

HW 8, ME 440 Intermediate Vibration, Fall 2017

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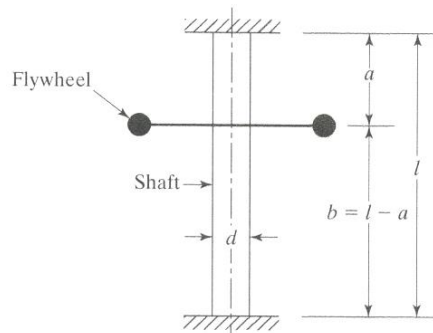
0.1 Problem 1

Problem 1

Download the ANSYS input file “*MODAL_pipe_flywheel.txt*” from HW7 on Canvas, run this input file in ANSYS and go through the file line by line to figure out what the system parameters are for this modal analysis. (Hint: When viewing the mode shapes within ANSYS, try plotting all 3 displacements and all 3 rotations (1 at a time) available under the “Nodal Solu” / “DOF Solution” option; this should be helpful in determining the type of displacement associated with each specific frequency.

- A) Modify the “*MODAL_pipe_flywheel.txt*” file to use ANSYS to predict the natural frequencies and mode shapes for the problem listed below (NOTE: you should remember this problem from HW3).

A flywheel is mounted on a vertical shaft, as shown below. The shaft has a diameter d and length l and is fixed at both ends. The flywheel has a weight of W and a radius of gyration of r . Find the natural frequency of the longitudinal, the transverse, and the torsional vibration of the system. For the parameters, assume that $d = 1.2$ in, $a = 2$ ft, $b = 4$ ft, $W = 100$ lbs and $r = 16$ in. (Assume the shaft is massless and the flywheel is rigid.)



For this problem, submit a hard copy of your modified .txt file and also create a table comparing the analytical and finite element frequencies (including % error) for the first longitudinal, first transverse and first torsional mode. Which mode has the most error? Which mode SHOULD have the most error? And why?

The APDL was modified to use solid pipe288 and put the mass element at the location as shown in the problem statement. The following are the four modes generated by ANSYS

set number	mode	frequency (Hz)
1	Torsion	10.437
2	First transverse (bending)	14.1815
3	Second transverse (bending)	35.384
4	First longitudinal (axial)	447.98

The following are the four plots showing the mode shapes for each of the above modes

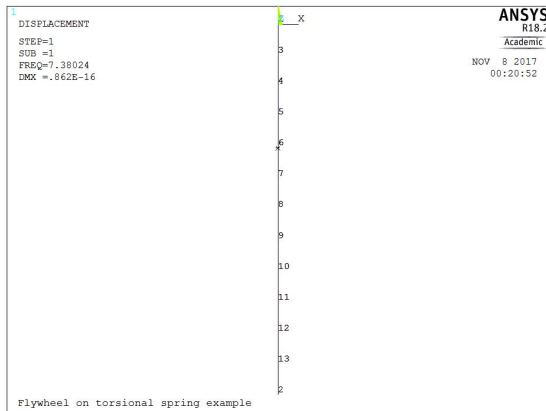


Figure 1: First mode: Torsion
7.3802 Hz

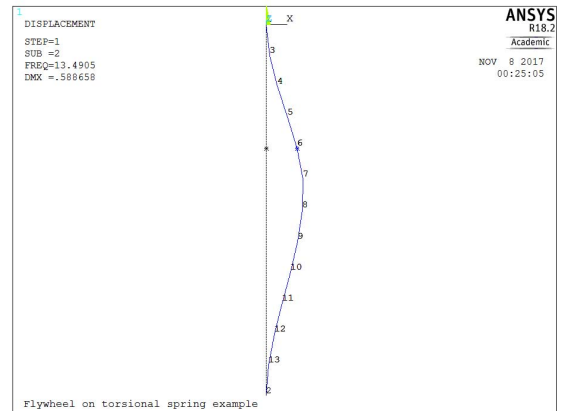


Figure 2: Second mode: Bending
13.491 Hz

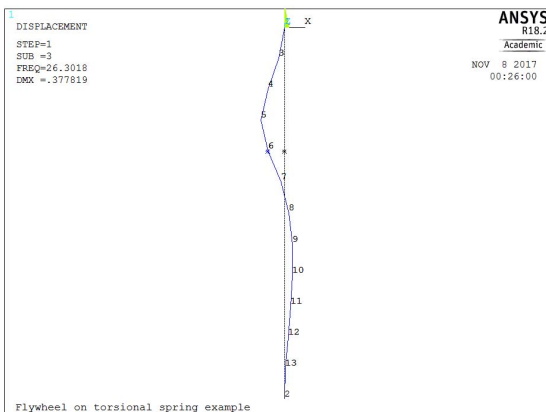


Figure 3: Third mode: Bending
26.302 Hz

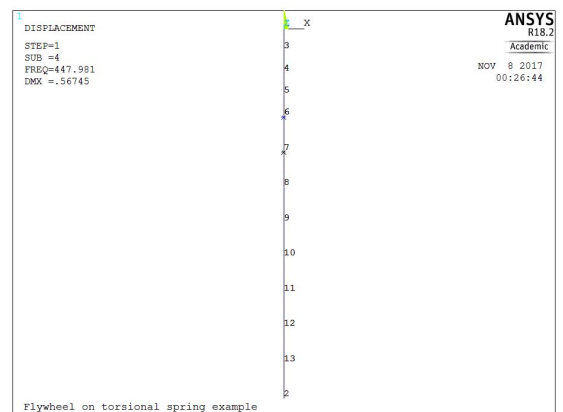


Figure 4: Fourth mode: Axial
447.98 Hz

The above result was next compared to the analytical result that was done in HW 3, by using the numerical value given in this problem. The numerical values for this problem are listed here

variable name	numerical value
L (length of pipe)	6 ft
a	2 ft
b	4 ft
d (diameter of pipe)	$1.2 \text{ in} = \frac{1.2}{12} = 0.1 \text{ ft}$
W (weight of flywheel)	100 lb
r (outer radius of flywheel)	$16 \text{ in} = \frac{16}{12} = 1.3333 \text{ ft}$
r_f (radius of gyration)	$\sqrt{\frac{r^2}{2}} = \sqrt{\frac{1.3333^2}{2}} = 0.94279 \text{ ft}$
E (Elastic modulus of pipe material, steel)	$29007547.546 \times 144 \text{ psf}$ (200 GPa)
G (shear modulus for pipe material, steel)	$11196913.353 \times 144 \text{ psf}$ (7.2 GPa)
Poisson's ratio for steel	0.295
I area moment of inertia for pipe section	$\frac{\pi}{4} \left(\frac{d}{2}\right)^4 = 4.90874 \times 10^{-6} \text{ ft}^4$
$I_{flywheel}$ mass moment of inertial of flywheel	$\frac{W}{g} r_f^2 = 5.52105 \text{ slug-ft}^2$

The above values were now used in the derivations from HW3 to obtain numerical values for the natural frequencies. The following are the results obtained (using analytical result from HW3 derivation)

mode	Analytical result	Numerical calculation ω_n in rad/sec	Hz
Torsion	$\omega_n = \sqrt{\frac{gG\pi d^4}{32Wr_f^2} \left(\frac{1}{a} + \frac{1}{b}\right)}$	$\sqrt{\frac{(32.2)(11196913.353 \times 144)\pi(0.1)^4}{32(100)(0.94279)^2} \left(\frac{1}{2} + \frac{1}{4}\right)} = 65.58$	10.437
bending (1)	$\omega_n = \sqrt{\frac{3gEI}{W} \left(\frac{L}{ab}\right)^3}$	$\sqrt{\frac{3(32.2)(29007547.546 \times 144)(4.90874 \times 10^{-6})}{100} \left(\frac{6}{(2)(4)}\right)^3} = 91.412$	14.549
axial	$\omega_n = \sqrt{\frac{gAE}{W} \left(\frac{1}{a} + \frac{1}{b}\right)}$	$\sqrt{\frac{(32.2)\left(\pi\left(\frac{0.1}{2}\right)^2\right)(29007547.546 \times 144)}{100} \left(\frac{1}{2} + \frac{1}{4}\right)} = 2814.8$	447.99

The following table compares the above analytical result with the ANSYS result shown earlier with the percentage error

mode	ANSYS result (Hz)	Analytical result (Hz)	error percentage
Torsion	10.437	10.437	0%
First bending	14.1815	14.549	$\left(\frac{14.1815 - 14.549}{14.1815}\right) \times 100 = 2.59\%$
First axial	447.98	447.99	$\frac{447.98 - 447.99}{447.98} \times 100 = 0.002\%$

The mode that has most error is the first bending (transverse) mode. This was the case also in HW7 ANSYS problem. ANSYS result is the more accurate one. The analytical result for this mode was derived The transverse case uses stiffness $3EI\left(\frac{L}{ab}\right)^3$ due to load at a distance from one end of fixed-free beam and b distance from the other end of the fixed beam. But this derivation does not account for any bending rotation in the beam as the ANSYS result would do.

0.1.1 Listing of modified APDL script

```

1  !-- Modified APDL script for HW 8, ME 440, Fall 2017
2  !
3
4  /filnam, pipe_flywheel_modal
5  /title, Flywheel on torsional spring example
6  /prep7
7
8
9  !-- give names for elements -----
10 MASS_ELEMENT=1
11 PIPE_ELEMENT=2
12
13 !-- define the mass element -----
14 ET,MASS_ELEMENT,mass21,,0,0 !element type no.1 is mass21 (",,,0" signifies
15 !that this is a 3-D mass with rotary inertia)
16 ! model parameters for MASS_ELEMENT
17 mass      = (100/32.2)      ! mass of flywheel (lb)
18 r_wheel   = (16/12)        ! radius of gyration (ft)
19 Iyy       = mass*(r_wheel*r_wheel)/2 ! mass moment of inertia
20 OUTER_DIAMETER = (1.2/12) ! outer diameter of pipe (ft)
21 wall_t    = OUTER_DIAMETER/2-0.0001 ! Solid pipe! This gives warning
22 ! but we can ignore it for now
23 SHAFT_LENGTH = 6 ! shaft length (ft)
24 n_modes     = 10 ! number of modes wanted, but ANSYS always gives 4
25
26 !real constants for MASS_ELEMENT
27 r,MASS_ELEMENT,mass,mass,mass,0.5*IYY,IYY,0.5*IYY
28
29 !-- define the shaft element as solid pipe -----
30 ET,PIPE_ELEMENT,pipe288
31
32 mp,ex,MASS_ELEMENT,29007547.546*144 ! (200e9 SI) elastic modulus PSF
33 mp,gxy,MASS_ELEMENT,11196913.35276*144 ! (77.2e9 SI) shear modulus PSF
34 mp,prxy,MASS_ELEMENT,0.295 ! poisson's ratio for steel is 0.295
35
36 KEYOPT,PIPE_ELEMENT,4,2 !Thick wall per ansys help
37
38 !SECTYPE, SECID, Type, Subtype, Name, REFINEKEY
39 ! Associates section type information with a section ID number.
40 sectype,1,pipe ! section type 1 is "pipe"

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41 | secdata,OUTER_DIAMETER,wall_t ! section data for pipe is outer
42 |                               ! diameter and wall thickness
43 |
44 |
45 | !-- key points -----
46 | k,1,0,0,0 ! keypoint 1 is at x=0, y=0, z=0, one fixed end of pipe
47 |
48 | ! keypoint 2 where fluwheel is located
49 | k,2,0,-SHAFT_LENGTH/2.0,0
50 |
51 | k,3,0,-SHAFT_LENGTH,0 ! keypoint 3 is other end of the fixd pipe
52 |
53 | !-- create elements -----
54 | TYPE,PIPE_ELEMENT ! element type of subsequently defined elements.
55 |
56 | !SECNUM, SECID
57 | ! Sets the element section attribute pointer.
58 | ! Defines the section ID number to be assigned to the
59 | ! subsequently-defined elements Defaults to 1. See SECTYPE for more
60 | ! information about the section ID number.
61 |
62 | secnum,1 !specify section type number of subsequently defined elements
63 |
64 |
65 | !-- create line -----
66 | !L, P1, P2
67 | !Defines a line between two keypoints.
68 |
69 | L,1,3 ! creates ONE line from keypoint 1 to keypoint 3
70 |
71 | !LESIZE, NL1, SIZE, ANGSIZ, NDIV, SPACE, KFORC, LAYER1, LAYER2, KYNDIV
72 | !Specifies the divisions and spacing ratio on unmeshed lines.
73 | ! NL1 Number of the line to be modified.
74 | ! SIZE If NDIV is blank, SIZE is the division (element edge) length.
75 | ! The number of divisions is automatically calculated from the
76 | ! line length (rounded upward to next integer). If SIZE is zero
77 | ! (or blank), use ANGSIZ or NDIV
78 | ! ANGSIZ The division arc (in degrees) spanned by the element edge
79 | ! NDIV If positive, NDIV is number of element divisions per line.
80 |
81 | lesize,1,,12 ! line 1 will consist of 12 elements when meshed
82 |
83 | !LMESH, NL1, NL2, NINC Generates nodes and line elements along lines
84 | ! Mesh lines from NL1 to NL2
85 |
86 | lmesh,ALL ! line 1 meshed, resulting in elements representing the pipe
87 |
88 | !-----
89 | type,MASS_ELEMENT ! element type of subsequently defined elements
90 | real,1 ! real constant set of subsequently defined element
91 |
92 | !E, I, J, K, L, M, N, O, P
93 | !Defines an element by node connectivity.
94 | ! I Number of node assigned to first nodal position (node I)
95 | E,6 ! create element to be created at node 6
96 |
97 | finish
98 |
99 | /solu !select static load solution
100 |
101 | !-- Set the boundary conditions -----
102 | nsel,all ! selects all nodes
103 | d,all,uz,0 ! sets the z displacements on selected nodes to be 0
104 | ! limiting our modal analysis to modes in the xy plane
105 | d,all,rotx ! sets the rotx displacements on selected nodes to be 0
106 |
107 | !displacement boundary conditions
108 | ! NSEL, Type, Item, Comp, VMIN, VMAX, VINC, KABS
109 | ! Type S Select a new set (default).
110 | ! Item LOC X,Y,Z X,Y, or Z location in active coordinate system
111 |
112 | nsel,S,NODE,,1 ! select node at x = 0

```

```

113 d,all,ux,0      ! displacement of selected node in x-dir is 0
114 d,all,uy,0      ! displacement of selected node in y-dir is 0
115 d,all,uz,0      ! displacement of selected node in z-dir is 0
116 d,all,rotx,0    ! rotations of selected node about x axis is 0
117 d,all,roty,0    ! rotations of selected node about y axis is 0
118 d,all,rotz,0    ! rotations of selected node about z axis is 0
119
120
121 nsel,A,NODE,,2  ! select node at x = -SHAFT_LENGTH
122 d,all,ux,0      ! displacement of selected node in x-dir is 0
123 d,all,uy,0      ! displacement of selected node in y-dir is 0
124 d,all,uz,0      ! displacement of selected node in z-dir is 0
125 d,all,rotx,0    ! rotations of selected node about x axis is 0
126 d,all,roty,0    ! rotations of selected node about y axis is 0
127 d,all,rotz,0    ! rotations of selected node about z axis is 0
128
129 allsel
130
131 antype,modal
132 modopt,lanb,20
133 solve
134 finish
135
136 /post1

```

0.2 Problem 2

Problem 2

A centrifugal pump, weighing 700 N and operating at 1000 rpm, is mounted on six springs of stiffness 6000 N/m each. Find the maximum permissible unbalance in order to limit the steady-state deflection to 5.0 mm peak-to-peak.

The first step is to determine the natural frequency of the system. Since the springs are in parallel then

$$k_{eq} = 6k$$

And the equivalent mass is $m_{eq} = \frac{W}{g}$ where $W = 700$ N. Hence

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{6k}{\frac{W}{g}}} = \sqrt{\frac{6(6000)}{\frac{700}{9.81}}} = 22.461 \text{ rad/sec}$$

Since this is undamped system, then the steady state solution (particular solution) is given by

$$y_p(t) = \frac{x_{st}}{\sqrt{(1-r^2)^2}} \cos \omega t \quad (1)$$

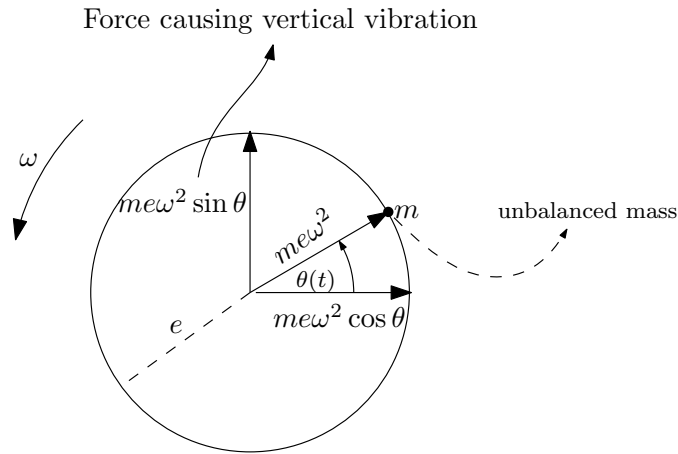
Where $r = \frac{\omega}{\omega_n}$ and ω is the driving frequency, which is

$$\omega = 1000 \left(\frac{2\pi}{rev} \right) \left(\frac{\min}{60} \right) = 1000 \left(\frac{2\pi}{60} \right) = 104.72 \text{ rad/sec}$$

From (1), we see that the maximum steady state response is

$$y_{ss} = \frac{x_{st}}{\sqrt{(1-r^2)^2}} \quad (2)$$

We now just need to determine x_{st} which is the static deflection. Let m_0 be the unbalanced mass which is spinning inside, and let e be the radius around the spin axis. Therefore, and assuming ω is constant, this mass will have only radial acceleration towards the center of $e\omega^2$ and therefore it will induce a centripetal force $m_0e\omega^2$.



From the above we see that the vertical force is

$$F(t) = \overbrace{m_0 e \omega^2}^{F_0} \sin \theta(t)$$

Hence the static deflection is

$$x_{st} = \frac{F_0}{k_{eq}} = \frac{m_0 e \omega^2}{6k}$$

Substituting this into (2) gives

$$y_{ss} = \frac{\frac{m_0 e \omega^2}{6k}}{\sqrt{(1-r^2)^2}} = \frac{m_0 e \omega^2}{6k \sqrt{(1-r^2)^2}} \quad (3)$$

But r is

$$r = \frac{\omega}{\omega_n} = \frac{104.72}{22.461} = 4.6623$$

Since $r > 1$ then we now can simplify $\sqrt{(1-r^2)^2} = r^2 - 1$ and (3) becomes

$$y_{ss} = \frac{m_0 e \omega^2}{6k (r^2 - 1)}$$

Since we want to limit deflection to 5 mm peak to peak, then we want to limit $y_{ss} = 2.5$ mm (which is half of the peak-to-peak). The above equation becomes

$$\begin{aligned} 2.5 \times 10^{-3} &= \frac{m_0 e (104.72)^2}{6 (6000) (4.6625^2 - 1)} \\ &= \frac{m_0 e (104.72)^2}{36000 (20.739)} \\ &= \frac{m_0 e (104.72)^2}{7.466 \times 10^5} \end{aligned}$$

Solving for unbalance $m_0 e$ gives

$$m_0 e = \frac{(2.5 \times 10^{-3}) (7.466 \times 10^5)}{(104.72)^2}$$

Or

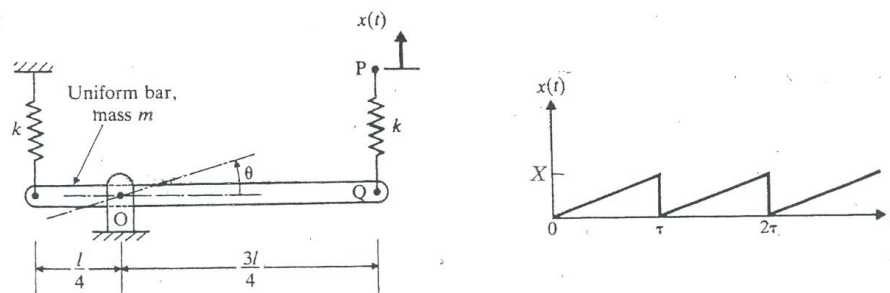
$$m_0 e = 0.1702 \text{ kg-meter}$$

This means to limit $m_0 e$ below this value in order to limit vibration to 5 mm, peak-to-peak.

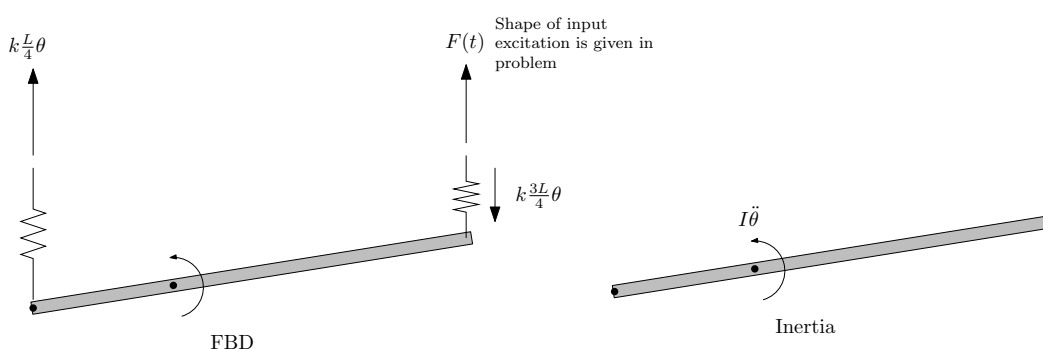
0.3 Problem 3

Problem 3

Determine the steady-state response of the system $\theta(t)$ due to the input excitation shown, using the system parameters given in the figure. (Use a trigonometric Fourier expansion of the input excitation.)



The first step is to make a FBD and corresponding inertia diagram Where it is assumed the left spring is in tension and the right side spring is in compression.



Taking moments around the pivot o where the bar is rotating around, and using anti-clockwise as positive gives (this assumes small angle approximation)

$$\sum M = I_o \ddot{\theta}$$

$$-k \left(\frac{L}{4} \theta \right) \frac{L}{4} - k \left(\frac{3L}{4} \theta \right) \frac{3L}{4} + kx(t) \left(\frac{3L}{4} \right) = I_o \ddot{\theta} \quad (1)$$

But I_o is the mass moment of inertia around o , which is

$$I_o = \overbrace{\frac{1}{12} mL^2}^{I_{cg}} + \overbrace{m \left(\frac{1}{4} L \right)^2}^{\text{parallel axis}}$$

$$= \frac{7}{48} L^2 m$$

Therefore the equation of motion (1) becomes

$$\frac{7}{48} L^2 m \ddot{\theta} = -k \left(\frac{L^2}{16} \theta + \frac{9L^2}{16} \theta \right) + k \frac{3L}{4} x(t)$$

$$L^2 m \ddot{\theta} + \theta \left(k \frac{10}{16} L^2 \right) \frac{48}{7} = k \frac{48}{7} \left(\frac{3L}{4} \right) x(t)$$

$$m \ddot{\theta} + \theta \left(\frac{30}{7} k \right) = k \frac{36}{7} \frac{1}{L} x(t) \quad (2)$$

Therefore

$$\omega_n = \sqrt{\frac{30}{7} \frac{k}{m}}$$

We now need to expand $x(t)$ in Fourier series. $x(t)$ has period of τ . This is not even and not odd function.

$$x(t) = \frac{X}{\tau}t$$

Hence

$$\begin{aligned} a_0 &= \frac{1}{\frac{\tau}{2}} \int_0^{\tau} \frac{X}{\tau} t dt = \frac{2}{\tau} \frac{X}{\tau} \left(\frac{t^2}{2} \right)_0^{\tau} = \frac{X}{\tau^2} \tau^2 = X \\ a_n &= \frac{1}{\frac{\tau}{2}} \int_0^{\tau} \frac{X}{\tau} t \cos\left(\frac{2\pi}{\tau}nt\right) dt \\ &= \frac{2}{\tau} \frac{X}{\tau} \int_0^{\tau} t \cos\left(\frac{2\pi}{\tau}nt\right) dt \\ &= \frac{2}{\tau} \frac{X}{\tau} (0) \\ &= 0 \end{aligned}$$

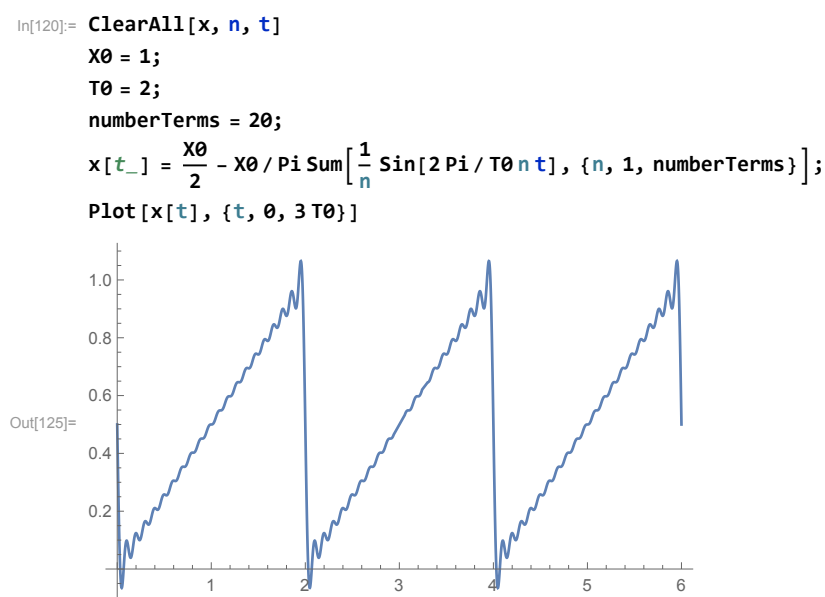
And

$$\begin{aligned} b_n &= \frac{1}{\frac{\tau}{2}} \int_0^{\tau} \frac{X}{\tau} t \sin\left(\frac{2\pi}{\tau}nt\right) dt \\ &= \frac{2}{\tau} \frac{X}{\tau} \int_0^{\tau} t \sin\left(\frac{2\pi}{\tau}nt\right) dt \\ &= \frac{2}{\tau} \frac{X}{\tau} \left(-\frac{\tau^2}{2n\pi} \right) \\ &= -\frac{X}{n\pi} \end{aligned}$$

Hence

$$\begin{aligned} x(t) &\approx \frac{a_0}{2} + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi}{\tau}nt\right) \\ &\approx \frac{X}{2} - \frac{X}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{2\pi}{\tau}nt\right) \\ &\approx \frac{X}{2} - \frac{X}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{2\pi}{\tau}nt\right) \end{aligned}$$

To verify this solution, the above is plotted for number of terms to see if it will approximate the original $x(t)$.



Now we go back to the original equation of motion (2), and replace $x(t)$ by its Fourier

series expansion

$$\begin{aligned}
 m\ddot{\theta} + \theta \left(\frac{30}{7}k \right) &= k \frac{36}{7} \frac{1}{L} \left(\frac{X}{2} - \frac{X}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \left(\frac{2\pi}{\tau} nt \right) \right) \\
 &= k \frac{18}{7} \frac{X}{L} - k \frac{1}{\pi} \frac{36}{7} \frac{X}{L} \left(\sin \left(\frac{2\pi}{\tau} t \right) + \frac{1}{2} \sin \left(\frac{2\pi}{\tau} 2t \right) + \frac{1}{3} \sin \left(\frac{2\pi}{\tau} 3t \right) + \dots \right) \\
 &= k \frac{18}{7} \frac{X}{L} - k \frac{1}{\pi} \frac{36}{7} \frac{X}{L} \left(\sin(\omega t) + \frac{1}{2} \sin(2\omega t) + \frac{1}{3} \sin(3\omega t) + \frac{1}{4} \sin(4\omega t) + \dots \right) \quad (3)
 \end{aligned}$$

Linearity is now used to find the solution to the above by adding the the steady state response to each of the terms. The steady state response to the first term above, which is $\frac{18}{7}k \frac{X}{mL}$ is the steady state response to the ODE

$$m\ddot{\theta} + \theta \left(\frac{30}{7}k \right) = \left(\frac{18}{7}k \frac{X}{L} \right)$$

Which Is given by

$$y_{ss} = \left(k \frac{18}{7} \frac{X}{L} \right) \frac{1}{k_{eq}}$$

But $k_{eq} = \frac{30}{7}k$, therefore

$$\begin{aligned}
 y_{ss} &= \left(\frac{18}{7}k \frac{X}{L} \right) \frac{7}{30k} \\
 &= \frac{9}{15} \frac{X}{L}
 \end{aligned}$$

This is the response to only the first term in (3). Now we do the same for each of the trig terms. But we only need to consider one general term. The ODE we will look at now is

$$\begin{aligned}
 m\ddot{\theta} + \theta \left(\frac{30}{7}k \right) &= k \frac{1}{\pi} \frac{36}{7} \frac{X}{L} \sum_{n=1}^{\infty} \frac{1}{n} \sin \left(\frac{2\pi}{\tau} nt \right) \\
 &= k \frac{1}{\pi} \frac{36}{7} \frac{X}{L} \left(\sin \left(\frac{2\pi}{\tau} t \right) + \frac{1}{2} \sin \left(\frac{2\pi}{\tau} 2t \right) + \frac{1}{3} \sin \left(\frac{2\pi}{\tau} 3t \right) + \dots \right) \\
 &= k \frac{1}{\pi} \frac{36}{7} \frac{X}{L} \left(\sin(\omega t) + \frac{1}{2} \sin(2\omega t) + \frac{1}{3} \sin(2\omega t) + \dots \right)
 \end{aligned}$$

Considering one general term

$$\begin{aligned}
 m\ddot{\theta} + \theta \left(\frac{30}{7}k \right) &= k \left(\frac{1}{\pi} \frac{36}{7} \frac{X}{L} \frac{1}{n} \right) \sin(n\omega t) \\
 &= F_0 \sin(n\omega t) \quad (4)
 \end{aligned}$$

Where

$$\begin{aligned}
 F_0 &= \left(k \frac{1}{\pi} \frac{36}{7} \frac{X}{L} \frac{1}{n} \right) \\
 x_{st} &= \frac{F_0}{k_{eq}} \\
 &= \frac{k \frac{1}{\pi} \frac{36}{7} \frac{X}{L} \frac{1}{n}}{\frac{30}{7}k} \\
 &= \frac{6}{5\pi L} \frac{X}{n} \quad (5)
 \end{aligned}$$

We know the steady state (particular) solution for (4) is

$$\theta_{ss}(t) = \frac{x_{st}}{(1 - (nr)^2)} \sin(n\omega t) \quad (6)$$

Where r is

$$r = \frac{\omega}{\omega_n} = \frac{\frac{2\pi}{\tau}}{\sqrt{\frac{30}{7} \frac{k}{m}}} = \frac{2\pi}{\tau \sqrt{\frac{30}{7} \frac{k}{m}}} \quad (7)$$

The above is the steady state response for the n^{th} term. So the total response is the sum of

all these responses. Putting all this together, we now obtain the steady state solution as

$$\theta_{ss}(t) = k \frac{9X}{15kL} - \sum_{n=1}^{\infty} \frac{x_{st}}{(1 - (nr)^2)} \sin(n\omega t) \quad (8)$$

Where x_{st} is given (5) and r is given by (7) and $\omega = \frac{2\pi}{\tau}$. To try verify the above, it is plotted using the following values $X = 1, L = 10$ meter, $k = 100$ N/m, $\tau = 3$ sec and $m = 5$ kg. This is the result (for 30 terms in Fourier sum)

