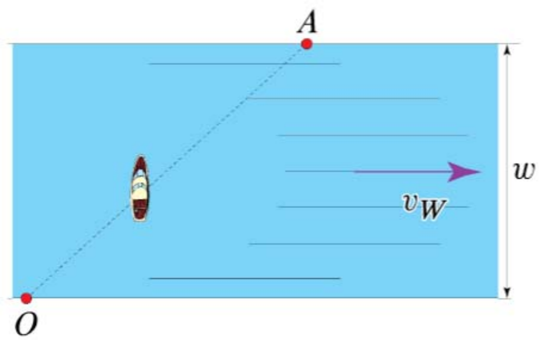
0.1 Problem 1

A remote control boat, capable of a maximum speed of 8 ft/s in still water, is made to cross a stream with a width$w$ = 37 ft that is flowing with a speed v_W = 7 ft/s. If the boat starts from point O and keeps its orientation parallel to the cross−stream direction, find the location of point A at which the boat reaches the other bank while moving at its maximum speed. Furthermore, determine how long the crossing requires.



t = [ ] s

x = [ ] ft

The time to reach the top edge of the river is

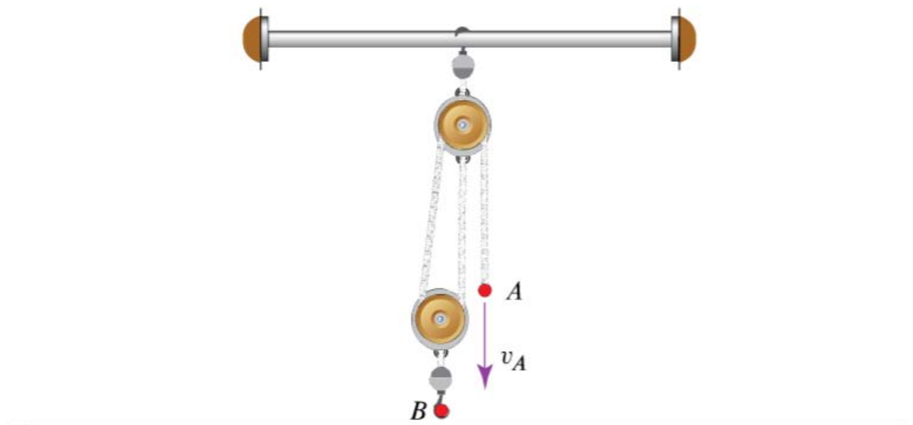
$$t = \frac{37}{8} = 4.625 \text{ sec}$$

The distance travelled in horizontal direction is therefore

$$x = (7)(4.625) = 32.375 \text{ ft}$$

0.2 Problem 2

The object in the figure is called a *gun tackle*, and it used to be very common on sailboats to help in the operation of front−loaded guns. If the end at is pulled down at a speed of 1.5 m/s, determine the velocity ofB. Neglect the fact that some portions of the rope are not vertically aligned.



v_B = [ ] m/s

Length of rope L is

$$L = 2x_B + x_A$$

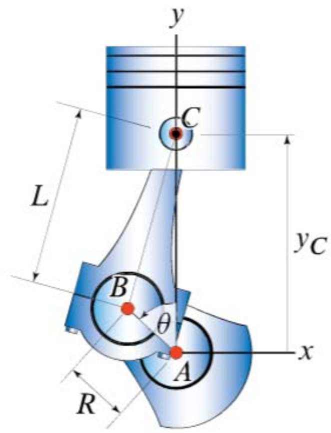
Where x_B is distance from top to B and x_A is distance from top to A. Taking derivatives gives

$$0 = 2v_B + v_A$$

$$v_B = -\frac{v_A}{2} = -\frac{-1.5}{2} = 0.75 \text{ m/s}$$

0.3 Problem 3

The piston head at C is constrained to move along the y axis. Let the crank AB be rotating counterclockwise at a constant angular speed $\dot{\theta}$ = 1,950 rpm, R = 3.4 in., and L = 5.7 in. Determine the velocity of C when θ = 40°.



$\dot{y}_C$ = [ ] ft/s

$$\frac{d\theta}{dt} = (1950)\left(\frac{2\pi}{\text{rotation}}\right)\left(\frac{\text{minute}}{60}\right)$$
$$= (1950)\,\frac{2\pi}{60}$$
$$= 204.2035 \text{ rad/sec}$$

But

$$L^2 = R^2 + y_c^2 - 2(R)\left(y_c\right)\cos(\theta)$$

$$y_c^2 - 2Ry_c\cos(\theta) + \left(R^2 - L^2\right) = 0$$

Or

$$y_c = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$= R\cos\theta \pm \frac{1}{2}\sqrt{4R^2\cos^2\theta - 4\left(R^2 - L^2\right)}$$
$$= R\cos\theta \pm \frac{1}{2}\sqrt{4R^2\cos^2\theta - 4R^2 + 4L^2}$$
$$= R\cos\theta \pm \frac{1}{2}\sqrt{4R^2\left(\cos^2\theta - 1\right) + 4L^2}$$
$$= R\cos\theta \pm \frac{1}{2}\sqrt{-4R^2\sin^2\theta + 4L^2}$$
$$= R\cos\theta \pm \sqrt{L^2 - R^2\sin^2\theta}$$

At $\theta = 0$, $y_c = R + L$, therefore we pick the plus sign

$$y_c = R\cos\theta + \sqrt{L^2 - R^2\sin^2\theta}$$

Taking derivative with time

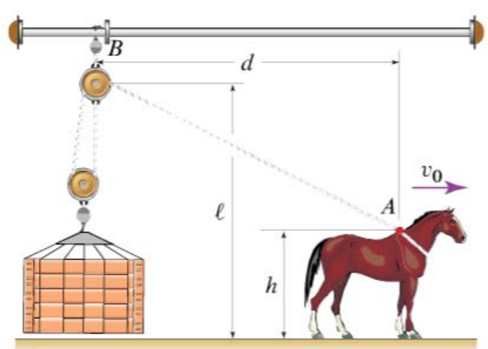
$$\dot{y}_c = -R\dot{\theta}\sin\theta + \frac{1}{2}\frac{1}{\sqrt{L^2 - R^2\sin^2(\theta)}}\left(-2R^2\sin\theta\left(\dot{\theta}\cos\theta\right)\right)$$
$$= -R\dot{\theta}\sin\theta - \frac{R^2\dot{\theta}\sin\theta\cos\theta}{\sqrt{L^2 - R^2\sin^2(\theta)}}$$

Plugging in values $R = \frac{3.4}{12}$ ft, $L = \frac{5.7}{12}$ ft and $\theta = 40^0$ and $\dot{\theta} = 204.2035$ gives

$$\dot{y}_c = -\left(\frac{3.4}{12}\right)(204.2035)\sin\left(\frac{40}{180}\pi\right) - \frac{\left(\frac{3.4}{12}\right)^2(204.2035)\sin\left(\frac{40}{180}\pi\right)\cos\left(\frac{40}{180}\pi\right)}{\sqrt{\left(\frac{5.7}{12}\right)^2 - \left(\frac{3.4}{12}\right)^2\left(\sin\left(\frac{40}{180}\pi\right)\right)^2}}$$
$$= -55.59 \text{ ft/sec}$$

0.4 Problem 4

A horse is lifting a 550 lb crate by moving to the right at a constant speed $v_0 = 3.2$ ft/s. Observing that B is fixed and letting $h = 6.4$ ft and $\ell = 14.5$ ft, determine the tension in the rope when the horizontal distance d between B and point A on the horse is 9.5 ft.



a_y = [ ] ft/s^2

T = [ ] lb

Resolving forces in vertical direction

$$mg - 2T = m\ddot{y} \tag{1}$$

To find y, since rope length is

$$L = 2y + \sqrt{x_A^2 + (l-h)^2}$$

Taking derivative gives

$$
\begin{aligned}
0 &= 2\dot{y} + \frac{x_A \dot{x}_A}{\sqrt{x_A^2 + (l-h)^2}} \\
\dot{y} &= \frac{-x_A \dot{x}_A}{\sqrt{x_A^2 + (l-h)^2}}
\end{aligned}
$$

Taking another derivative

$$
\ddot{y} = \frac{-\dot{x}_A^2}{2\sqrt{x_A^2 + (l-h)^2}} + \frac{x_A^2 \dot{x}_A^2}{2\left(x_A^2 + (l-h)^2\right)^{\frac{3}{2}}}
$$

But $\dot{x}_A = v_0 = 3.2$ ft/sec. Hence

$$
\ddot{y} = \frac{-v_0^2}{2\sqrt{x_A^2 + (l-h)^2}} + \frac{x_A^2 v_0^2}{2\left(x_A^2 + (l-h)^2\right)^{\frac{3}{2}}}
$$

When $l = 14.5, h = 6.4, x_A = 9.5$ then

$$
\begin{aligned}
\ddot{y} &= \frac{-(3.2)^2}{2\sqrt{9.5^2 + (14.5-6.4)^2}} + \frac{(9.5)^2 (3.2)^2}{2\left(9.5^2 + (14.5-6.4)^2\right)^{\frac{3}{2}}} \\
&= -0.172\,64 \text{ ft/sec}^2
\end{aligned}
$$

From (1) we solve for tension

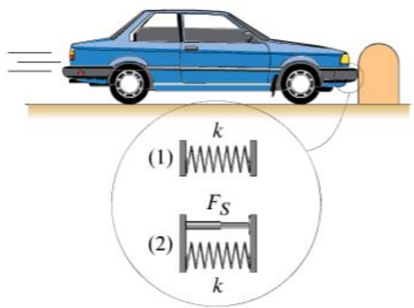
$$
\begin{aligned}
T &= \frac{mg - m\ddot{y}}{2} \\
&= \frac{m\left(g - \ddot{y}\right)}{2} \\
&= \frac{550\,(32.2 - (-0.172\,64))}{2} \\
&= 8902.476 \text{ lb force}
\end{aligned}
$$

Hence

$$
\begin{aligned}
T &= \frac{8902.476}{32.2} \\
&= 276.474 \text{ lb}
\end{aligned}
$$

0.5 Problem 5

Car bumpers are designed to limit the extent of damage to the car in the case of low−velocity collisions. Consider a3,340 lb passenger car impacting a concrete barrier while traveling at a speed of4.3 mph. Model the car as a particle, and consider two types of bumper: (1) a simple linear spring withconstant *k* and (2) a linear spring of constant *k* in parallel with a shock absorbing unit generating a nearly constant force of710 lb over 0.21 ft. If the bumper is of type 1, find the value of *k* necessary to stop the car in a distance of 0.21 ft.



k = [   ] $\times 10^4$ lb/ft

Resolving forces in the x direction gives equation of motion

$$m\ddot{x} + kx = 0$$

$$\ddot{x} = -\frac{k}{m}x$$

Let $\frac{d^2x}{dt^2} = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$ and the above becomes

$$v\frac{dv}{dx} = -\frac{k}{m}x$$

This is now separable

$$\int_{v_i}^{v_{stop}} v\,dv = -\int_0^{x_i} \frac{k}{m}x\,dx$$

But $v_{stop} = 0$ and the above becomes

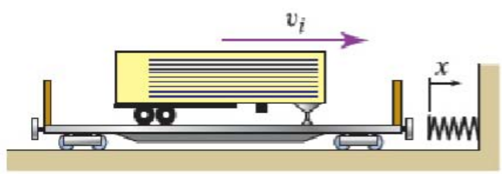
$$v_i^2 = \frac{k}{m}x_i^2$$

For $v_i = 4.3$ mph and $x_i = 0.21$ ft, we solve for k from the above

$$\begin{aligned} k &= \frac{mv_i^2}{x_i^2} = \frac{\left(\frac{3340}{32.2}\right)\left((4.3)\left(\frac{5280}{3600}\right)\right)^2}{(0.21)^2} \\ &= 93551.7 \text{ lb/ft} \\ &= 9.355 \times 10^{-4} \text{ lb/ft} \end{aligned}$$

Question, why had to divide by g in above to get correct answer? Problem said *lb* in statement?

A railcar with an overall mass of 74,000 kg traveling with a speed v_i is approaching a barrier equipped with a bumper constisting of a nonlinear spring whose force vs. compression law is given by $F_S = \beta x^3$, where $\beta = 650 \times 10^6$ N/m^3 and x is the compression of the bumper. Treating the system as a particle, assuming that the contact between railcar and rails is frictionless, and letting $v_i = 3$ km/h, determine the bumper compression necessary to bring the railcar to a stop.



x_{stop} = ☐ m

0.6 Problem 6

Resolving forces in the x direction gives equation of motion

$$m\ddot{x} + \beta x^3 = 0$$
$$\ddot{x} = -\frac{\beta x^3}{m}$$

Let $\frac{d^2x}{dt^2} = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$ and the above becomes

$$v\frac{dv}{dx} = -\frac{\beta x^3}{m}$$

This is now separable

$$\int_{v_i}^{v_{stop}} vdv = -\frac{\beta}{m}\int_0^{x_{stop}} x^3 dx$$

But $v_{stop} = 0$ and the above becomes

$$v_i^2 = \frac{1}{2}\frac{\beta}{m}x_{stop}^4$$

For $v_i = 3$ km/h and $\beta = 650 \times 10^6$ and $m = 75000$ kg, we solve for x_{stop} from the above

$$\begin{aligned} x_{stop} &= \left(\frac{2mv^2}{\beta}\right)^{\frac{1}{4}} \\ &= \left(\frac{2\,(75000)\left(3\left(\frac{1000}{3600}\right)\right)}{650 \times 10^6}\right)^{\frac{1}{4}} \\ &= 0.11776 \text{ m} \end{aligned}$$