HW 5,ME 240 Dynamics, Fall 2017

Nasser M. Abbasi

December 30, 2019

0.1 Problem 1

A 70 kg skydiver is falling at a speed of 241 km/h when the parachute is deployed, allowing the skydiver to land at a speed of 4 m/s. Modeling the skydiver as a particle, determine the total work done on the skydiver from the moment of parachute deployment until landing.

U_{1-2} = -156.295 kJ

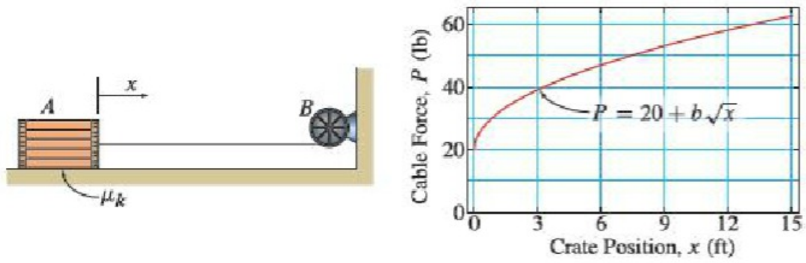
Landing speed is $v_2 = 4$ m/sec.

$$
\begin{aligned}
U_{12} &= T_2 - T_1 \\
&= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \\
&= \frac{1}{2}m\left(4^2 - \left(241\frac{1000}{km}\frac{hr}{3600}\right)^2\right) \\
&= \frac{1}{2}70\left((4)^2 - \left(241\left(\frac{1000}{3600}\right)\right)^2\right) \\
&= -156294.6
\end{aligned}
$$

Hence work on person is -156.295 kJ

0.2 Problem 2

The crate A of weight $W = 32$ lb is being pulled to the right by the winch at B. The crate starts at $x = 0$ and is pulled a total distance of 15 ft over the rough surface for which the coefficient of kinetic friction is $\mu_k = 0.4$. The force P in the cable due to the winch varies according to the plot, where P is in lb, b is in lb/$\sqrt{\text{ft}}$, and x is in ft. The coefficient of static friction is insufficient to prevent slipping. Using the work–energy principle, determine the speed of the block when $b = 12$ lb/$\sqrt{\text{ft}}$ and $x = 15$ ft.



$v_2 =$

ft/s

Force in the x direction is

$$
\begin{aligned}
F &= F_p - F_{friction} \\
&= (20 + 12x) - \mu_k N \\
&= (20 + 12x) - (0.4)(32)
\end{aligned}
$$

Hence

$$
\begin{aligned}
U_{12} &= T_2 - T_1 \\
\int_0^{15} \bar{F} \cdot d\bar{r} &= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \\
\int_0^{15} \left(\left(20 + 12x^{\frac{1}{2}}\right) - (0.4)(32) \right) dx &= \frac{1}{2} \frac{32}{32.2} v_2^2
\end{aligned}
$$

Since $v_1 = 0$ then above becomes

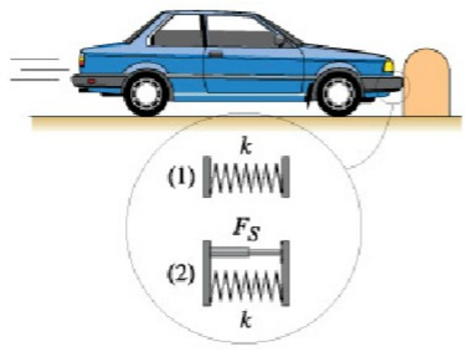
$$
\begin{aligned}
\int_0^{15} \left(\left(20 + 12x^{\frac{1}{2}}\right) - (0.4)(32) \right) dx &= \frac{1}{2} \left(\frac{32}{32.2} \right) v_2^2 \\
\int_0^{15} 12x^{\frac{1}{2}} + 7.2dx &= 0.49689 v_2^2 \\
\left(\frac{(12)(2)}{3} x^{\frac{3}{2}} + 7.2x \right)_0^{15} &= 0.49689 v_2^2 \\
\frac{(12)(2)}{3} (15)^{\frac{3}{2}} + 7.2(15) &= 0.49689 v_2^2 \\
572.758 &= 0.49689 v_2^2 \\
v_2^2 &= \frac{572.758}{0.49689} \\
&= 1152.686
\end{aligned}
$$

Hence

$$
\begin{aligned}
v_2 &= \sqrt{1152.686} \\
&= 33.951 \text{ ft/sec}
\end{aligned}
$$

0.3 Problem 3

Car bumpers are designed to limit the extent of damage to the car in the case of low–velocity collisions. Consider a 1,324 kg passenger car impacting a concrete barrier while traveling at a speed of 5.5 km/h. Model the car as a particle and consider two bumper models: (1) a simple linear spring with constant k and (2) a linear spring of constant k in parallel with a shock absorbing unit generating a nearly constant force $F_S = 2{,}010$ N over 10 cm. If the bumper is of type (1) and if $k = 9.4 \times 10^4$ N/m, find the spring compression (distance) necessary to stop the car.



The spring compression necessary to stop the car is ______ m.

Since all forces we can use conservation of energy $T_1 + V_1 = T_2 + V_2$ Where $V_1 = 0$ since spring is not compressed yet and $T_2 = 0$ since the car would be stopped by then. Hence

$$
\begin{aligned}
\frac{1}{2} m v_2^2 &= \frac{1}{2} k \Delta^2 \\
\Delta^2 &= \frac{m v_2^2}{k} \\
&= \frac{(1324)\left(5.5\left(\frac{1000}{km}\right)\left(\frac{hr}{3600}\right)\right)^2}{9.4 \times 10^4} \\
&= \frac{(1324)\left(5.5\left(\frac{1000}{3600}\right)\right)^2}{9.4 \times 10^4} \\
\Delta^2 &= 0.033
\end{aligned}
$$

Hence

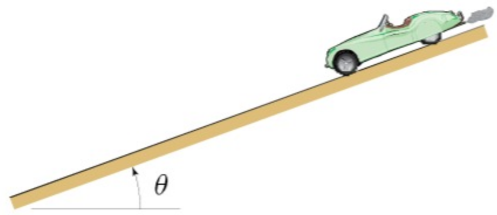
$$\Delta = 0.182 \text{ meter}$$

We could also have solved this using work-energy. Force on car is $-kx$, hence $U_{12} = \int_0^x \bar{F} \cdot d\bar{r}$ and therefore

$$
\begin{aligned}
\int_0^x \bar{F} \cdot d\bar{r} &= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \\
\int_0^x -kx dx &= -\frac{1}{2} m v_1^2 \\
\frac{1}{2} k \left(x^2\right)_0^x &= \frac{1}{2} 1324 \left(5.5\left(\frac{1000}{3600}\right)\right)^2 \\
9.4 \times 10^4 x^2 &= 1324 \left(5.5\left(\frac{1000}{3600}\right)\right)^2 \\
x^2 &= \frac{1324\left(5.5\left(\frac{1000}{3600}\right)\right)^2}{9.4 \times 10^4} \\
x &= 0.182 \text{ meter}
\end{aligned}
$$

0.4 Problem 4

A classic car is driving down an incline at 58 km/h when its brakes are applied. Treating the car as a particle, neglecting all forces except gravity and friction, and assuming that the tires slip, determine the coefficient of kinetic friction if the car comes to a stop in 51 m and $\theta = 21°$.



μ_k =

Distance is $L = 51$ meter (not clear in problem image).

Taking zero PE at horizontal datum when car comes to a stop at the bottom of hill, then using

$$T_1 + V_1 + U_{12} = T_2 + V_2$$

Where T_1 is KE in state 1, when the car just hit the brakes, and V_1 is its gravitational PE and U_{12} is work by non-conservative forces, which is friction here. $T_2 = 0$ since car stops in state 2 and V_2 is PE in state 2 which is zero. Hence we have

$$\frac{1}{2}mv_1^2 + mgL\sin\theta + \int_0^L -\mu N dx = 0$$

$$\frac{1}{2}mv_1^2 + mgL\sin\theta - \int_0^L \mu\left(mg\cos\theta\right)dx = 0$$

$$\frac{1}{2}mv_1^2 + mgL\sin\theta - L\mu mg\cos\theta = 0$$

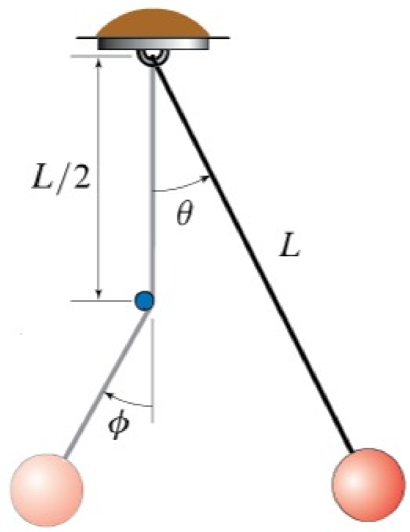
$$\mu = \frac{\frac{1}{2}v_1^2 + gL\sin\theta}{Lg\cos\theta}$$

Hence

$$\begin{aligned}\mu &= \frac{\frac{1}{2}\left(58\left(\frac{1000}{3600}\right)\right)^2 + 9.81\,(51)\sin\left(21\left(\frac{\pi}{180}\right)\right)}{(51)\,(9.81)\cos\left(21\left(\frac{\pi}{180}\right)\right)} \\ &= \frac{129.7840 + 179.2951}{467.0796} \\ &= 0.662\end{aligned}$$

0.5 Problem 5

A pendulum with mass $m = 1.3$ kg and length $L = 1.86$ m is released from rest at an angle θ_i. Once the pendulum has swung to the vertical position (i.e., $\theta = 0$), its cord runs into a small fixed obstacle. In solving this problem, neglect the size of the obstacle, model the pendulum's bob as a particle, model the pendulum's cord as massless and inextensible, and let gravity and the tension in the cord be the only relevant forces. What is the maximum height, measured from its lowest point, reached by the pendulum if $\theta_i = 16°$?



h_{max} = [ ] m

$$T_1 + V_1 = T_2 + V_2$$

Where state 1 is initial state, and state 2 is when bob at bottom. Datum is taken when bob at bottom. Hence

$$0 + mg(L - L\cos\theta) = \frac{1}{2}mv_2^2 + 0$$
$$Lmg(1 - \cos\theta) = \frac{1}{2}mv_2^2$$

Now let state 3 be when bob is up again on the other side. Hence we have

$$T_2 + V_2 = T_3 + V_3$$

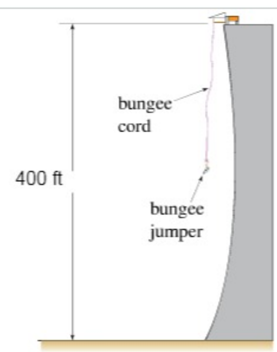
But $T_3 = 0$ and $V_3 = mgh_{\max}$, therefore

$$\frac{1}{2}mv_2^2 = mgh_{\max}$$

Or, since $\frac{1}{2}mv_2^2 = Lmg(1 - \cos\theta)$, then the above becomes

$$\begin{aligned} h_{\max} &= L(1 - \cos\theta_i) \\ &= 1.86\left(1 - \cos\left(16\left(\frac{\pi}{180}\right)\right)\right) \\ &= 0.0721 \text{ m} \end{aligned}$$

0.6 Problem 6



While the stiffness of an elastic cord can be nearly constant (i.e., the force versus displacement curve is a straight line) over a large range of stretch, as a bungee cord is stretched, it softens; that is, the cord tends to get less stiff as it gets longer. Assuming a softening force−displacement relation of the form $k\delta - \beta\delta^3$, where $k = 2.60$ lb/ft and $\beta = 0.000014$ lb/ft^3 and where δ (measured in ft) is the displacement of the cord from its unstretched length, and considering a bungee cord whose unstretched length is 150 ft,

Part 1 out of 3

(a) the expression of the cord's potential energy as a function of δ;

$V = \dfrac{\text{(select)}\ \text{(select)}^2}{\square} - \dfrac{\text{(select)}\ \text{(select)}^4}{\square}$

0.6.1 Part (a)

$$
\begin{aligned}
V &= \int^x k\delta - \beta\delta^3 d\delta \\
&= \frac{kx^2}{2} - \frac{\beta x^4}{4}
\end{aligned}
$$

0.6.2 Part (b)

Let datum be at top. Hence

$$
\begin{aligned}
T_1 + V_{1,gravity} + V_{1,rope} &= T_2 + V_{2,gravity} + V_{2,rope} \\
0 + 0 + 0 &= \frac{1}{2}mv_2^2 - mgh + \left(\frac{k\delta^2}{2} - \frac{\beta\delta^4}{4}\right) \\
v &= \sqrt{2gh - \frac{2}{m}\left(\frac{k\delta^2}{2} - \frac{\beta\delta^4}{4}\right)} \\
&= \sqrt{\frac{\beta\delta^4 - 2kv^2 + 4mgh}{2m}} \\
&= \sqrt{\frac{(0.000014)(150)^4 - 2(2.6)(150)^2 + 4(170)(150)}{2\left(\frac{170}{32.2}\right)}} \\
&= \sqrt{-749.3603} \\
&= \sqrt{\frac{(0.000013)(250)^4 - 2(2.58)(250)^2 + 4(170)(250)}{2\left(\frac{170}{32.2}\right)}}
\end{aligned}
$$

But $\delta = h - 150 = 400 - 150 = 250$, hence

$$
\begin{aligned}
v &= \sqrt{\frac{(0.000014)(250)^4 - 2(2.6)(250)^2 + 4(170)(400)}{2\left(\frac{170}{32.2}\right)}} \\
&= 12.64184 \text{ ft/sec}
\end{aligned}
$$

0.6.3 Part (c)

$$\delta = \sqrt{\frac{k}{3\beta}} = \sqrt{\frac{2.6}{3\,(0.000014)}} = 248.806\,7$$

Hence

$$\begin{aligned} a &= \left| g\left(1 - \frac{k\delta - \beta\delta^3}{W}\right)\right| \\ &= \left|32.2\left(1 - \frac{(2.6)\,(250) - (0.000014)\,(250)^3}{170}\right)\right| \\ &= 49.48382 \text{ ft/s}^2 \\ &= \frac{49.48382}{32.2} \\ &= 1.537 \text{ g} \end{aligned}$$