# HW 2,ME 240 Dynamics, Fall 2017

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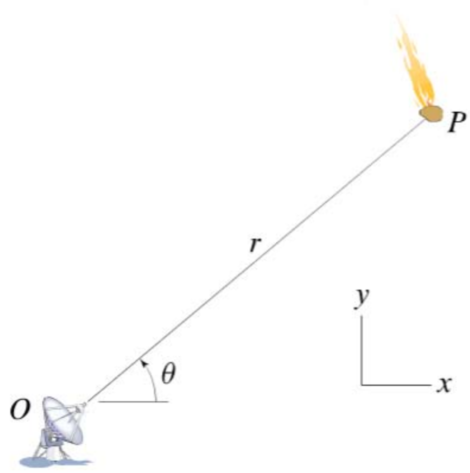
## 0.1 Problem 1

**The radar station at $O$ is tracking the meteor $P$ as it moves through the atmosphere. At the instant shown, the station measures the following data for the motion of the meteor: $r$ = 21,000 ft, $\theta = 38°$, $\dot{r} = -22{,}490$ ft/s, and $\dot{\theta} = -2.933$ rad/s. Determine the magnitude and direction (relative to the $xy$ coordinate system shown) of the velocity vector at this instant.**




**The magnitude of the velocity is** ☐ **ft/s.**

**Direction of the velocity relative to the $x$ axis is** ☐ **°.**

$$\bar{v} = \dot{r}\hat{u}_r + r\dot{\theta}\hat{u}_\theta$$

Hence

$$\begin{aligned}
|\bar{v}| &= \sqrt{\dot{r}^2 + \left(r\dot{\theta}\right)^2} \\
&= \sqrt{(-22490)^2 + (21000 \times (-2.933))^2} \\
&= 65571 \text{ ft/sec}
\end{aligned}$$

Since

$$\begin{aligned}
\hat{u}_r &= \hat{\imath}\cos\theta + \hat{\jmath}\sin\theta \\
\hat{u}_\theta &= -\hat{\imath}\sin\theta + \hat{\jmath}\cos\theta
\end{aligned}$$

Then the velocity vector in Cartesian is

$$\begin{aligned}
\bar{v} &= \dot{r}\left(\hat{\imath}\cos\theta + \hat{\jmath}\sin\theta\right) + r\dot{\theta}\left(-\hat{\imath}\sin\theta + \hat{\jmath}\cos\theta\right) \\
&= \hat{\imath}\left(\dot{r}\cos\theta - r\dot{\theta}\sin\theta\right) + \hat{\jmath}\left(\dot{r}\sin\theta + r\dot{\theta}\cos\theta\right)
\end{aligned}$$

Plug-in numerical values

$$\begin{aligned}
\bar{v} = \hat{\imath}&\left(-22490\cos\left(38\left(\frac{\pi}{180}\right)\right) - (21000)(-2.933)\sin\left(38\left(\frac{\pi}{180}\right)\right)\right) \\
+ \hat{\jmath}&\left((-22490)\sin\left(38\left(\frac{\pi}{180}\right)\right) + (21000)(-2.933)\cos\left(38\left(\frac{\pi}{180}\right)\right)\right)
\end{aligned}$$

Or

$$\bar{v} = (20198.08)\,\hat{\imath} - \hat{\jmath}\,(62382.17)$$

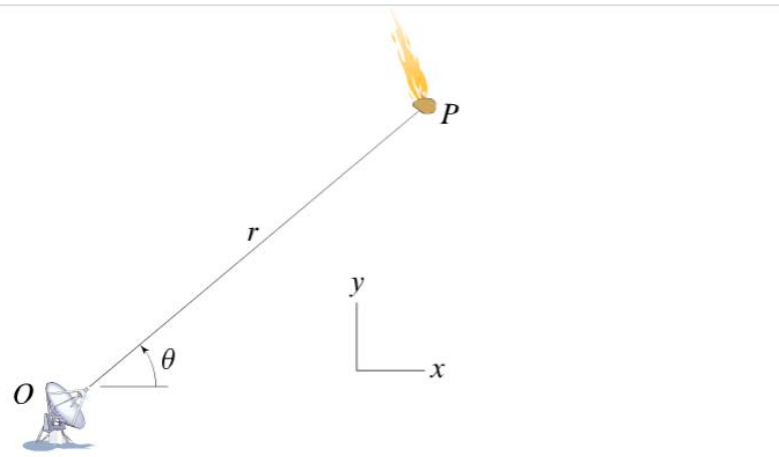
To check, we find magnitude of $\bar{v}$ in Cartesian

$$|\bar{v}| = \sqrt{(20198.08)^2 + (62382.17)^2} = 65571$$

Which is the same as before. Hence the velocity vector makes angle $\tan^{-1}\left(\frac{-62382}{20198}\right) = -1.2577$ rad or $-1.2577\left(\frac{180}{\pi}\right) = -72.061$ degrees with the x-axis.

## 0.2 Problem 2

**The radar station at $O$ is tracking the meteor $P$ as it moves through the atmosphere. At the instant shown, the station measures the following data for the motion of the meteor: $r = 21{,}400$ ft, $\theta = 44°$, $\dot{r} = -22{,}490$ ft/s, $\dot{\theta} = -2.944$ rad/s, $\ddot{r} = 187{,}300$ ft/s$^2$, and $\ddot{\theta} = -5.407$ rad/s$^2$. Determine the magnitude and direction (relative to the $xy$ coordinate system shown) of the acceleration vector at this instant.**




**The magnitude of the acceleration is ______ ft/s$^2$.**

**Direction of the acceleration relative to the $x$ axis is ______ ° CCW from the horizontal.**

$$\bar{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{u}_\theta$$

Hence

$$\begin{aligned}
|\bar{a}| &= \sqrt{\left(\ddot{r} - r\dot{\theta}^2\right)^2 + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)^2} \\
&= \sqrt{\left(187300 - (21400)(-2.944)^2\right)^2 + ((21400)(-5.407) + 2(-22490)(-2.944))^2} \\
&= 16810.49 \text{ ft/sec}^2
\end{aligned}$$

Since

$$\begin{aligned}
\hat{u}_r &= \hat{\imath}\cos\theta + \hat{\jmath}\sin\theta \\
\hat{u}_\theta &= -\hat{\imath}\sin\theta + \hat{\jmath}\cos\theta
\end{aligned}$$

Then the acceleration vector in Cartesian is

$$\begin{aligned}
\bar{a} &= \left(\ddot{r} - r\dot{\theta}^2\right)\left(\hat{\imath}\cos\theta + \hat{\jmath}\sin\theta\right) + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\left(-\hat{\imath}\sin\theta + \hat{\jmath}\cos\theta\right) \\
&= \hat{\imath}\left(\left(\ddot{r} - r\dot{\theta}^2\right)\cos\theta - \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\sin\theta\right) + \hat{\jmath}\left(\left(\ddot{r} - r\dot{\theta}^2\right)\sin\theta + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\cos\theta\right) \qquad (1)
\end{aligned}$$

But

$$\begin{aligned}
\left(\ddot{r} - r\dot{\theta}^2\right) &= \left(187300 - (21400)(-2.944)^2\right) = 1823.290 \\
\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right) &= ((21400)(-5.407) + 2(-22490)(-2.944)) = 16711.32
\end{aligned}$$

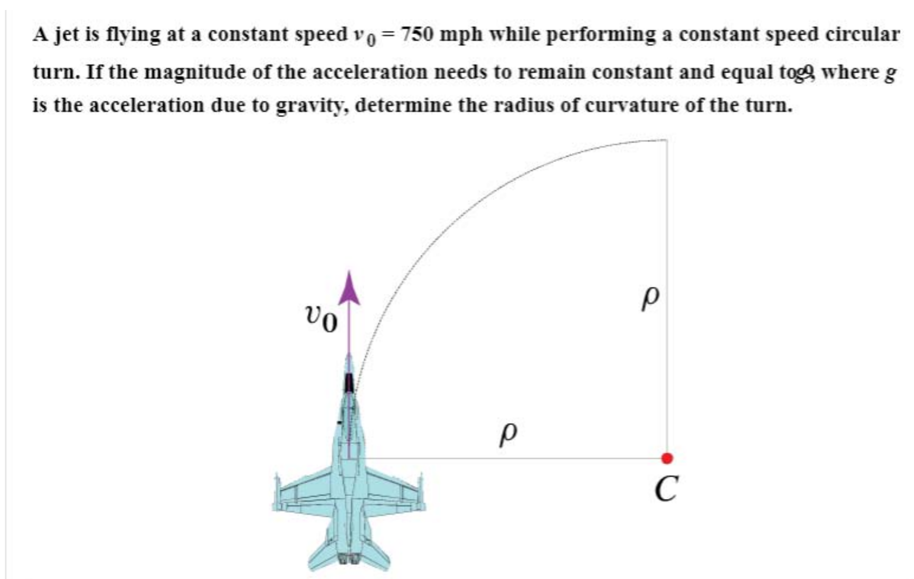
Hence (1) becomes

$$\begin{aligned}\bar{a} &= \hat{\imath}\left(1823.290\cos\left(44\frac{\pi}{180}\right) - (16711.32)\sin\left(44\frac{\pi}{180}\right)\right)\\ &+ \hat{\jmath}\left(1823.290\sin\left(44\frac{\pi}{180}\right) + (16711.32)\cos\left(44\frac{\pi}{180}\right)\right)\\ \\ &= \hat{\imath}\,(-10297.09) + \hat{\jmath}\,(13287.68)\end{aligned}$$

To verify things, we check the magnitude of $\bar{a}$ is the same as found above (since the magnitude of vector does not depend on coordinates. We see that $|\bar{a}| = \sqrt{(-10297.09)^2 + (13287.68)^2} = 16810.49$ which is the same as before.

Hence the acceleration vector makes angle $\tan^{-1}\left(\frac{13287.68}{-10297.09}\right) = 127.7$ degrees with the x-axis.

## 0.3 Problem 3



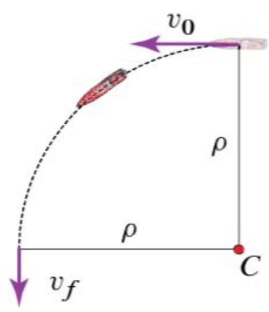

$$\bar{a} = \dot{V}\hat{u}_t + \frac{V^2}{\rho}\hat{u}_n$$

But $\dot{V} = 0$ since velocity is constant. Hence $|\bar{a}| = \frac{V^2}{\rho}$. Therefore

$$\begin{aligned}\frac{V^2}{\rho} &= 9g\\ \frac{\left(750\left(\frac{5280}{3600}\right)\right)^2}{9\,(32.2)} &= \rho\\ \rho &= 4175.3\text{ ft}\end{aligned}$$

## 0.4 Problem 4

A race boat is traveling at a constant speed $v_0 = 85$ mph when it performs a turn with constant radius $\rho$ to change its course by 90° as shown. The turn is performed while losing speed uniformly in time so that the boat's speed at the end of the turn is $v_f = 80$ mph. If the maximum allowed normal acceleration is equal to $2g$, where $g$ is the acceleration due to gravity, determine the tightest radius of curvature possible and the time needed to complete the turn.




$\rho_{\min}$ = [ ] ft

$t_f$ = [ ] s

$$\bar{a} = \dot{V}\hat{u}_t + \frac{V^2}{\rho}\hat{u}_n$$

Maximum normal acceleration is $\frac{V^2}{\rho}$. Hence it occurs when $V$ is maximum, which is at start of turn. Then we want

$$\frac{V_{\max}^2}{\rho_{\min}} = 2g$$

$$\rho_{\min} = \frac{V_{\max}^2}{g} = \frac{\left(85\left(\frac{5280}{3600}\right)\right)^2}{2\,(32.2)} = 241.33 \text{ ft}$$

Now since

$$v_f^2 = v_o^2 + 2a_t s$$

Where $a_t$ is tangential acceleration and $s$ is distance travelled which is $\frac{1}{4}$ of circumference of circle or $\frac{1}{4}\left(2\pi\rho_{\min}\right)$, therefore

$$a_t = \frac{v_f^2 - v_o^2}{2\left(\frac{1}{2}\pi\rho_{\min}\right)} = \frac{\left(80\left(\frac{5280}{3600}\right)\right)^2 - \left(85\left(\frac{5280}{3600}\right)\right)^2}{\pi\,(241.33)} = -2.341 \text{ ft/sec}^2$$

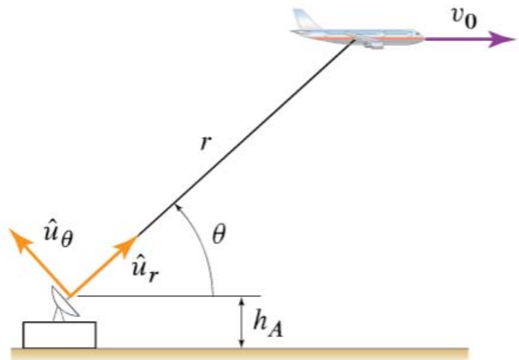
Therefore to find time of travel, since acceleration is constant

$$v_f = v_0 + a_t t$$

$$t = \frac{v_f - v_0}{a_t} = \frac{80\left(\frac{5280}{3600}\right) - 85\left(\frac{5280}{3600}\right)}{-2.341} = 3.133 \text{ sec}$$

## 0.5 Problem 5

**A radar station is tracking a plane flying at a constant altitude with a speed $v_0 = 560$ mph. If, at a given instant $r = 6.9$ mi and $\theta = 31°$, determine the corresponding values of $\dot{r}$, $\dot{\theta}$, $\ddot{r}$, $\ddot{\theta}$.**




$\dot{r}$ = [ ] **mph**

$\dot{\theta}$ = [ ] **rad/h**

$\ddot{r}$ = [ ] **mi/h$^2$**

$\ddot{\theta}$ = [ ] **rad/h$^2$**

Since velocity is horizontal, then

$$\bar{v} = v_0 \hat{\imath}$$

But $\hat{\imath} = \cos\theta \hat{u}_r - \sin\theta \hat{u}_\theta$, hence

$$\begin{aligned}\bar{v} &= v_0 \cos\theta \hat{u}_r - v_0 \sin\theta \hat{u}_\theta \\ &= \dot{r}\hat{u}_r + r\dot{\theta}\hat{u}_\theta\end{aligned}$$

Therefore

$$\begin{aligned}\dot{r} &= v_0 \cos\theta \\ r\dot{\theta} &= -v_0 \sin\theta\end{aligned}$$

We are given that $v_0 = 560$ mph and $\theta = 31$, hence solving gives

$$\begin{aligned}\dot{r} &= 560 \cos\left(31 \frac{\pi}{180}\right) \\ (6.9)\,\dot{\theta} &= -(560) \sin\left(31 \frac{\pi}{180}\right)\end{aligned}$$

Or

$$\begin{aligned}\dot{r} &= 480.014 \text{ mph} \\ \dot{\theta} &= -41.8 \text{ rad/h}\end{aligned}$$

The acceleration vector is

$$\bar{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{u}_\theta$$

Since constant velocity, then acceleration is zero. This gives us two equations to solve for $\ddot{\theta}, \ddot{r}$

$$\begin{aligned}\ddot{r} - r\dot{\theta}^2 &= 0 \\ r\ddot{\theta} + 2\dot{r}\dot{\theta} &= 0\end{aligned}$$

Or

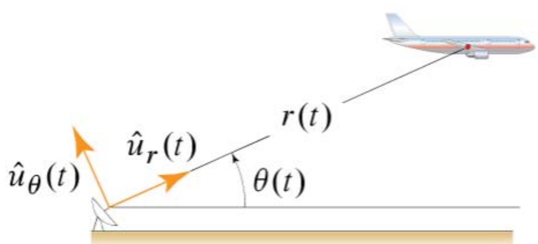
$$\begin{aligned}\ddot{r} - (6.9)(-41.8)^2 &= 0 \\ (6.9)\,\ddot{\theta} + 2\,(480.014)(-41.8) &= 0\end{aligned}$$

Solving gives

$$\begin{aligned}\ddot{r} &= 12054.23 \text{ mph} \\ \ddot{\theta} &= 5815.477 \text{ rad/h}^2\end{aligned}$$

## 0.6 Problem 6

**During a given time interval, a radar station tracking an airplane records the readings**
**$\dot{r}(t) = [448.1 \cos\theta(t) + 13.17 \sin\theta(t)]$ mph,**
**$r(t)\dot{\theta}(t) = [13.17 \cos\theta(t) - 448.1 \sin\theta(t)]$ mph,**
**where $t$ denotes time. Determine the speed of the plane. Furthermore, determine whether the plane being tracked is ascending or descending and the corresponding climbing rate (i.e., the rate of change of the plane's altitude) expressed in ft/s.**




**$v$ = [ ] mi/h**

**The airplane is (select) at a rate of [ ] ft/s.**

$$
\begin{aligned}
\bar{v} &= \dot{r}\hat{u}_r + r\dot{\theta}\hat{u}_\theta \\
&= (448.1 \cos\theta + 13.17 \sin\theta)\,\hat{u}_r + (13.17 \cos\theta - 448.1 \sin\theta)\,\hat{u}_\theta
\end{aligned}
$$

Hence

$$
\begin{aligned}
|\bar{v}|^2 &= (448.1 \cos\theta + 13.17 \sin\theta)^2 + (13.17 \cos\theta - 448.1 \sin\theta)^2 \\
&= (448.1)^2 \cos^2\theta + (13.17)^2 \sin^2\theta + 2\left((448.1)(13.17)\cos\theta\sin\theta\right) \\
&\quad + (13.17)^2 \cos^2\theta + (-448.1)^2 \sin^2\theta - 2\left((448.1)(13.17)\cos\theta\sin\theta\right)
\end{aligned}
$$

Which simplifies to

$$
\begin{aligned}
|\bar{v}|^2 &= (448.1)^2 \cos^2\theta + (13.17)^2 \sin^2\theta + (13.17)^2 \cos^2\theta + (-448.1)^2 \sin^2\theta \\
&= (448.1)^2 \left(\cos^2\theta + \sin^2\theta\right) + (13.17)^2 \left(\cos^2\theta + \sin^2\theta\right) \\
&= (448.1)^2 + (13.17)^2 \\
&= 200967.1
\end{aligned}
$$

Hence

$$
|\bar{v}| = 448.294 \text{ mph}
$$

Let $y$ be vertical distance. Hence $y = r\sin\theta$ and

$$
\begin{aligned}
\dot{y} &= \dot{r}\sin\theta + r\dot{\theta}\cos\theta \\
&= (448.1 \cos\theta + 13.17 \sin\theta)\sin\theta + (13.17 \cos\theta - 448.1 \sin\theta)\cos\theta \\
&= 448.1 \cos\theta\sin\theta + 13.17 \sin^2\theta + 13.17 \cos^2\theta - 448.1 \sin\theta\cos\theta \\
&= 13.17 \text{ mph}
\end{aligned}
$$

Hence it is ascending. Convert to ft/sec

$$
\begin{aligned}
\dot{y} &= 13.17 \frac{5280}{mile} \frac{hr}{3600} \\
&= 13.17 \frac{5280}{3600} \\
&= 19.316 \text{ ft/sec}
\end{aligned}
$$