(\bar{v}_1+\bar{v}_2\right)+b\left(2\bar{v}_1+3\bar{v}_2\right)&=\bar{0}\\\bar{v}_1\left(a+2b\right)+\bar{v}_2\left(a+3b\right)&=\bar{0}\end{aligned}$$

But since we are told that $\bar{v}_1,\bar{v}_2$ are L.I., then this implies that $a+2b=0$ and $a+3b=0$. These two equations we solve now for a,b. These two equations show that $2b=3b$, which means $b=0$. Hence $a=0$ as well. Therefore only solution for $a\bar{u}_1+b\bar{u}_2=\bar{0}$ is that $a=b=0$. This is the same as saying $\bar{u}_1,\bar{u}_2$ are linearly independent.

QED