Design project report, EMA 545, team 4, Spring 2013

Spring 2013
University of Wisconsin, Madison

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Spring 2013 Compiled on April 19, 2021 at 3:06am [public]

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1 Introduction

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This report outlines a simple passive vibration isolation system design for use in the first class cabin of a Boeing 757-200 airplane with the goal of reducing the vibrations felt by the passengers in the first class cabin. This was done by simulation in order to select suitable design parameters that produced an acceptable absolute acceleration time history compared the rest of the airplane during a turbulent flight.

2 Discussion and results

2.1 Notations used in the report

M	mass of first class cabin
k	spring constant
ζ	critical damping constant
r	ratio of external load frequency to the natural frequency of first class cabin $\frac{\varpi}{\omega_{natural}}$
r_n	ratio of external load n^{th} harmonic frequency to the natural frequency of first class cabin $\frac{\varpi_n}{\omega_{natural}}$
T_r	Transmissibility. The ratio of cabin absolute displacement to base absolute displacement
ω_{nat}	Natural frequency of first class cabin
ω_1	Fundamental frequency of the external load frequency.
EOM	Equation Of Motion
c	damping constant for damper under first class cabin
Z_n^{acc}	the complex amplitude of the term associated with the n^{th} harmonic of the frequency $z''(t)$
Z_n^{disp}	the complex amplitude of the term associated with the n^{th} harmonic of the displacement $z(t)$
Y_n	the complex amplitude of the term associated with the n^{th} harmonic of the displacement of $y(t)$

Table 1. Description of mathematical notations used in report

2.2 Mathematical model

Reducing the vibration effect felt by the passengers in the first class cabin was based on reducing the transmissibility ratio (T_r) of the *absolute acceleration* of the airplane to that of the first class cabin. A passive vibration isolation system was used for its ease of implementaion and its low cost. The model is based on figure 1 below

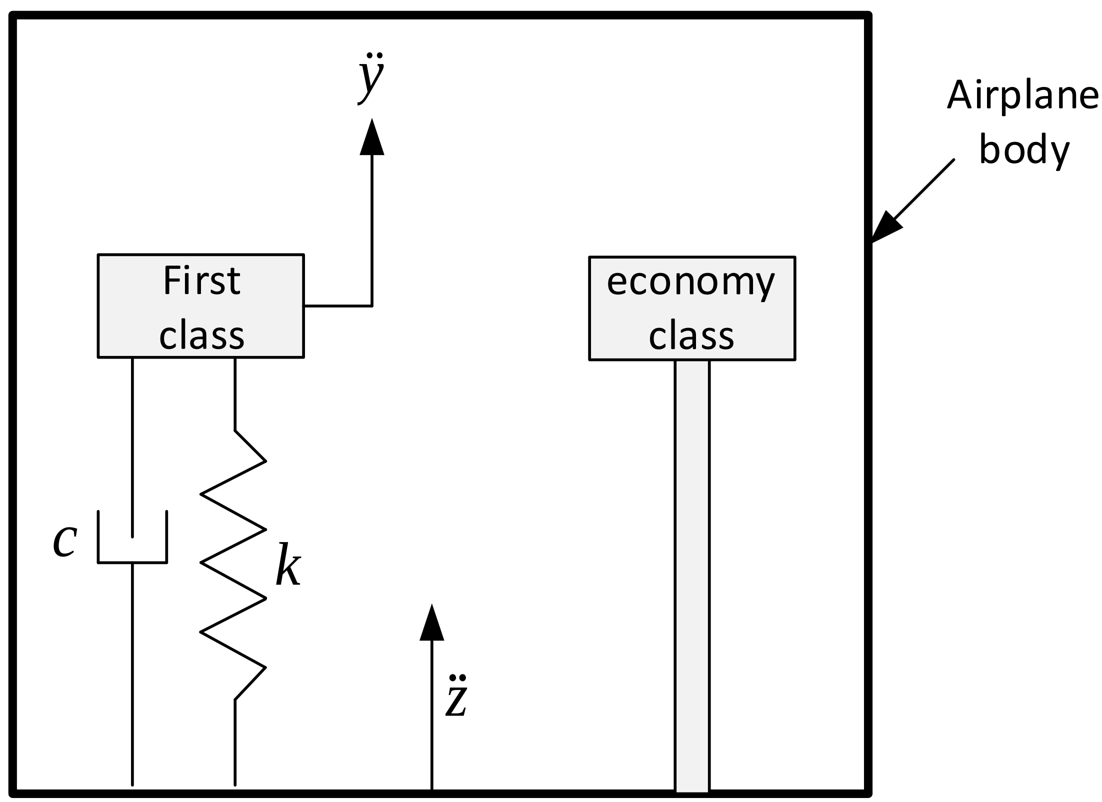

Figure 1. Mechanical model view of vibration isolation system in place.

The absolute acceleration of the first class cabin, $y''(t)$, was calculated with the vibration isolation system in place and then compared to the absolute acceleration, $z''(t)$, of the rest of the airplane. The goal was to produce a smooth absolute acceleration time history when compared to the rest of the airplane. This was done by adjusting M, ζ and K and running a simulation of the motion of the plane with our vibration isolation system in place. A plot of T_r vs. r was also used to insure that the maximum T_r remained small as the frequency ratio r was increased.

Assuming the mass of cabin is M, which includes the live load (passengers), then applying Newton's laws the the first class cabin results in the equation of motion

$$my'' + c\left(y' - z'\right) + k\left(y - z\right) = 0$$

$$my'' + cy' + ky = cz' + kz$$

The transfer function between $y(t)$ and $z(t)$ in the frequency domain can now be derived (Appendix contains complete derivation) resulting in

$$T(r)=\left|\frac{Y_n}{Z_n^{disp}}\right|=\frac{\sqrt{1+(2\zeta r_n)^2}}{\sqrt{\left(1-r_n^2\right)^2+(2\zeta r_n)^2}}$$

To compare the absolute acceleration of the first class cabin with the rest of the airplane, the absolute acceleration, $y''(t)$, is now found from Y_n. Since $y(t)=\mathrm{Re}\left\{Y_n e^{i\varpi_n t}\right\}$ then $y''(t)=\mathrm{Re}\left\{-\varpi_n^2 Y_n e^{i\varpi_n t}\right\}$.

2.3 Design results

$z''(t)$ (given) and $y''(t)$ (computed) are now plotted on the same plot in order to compare the effect of our vibration isolation system to the comfort of the first class passengers. The final design parameters used are (Appendix 5.1)

M Mass of first class cabin (dead+live)	3050 kg
ζ	0.7
k	9700 N/M

Table 3. Final values of design parameters

Figure 2 below shows the result using the above parameters

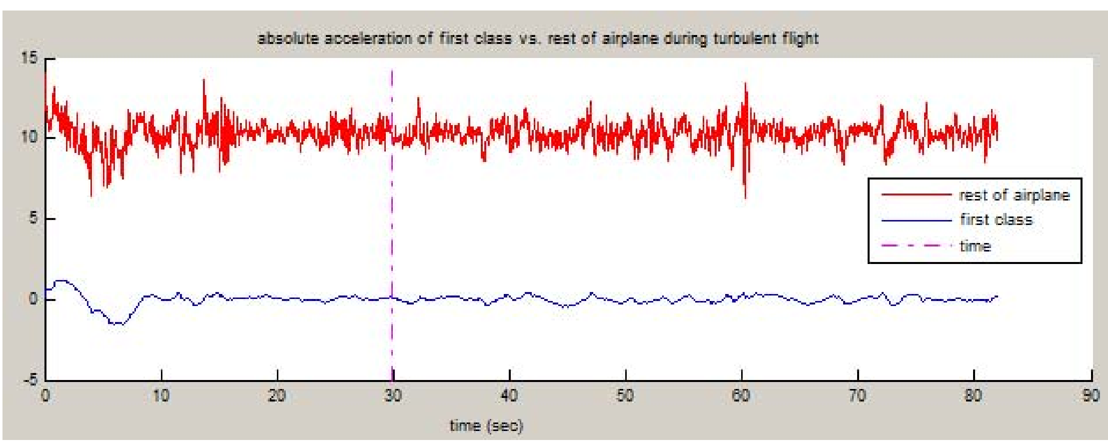

Figure 2. First class cabin absolute acceleration compared to rest of airplane.

We see from figure 2 that the absolute acceleration of the first class cabin has much less variation and is much smoother than the absolute acceleration of the rest of the airplane. From this we can see that the first class passengers experience a much more comfortable flight than the rest of the airplane. In addition, the transmissibility plot was found to be acceptable since T_r decreases with increasing r

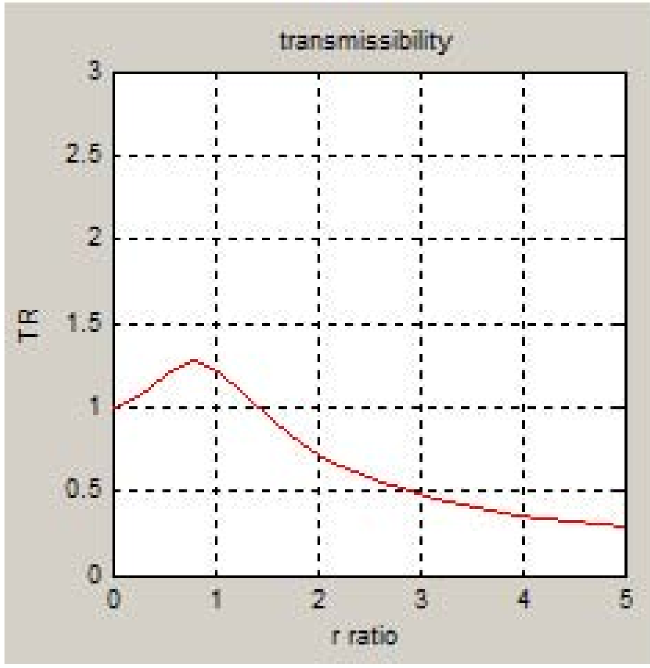

Figure 3. Transmissibility plot of first class cabin.

In addition to producing a smooth absolute acceleration time history, the goal was also to insure that T_r decreased as r increased. This implies that at higher external acceleration relative to the natural frequency, our vibration isolation system remained effective. The simulation program generated a mechanical view showing the absolute position of the first class cabin, with an offset, and the absolute position of the airplane during the flight as shown in figure 4 below.

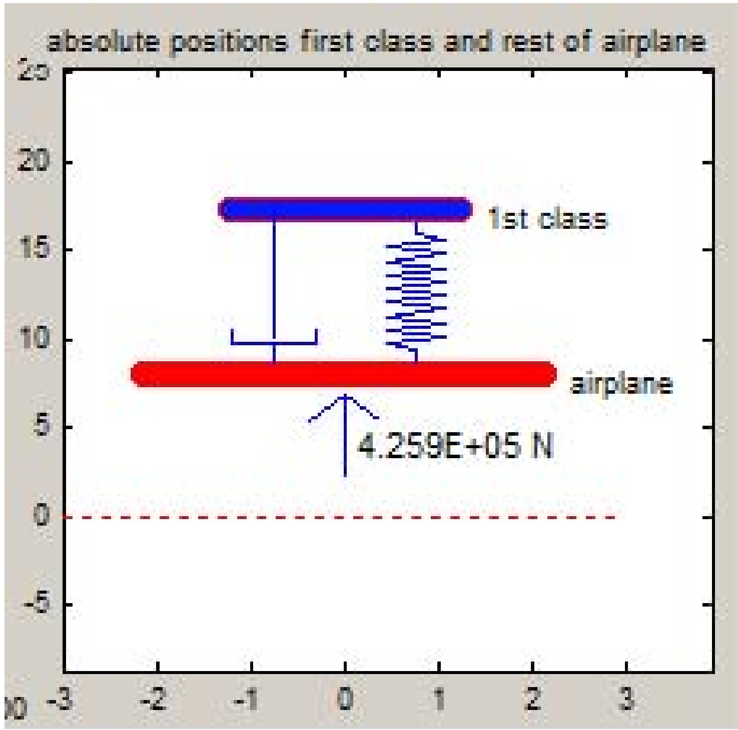

Figure 4. Animation of vibration isolation during flight.

The force shown in Figure 4. below the airplane is the numerical value of $-Mz''$ where z'' is the absolute acceleration of the airplane and M is the total mass of the first class cabin.

3 Implementation of the vibration isolation system

The vibration dampening system proposed for the first class cabin is a simple spring dashpot system that utilizes the additive properties of springs and dashpots to dampen the vibration of the first class cabin in the Boeing 757-200.

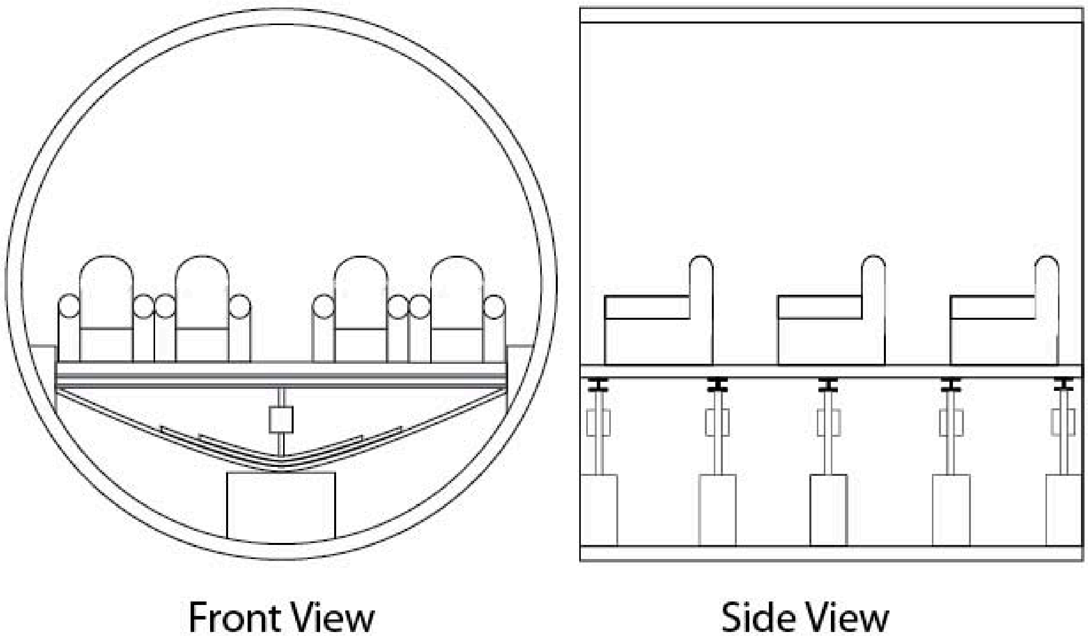

Figure 5. Schematic diagram of vibration isolation system in place

The design of our passive vibration isolation system is simple and effective with a minimal costs. It starts by defining the area that represents the first class cabin, which is at the front of the plane right behind the cockpit.

The cabin spans the entire inside width of the airplane body, which is 3.53 m (11.58 ft), and then extends down the body of the plane roughly 3.35 m (11 ft) giving the first class cabin a total area of $3.53 \times 3.35\ m^2$ ($11.58 \times 11\ ft^2$).

The next step in our design is to define the area that will actually be part of the vibration isolation system. We cannot use the whole floor of the first class cabin because the rounded body of the airplane would not allow the floor to travel up and down rendering our whole system ineffective. To solve this problem we started at the center of the plane's cross section and went out 1.524 m (5 ft) in either direction giving a total area of the platform used in our vibration isolation system $3.048 \times 3.35\ m^2$ ($10 \times 11\ ft^2$) as seen above in figure 5.

To begin the actual design, additional support must be given to the aluminum floor of the cabin. The use of 6061 T6 Aluminum I-beams (specifications are given in appendix 5) spanning the width of the platform provides the needed support. In addition the I-beams provide a sturdy surface for the spring and dashpot system to contact the cabin floor.

The key component of the vibration dampening system is the use of carbon fiber leaf springs. We chose carbon fiber leaf springs in place of steel for several reasons. They provide a softer ride at a lower noise level and excellent stability due to better damping characteristics than steel. Placed in series, the use of 5 carbon fiber leaf springs provides the spring constant required (9700 N/m) and a low increase in weight.

The dashpots needed for our design, 2*K*325 Dashpots, can be purchased from many manufactures. When added in parallel they provided the necessary damping coefficient of 7425 N*s/m needed when the first class cabin is full and 5800 N/s*m when the cabin is empty.

Our design for this passive vibration isolation system works whether the first class cabin is full, empty, or half way in-between. The system works best when the cabin is fully loaded with passengers, and has almost identical results with no passengers on board. Even though the results are slightly diminished with fewer passengers, the system still creates a noticeably smoother flight.

4 Cost estimate of the vibration isolation system

The total cost of our vibration isolation system is around $16500 (appendix 5). The cost of the aluminum support beams, dashpots and carbon fiber leaf springs make up the majority of the material cost totaling only about $3000. The majority of the total cost comes from the additional weight of the system and the resulting price of fuel used during the planes lifetime. The additional weight results in an expected cost of about $13500 over the lifetime of the plane.

The damping effects of the system could be improved if weight were added to the cabin. However the additional cost of the added weight over the lifetime of the plane would outweigh the benefits for the passengers. If however some heavy components of the plane were to be attached to the first class cabin, the system could be redesigned for an even better ride. This would require further investigation into the balance of the plane, flight dynamics and a deeper knowledge of the various components of the plane so it falls out of the scope of this project.

5 Appendix

5.1 Design values

weight

This table shows the design values based on weight

Description of item	Mass (kg)
15 Chairs @ 100 kg/chair	1500
10 ft by 11 ft aluminum flooring	200
5 Aluminum I-beams @ 20 kg/beam	100
5 Carbon Fiber leaf springs @ 5 kg/spring	25
Miscellaneous weight	25
15 Passengers @ 80 kg	1200
Weight of First Class Cabin before vibration isolation system	1700
Weight of First Class Cabin after vibration isolation system	1850
Weight of First Class Cabin with maximum passengers	3050

Table 3. Mass of items used in design calculations

The total mass M has the value of 3050 kg. For our ζ value we choose the value 0.7 as it worked well in simulations to provide a smooth ride for the passengers while still keeping T_r small.

Leaf springs and spring K value

The most important aspect of picking a k value is the total allowed clearance the first class cabin floor has to move. The first class cabin's floor has a displacement relative to the body of the aircraft and if that gets too large the floor will make contact with the body of the airplane. The lower the k value we choose, the larger the displacement of the first class cabin relative to the body of the airplane will become. The maximum travel distance of the first class cabin is around 20 cm (7.87 in) and we can use this value to pick an appropriate k value. A k value around 10000 N/m keeps the first class cabin floor within this tolerance. The following plot shows the absolute acceleration of the cabin vs. the rest of the airplane during the turbulent flight [1].

[1] absolute position of the first class cabin was computed from the absolute acceleration of the cabin in the frequency domain. Hence the average value was not used due to the division by zero problem with this method. We do not have another method to find absolute position from absolute acceleration (unless we use more advanced numerical integration method in time domain, which is beyond the scope of this course)

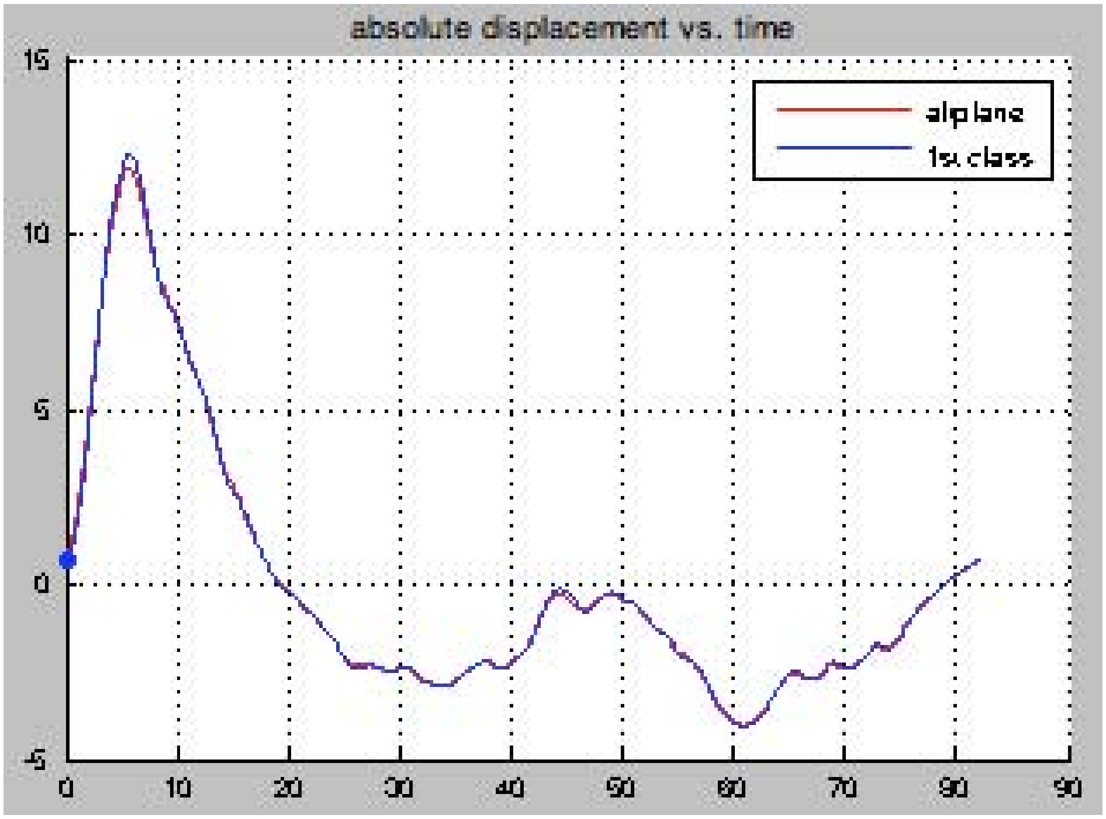

Figure 6. absolute acceleration of first class cabin compared to rest of airplane

To keep the weight of our vibration isolation system as small as possible we opted to use carbon fiber leaf springs. The k value of any leaf spring system can be calculated by the equation

$$k = \frac{8Enbt^3}{3L^3}$$

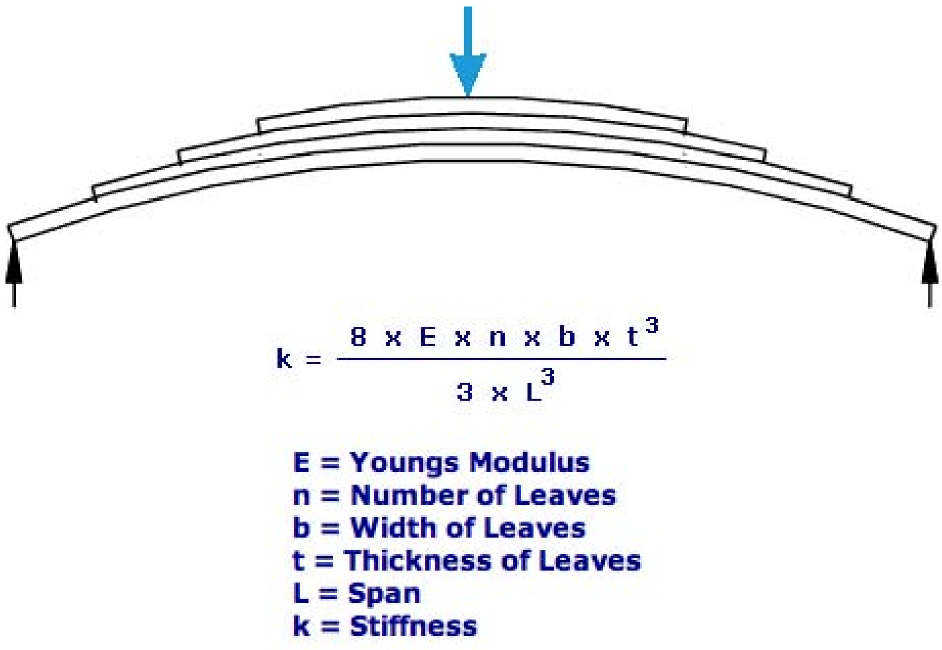

Figure 7. Leaf spring design used in vibration isolation system

Since our springs are in parallel, the k values add to give a total equivalent k. We took the k value selected (10000 N/m) and divided it by 5 giving us an individual k value of 2000 N/m. Using the following dimensions for the leaf spring resulted in a k value of 1940 N/m for a total k value of 9700 N/m.

- $E = 17$ Gpa
- $n = 3$
- $L = 3.048$ meter (10 ft)
- $b = 0.1016$ meter (4 in)
- $t = 0.015875$ meter (5/8 in)

5.2 Cost values

After finding the materials we needed, the following describes how we calculated the total cost of our vibration isolation system.

- 5 @ 10 ft 6061 T6 Aluminum I-beams @ \$180/beam results in \$900.
  Width 6 in, Flange 4 in, Web 0.19 in, Thickness 0.28 in.
- 5 Carbon Fiber leaf Springs @ \$300/spring results in \$1500.
- 150 kg of extra weight, total weight of the airplane is 59350 kg.
  Fuel costs for this aircraft was estimated to be \$3,500/hour and a typical aircraft operates 3000 hours per year. An increase of 1% in the weight of the aircraft is

expected to increase fuel costs by 0.5%
$\frac{150\text{ kg}}{2*59350\text{ kg}} \times \$3500 \times 3000 = \$13270$

- 5 2k325 Dashpots @ \$100/dashpot = \$500
Needed c is around 2000 N*s/m. These dashpots have an adjustable c from 0 to 7000 N*s/m

- Total cost estimate \$16500

5.3 Simulation program description

The simulation program was a GUI program written in Matlab version 2013a, which made it easier to determine the parameters to use for the design. The following is a screen shot of the program. The program can be downloaded from the project web site

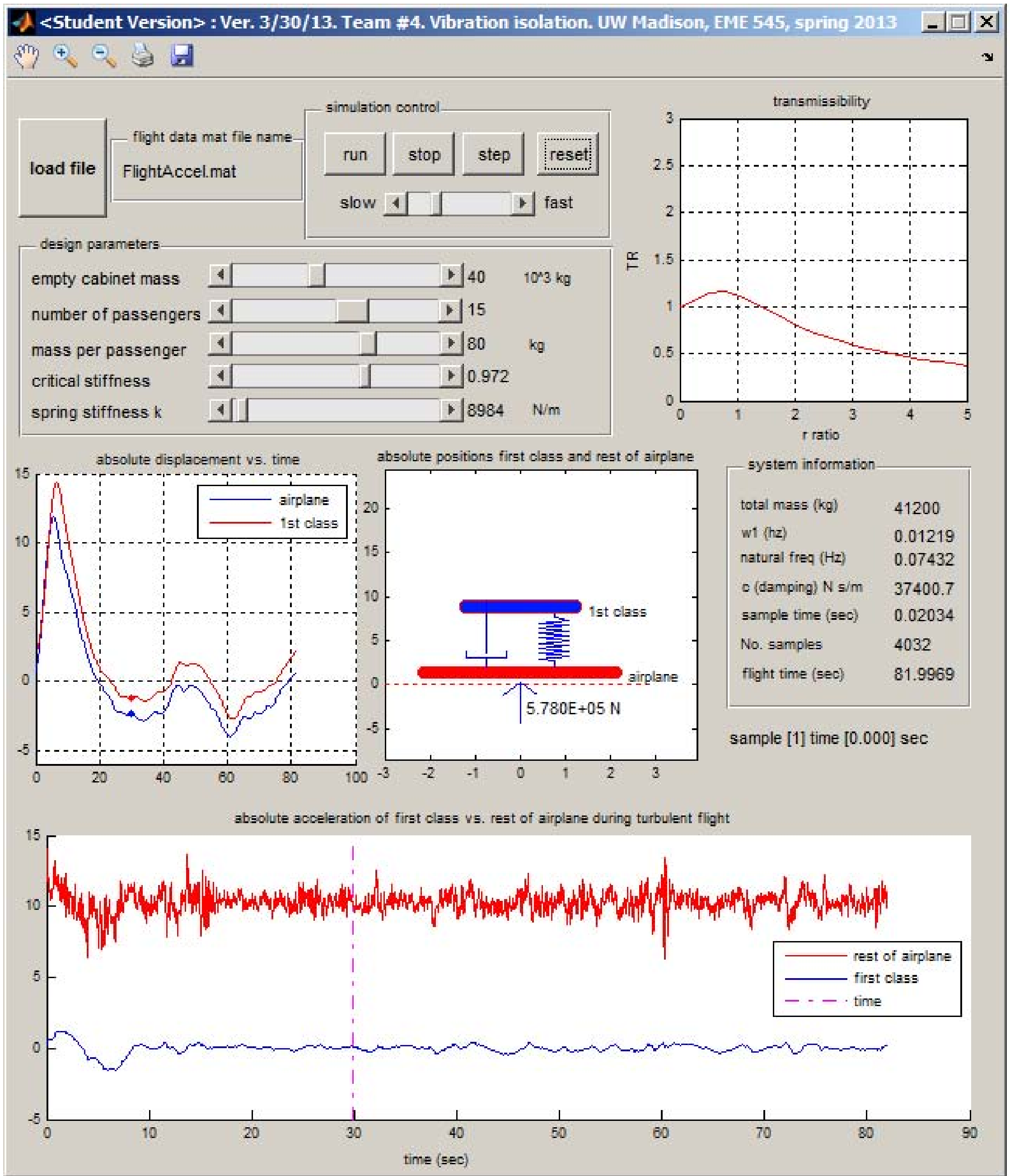

Figure 6. Simulation Matlab program used for obtaining the design parameters.

The first step is to load the Matlab .mat file which contains the acceleration time history. Then one can use the sliders to adjust the system parameters and see the effect on the absolute acceleration of the first class cabin. Computation was done in the FFT domain using the functions $fft\_easy()$ and $ifft\_easy()$ in the class web site. The absolute displacement was found from the absolute acceleration in the frequency domain. Due to the problem of division by zero for the first component in the frequency vector, this was set to zero before using $ifft\_easy()$.

5.4 Derivation of the transfer function

Assuming the mass of cabinet is M which includes passengers weight, by applying Newton's laws the EOM for the first class cabin is

$$\begin{aligned} my'' + c\left(y' - z'\right) + k\left(y - z\right) &= 0 \\ my'' + cy' + ky &= cz' + kz \end{aligned} \tag{1}$$

The time history of the turbulent acceleration $z''(t)$ was given to us in the matlab mat file. Therefore in the frequency domain, and assuming the time history represents one period we can write

$$z'' = \operatorname{Re}\left\{Z_n^{acc} e^{i(\omega_1 n)t}\right\}$$

Substituting back into Eq 1 and simplifying, the magnitude of the absolute displacement of the first class cabin relative to absolute displacement of airplane is found to be

$$\left|\frac{Y_n}{Z_n^{disp}}\right| = \frac{\sqrt{1 + (2\zeta r_n)^2}}{\sqrt{\left(1 - r_n^2\right)^2 + (2\zeta r_n)^2}}$$

Where Z_n^{acc} is the complex amplitude of the n^{th} harmonic component in the acceleration data. Letting $\omega_1 n \equiv \varpi_n$ then in the frequency domain Eq 1 becomes

$$\begin{aligned} \operatorname{Re}\left\{\left(-m\varpi_n^2 + i\varpi_n c + k\right) Y_n e^{i\varpi_n t}\right\} &= \operatorname{Re}\left\{\left(c\frac{Z_n^{acc}}{i\varpi_n} + k\frac{Z_n^{acc}}{-\varpi_n^2}\right) e^{i\varpi_n t}\right\} \\ Y_n &= \left(\frac{\frac{c}{i\varpi_n} - \frac{k}{\varpi_n^2}}{-m\varpi_n^2 + i\varpi_n c + k}\right) Z_n^{acc} \\ &= -\frac{Z_n^{acc}}{\varpi_n^2}\frac{1 + i2\zeta r_n}{\left(1 - r_n^2\right) + 2i\zeta r_n} \end{aligned}$$

Where

$$r_n = \frac{\varpi_n}{\omega_{natural}}$$

But $-\frac{Z_n^{acc}}{\varpi_n^2}$ is the absolute displacement of the airplane, say Z_n^{disp}, hence the transfer function between the absolute displacement of first class cabin and the absolute displacement of the airplane is

$$Y_n = \frac{1 + i2\zeta r_n}{\left(1 - r_n^2\right) + 2i\zeta r_n} Z_n^{disp}$$

The magnitude of the absolute displacement of first class cabinet relative to absolute displacement of the airplane is

$$\left|\frac{Y_n}{Z_n^{disp}}\right| = \frac{\sqrt{1 + (2\zeta r_n)^2}}{\sqrt{\left(1 - r_n^2\right)^2 + (2\zeta r_n)^2}}$$

5.5 References

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