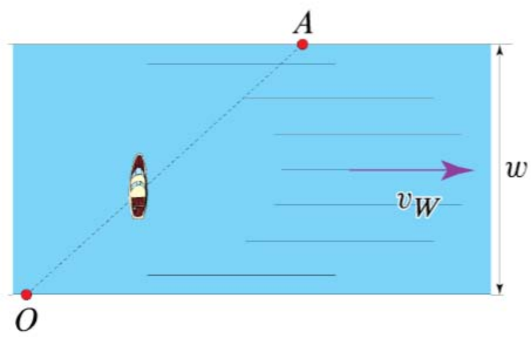
## 0.1 Problem 1

**A remote control boat, capable of a maximum speed of 8 ft/s in still water, is made to cross a stream with a width $w$ = 37 ft that is flowing with a speed $v_W$ = 7 ft/s. If the boat starts from point $O$ and keeps its orientation parallel to the cross–stream direction, find the location of point $A$ at which the boat reaches the other bank while moving at its maximum speed. Furthermore, determine how long the crossing requires.**




$t$ = [ ] s

$x$ = [ ] ft

The time to reach the top edge of the river is

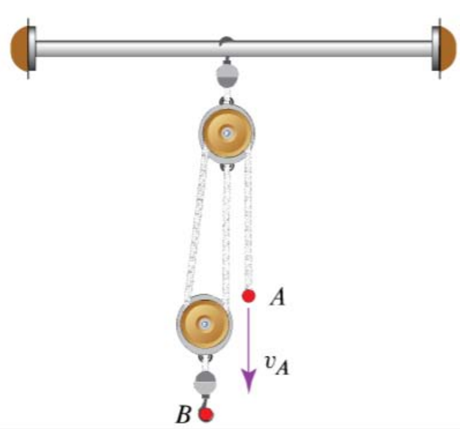
$$t = \frac{37}{8} = 4.625 \text{ sec}$$

The distance travelled in horizontal direction is therefore

$$x = (7)(4.625) = 32.375 \text{ ft}$$

## 0.2 Problem 2

**The object in the figure is called a *gun tackle*, and it used to be very common on sailboats to help in the operation of front–loaded guns. If the end at $A$ is pulled down at a speed of 1.5 m/s, determine the velocity of $B$. Neglect the fact that some portions of the rope are not vertically aligned.**




$v_B$ = [ ] m/s

Length of rope $L$ is

$$L = 2x_B + x_A$$

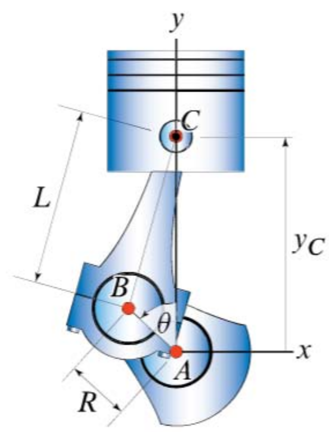
Where $x_B$ is distance from top to $B$ and $x_A$ is distance from top to $A$. Taking derivatives gives

$$0 = 2v_B + v_A$$

$$v_B = -\frac{v_A}{2} = -\frac{-1.5}{2} = 0.75 \text{ m/s}$$

## 0.3 Problem 3

**The piston head at $C$ is constrained to move along the $y$ axis. Let the crank $AB$ be rotating counterclockwise at a constant angular speed $\dot{\theta}$ = 1,950 rpm, $R$ = 3.4 in., and $L$ = 5.7 in. Determine the velocity of $C$ when $\theta$ = 40°.**




$\dot{y}_C$ = [ ] ft/s

$$\frac{d\theta}{dt} = (1950)\left(\frac{2\pi}{\text{rotation}}\right)\left(\frac{\text{minute}}{60}\right)$$
$$= (1950)\frac{2\pi}{60}$$
$$= 204.2035 \text{ rad/sec}$$

But

$$L^2 = R^2 + y_c^2 - 2(R)\left(y_c\right)\cos(\theta)$$

$$y_c^2 - 2Ry_c\cos(\theta) + \left(R^2 - L^2\right) = 0$$

Or

$$y_c = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$= R\cos\theta \pm \frac{1}{2}\sqrt{4R^2\cos^2\theta - 4\left(R^2 - L^2\right)}$$
$$= R\cos\theta \pm \frac{1}{2}\sqrt{4R^2\cos^2\theta - 4R^2 + 4L^2}$$
$$= R\cos\theta \pm \frac{1}{2}\sqrt{4R^2\left(\cos^2\theta - 1\right) + 4L^2}$$
$$= R\cos\theta \pm \frac{1}{2}\sqrt{-4R^2\sin^2\theta + 4L^2}$$
$$= R\cos\theta \pm \sqrt{L^2 - R^2\sin^2\theta}$$

At $\theta = 0$, $y_c = R + L$, therefore we pick the plus sign

$$y_c = R\cos\theta + \sqrt{L^2 - R^2\sin^2\theta}$$

Taking derivative with time

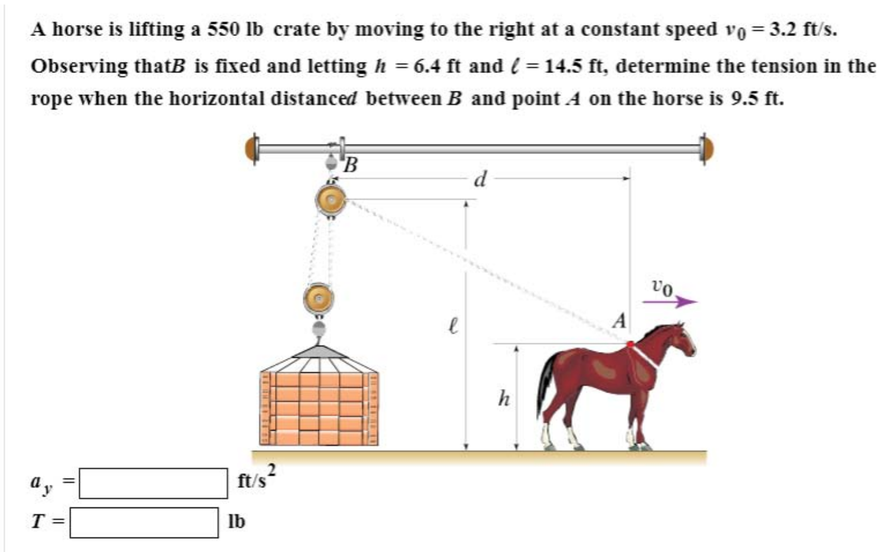
$$\dot{y}_c = -R\dot{\theta}\sin\theta + \frac{1}{2}\frac{1}{\sqrt{L^2 - R^2\sin^2(\theta)}}\left(-2R^2\sin\theta\left(\dot{\theta}\cos\theta\right)\right)$$
$$= -R\dot{\theta}\sin\theta - \frac{R^2\dot{\theta}\sin\theta\cos\theta}{\sqrt{L^2 - R^2\sin^2(\theta)}}$$

Plugging in values $R=\frac{3.4}{12}$ ft, $L=\frac{5.7}{12}$ ft and $\theta=40^0$ and $\dot{\theta}=204.2035$ gives

$$\dot{y}_c=-\left(\frac{3.4}{12}\right)(204.2035)\sin\left(\frac{40}{180}\pi\right)-\frac{\left(\frac{3.4}{12}\right)^2(204.2035)\sin\left(\frac{40}{180}\pi\right)\cos\left(\frac{40}{180}\pi\right)}{\sqrt{\left(\frac{5.7}{12}\right)^2-\left(\frac{3.4}{12}\right)^2\left(\sin\left(\frac{40}{180}\pi\right)\right)^2}}$$

$$=-55.59 \text{ ft/sec}$$

## 0.4 Problem 4




Resolving forces in vertical direction

$$mg-2T=m\ddot{y} \tag{1}$$

To find $y$, since rope length is

$$L=2y+\sqrt{x_A^2+(l-h)^2}$$

Taking derivative gives

$$0=2\dot{y}+\frac{x_A\dot{x}_A}{\sqrt{x_A^2+(l-h)^2}}$$

$$\dot{y}=\frac{-x_A\dot{x}_A}{\sqrt{x_A^2+(l-h)^2}}$$

Taking another derivative

$$\ddot{y}=\frac{-\dot{x}_A^2}{2\sqrt{x_A^2+(l-h)^2}}+\frac{x_A^2\dot{x}_A^2}{2\left(x_A^2+(l-h)^2\right)^{\frac{3}{2}}}$$

But $\dot{x}_A=v_0=3.2$ ft/sec. Hence

$$\ddot{y}=\frac{-v_0^2}{2\sqrt{x_A^2+(l-h)^2}}+\frac{x_A^2v_0^2}{2\left(x_A^2+(l-h)^2\right)^{\frac{3}{2}}}$$

When $l=14.5, h=6.4, x_A=9.5$ then

$$\ddot{y}=\frac{-(3.2)^2}{2\sqrt{9.5^2+(14.5-6.4)^2}}+\frac{(9.5)^2(3.2)^2}{2\left(9.5^2+(14.5-6.4)^2\right)^{\frac{3}{2}}}$$

$$=-0.17264 \text{ ft/sec}^2$$

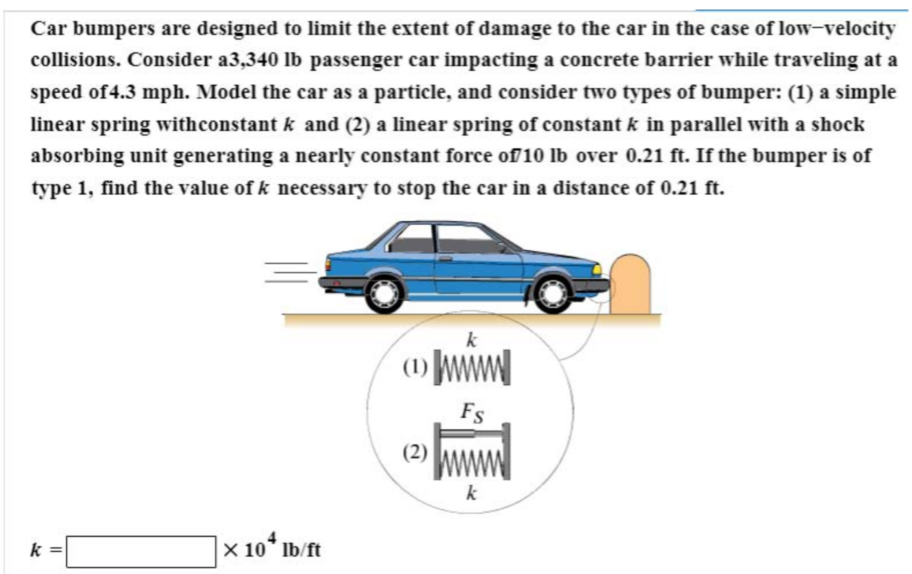
From (1) we solve for tension

$$
\begin{aligned}
T &= \frac{mg - m\ddot{y}}{2} \\
&= \frac{m\left(g - \ddot{y}\right)}{2} \\
&= \frac{550\,(32.2 - (-0.172\,64))}{2} \\
&= 8902.476 \text{ lb force}
\end{aligned}
$$

Hence

$$
\begin{aligned}
T &= \frac{8902.476}{32.2} \\
&= 276.474 \text{ lb}
\end{aligned}
$$

## 0.5 Problem 5


Car bumpers are designed to limit the extent of damage to the car in the case of low–velocity collisions. Consider a3,340 lb passenger car impacting a concrete barrier while traveling at a speed of4.3 mph. Model the car as a particle, and consider two types of bumper: (1) a simple linear spring withconstant $k$ and (2) a linear spring of constant $k$ in parallel with a shock absorbing unit generating a nearly constant force of710 lb over 0.21 ft. If the bumper is of type 1, find the value of $k$ necessary to stop the car in a distance of 0.21 ft.


Resolving forces in the $x$ direction gives equation of motion

$$
\begin{aligned}
m\ddot{x} + kx &= 0 \\
\ddot{x} &= -\frac{k}{m}x
\end{aligned}
$$

Let $\frac{d^2x}{dt^2} = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$ and the above becomes

$$
v\frac{dv}{dx} = -\frac{k}{m}x
$$

This is now separable

$$
\int_{v_i}^{v_{stop}} v dv = -\int_0^{x_i} \frac{k}{m} x dx
$$

But $v_{stop} = 0$ and the above becomes

$$
v_i^2 = \frac{k}{m}x_i^2
$$

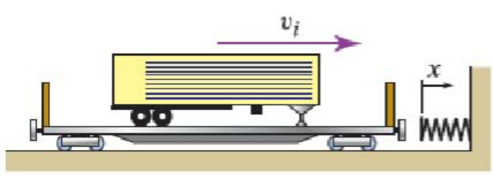
For $v_i = 4.3$ mph and $x_i = 0.21$ ft, we solve for $k$ from the above

$$
\begin{aligned}
k = \frac{mv_i^2}{x_i^2} &= \frac{\left(\frac{3340}{32.2}\right)\left((4.3)\left(\frac{5280}{3600}\right)\right)^2}{(0.21)^2} \\
&= 93551.7 \text{ lb/ft} \\
&= 9.355 \times 10^{-4} \text{ lb/ft}
\end{aligned}
$$

Question, why had to divide by $g$ in above to get correct answer? Problem said $lb$ in statement?

## 0.6 Problem 6

**A railcar with an overall mass of 74,000 kg traveling with a speed $v_i$ is approaching a barrier equipped with a bumper constisting of a nonlinear spring whose force vs. compression law is given by$F_S = \beta x^3$, where $\beta = 650 \times 10^6$ N/m$^3$ and $x$ is the compression of the bumper. Treating the system as a particle, assuming that the contact between railcar and rails is frictionless, and letting$v_i = 3$ km/h, determine the bumper compression necessary to bring the railcar to a stop.**




$x_{stop}$ = ☐ m

Resolving forces in the $x$ direction gives equation of motion

$$m\ddot{x} + \beta x^3 = 0$$
$$\ddot{x} = -\frac{\beta x^3}{m}$$

Let $\frac{d^2x}{dt^2} = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$ and the above becomes

$$v\frac{dv}{dx} = -\frac{\beta x^3}{m}$$

This is now separable

$$\int_{v_i}^{v_{stop}} v dv = -\frac{\beta}{m}\int_0^{x_{stop}} x^3 dx$$

But $v_{stop} = 0$ and the above becomes

$$v_i^2 = \frac{1}{2}\frac{\beta}{m}x_{stop}^4$$

For $v_i = 3$ km/h and $\beta = 650 \times 10^6$ and $m = 75000$ kg, we solve for $x_{stop}$ from the above

$$\begin{aligned} x_{stop} &= \left(\frac{2mv^2}{\beta}\right)^{\frac{1}{4}} \\ &= \left(\frac{2\,(75000)\left(3\left(\frac{1000}{3600}\right)\right)}{650 \times 10^6}\right)^{\frac{1}{4}} \\ &= 0.11776 \text{ m} \end{aligned}$$