Homework #8
## EMA 545, Spring 2013

Problem 1: (40 points)

a.) Find the nonlinear equation of motion for the system pictured below. The block has mass m and the guide can be approximated as frictionless. In the position shown the spring is unstretched and the angle between the spring and guide bar is θ_0.

b.) Linearize your equation of motion for small deflections from the position shown (i.e. using a Taylor series expansion on $k(x)$ about $x=0$). Use a computer to plot $k(x)$ versus the linear approximation for L=1 m, k=1000 N/m and θ_0 = 45 degrees for x ranging from -1 m to +1 m.

c.) Find the equations of motion for the system using the stiff spring approximation and assuming small displacements from an equilibrium position defined by L=1 m, k=1000 N/m and θ_0 = 45 degrees. Compare your result with your linearized result from part (b).

d.) Using m=1, find the response of the nonlinear system (in part a) using ode45 and plot the displacement of the mass over a few cycles when it is released from rest at x(0)=0.1 and also at x(0)=0.5 meters. Overlay both curves on the same set of axes. How does the period of the response compare with the linearized natural frequency in each case? In what other way(s) does the nonlinearity manifest itself in the response of the system when x(0)=0.5?
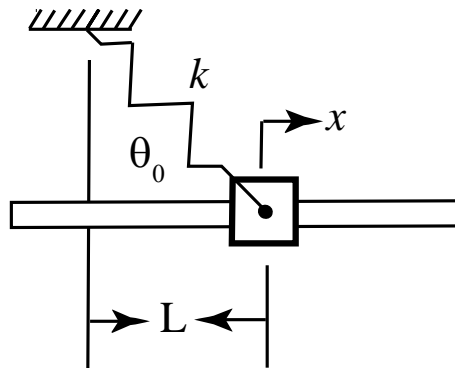
Problem 2: Exercise 1.27 from Ginsberg.

A standard model for a wing has a translational spring k_y and a torsional spring k_T representing the elastic rigidity. Point E represents the elastic center because static application of a vertical force at that point results in upward displacement without an associated rotation. The design of the wing is such that horizontal movement of point E is negligible. The lift force L acts at point P, which is called the center of pressure. The lift force may be treated as known. When the wing is in its static equilibrium position, points G, E and P form a horizontal line. Point G is the center of mass, and the radius of gyration of the wing about that point is r_G. Denote the mass of the wing m. Derive the equations of motion for the wing, <u>assuming small displacements (and small rotational displacements)</u>. Put the equations in matrix form and <u>check the units and sign of each</u>

term in your EOM. (Hint: use the displacement of the center of gravity and the rotation of the wing as generalized coordinates.)
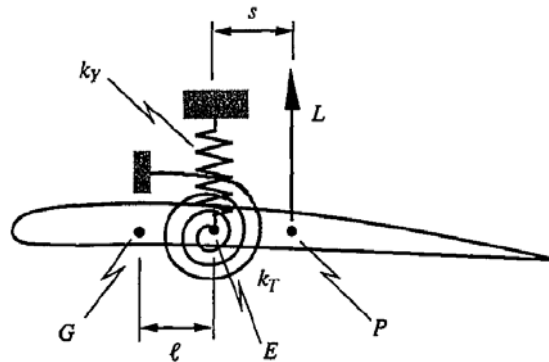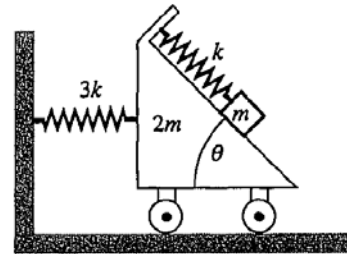

EXERCISE 1.27

Problem 3: Use the power balance method and the stiff spring approximation to find the equation of motion of the system pictured in Problem 1.16.

Problem 4: Exercise 1.33 from Ginsberg: (be very careful to write a correct expression for the acceleration of the small block.) Check the unit and sign of each term in your EOM.

1.33 Determine the equations of motion governing a pair of generalized coordinates that locate the position of the cart and the sliding block. Friction is negligible.

EXERCISE 1.33

Problem 5: Exercise 1.30 from Ginsberg: Use the stiff spring approximation and assume small deflections of both bars. Check the units and sign of each term in your EOM. Gravity acts downward (same direction as the force, F).
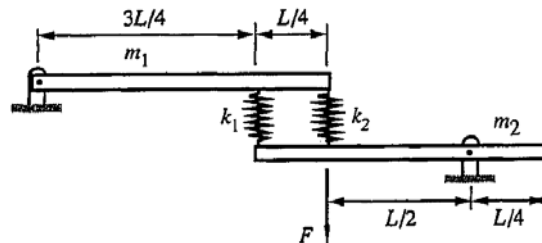
1.30 Both bars in the linkage are horizontal, as shown, when the system is in static equilibrium. Determine the linearized equations of motion for this system.

EXERCISE 1.30

Problem 6: Exercise 4.1 in Ginsberg. Solve the eigenvalue problem by hand to get the natural frequencies and mode shapes. You may check your answers with Matlab.