HW 13,ME 240 Dynamics, Fall 2017

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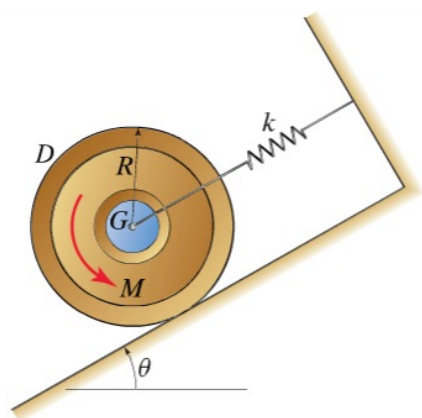
0.1 Problem 1



The disk D, which has weight W, mass center G coinciding with the disk's geometric center, and radius of gyration k_G, is at rest on an incline when the constant moment M is applied to it. The disk is attached at its center to a wall by a linear elastic spring of constant k. The spring is unstretched when the system is at rest. Assuming that the disk rolls without slipping and that it has not yet come to a stop, determine the angular velocity of the disk after its center G has moved a distance d down the incline. After doing so, using $k = 4$ lb/ft, $R = 1.4$ ft, $W = 10$ lb, and $\theta = 28°$, determine the value of the moment M for the disk to stop after rolling $d_s = 4$ ft down the incline.

$\omega_d =$ ◯ **A.** $\sqrt{\frac{g}{W(R^2+k_G^2)}}\sqrt{d[(M/R)+W\sin\theta - dk]}$

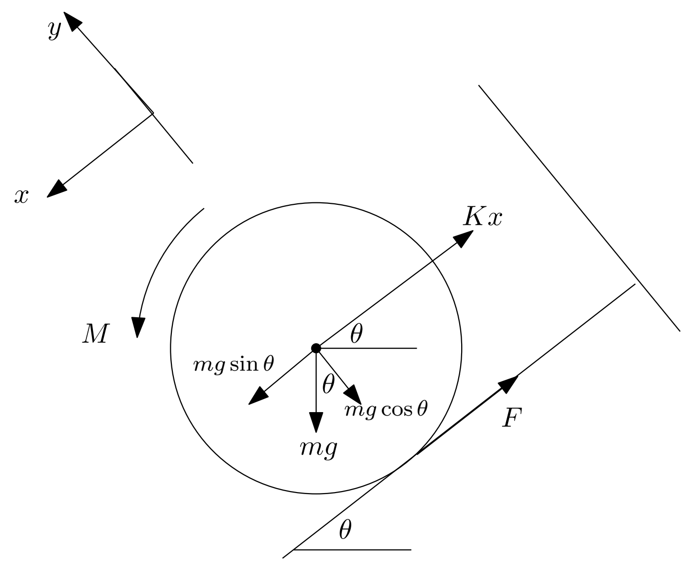
◯ **B.** $\sqrt{\frac{g}{W(R^2+k_G^2)}}\sqrt{d[(2M/R)+2W\sin\theta - dk]}$

◯ **C.** $\sqrt{\frac{g}{W(R^2+k_G^2)}}\sqrt{d[(2M/R)+2W\cos\theta + dk]}$

◯ **D.** $\sqrt{\frac{2g}{W(R^2-k_G^2)}}\sqrt{d[(2M/R)+2W\sin\theta - dk]}$

$M =$ ______ ft · lb

Free body diagram is



Method one, using work-energy

Applying work energy

$$T_1 + U_1 + \int_0^{\theta_{final}} M d\theta = T_2 + U_2 \tag{1}$$

But $T_1 = 0$ and $U_1 = 0$ (using initial position as datum).

$$\int_0^{\theta_{final}} M d\theta = M\theta_{final} = M\frac{d}{R}$$

Where d is distance travelled (since no slip, we use $d = R\theta$).

$$\begin{aligned} T_2 &= \frac{1}{2}mv_{cg}^2 + \frac{1}{2}I_{cg}\omega^2 \\ &= \frac{1}{2}m(R\omega)^2 + \frac{1}{2}\left(mk_G^2\right)\omega^2 \end{aligned}$$

And

$$U_2 = \frac{1}{2}kd^2 - Wd\sin\theta$$

Hence (1) becomes

$$\begin{aligned} M\frac{d}{R} &= \frac{1}{2}m(R\omega)^2 + \frac{1}{2}\left(mk_G^2\right)\omega^2 + \frac{1}{2}kd^2 - Wd\sin\theta \\ M\frac{d}{R} - \frac{1}{2}kd^2 + Wd\sin\theta &= \omega^2\left(\frac{1}{2}mR^2 + \frac{1}{2}mk_G^2\right) \\ \omega^2 &= \frac{M\frac{d}{R} - \frac{1}{2}kd^2 + Wd\sin\theta}{\frac{1}{2}mR^2 + \frac{1}{2}mk_G^2} \\ &= \frac{2\left(M\frac{d}{R} - \frac{1}{2}kd^2 + Wd\sin\theta\right)}{\frac{W}{g}\left(R^2 + k_G^2\right)} \\ &= \frac{g}{W\left(R^2 + k_G^2\right)}\left(2M\frac{d}{R} - kd^2 + 2Wd\sin\theta\right) \end{aligned}$$

Or

$$\omega = \sqrt{\frac{g}{W\left(R^2 + k_G^2\right)}}\sqrt{d\left(2\frac{M}{R} + 2W\sin\theta - kd\right)} \tag{2}$$

Hence choice B. Plug-in numerical values gives $k = 4, R = 1.4, W = 10, \theta = 28^0$, and since

$I_{disk} = \frac{1}{2}mR^2 = mk_G^2$ then $k_G^2 = \frac{R^2}{2} = \frac{1.4^2}{2} = 0.98$, then (2) becomes for $\omega = 0$

$$0 = \sqrt{\frac{32.2}{10\left(1.4^2 + 0.98\right)}}\sqrt{(4)\left(2\frac{M}{1.4} + 2\,(10)\sin\left(28\left(\frac{\pi}{180}\right)\right) - (4)\,(4)\right)}$$
$$0 = 1.047\sqrt{5.714\,M - 26.442}$$

Solving for moment M gives

$$M = 4.627 \text{ ft-lb}$$

<u>Method two, using Newton methods</u>

$\sum F_x$ gives (where positive x is as shown in diagram, going down the slope).

$$W\sin\theta - kx - F = m\ddot{x} = mR\ddot{\theta} \tag{1}$$

Taking moment about CG of disk. But note that now anti-clock wise is negative and not positive, due to right-hand rule)

$$-M - FR = -I_{cg}\ddot{\theta} \tag{2}$$

From (2) we solve for F and use (1) to find $\ddot{\theta}$. From (2)

$$F = \frac{I_{cg}\ddot{\theta} - M}{R}$$

Plug the above into (1)

$$W\sin\theta - kx - \frac{I_{cg}\ddot{\theta} - M}{R} = mR\ddot{\theta}$$
$$W\sin\theta - kx = mR\ddot{\theta} + \left(\frac{I_{cg}\ddot{\theta} - M}{R}\right)$$
$$W\sin\theta - kx = mR\ddot{\theta} + \frac{I_{cg}\ddot{\theta}}{R} - \frac{M}{R}$$
$$W\sin\theta - kx = \ddot{\theta}\left(mR + \frac{I_{cg}}{R}\right) - \frac{M}{R}$$
$$\frac{M}{R} + mg\sin\theta - kx = \ddot{\theta}\left(mR + \frac{mk_G^2}{R}\right)$$

Hence

$$\ddot{\theta} = \frac{\frac{M}{R} + W\sin\theta - kx}{mR + \frac{mk_G^2}{R}}$$
$$= \frac{M + WR\sin\theta - kRx}{\frac{W}{g}\left(R^2 + k_G^2\right)}$$
$$= \frac{g}{W\left(R^2 + k_G^2\right)}\left(M + WR\sin\theta - kRx\right)$$

The above shows that $\ddot{\theta}$ is not constant. To find ω we need to integrate both sides. Since $\ddot{\theta} = \frac{d\omega}{dt} = \frac{d\omega}{dx}\frac{dx}{dt} = \frac{d\omega}{dx}R\omega$ then the above can be written as

$$R\omega d\omega = \frac{g}{W\left(R^2 + k_G^2\right)}\left(M + WR\sin\theta - kRx\right)dx$$

Integrating

$$\frac{R}{2}\omega^2 = \frac{g}{W\left(R^2 + k_G^2\right)}\left(Mx + WRx\sin\theta - kR\frac{x^2}{2}\right)$$

When $x = d$, the above becomes

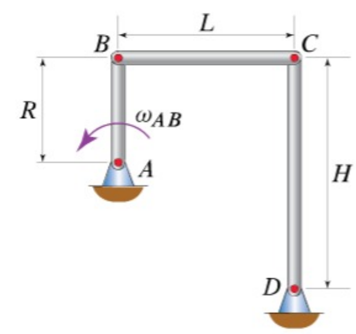
$$\frac{R}{2}\omega^2 = \frac{g}{W\left(R^2 + k_G^2\right)}\left(Md + WRd\sin\theta - kR\frac{d^2}{2}\right) \tag{3}$$

Hence

$$\omega^2 = 2\left(\frac{g}{W\left(R^2+k_G^2\right)}\left(\frac{Md}{R}+Wd\sin\theta-k\frac{d^2}{2}\right)\right)$$
$$= \frac{g}{W\left(R^2+k_G^2\right)}d\left(\frac{2M}{R}+2W\sin\theta-kd\right) \tag{4}$$

Compare (4) to (2) in first method, we see they are the same.

0.2 Problem 2



The uniform thin pin−connected bars AB, BC, and CD have masses $m_{AB}=2.2$ kg, $m_{BC}=3.4$ kg, and $m_{CD}=5.2$ kg, respectively. Letting $R=0.76$ m, $L=1.2$ m, and $H=1.54$ m, and knowing that bar AB rotates at a constant angular velocity $\omega_{AB}=5$ rad/s, compute the angular momentum of bar AB about A, of bar BC about A, and bar CD about D at the instant shown.

$(\vec{h}_A)_{AB} =$ (______ kg·m²/s) $\hat{k}$

$(\vec{h}_A)_{BC} =$ (______ kg·m²/s) $\hat{k}$

$(\vec{h}_D)_{CD} =$ (______ kg·m²/s) $\hat{k}$

$$\begin{aligned}
h_{AB} &= I_A\omega_{AB}\\
&= \left(\frac{1}{3}m_{AB}R^2\right)\omega_{AB}\\
&= \frac{1}{3}(2.2)(0.76)^2(5)\\
&= 2.119 \text{ kg m}^2/s
\end{aligned}$$

For bar BC, it has zero ω_{BC} at this instance. Therefore the only angular momentum comes from translation. WHich is

$$h_{BC} = m_{BC}v_{cg}R$$

But v_{cg} for bar BC is $R\omega_{AB}$, hence

$$
\begin{aligned}
h_{BC} &= m_{BC} R^2 \omega_{AB} \\
&= (3.4)\,(0.76)^2\,(5) \\
&= 9.819 \;\; \text{kg m}^2/s
\end{aligned}
$$

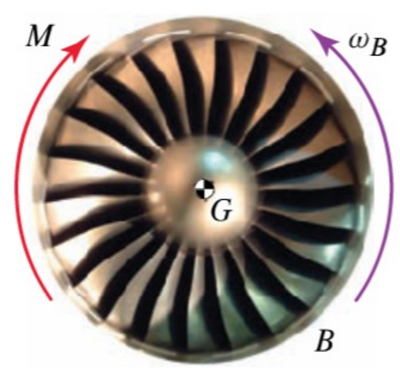
Finally, for bar DC, since it point C moves with speed $v = R\omega_{AB}$, then

$$
\begin{aligned}
R\omega_{AB} &= H\omega_{CD} \\
\omega_{CD} &= \frac{R}{H}\omega_{AB}
\end{aligned}
$$

Therefore

$$
\begin{aligned}
h_{CD} &= I_{CD}\omega_{CD} \\
&= \frac{1}{3} m_{CD} H^2 \frac{R}{H}\omega_{AB} \\
&= \frac{1}{3} m_{CD} H R \omega_{AB} \\
&= \frac{1}{3}\,(5.2)\,(1.54)\,(0.76)\,(5) \\
&= 10.143 \;\; \text{kg m}^2/s
\end{aligned}
$$

0.3 Problem 3



A rotor, B, with center of mass G, weight W = 3,400 lb, and radius of gyration k_G = 15.1 ft is spinning with an angular speed of ω_B = 1,150 rpm when a braking system is applied to it, providing a time−dependent torque $M = M_0(1 + ct)$, with M_0 = 3,400 ft·lb and c = 0.012 s^{-1}. If G is also the geometric center of the rotor and is a fixed point, determine the time, t_s, that it takes to bring the rotor to a stop.

$$
\begin{aligned}
torque &= I_{cg}\ddot{\theta} \\
-M_0\,(1 + ct) &= m k_G^2 \ddot{\theta} \\
\frac{d\dot{\theta}}{dt} &= -\frac{M_0}{m k_G^2}\,(1 + ct) \\
\int_{\omega_{AB}}^{0} d\dot{\theta} &= -\frac{M_0}{m k_G^2}\int_0^{t_s}(1 + ct)\,dt \\
-\omega_{AB} &= -\frac{M_0}{m k_G^2}\left(t_s + \frac{c}{2}t_s^2\right) \\
\omega_{AB} &= \frac{M_0}{m k_G^2}\left(t_s + \frac{c}{2}t_s^2\right) \qquad (1)
\end{aligned}
$$

Hence

$$(1150)\frac{2\pi}{60} = \frac{3400}{\frac{3400}{32.2}(15.1)^2}\left(t_s + \frac{0.012}{2}t_s^2\right)$$
$$120.428 = 0.141\left(t_s + 0.006t_s^2\right)$$

Solving

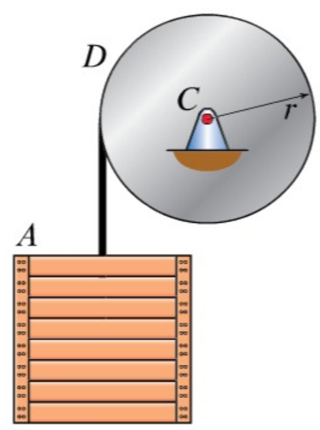
$$t_s = 302.763 \text{ seconds}$$

Another way to solve this is to use conservation of angular momentum.

$$h_1 + \int \tau dt = h_2$$
$$I_{cg}\omega_B + \int_0^{t_s}(-M)\,dt = 0$$
$$\left(mk_G^2\right)\omega_B - \int_0^{t_s} M_0(1+ct)\,dt = 0$$
$$\left(mk_G^2\right)\omega_B - M_0\left(t_s + \frac{c}{2}t_s^2\right) = 0$$
$$\omega_B = \frac{M_0}{mk_G^2}\left(t_s + \frac{c}{2}t_s^2\right)$$

Which is the same as (1).

0.4 Problem 4



A crate, A, with weight $W_A = 325$ lb is hanging from a rope wound around a uniform drum, D, of radius $r = 1.2$ ft, weight $W_D = 117$ lb, and center C. The systems is initially at rest when the restraining system holding the drum stationary fails, thus causing the drum to rotate, the rope to unwind, and, consequently, the crate to fall. Assuming that the rope does not stretch or slip relative to the drum and neglecting the inertia of the rope, determine the speed of the crate2.5 s after the system starts to move.

v = ☐ ft/s downward

It is easier to solve this using conservation of angular and linear momentum. There are two bodies in this problem. One has angular momentum and the second (cart) has linear momentum. So we need to apply

$$p_1 + \int_0^t f\,dt = p_1 \qquad (1)$$

Where $p = mv$, the linear momentum. The above is applied to the cart. And also apply

$$h_1 + \int_0^t \tau dt = h_1 \tag{2}$$

Where $h = I\omega$, the angular momentum, and this is applied to the drum. Using the above two equations we will find final velocity of cart. We break the system to 2 bodies, using free body diagram. Let tension in cable be T. And since in state (1), $v_A = 0$, then equation (1) becomes

$$\begin{aligned} \int_0^t (T - W_A)\, dt &= m_A v_A \\ \int_0^t T dt &= W_A t + \frac{W_A}{g} v_A \end{aligned} \tag{3}$$

In the above, $\int_0^t (T - W_A)\, dt$ is the impulse, and v_A is the final speed we want to find. We do not know the tension T.

Equation (2) becomes ($h_1 = 0$, since drum is not spining then)

$$\int_0^t Tr dt = I_{cg}\omega_D$$

Where Tr is the torque, caused by the tension T in cable. But $v_A = -r\omega_D$, where the minus sign since it is moving downwards. Hence the above becomes

$$\begin{aligned} \int_0^t T dt &= -\left(\frac{W_D}{g}\frac{r^2}{2}\right)\frac{v_A}{r^2} \\ &= -\left(\frac{W_D}{2g}\right) v_A \end{aligned} \tag{4}$$

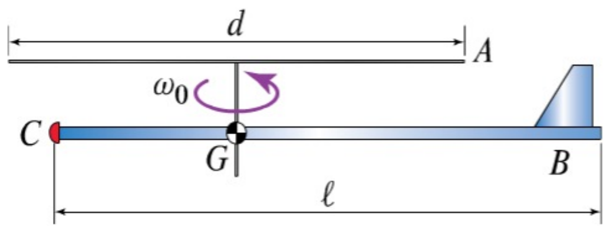
Comparing (3,4) we see that

$$\begin{aligned} -\left(\frac{W_D}{2g}\right) v_A &= W_A t + \frac{W_A}{g} v_A \\ W_A t + \frac{W_A}{g} v_A + \left(\frac{W_D}{2g}\right) v_A &= 0 \\ v_A\left(\frac{W_A}{g} + \frac{W_D}{2g}\right) + W_A t &= 0 \\ v_A &= \frac{-2W_A g t}{2W_A + W_D} \end{aligned}$$

Therefore

$$\begin{aligned} v_A &= \frac{-2\,(325)\,(32.2)\,(2.5)}{2\,(325) + (117)} \\ &= -68.22 \text{ ft/sec} \end{aligned}$$

0.5 Problem 5



A toy helicopter consists of a rotor, A, with diameter d = 11.7 in. and weight $W_A = 0.090 \times 10^{-3}$ oz, a thin body, B, of length ℓ = 14.3 in. and weight $W_B = 0.139 \times 10^{-3}$ oz, and a small ballast, C, placed at the front end of the body with weight $W_C = 0.0683 \times 10^{-3}$ oz. The ballast's weight is such that the axis of rotation of the rotor goes through G, which is the center of mass of the body and ballast. While holding the body (and ballast) fixed, the rotor is spun as shown with ω_0 = 170 rpm. Neglecting aerodynamic effects, the weights of the rotor's shaft and the body's tail, and assuming there is friction between the helicopter's body and the rotor's shaft, determine the angular velocity of the body once the toy is released and the angular velocity of the rotor decreases to 145 rpm. Model the body as a uniform thin rod and the ballast as a particle. Assume that the rotor and the body remain horizontal after release.

ω_{Bf} = [ ] rpm