Homework #10
EMA 545, Spring 2013

For all of these problems you may use Matlab or some other package to find the natural frequencies and mode vectors and to mass normalize the mode vectors (if needed).

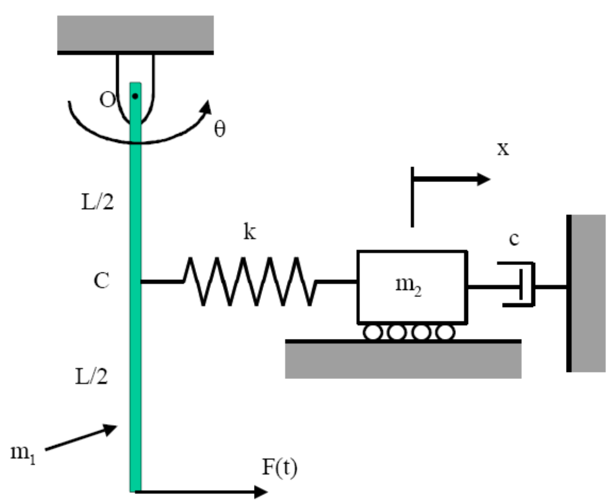
1.) Exercise 4.8 from Ginsberg. (Note: the spring constants are defined such that the frequencies given are the natural frequencies that each spring-mass system would have if it were attached to a rigid base. Notice that there is not a simple relationship between those frequencies and the natural frequencies of the system as a whole.)

2.) Exercise 4.30 as given in the text. Repeat the analysis for $k=2mg/L$ and graph that response as well. (Questions to consider: What do you notice about the natural frequencies of this system in each case? How does that affect the way the response looks? Why?)

3.) Exercise 4.43 from Ginsberg. How does the time required to reach steady state compare with $\tau_r=1/(\zeta_r\omega_r)$ for each mode, r=1,2,3?

4.) Exercise 4.47 from Ginsberg.

5.) (Covering material from Chapter 5)
A uniform rod of length L and mass m_1 is attached to a cart having mass m_2 by means of a spring k. A viscous damper c resists the motion of the cart.



a.) Let $F(t)=Re[F\exp(i\omega t)]$, $x(t) = Re[X\exp(i\omega t)]$ and $\theta(t)=Re[Y\exp(i\omega t)]$. Find analytical expressions for the complex transfer functions X/F and Y/F.
b.) Find the magnitude and phase of the response of x and θ when the system is forced at its natural frequencies $\omega=\omega_1$ and $\omega=\omega_2$. Compare these values to the eigenvectors for modes 1 and 2. Use the following numerical values: $m_1=m_2=1$ kg, k=3 N/m, L=1 m, g=9.81 m/s^2, and c=0.1 N-s/m.

c.) Plot the transfer functions Y/F and X/F over a range of frequencies encompassing both modes of vibration. Use the plot to determine at what frequency m_2 acts as a vibration absorber for the rod. How does that frequency compare with the natural frequency that the system would have if the rod were held fixed: $\omega_{cart} = (k/m_2)^{1/2}$?

6.) Consider **Exercise 3.45 and 3.46** in the text (you solved this in problem #3 in HW#6). Use the steady-state displacement that you computed using FFT techniques for $\tau = 3\pi/\omega_n$ to compute the maximum stress in the spring. Assume that the spring is a cantilever beam (in bending) modeled after one of the pillars supporting the ERB, which have length L=40m, rectangular cross section with height h, equal to the width b=h=0.6m, and is constructed from a material with modulus E=30 GPa and ultimate tensile strength σ=40 MPa. (The mass of the beam is assumed to be included in *m*, so its density is not needed.) Let the mass *m* be such that the natural frequency of the mass-spring system is ω_n=0.2Hz. What is the amplitude of the force, P, such that the beam fails due to the dynamic load? Compare that to the static load required to cause the beam to fail (also in bending).