HW 11,ME 240 Dynamics, Fall 2017

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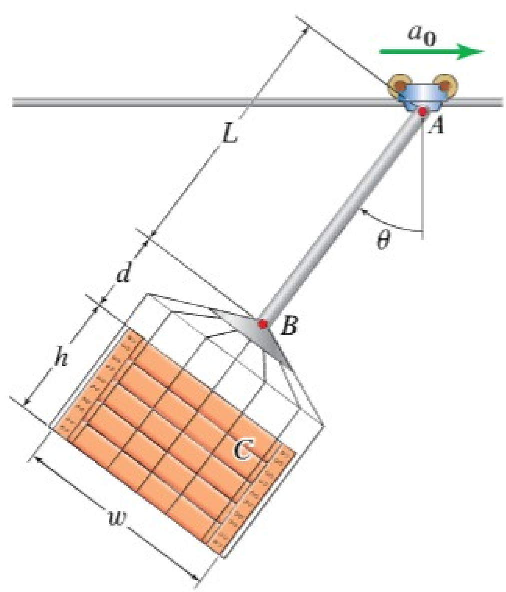
0.1 Problem 1

The uniform slender bar AB has a weight $W_{AB} = 128$ lb while the crate's weight is $W_C = 462$ lb. The bar AB is rigidly attached to the cage containing the crate. Neglect the mass of the cage, and assume that the mass of the crate is uniformly distributed. Furthermore, let $L = 8.8$ ft, $d = 2.6$ ft, $h = 3.9$ ft, and $w = 6.3$ ft. If the trolley is accelerating with $a_0 = 9$ ft/s^2, determine θ so that the bar−crate system translates with the trolley.



$\theta =$ ☐ °

Let us assume the center of mass of the overall system is at some distance z from point A somewhere between A and C. It does not matter where it is. Therefore the rotational equation of motion for the hanging system is

$$M_{cg} = I_A \alpha$$

Where M is the moment of external forces around this center of mass and I_A is the mass moment of inertia around A. But since we want the system to be translating, then $\alpha = 0$. Therefore

$$\begin{aligned} M &= 0 \\ F_y z \sin\theta - F_x z \cos\theta &= 0 \end{aligned} \tag{1}$$

Notice the weights do not come into play, since we are taking moments about center of mass of the overall system.

So we just need to find F_x, F_y. These forces are the reactions on point A where it is connected. These can be found by resolving forces in the horizontal and vertical direction. In horizontal direction

$$F_x = (m_{AB} + m_C)\, a_0 \tag{2}$$

In vertical direction (where these is no acceleration)

$$\begin{aligned} F_y - W_{AB} - W_C &= 0 \\ F_y &= W_{AB} + W_C \end{aligned} \tag{3}$$

Plugging (2,3) into (1) and canceling z (as we see, we really did not need to find where z is), gives

$$\begin{aligned} (W_{AB} + W_C)\sin\theta - (m_{AB} + m_C)\, a_0 \cos\theta &= 0 \\ \tan\theta &= \frac{(m_{AB} + m_C)\, a_0}{(W_{AB} + W_C)} \end{aligned}$$

Plugging the numerical values

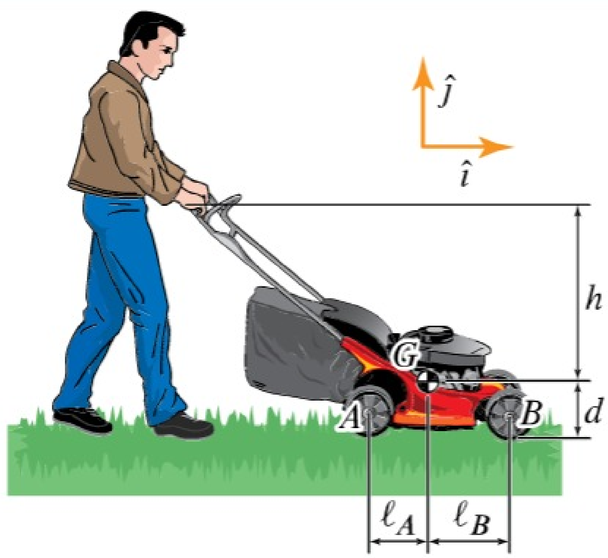
$$\begin{aligned} \tan\theta &= \frac{\left(\frac{128}{32.2} + \frac{462}{32.2}\right) 9}{(128 + 462)} \\ &= 0.279 \end{aligned}$$

Hence

$$\begin{aligned} \theta &= \arctan(0.279) \\ &= 0.272 \\ &= 15.616^0 \end{aligned}$$

0.2 Problem 2

A person is pushing a lawn mower of mass m = 37 kg and with h = 0.71 m, d = 0.22 m, ℓ_A = 0.29 m, and ℓ_B = 0.35 m. Assuming that the force exerted on the lawn mower by the person is completely horizontal, the mass center of the lawn mower is at G, and neglecting the rotational inertia of the wheels, determine the minimum value of this force that causes the rear wheels (labeled A) to lift off the ground. In addition, determine the corresponding acceleration of the mower.



F = [ ] N and a_{G_x} = [ ] m/s²

Taking moments about G (and assuming no friction from the ground as problems says to neglect rotational inertia of wheels, which seems to imply this).

$$-Fh + N_B L_B - N_A L_A = I\alpha$$

For $\alpha = 0$

$$-Fh + N_B L_B - N_A L_A = 0$$

And when $N_A = 0$

$$F = \frac{N_B L_B}{h}$$

But $N_A + N_B = mg$ or since $N_A = 0$ then $N_B = mg$ and the above becomes

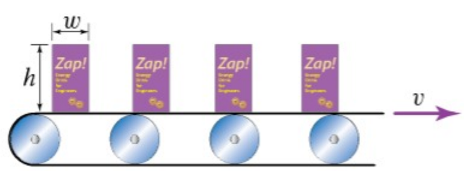
$$
\begin{aligned}
F_{\min} &= \frac{mgL_B}{h} \\
&= \frac{(37)(9.81)(0.35)}{(0.71)} \\
&= 178.929 \text{ N}
\end{aligned}
$$

And the acceleration is

$$
\begin{aligned}
F &= ma \\
178.929 &= 37a \\
a &= \frac{178.929}{37} \\
&= 4.836 \text{ m/s}^2
\end{aligned}
$$

0.3 Problem 3

A conveyor belt must accelerate the cans from rest to v = 18.2 ft/s as quickly as possible. Treating each can as a uniform circular cylinder weighing 1.4 lb, find the minimum possible time to reach v so that the cans do not tip or slip on the conveyor. Assume that acceleration is uniform and use w = 4.9 in., h = 5.1 in., and μ_s = 0.51.



$t_{\min}$ = [ ] s

$$
\begin{aligned}
F &= ma \\
\mu N &= ma \\
a &= \frac{\mu N}{m} \\
&= \frac{(0.51)\left(mg\right)}{m} \\
&= (0.51)(32.2) \\
&= 16.422 \text{ ft/s}^2
\end{aligned}
$$

Hence

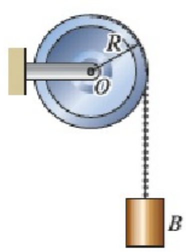
$$v = at$$
$$t = \frac{v}{a}$$
$$= \frac{18.2}{16.422}$$
$$= 1.108 \text{ sec}$$

0.4 Problem 4

The spool is pinned at its center at O, about which it can spin freely. The radius of the spool is $R = 0.18$ m, its radius of gyration is $k_O = 0.11$ m, and the mass of the spool is $m_s = 4$ kg.

The mass B is suspended from the periphery of the spool by a chain of negligible mass that moves over the spool without slip. The mass of B is $m_B = 6$ kg.

If the system is released from rest, determine the angular acceleration of the spool and the tension in the chain.



$T =$ [ ] N

$\vec{\alpha}_S =$ [ ] $\hat{k}$ rad/s^2

Resolve forces in vertical direction for hanging mass

$$T - m_B g = m_B a_y$$

But $a_y = R\alpha$ where α is angular acceleration of spool. Hence

$$T - m_B g = m_B R\alpha \tag{1}$$

For the spool, the equation of motion is $M = I\alpha$ or

$$-TR = m r_G^2 \alpha \tag{2}$$

Where r_G is radius of gyration. We have two equations and two unknowns $\alpha, T.$, solving gives

$$\alpha = \frac{-m_B g R}{m r_G^2 + m_B R^2}$$
$$T = m_B R\alpha + m_B g$$

Hence

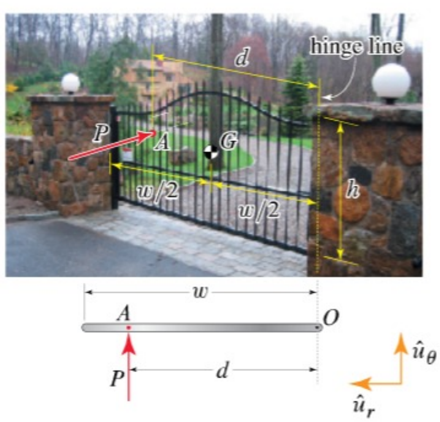
$$\alpha = \frac{-(6)(9.81)(0.18)}{(4)(0.11)^2 + (6)(0.18)^2} = -43.636 \text{ rad/sec}^2$$

And

$$\begin{aligned} T &= (6)(0.18)(-43.636) + (6)(9.81) \\ &= 11.733 \text{ N} \end{aligned}$$

0.5 Problem 5

The driveway gate is hinged at its right end and can swing freely in the horizontal plane. The gate is pushed open by the force P that always acts perpendicular to the plane of the gate at point A, which is a horizontal distance d from the gate hinge. The weight of the gate is $W = 213$ lb, and its mass center is at G, which is a distance $w/2$ from each end of the gate, where $w = 14$ ft. Assume that the gate is initially at rest and model the gate as a uniform thin bar as shown below in the photo. Given that a force of $P = 21$ lb is applied at the center of mass of the gate (i.e., $d = w/2$), determine the reaction at the hinge O after the force P has been continuously applied for 2.2 s.



O_r = [ ] lb and O_θ = [ ] lb

I will use L for w so not to confuse it with ω. Resolving forces in x direction

$$-O_x = ma_{Gx}$$

in the y direction

$$P + O_y = ma_{Gy}$$

But $a_{Gx} = \frac{L}{2}\omega^2$ and $a_{G_y} = -\frac{L}{2}\alpha$ where α is angular acceleration of gate. Hence the above becomes

$$-O_x = m\frac{L}{2}\omega^2 \tag{1}$$

$$P + O_y = -m\frac{L}{2}\alpha \tag{2}$$

Now the angular acceleration equation for the gate is, taking moments around center of mass

$$\begin{aligned} O_y\frac{L}{2} &= I_{cg}\alpha \\ &= \frac{mL^2}{12}\alpha \end{aligned} \tag{3}$$

From (2) $O_y = -m\frac{L}{2}\alpha - P$, plug this in (3) gives

$$\begin{aligned} \left(-m\frac{L}{2}\alpha - P\right)\frac{L}{2} &= \frac{mL^2}{12}\alpha \\ -P\frac{L}{2} &= \frac{mL^2}{12}\alpha + m\frac{L^2}{4}\alpha \\ -P\frac{L}{2} &= \alpha\left(\frac{mL^2}{12} + \frac{mL^2}{4}\right) \\ \alpha &= -\frac{P\frac{L}{2}}{\frac{1}{3}L^2m} \\ &= -\frac{3}{2}\frac{P}{Lm} \end{aligned}$$

Plug-in numerical values

$$\alpha = -\frac{3}{2}\frac{(21)}{(14)\left(\frac{213}{32.2}\right)} = -0.340$$

From (3)

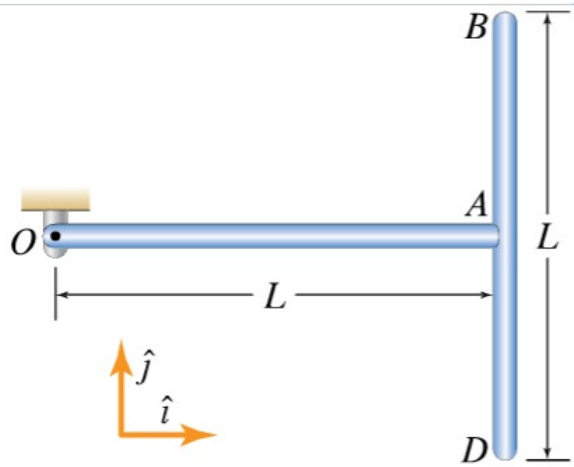
$$\begin{aligned} O_y &= \frac{mL}{6}\alpha \\ &= \frac{\frac{213}{32.2}(14)}{6}(-0.3408) \\ &= -5.25 \text{ N} \end{aligned}$$

To find ω, from $\omega = \alpha t = -0.340\ (2.2) = -0.748$ rad/sec, hence from (1)

$$-O_x = m\frac{L}{2}\omega^2$$
$$= \frac{213}{32.2}\left(\frac{14}{2}\right)(-0.748)^2$$
$$O_x = -25.929 \text{ N}$$

0.6 Problem 6

The T bar consists of two thin rods, OA and BD, each of length $L = 1.88$ m and mass $m = 10$ kg, that are connected to the frictionless pin at O. The rods are welded together at A and lie in the vertical plane. If, at the instant shown, the system is rotating clockwise with angular velocity $\omega_0 = 6.9$ rad/s, determine the force on the pin at O as well as the angular acceleration of the rods.



The force on pin O is ([ ] $\hat{\imath}$ + [ ] $\hat{\jmath}$) N.

The angular acceleration of the rods is ([ ] rad/s^2) $\hat{k}$.

Resolving forces in x direction, where F_x, F_y are forces in hinge

$$F_x = -m\left(\frac{3}{2}L\right)\omega^2 \tag{1}$$

In y direction

$$F_y - 2mg = m\left(\frac{3}{2}L\right)\alpha \tag{2}$$

Taking moments about the hinge O

$$\left(-mg\frac{L}{2} - mgL\right) = \left(\left(m\frac{L^2}{3}\right) + \left(\frac{1}{12}mL^2 + mL^2\right)\right)\alpha \tag{3}$$

Solving (2,3) for F_y, α gives

$$F_y = 40.394 \text{ N}$$
$$\alpha = -5.52503 \text{ rad/sec}^2$$

We are given $\omega = 6.9$ rad/sec., hence from (1)

$$F_x = -1342.6 \text{ N}$$