cheat sheet ME440 written by Nasser M Abbasi

odd $f(-t)=-f(t)$

even $f(-t)=f(t)$

Fourier Series $f(t)\approx \frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi}{T}nt\right)\sin\left(\frac{2\pi}{T}nt\right)$

$a_0=\frac{1}{T/2}\int_{-T/2}^{T/2} f(t)\,dt$; $a_n=\frac{1}{T/2}\int_{-T/2}^{T/2} f(t)\cos\left(\frac{2\pi}{T}nt\right)dt$ …

$X\cos(\omega t-\theta)=A\cos(\omega t)+B\sin(\omega t)$

$X\cos(\omega t-\theta)=A\cos(\omega t)+B\cos(\omega t-\pi/2)$

$A=X\cos\theta$, $B=X\sin\theta$; $\theta=\tan^{-1}\frac{B}{A}$

$\cos x=1-\frac{x^2}{2}+\frac{x^4}{4!}-\cdots$; $\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots$

$\sin^2 x=\frac{1}{2}-\frac{1}{2}\cos 2x$	$\sin u\sin v=\frac{1}{2}[\cos(u-v)-\cos(u+v)]$
$\cos^2 x=\frac{1}{2}+\frac{1}{2}\cos 2x$	$\cos u\cos v=\frac{1}{2}[\cos(u-v)+\cos(u+v)]$
$\sin 2x=2\sin x\cos x$	$\sin u\cos v=\frac{1}{2}[\sin(u+v)+\sin(u-v)]$
$\cos 2x=1-2\sin^2 x$	

complex F.S. $f(t)\sim\sum_{n=-\infty}^{\infty} C_n e^{j\frac{2\pi}{T}nt}=\sum C_n e^{jn\omega t}$; $C_n=\frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-j\omega nt}\,dt$

$a_n=C_n+C_{-n}$

$b_n=j(C_n-C_{-n})$

$a_0=2C_0$

critical damping $x(t)=e^{-\omega_n t}(C_1+C_2 t)$. see slide 165

$C_1=x_0$

$C_2=\dot{x}_0+\omega_n x_0$

$C_{critical}=2m\omega_n$; $\xi=\frac{c}{c_r}=\frac{c}{2m\omega_n}$

$\ddot{x}+2\xi\omega_n\dot{x}+\omega_n^2 x=F$, $\boxed{\omega_n=\sqrt{\frac{k}{m}}}$ $2\xi\omega_n=\frac{c}{m}$ or $\boxed{\xi=\frac{c}{2\sqrt{km}}}$

$\xi=0$ undamped: $x(t)=x_0\cos\omega_n t+\frac{\dot{x}_0}{\omega_n}\sin\omega_n t$. or $x(t)=X\cos(\omega_n t-\theta)$

$X=\sqrt{x_0^2+\left(\frac{\dot{x}_0}{\omega_n}\right)^2}$; $\theta=\tan^{-1}\left(\frac{\dot{x}_0}{\omega_n x_0}\right)$

damped: $\xi>1$

$\xi=1$ $x(t)=e^{-\omega_n t}[x_0+(\dot{x}_0+\omega_n x_0)t]$

$\xi<1$ $x(t)=e^{-\xi\omega_n t}(A\cos(\omega_d t)+B\sin\omega_d t)$. $A=x_0$; $B=\frac{\dot{x}_0+\xi\omega_n x_0}{\omega_d}$

$\boxed{\omega_d=\omega_n\sqrt{1-\xi^2}}$ or $x(t)=Xe^{-\xi\omega_n t}\cos(\omega_d t-\phi)$; $X=\sqrt{A^2+B^2}$, $\phi=\tan^{-1}\left(\frac{B}{A}\right)$

Log decrement: $\boxed{\ln\left(\frac{X_1}{X_2}\right)=\xi\omega_n\tau_d}$ must be over one period

$=\frac{2\pi\xi}{\sqrt{1-\xi^2}}=\delta$. if more than one cycle is given then

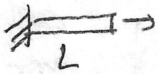
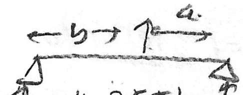
$\boxed{\delta=\frac{1}{n}\ln\left(\frac{X_1}{X_{n+1}}\right)}$ Find δ then to find ξ. Then find $\omega_n=\frac{2\pi}{\tau_d\sqrt{1-\xi^2}}$. find ω_n

For rotation $\boxed{C_{critical}=2I_0\omega_n}$

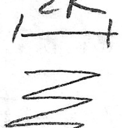
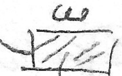
For critical damping, $x_{max}=\frac{\dot{x}_0 e^{-1}}{\omega_n}$

For bending use $m=0.229$ column mass

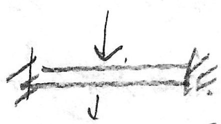
for longitudinal I use $\frac{1}{3}$ mass of spring

Stiffness  $K = \frac{A \cdot E}{L}$;  $K = 3EI\left(\frac{L}{ab}\right)^3$  $K = \frac{3EIL}{(ba)^2}$

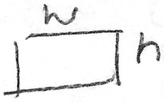
1 ounce = 0.0625 lb.

Slide 24  $K = \frac{Gd^4}{64nR^3}$  $K = \frac{E}{L^3}(wh^3)$  cross section

 $\rightarrow \frac{EA}{L} = K$

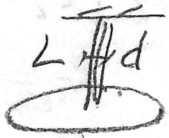
 $K = \frac{16E}{L^3}(wh^3)$

 $K = \frac{4E}{L^3}(wh^3)$

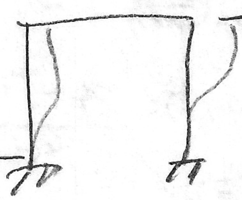


$$I_A = \frac{1}{12} wh^3$$

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}I_{cg}\dot{\theta}^2 \quad \text{or} \quad \left(\frac{1}{2}I_0\dot{\theta}^2\right)$$

Torsion  $K_t = \frac{T}{\Theta} = \frac{GI_0}{L} = \frac{\pi G d^4}{32L}$. $I_0 = \frac{\pi d^4}{32}$ (Torque; Polar area)

 $K = \frac{3EI}{L^3}$

 $\rightarrow K = \frac{12EI}{L^3}$

parallel

$M = \frac{6EI}{L^2}\Delta$

$\sigma = \frac{6EI}{L^2 S}$ (max stress)

unbalance

$m\ddot{x} + kx = m_0 e\omega^2 \sin\omega t$

for damped resonance $\zeta < 0.707$ only

$\omega_r = \omega_n\sqrt{1-2\zeta^2}$

$\omega_r < \omega_d < \omega_n$

$\frac{1}{\sqrt{(1-r^2)^2}} = \frac{1}{r^2-1}$ $r>1$

$\frac{1}{1-r^2}$ $r<1$

$X_p = \frac{1}{1-r^2}\sin\omega t$ for $r<1$

$X_p = \frac{1}{r^2-1}\sin\omega t$ for $r>1$.

impulse response

$x(t) = \frac{\hat{F}\Delta t}{m\omega_n}\sin\omega_n t$ ($\rightarrow \dot{x}(0)$)

use $mv_0 = \int_0^t F\,dt$ to find $\dot{x}(0)$

$F\,dt = m\,dv$.

$\int_0^T F\,dt = mv(0^+)$ assume $v(0) = 0$.

 $I_0 = \frac{\pi}{32}d^4$. Polar area moment of inertia for torsion.

$\Theta = \frac{(\text{Torque})(L)}{G I_0}$

given $m\ddot{x}+KX=f$.
① Find $A=M^{-1}K$. ② Find λ_1, λ_2.
③ find eigenvectors $u_1, u_2 \Rightarrow [u]$
④ write $\{\ddot{q}\}+\begin{bmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{bmatrix}[K] = u^{-1} M^{-1}\{f\}$.
⑤ solve these.
⑥ let $X=[u]\{q\}$. $\Rightarrow q(0)=u^{-1}X(0)$.
⑦ transfer back to normal coordinate.

for damped system. use slide 393.

$$[M]\ddot{q}+\beta[M]\Omega\dot{q}+M\Omega q=u^Tf.$$

where $[M]=u^T m u$, and $\Omega=\begin{bmatrix} \omega_{n(1)}^2 & 0 \\ 0 & \omega_{n(2)}^2 \end{bmatrix}$

For Beam problems, easier to use Flexibility
For spring problems, easier to use stiffness

$a\lambda^2+b\lambda+C=0 \Rightarrow \lambda=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ eigenvalue

One mile = 5280 ft.
one mile = 1609.34 meters

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}=\frac{1}{\det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$