HW 76,ME 240 Dynamics, Fall 2017

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0.1 Problem 1

A 181 gr (7,000 gr = 1 lb) bullet goes from rest to 3,347 ft/s in 0.0011 s. Determine the magnitude of the impulse imparted to the bullet during the given time interval. In addition, determine the magnitude of the average force acting on the bullet.

The magnitude of the impulse imparted to the bullet is [ ] lb·s and the magnitude of the average force acting on the bullet is [ ] lb.

Using impulse momentum

$$p_1 + \int_0^t F_{av}(t)\, dt = p_2$$

But $p_1 = mv_1 = 0$ since starting from rest and $p_2 = mv_2$, therefore

$$\begin{aligned}\int_0^t F_{av}(t)\, dt &= \frac{\left(\frac{181}{7000}\right)}{32.2}(3347) \\ &= 2.688 \text{ lb-sec}\end{aligned}$$

Therefore

$$\begin{aligned}F_{av}(0.0011) &= 2.688 \\ F_{av} &= \frac{2.688}{0.0011} \\ &= 2443.636 \text{ lb}\end{aligned}$$

0.2 Problem 2

The takeoff runway on carriers is much too short for a modern jetplane to take off on its own. For this reason, the takeoff of carrier planes is assisted by *hydraulic catapults* (Fig. A). The catapult system is housed below the deck except for a relatively small *shuttle* that slides along a rail in the middle of the runway (Fig. B). The front landing gear of carrier planes is equipped with a *tow bar* that, at takeoff, is attached to the catapult shuttle (Fig. C). When the catapult is activated, the shuttle pulls the airplane along the runway and helps the plane reach its takeoff speed. The takeoff runway is approximately 310 ft long, and most modern carriers have three or four catapults. If the carrier takeoff of a 45,500 lb plane subject to the 33,000 lb thrust of its engines were not assisted by a catapult, estimate how long it would take for a plane to safely take off, i.e., to reach a speed of 162 mph starting from rest. Also, how lon a runway would be needed under these conditions?

A B C

Photo credit (A): U.S. Navy photo by Photographer's Mate 2nd Class H. Dwain Willis
Photo credit (B): PHAN James Farrally II, U.S. Navy

$$p_1 + \int_0^t T dt = p_2$$

Where T is the thrust. But $p_1 = 0$, therefore

$$\begin{aligned} Tt &= mv_2 \\ t &= \frac{mv_2}{T} \\ &= \frac{\left(\frac{45500}{32.2}\right)\left(162\left(\frac{5280}{3600}\right)\right)}{33000} \\ &= 10.174 \text{ sec} \end{aligned}$$

To find how long a runway is needed

$$x_f = x_1 + v_1 t + \frac{1}{2} a t^2$$

But $x_1 = 0$ and $a = \frac{v_2 - v_1}{t}$, and $v_1 = 0$ since starting from rest, hence

$$\begin{aligned} x_f &= \frac{1}{2}at^2 \\ &= \frac{1}{2}\left(\frac{v_2}{t}\right)t^2 \\ &= \frac{1}{2}v_2 t \\ &= \left(\frac{1}{2}\right)162\left(\frac{5280}{3600}\right)(10.174) \\ &= 1208.671 \text{ ft} \end{aligned}$$

This is 4 times as long as without the catapults.

0.3 Problem 3

A $5\frac{1}{8}$ oz baseball traveling at 89 mph rebounds off a bat with a speed of 160 mph. The ball is i[n] contact with the bat for roughly 10^{-3} s. The incoming velocity of the ball is horizontal, and the outgoing trajectory forms an angle $\alpha = 32°$ with respect to the incoming trajectory.

Part 1

(a) Determine the impulse provided to the baseball by the bat.

Impulse = (3.28 $\hat{\imath}$ + 1.24 $\hat{\jmath}$) lb·s

Part 2

(b) Determine the average force exerted by the bat on the ball.

$(\vec{F}_b)_{avg}$ = (3280 $\hat{\imath}$ + 1240 $\hat{\jmath}$) lb.

Part 3

(c) Determine how much the angle α would change (with respect to 32°) if we were to neglect the effects of the force of gravity on the ball.

The angle α would change by 0 **° with respect to 32°.**

$$\bar{p}_1 + \int_0^t \bar{F} dt = \bar{p}_2$$

$$-mv_1\hat{\imath} + \int_0^t \left(F_x\hat{\imath} + F_y\hat{\jmath}\right) dt = mv_2\cos\alpha\hat{\imath} + mv_2\sin\alpha\hat{\jmath}$$

$$\hat{\imath}\left(-mv_1 + F_x t\right) + \hat{\jmath}\left(F_y t\right) = mv_2\cos\alpha\hat{\imath} + mv_2\sin\alpha\hat{\jmath}$$

Hence we obtain two equations

$$\begin{aligned} -mv_1 + F_x t &= mv_2\cos\alpha \\ F_y t &= mv_2\sin\alpha \end{aligned}$$

Or

$$\begin{aligned} F_x t &= mv_2\cos\alpha + mv_1 \\ F_y t &= mv_2\sin\alpha \end{aligned}$$

Now $m = \frac{\frac{5.125}{16}}{32.2} = 0.00994$ slug, and $v_1 = 89\left(\frac{5280}{3600}\right) = 130.533$ ft/sec and $v_2 = 160\left(\frac{5280}{3600}\right) = 234.667$ ft/sec. Hence

$$\begin{aligned} F_x t &= (0.00994)(234.667)\cos\left(32\left(\frac{\pi}{180}\right)\right) + (0.00994)(130.5333) \\ F_y t &= (0.00994)(234.667)\sin\left(32\left(\frac{\pi}{180}\right)\right) \end{aligned}$$

Or

$$\begin{aligned} F_x t &= 1.978 + 1.298 = 3.276 \\ F_y &= 1.236 \end{aligned}$$

Hence impulse is

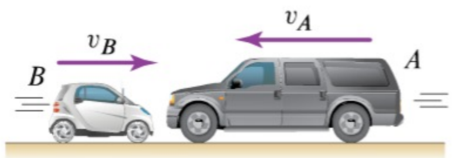
$$\bar{I} = 3.276\hat{\imath} + 1.236\hat{\jmath}$$

To find average force, we divide by time

$$\begin{aligned} \bar{F}_{av} &= \frac{3.276}{0.001}\hat{\imath} + \frac{1.236}{0.001}\hat{\jmath} \\ &= 3276\hat{\imath} + 1236\hat{\jmath} \end{aligned}$$

0.4 Problem 4

An 8,110 lb vehicle A traveling with a speed v_A = 57 mph collides head–on with a 2,070 lb vehicle B traveling in the opposite direction with a speed v_B = 32 mph. Determine the postimpact velocity of the two cars if the impact is perfectly plastic.



The postimpact velocity of car A is (______ ft/s) $\hat{\imath}$ and the postimpact velocity of car B is (______ ft/s) $\hat{\imath}$.

Since there is no external force, then $p_1 = p_2$ or

$$m_B v_B^- + m_A v_A^- = m_B v_B^+ + m_A v_A^+ \tag{1}$$

Where + means after impact and − means before impace. Therefore (using positive going to the right)

$$v_A^- = -57\left(\frac{5280}{3600}\right) = -83.6 \text{ ft/sec}$$
$$v_B^- = 32\left(\frac{5280}{3600}\right) = 46.933 \text{ ft/sec}$$
$$m_A = \frac{8110}{32.2} = 251.8634 \text{ slug}$$
$$m_B = \frac{2070}{32.2} = 64.2857 \text{ slug}$$

Hence (1) becomes

$$(64.2857)(46.933) - (251.8634)(83.6) = \frac{2070}{32.2} v_B^+ + \frac{8110}{32.2} v_A^+$$
$$-18038.66 = 64.286 v_B^+ + 251.8634 v_A^+ \tag{2}$$

And since $e = 0$, then

$$e = 0 = \frac{v_B^+ - v_A^+}{v_A^- - v_B^-}$$
$$v_B^+ = v_A^+ \tag{3}$$

Using (2,3) we solve for v_B^+, v_A^+. Plug (3) into (2) gives

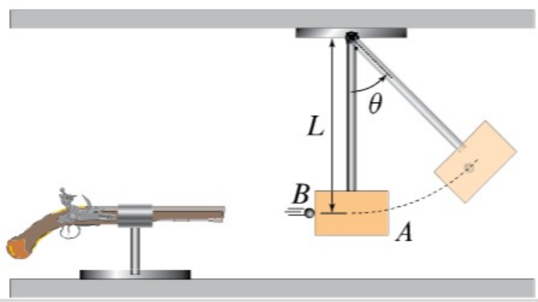
$$-18038.66 = 64.286 v_A^+ + 251.8634 v_A^+$$
$$-18038.66 = 316.1494 v_A^+$$
$$v_A^+ = \frac{-18038.66}{316.1494}$$
$$= -57.05739 \text{ ft/sec}$$

Hence

$$v_B^+ = -57.05739 \text{ ft/sec}$$

0.5 Problem 5

The ballistic pendulum used to be a common tool for the determination of the muzzle velocity of bullets as a measure of the performance of firearms and ammunition (nowadays, the ballistic pendulum has been replaced by the ballistic chronograph, an electronic device). The ballistic pendulum is a simple pendulum that allows one to record the maximum swing angle of the pendulum arm caused by the firing of a bullet into the pendulum bob. Let $L = 1.7$ m and $m_A = 5.8$ kg. For a certain historical pistol, which fired a roundball of mass $m_B = 90$ g, it is found that the maximum swing angle of the pendulum is $\theta_{max} = 51°$. Determine the preimpact speed of the bullet B.



Let v_B^- be speed of bullet befor impact. Assume that after imapct bullet and mass A are stuck togother with speed v^+. Hence

$$m_B v_B^- = (m_B + m_A) v^+ \tag{1}$$

Now we apply work-energy. Hence

$$\frac{1}{2}(m_B + m_A)(v^+)^2 = (m_B + m_A) g (L - L\cos\theta) \tag{2}$$

Where datum is taken at the horizontal level. From (2) we solve for v^+ and use it in (1) to find v_B^-. (2) becomes

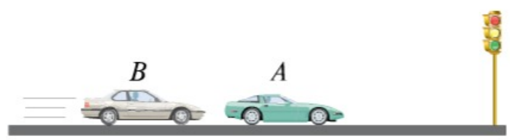
$$\begin{aligned}
\frac{1}{2}(0.09 + 5.8)(v^+)^2 &= (0.09 + 5.8)(9.81)(1.7)\left(1 - \cos\left(51\left(\frac{\pi}{180}\right)\right)\right) \\
2.945 (v^+)^2 &= 36.41094 \\
v^+ &= \sqrt{\frac{36.41094}{2.945}} \\
&= 3.516 \text{ m/sec}
\end{aligned}$$

Then (1) becomes

$$0.09v_B^- = (0.09 + 5.8)(3.516)$$
$$v_B^- = \frac{(0.09 + 5.8)(3.516)}{0.09}$$
$$= 230.103 \text{ m/sec}$$

0.6 Problem 6

Car A, with $m_A = 1{,}524$ kg, is stopped at a red light. Car B, with $m_B = 1{,}860$ kg and a speed of 38 km/h, fails to stop before impacting car A. After impact, cars A and B slide over the pavement with a coefficient of friction $\mu_k = 0.67$. How far will the cars slide if the cars become entangled?



d = ☐ m

Applying impulse momentum

$$m_B v_B^- = (m_B + m_A) v^+$$

Solving or v^+

$$v^+ = \frac{m_B v_B^-}{m_B + m_A}$$
$$= \frac{(1860)\left(38\left(\frac{1000}{3600}\right)\right)}{1860 + 1524}$$
$$= 5.802 \text{ m/sec}$$

Now applying work-energy

$$T_1 + U^{12} = T_2$$
$$\frac{1}{2}(m_B + m_A)(v^+)^2 - \int_0^d \mu (m_B + m_A) g dx = 0$$

We now solve for d

$$\frac{1}{2}(1860 + 1524)(5.802)^2 - (0.67)(1860 + 1524)(9.81)d = 0$$
$$d = \frac{\frac{1}{2}(1860 + 1524)(5.802)^2}{(0.67)(1860 + 1524)(9.81)}$$
$$= 2.561 \text{ meter}$$