Some notes on BO 3.35 (to get the leading behavior):

- a consistent balance at lowest order seems to be

$$(S_o')^2 \sim \exp(2/x), \quad x \to 0$$

- taking the square root and integrating leads to

$$S_o \sim \pm \int_1^x \exp(1/s) ds$$

- then change variables $s = 1/t$ to find

$$S_o \sim \pm \int_1^{(1/x)} \exp(t) \left(-\frac{1}{t^2} \right) dt$$

- integration by parts gives

$$S_o \sim \mp x^2 \exp(1/x) + \pm a_o + \mp \int_1^{(1/x)} \frac{2}{t^3} \exp(t) dt$$

$$\sim \mp x^2 \exp(1/x)$$

and subdominant terms have been dropped. [Why are these terms subdominant and why is this the only consistent integration by parts?]