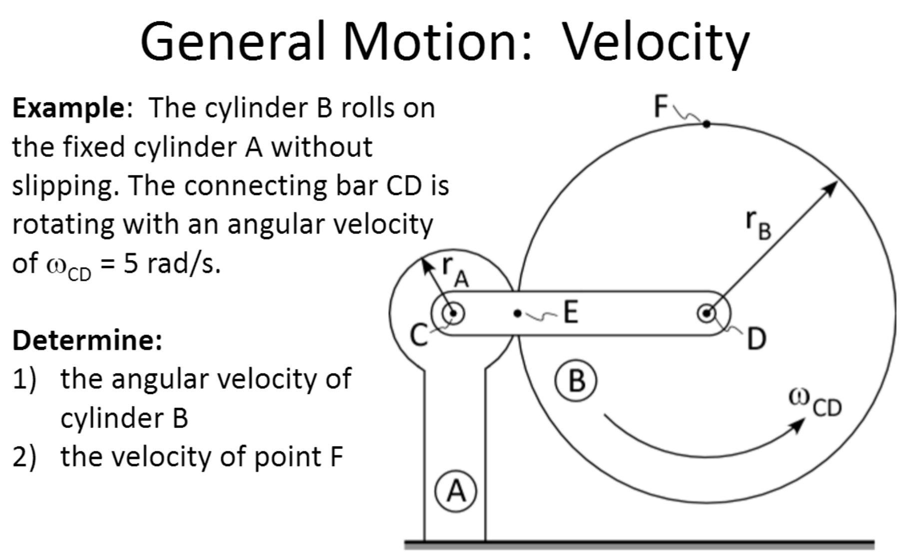
my solution to discussion problem Nov 10 2017,ME 240 Dynamics, Fall 2017

Nasser M. Abbasi

December 30, 2019

My solution is below

0.1 Problem 1



0.1.1 Part 1

Notice that the point E is not on the bar CD. It is the point where the disks meet at this instance shown.

$$\begin{aligned}\bar{V}_D &= \bar{V}_C + \bar{\omega}_{CD} \times \bar{r}_{D/C}\\ &= 0 + \omega_{CD}\hat{k} \times (r_A + r_B)\,\hat{\imath}\\ &= \omega_{CD}\,(r_A + r_B)\,\hat{\jmath} \end{aligned} \tag{1}$$

But we also see that $\bar{V}_D$ can be written as

$$\begin{aligned}\bar{V}_D &= \bar{V}_E + \bar{\omega}_{disk} \times \bar{r}_{D/E}\\ &= 0 + \omega_{disk}\hat{k} \times r_B\hat{\imath}\\ &= \omega_{disk} r_B\hat{\jmath} \end{aligned} \tag{2}$$

Where in the above we used the fact that $\bar{V}_E = \bar{V}_C = 0$ at the instance shown. Equating (1) and (2)

$$\begin{aligned}\omega_{CD}\,(r_A + r_B) &= \omega_{disk} r_B\\ \omega_{disk} &= \omega_{CD}\frac{r_A + r_B}{r_B} \end{aligned} \tag{3}$$

0.1.2 Part 2

$$\begin{aligned}\bar{V}_F &= V_D + \bar{\omega}_{disk} \times \bar{r}_{F/D}\\ &= \omega_{CD}\,(r_A + r_B)\,\hat{\jmath} + \omega_{disk}\hat{k} \times r_B\hat{\jmath} \end{aligned}$$

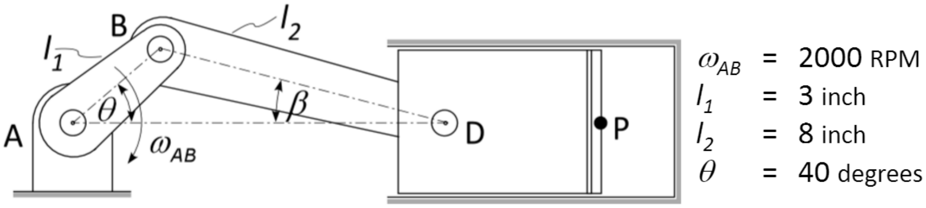
Hence

$$\begin{aligned}\bar{V}_F &= -\omega_{disk} r_B\hat{\imath} + \omega_{CD}\,(r_A + r_B)\,\hat{\jmath}\\ &= -\omega_{CD}\frac{r_A + r_B}{r_B} r_B\hat{\imath} + \omega_{CD}\,(r_A + r_B)\,\hat{\jmath}\\ &= \omega_{CD}\,(r_A + r_B)\,\hat{\imath} + \omega_{CD}\,(r_A + r_B)\,\hat{\jmath} \end{aligned}$$

0.2 Problem 2

General Motion - Velocity

Example: In the piston system shown, the crank AB has a constant clockwise angular velocity of 2000 RPM.



Determine the velocity of point P on the piston for the configuration parameters given above

$$\begin{aligned}\bar{V}_D &= \bar{V}_B + \bar{V}_{D/B} \\ &= \bar{V}_B + \bar{\omega}_{BD} \times \bar{r}_{D/B} \qquad (1)\end{aligned}$$

But

$$\begin{aligned}\bar{V}_B &= \bar{V}_A + \bar{\omega}_{AB} \times \bar{r}_{B/A} \\ &= 0 - \omega_{AB}\hat{k} \times \left(L_1 \cos\theta\hat{\imath} + L_1 \sin\theta\hat{\jmath}\right) \\ &= -\omega_{AB} L_1 \cos\theta\hat{\jmath} + \omega_{AB} L_1 \sin\theta\hat{\imath} \qquad (2)\end{aligned}$$

Where

$$\omega_{AB} = 2000\left(\frac{2\pi}{60}\right) = \frac{200}{3}\pi = 209.4395 \text{ rad/sec}$$

The angle β can be found as follows

$$\begin{aligned}\frac{\sin\theta}{L_2} &= \frac{\sin\beta}{L1} \\ \sin\beta &= \frac{L_1}{L_2}\sin\theta = \frac{3}{8}\sin\left(40\left(\frac{\pi}{180}\right)\right) = 0.241 \text{ radians} \\ &= 13.808^0\end{aligned}$$

Now we know everything to evaluate (1). Therefore

$$\begin{aligned}
\bar{V}_D &= \bar{V}_B + \bar{\omega}_{BD} \times r_{D/B} \\
&= \left(-\omega_{AB} L_1 \cos\theta \hat{\jmath} + \omega_{AB} L_1 \sin\theta \hat{\imath}\right) + \omega_{BD} \hat{k} \times \left(L_2 \cos\beta \hat{\imath} - L_2 \sin\beta \hat{\jmath}\right) \\
&= \left(-\omega_{AB} L_1 \cos\theta \hat{\jmath} + \omega_{AB} L_1 \sin\theta \hat{\imath}\right) + \omega_{BD} L_2 \cos\beta \hat{\jmath} + \omega_{BD} L_2 \sin\beta \hat{\imath} \\
&= \hat{\imath}\left(\omega_{AB} L_1 \sin\theta + \omega_{BD} L_2 \sin\beta\right) + \hat{\jmath}\left(-\omega_{AB} L_1 \cos\theta + \omega_{BD} L_2 \cos\beta\right) \qquad (3)
\end{aligned}$$

But the y componenent of $\bar{V}_D = 0$ since D can only move in x direction. Therefore from the above

$$\begin{aligned}
-\omega_{AB} L_1 \cos\theta + \omega_{BD} L_2 \cos\beta &= 0 \\
\omega_{BD} &= \omega_{AB} \frac{L_1 \cos\theta}{L_2 \cos\beta} \qquad (4)
\end{aligned}$$

Substituting (4) into the x component of (3) gives the answer we want

$$\begin{aligned}
\bar{V}_D &= \hat{\imath}\left(\omega_{AB} L_1 \sin\theta + \left(\frac{L_1 \cos\theta}{L_2 \cos\beta}\omega_{AB}\right) L_2 \sin\beta\right) \\
&= \hat{\imath}\left(L_1 \sin\theta + \left(\frac{L_1 \cos\theta}{L_2 \cos\beta}\right) L_2 \sin\beta\right)\omega_{AB} \\
&= \hat{\imath}\left(3 \sin\left(40\left(\frac{\pi}{180}\right)\right) + \frac{3\cos\left(40\left(\frac{\pi}{180}\right)\right)\sin\left(13.808\left(\frac{\pi}{180}\right)\right)}{\cos\left(13.808\left(\frac{\pi}{180}\right)\right)}\right) 209.4395 \\
&= 522.170\hat{\imath} \text{ inch/sec} \\
&= 43.514\hat{\imath} \text{ ft/sec}
\end{aligned}$$

And $\omega_{BD} = \omega_{AB} \frac{L_1 \cos\theta}{L_2 \cos\beta} = 209.4395 \frac{3\cos\left(40\left(\frac{\pi}{180}\right)\right)}{8\cos\left(13.808\left(\frac{\pi}{180}\right)\right)} = 61.955$ rad/sec