HW 9,ME 240 Dynamics, Fall 2017

Nasser M. Abbasi

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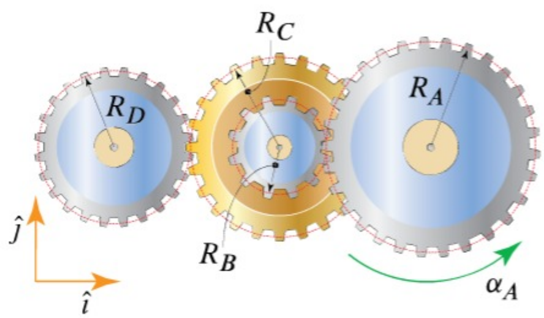
0.1 Problem 1

Letting $R_A = 201$ mm, $R_B = 114$ mm, $R_C = 162$ mm, and $R_D = 133$ mm, determine the angular acceleration of gears B, C, and D when gear A has an angular acceleration with magnitude $|\alpha_A| = 51$ rad/s^2 in the direction shown. Note that gears B and C are mounted on the same shaft and they rotate as a unit.



$\alpha_B =$

$\alpha_C =$

The tangential acceleration at the point where disk A and disk B meet is $R_A\alpha_A$. But this is also must be the same as $R_B\alpha_B$ since the gears assumed not to slip against each others. Therefore

$$\alpha_B = -\frac{R_A}{R_B}\alpha_A$$

The minus sign, is because gear A moves anti-clockwise, but B moves clockwise, hence in negative direction. Therefore

$$\begin{aligned}\alpha_B &= -\frac{201}{114}(51)\\ &= -89.921 \text{ rad/sec}^2\end{aligned}$$

Since C moves with B as one body, then $\alpha_B = \alpha_C$ and then

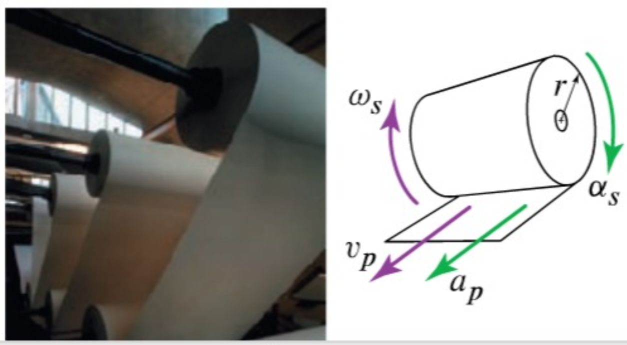
$$\alpha_C = -89.921 \text{ rad/sec}^2$$

Similarly

$$\begin{aligned}\alpha_D &= -\frac{R_C}{R_D}\alpha_C \\ &= -\frac{162}{133}(-89.921) \\ &= 109.528 \text{ rad/sec}^2\end{aligned}$$

0.2 Problem 2

At the instant shown the paper is being unrolled with a speed $v_P = 9.2$ m/s and an acceleration $a_P = 1$ m/s^2. If at this instant the outer radius of the roll is $r = 1.31$ m. determine the angular velocity ω_s and acceleration α_s of the roll.



Since $a_p = r\alpha_s$ then

$$\alpha_s = -\frac{a_p}{r} = -\frac{1}{1.31} = -0.763 \text{ rad/sec}^2$$

Since $v_p = r\omega_s$ then

$$\omega_s = -\frac{v_p}{r} = -\frac{9.2}{1.31} = -7.023 \text{ rad/sec}$$

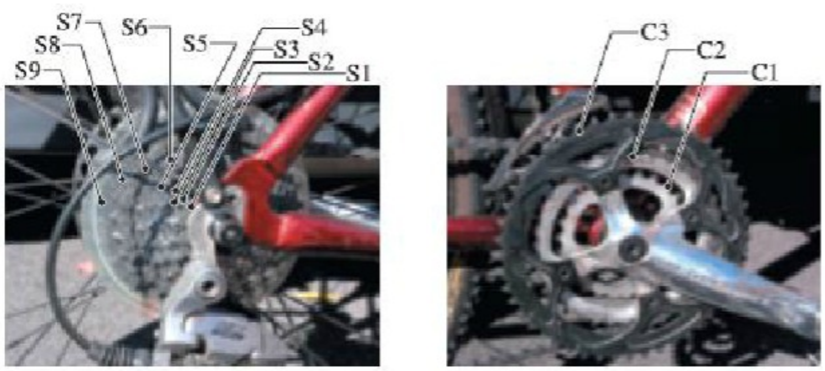
0.3 Problem 3

A bicycle has wheels 720 mm in diameter and a gear set with the dimensions given in the table below.

Crank			
Sprocket	C1	C2	C3
No. of Cogs	26	36	48
Radius (mm)	57.1	79.1	105.5

Cassette (9 speeds)									
Sprocket	S1	S2	S3	S4	S5	S6	S7	S8	S9
No. of Cogs	11	12	14	16	18	21	24	28	34
Radius (mm)	24.2	26.4	30.8	35.2	39.6	46.2	52.7	61.5	74.7

If a cyclist has a cadence of 1 Hz, determine the angular speed of the rear wheel in rpm when using the combination of C3 and S2. In addition, knowing that the speed of the cyclist is equal to the speed of a point on the tire relative to the wheel's center, determine the cyclist's speed in m/s.



ω_w = [ ] **rpm**

$v_{cyclist}$ = [ ] **m/s**

$$\begin{aligned} RC &= 105.5 \text{ mm} \\ RS &= 26.4 \text{ mm} \\ RW &= \frac{720}{2} = 360 \text{ mm} \\ \omega_C &= 2\pi \end{aligned}$$

Hence

$$\begin{aligned} \omega_{wheel} &= \frac{RC}{RS}\omega_C \\ &= \frac{105.5}{26.4}2\pi \\ &= 25.109 \text{ rad/sec} \end{aligned}$$

Or

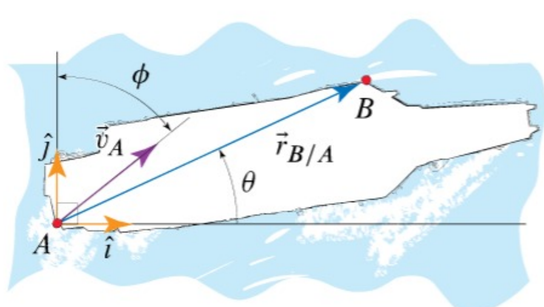
$$\begin{aligned} \omega_{wheel} &= \frac{25.109}{0.104719775} \\ &= 239.773 \text{ RPM} \end{aligned}$$

Hence

$$\begin{aligned} V &= \omega_{wheel}(RW) \\ &= 25.109\left(360\left(10^{-3}\right)\right) \\ &= 9.039 \text{ m/s} \end{aligned}$$

0.4 Problem 4

A carrier is maneuvering so that, at the instant shown, $|\vec{v}_A| = 30$ knots (1 kn is exactly equal to 1.852 km/h) and $\varphi = 34°$. Letting the distance between A and B be 231 m and $\theta = 20°$, determine $\vec{v}_B$ at the given instant if the ship's turning rate at this instant is $\dot{\theta} = 2°$/s clockwise.



$\vec{v}_B$ = ([ ] $\hat{\imath}$ + [ ] $\hat{\jmath}$)m/s

$$\begin{aligned}\bar{v}_B &= \bar{v}_A + \bar{\omega} \times \bar{r}_{B/A} \\ &= v_A \left(\sin\phi\hat{\imath} + \cos\phi\hat{\jmath}\right) + \bar{\omega} \times \bar{r}_{B/A}\end{aligned} \tag{1}$$

And

$$\bar{\omega} = -2^0\hat{k} = -0.03491\hat{k}$$

And

$$\bar{r}_{B/A} = d\cos\theta\hat{\imath} + d\sin\theta\hat{\jmath}$$

Hence (1) becomes

$$\begin{aligned}\bar{v}_B &= v_A \left(\sin\phi\hat{\imath} + \cos\phi\hat{\jmath}\right) + \omega\hat{k} \times \left(d\cos\theta\hat{\imath} + d\sin\theta\hat{\jmath}\right) \\ &= v_A \left(\sin\phi\hat{\imath} + \cos\phi\hat{\jmath}\right) + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & \omega \\ d\cos\theta & d\sin\theta & 0 \end{vmatrix} \\ &= v_A \left(\sin\phi\hat{\imath} + \cos\phi\hat{\jmath}\right) + \left(-\omega d\sin\theta\hat{\imath} - \hat{\jmath}\left(-\omega d\cos\theta\right)\right) \\ &= \hat{\imath}\left(v_A\sin\phi - \omega d\sin\theta\right) + \hat{\jmath}\left(v_A\cos\phi + \omega d\cos\theta\right)\end{aligned} \tag{2}$$

But

$$\begin{aligned}v_A &= 30\,(1.852)\left(\frac{1000}{km}\right)\left(\frac{hr}{3600}\right) \\ &= 30\,(1.852)\left(\frac{1000}{3600}\right) \\ &= 15.433\ \text{ m/s}\end{aligned}$$

And $d = 231$, hence (2) becomes

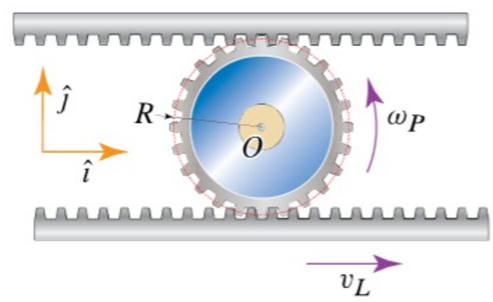
$$\begin{aligned}\bar{v}_B = \hat{\imath}&\left((15.433)\sin\left(34\left(\frac{\pi}{180}\right)\right) - (-0.0349)\,(231)\sin\left(20\left(\frac{\pi}{180}\right)\right)\right) \\ + \hat{\jmath}&\left((15.433)\cos\left(34\left(\frac{\pi}{180}\right)\right) + (-0.0349)\,(231)\cos\left(20\left(\frac{\pi}{180}\right)\right)\right)\end{aligned}$$

Or

$$\bar{v}_B = 11.388\ \hat{\imath} + 5.217\hat{\jmath}$$

0.5 Problem 5

At the instant shown the lower rack is moving to the right with a speed of $v_L = 5$ ft/s, while the upper rack is fixed. If the nominal radius of the pinion is $R = 2.3$ in, determine ω_P, the angular velocity of the pinion, as well as the velocity of point O, i.e., the center of the pinion.



$\vec{\omega}_P =$ ____ $\hat{k}$ rad/s.

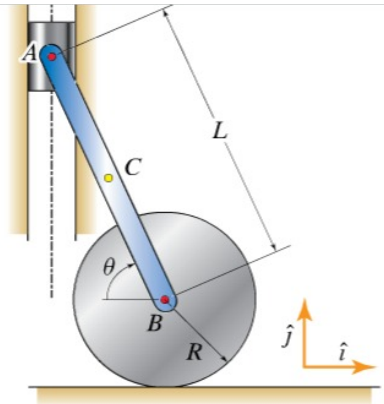
$\vec{v}_O =$ ____ $\hat{\imath}$ ft/s.

$$\begin{aligned}
\omega_p\,(2R) &= v_L \\
\omega_p &= \frac{v_L}{2R} \\
&= \frac{5}{2\left(\frac{2.3}{12}\right)} = 13.043 \text{ rad/sec}
\end{aligned}$$

And

$$\begin{aligned}
v_o &= \omega_p R \\
&= 13.043\,48\left(\frac{2.3}{12}\right) \\
&= 2.5 \text{ m/s}
\end{aligned}$$

0.6 Problem 6



The system shown consists of a wheel of radius R = 12 in. rolling on a horizontal surface. A bar AB of length L = 35 in. is pin−connected to the center of the wheel and to a slider A that is constrained to move along a vertical guide. Point C is the bar's midpoint. If, when θ = 77°, the wheel is moving to the right so that v_B = 5 ft/s, determine the angular velocity of the bar as well as the velocity of the slider A.

$\vec{\omega}_{AB} = \boxed{}\,\hat{k}$ **rad/s** $\hat{k}$

$\vec{v}_A = \boxed{}\,\hat{j}$ **ft/s** $\hat{j}$

Since

$$\omega L \sin\theta = v_B$$

Then

$$\begin{aligned}\omega &= \frac{v_B}{L\sin\theta}\\ &= \frac{5}{\left(\frac{35}{12}\right)\sin\left(77\left(\frac{\pi}{180}\right)\right)}\\ &= 1.759 \text{ rad/sec}\end{aligned}$$

And

$$\omega L \cos\theta = v_A$$

Then

$$\begin{aligned}
v_A &= -(1.759)\left(\frac{35}{12}\right)\cos\left(77\frac{\pi}{180}\right) \\
&= -1.154 \text{ m/s}
\end{aligned}$$

The minus sign since A moves down.

This can also be solved using vector method as follows

$$\vec{v}_A = \vec{v}_B + \vec{\omega}_{AB} \times \vec{r}_{A/B}$$

Where $\vec{r}_{A/B} = -L\cos\theta\hat{\imath} + L\sin\theta\hat{\jmath}$ and $\vec{v}_B = 5\hat{\imath}$ is given. Hence the above becomes

$$\begin{aligned}
\vec{v}_A &= 5\hat{\imath} + \omega_{AB}\hat{k} \times \left(-L\cos\theta\hat{\imath} + L\sin\theta\hat{\jmath}\right) \\
&= 5\hat{\imath} + \left(-\omega_{AB}L\cos\theta\hat{\jmath} - \omega_{AB}L\sin\theta\hat{\imath}\right) \\
&= \hat{\imath}\left(5 - \omega_{AB}L\sin\theta\right) + \hat{\jmath}\left(-\omega_{AB}L\cos\theta\right) \qquad (1)
\end{aligned}$$

And now comes the main point. We argue that A can only move in vertical direction, hence the $\hat{\imath}$ component above must be zero. Therefore

$$5 - \omega_{AB}L\sin\theta = 0$$

There is only one unknown in the above. SOlving for ω_{AB} gives

$$\omega_{AB} = 1.759 \text{ rad/sec}$$

Now we go back to (1) and find $\vec{v}_A$

$$\begin{aligned}
\vec{v}_A &= \hat{\imath}(0) - \hat{\jmath}\left(1.759\left(\frac{35}{12}\right)\cos\left(77\frac{\pi}{180}\right)\right) \\
&= \hat{\imath}(0) - \hat{\jmath}(1.154)
\end{aligned}$$

Which is the same as method earlier. Notice we did not need to use R, the radius of the disk.