Define the ODE

```
> assume(a>0 and 'real');
  ode:=epsilon^2*diff(y(x),x$2)=(x^2+a*x)*y(x);
```

$$ode := \epsilon^2 \left(\frac{d^2}{dx^2}\, y(x)\right) = \left(a\sim x + x^2\right) y(x)$$

Solve without giving any B.C.

```
> sol:=dsolve(ode,y(x));
```

$$sol := y(x) = \_C1 \operatorname{hypergeom}\left(\left[-\frac{1}{16}\,\frac{a\sim^2 - 4\,\epsilon}{\epsilon}\right], \left[\frac{1}{2}\right], \frac{1}{4}\,\frac{(a\sim + 2\,x)^2}{\epsilon}\right) e^{-\frac{1}{2}\,\frac{x\,(a\sim + x)}{\epsilon}} + \_C2\,(a\sim + 2\,x) \operatorname{hypergeom}\left(\left[-\frac{1}{16}\,\frac{a\sim^2 - 12\,\epsilon}{\epsilon}\right], \left[\frac{3}{2}\right], \frac{1}{4}\,\frac{(a\sim + 2\,x)^2}{\epsilon}\right) e^{-\frac{1}{2}\,\frac{x\,(a\sim + x)}{\epsilon}}$$

Solve with one B.C. at infinity given

```
> sol:=dsolve({ode,y(infinity)=0},y(x));
```

$$sol := y(x) = \lim_{\_a \to \infty} \left(-\left(\_C2\,(a\sim + 2\,\_a) \operatorname{hypergeom}\left(\left[-\frac{1}{16}\,\frac{a\sim^2 - 12\,\epsilon}{\epsilon}\right], \left[\frac{3}{2}\right], \frac{1}{4}\,\frac{(a\sim + 2\,\_a)^2}{\epsilon}\right) \operatorname{hypergeom}\left(\left[-\frac{1}{16}\,\frac{a\sim^2 - 4\,\epsilon}{\epsilon}\right], \left[\frac{1}{2}\right], \frac{1}{4}\,\frac{(a\sim + 2\,x)^2}{\epsilon}\right) e^{-\frac{1}{2}\,\frac{x\,(a\sim + x)}{\epsilon}}\right) \Big/ \left(\operatorname{hypergeom}\left(\left[-\frac{1}{16}\,\frac{a\sim^2 - 4\,\epsilon}{\epsilon}\right], \left[\frac{1}{2}\right], \frac{1}{4}\,\frac{(a\sim + 2\,\_a)^2}{\epsilon}\right)\right)\right) + \_C2\,(a\sim + 2\,x) \operatorname{hypergeom}\left(\left[-\frac{1}{16}\,\frac{a\sim^2 - 12\,\epsilon}{\epsilon}\right], \left[\frac{3}{2}\right], \frac{1}{4}\,\frac{(a\sim + 2\,x)^2}{\epsilon}\right) e^{-\frac{1}{2}\,\frac{x\,(a\sim + x)}{\epsilon}}$$

Now solve giving B.C. at -infinity

```
> sol:=dsolve({ode,y(-infinity)=0},y(x));
```

$$sol := y(x) = \lim_{\_a \to \infty} \left(\left(\operatorname{hypergeom}\left(\left[-\frac{1}{16}\,\frac{a\sim^2 - 12\,\epsilon}{\epsilon}\right], \left[\frac{3}{2}\right], \frac{1}{4}\,\frac{(-a\sim + 2\,\_a)^2}{\epsilon}\right) (-a\sim + 2\,\_a)\,\_C2 \operatorname{hypergeom}\left(\left[-\frac{1}{16}\,\frac{a\sim^2 - 4\,\epsilon}{\epsilon}\right], \left[\frac{1}{2}\right], \frac{1}{4}\,\frac{(a\sim + 2\,x)^2}{\epsilon}\right) e^{-\frac{1}{2}\,\frac{x\,(a\sim + x)}{\epsilon}}\right) \Big/ \left(\operatorname{hypergeom}\left(\left[-\frac{1}{16}\,\frac{a\sim^2 - 4\,\epsilon}{\epsilon}\right], \left[\frac{1}{2}\right], \frac{1}{4}\,\frac{(-a\sim + 2\,\_a)^2}{\epsilon}\right)\right)\right) + \_C2\,(a\sim + 2\,x) \operatorname{hypergeom}\left(\left[-\frac{1}{16}\,\frac{a\sim^2 - 12\,\epsilon}{\epsilon}\right], \left[\frac{3}{2}\right], \frac{1}{4}\,\frac{(a\sim + 2\,x)^2}{\epsilon}\right) e^{-\frac{1}{2}\,\frac{x\,(a\sim + x)}{\epsilon}}$$

Now solve by giving both B.C.at both ends

```
> sol:=dsolve({ode,y(infinity)=0,y(-infinity)=0},y(x));
```

$$sol := y(x) = 0$$