HW 10,ME 240 Dynamics, Fall 2017

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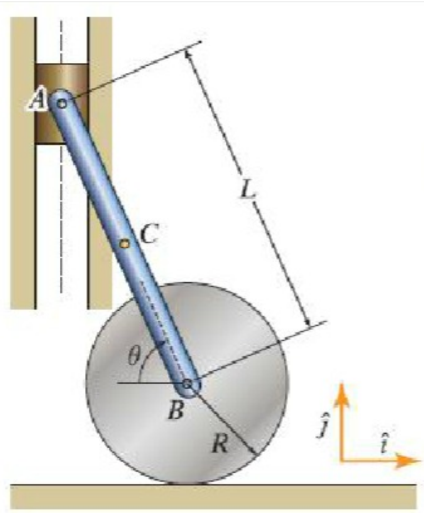
0.1 Problem 1

The system shown consists of a wheel of radius R = 5 in. rolling on a horizontal surface. A bar AB of length R = 33 in. is pin−connected to the center of the wheel and to a slider A that is constrained to move along a vertical guide. Poin$C$ is the bar's midpoint. If the wheel rolls without slip with a constant counterclockwise angular velocity of 15 rad/s, determine the velocity of the slider$A$ when θ = 48°.



$\vec{v}_A$ = ☐ $\hat{j}$ ft/s

Since the wheel rolls without slip with angular velocity $\omega_{disk} = 15$ rad/sec and its radius is $r = \frac{5}{12}$ ft, then the center of the wheel moves to the left (since disk is rolling with counter clock wise) with velocity

$$\begin{aligned} V_B &= r\omega_{disk} \\ &= \left(\frac{5}{12}\right)(15) \\ &= 6.25 \text{ ft/sec} \end{aligned}$$

In vector format

$$\vec{V}_B = -6.25\hat{\imath} + 0\hat{\jmath}$$

For the point A

$$\begin{aligned} \vec{V}_A &= \vec{V}_B + \vec{\omega}_{AB} \times \vec{r}_{A/B} \\ &= \left(-6.25\hat{\imath} + 0\hat{\jmath}\right) + \omega_{AB}\hat{k} \times \left(-L\cos\theta\hat{\imath} + L\sin\theta\hat{\jmath}\right) \\ &= -6.25\hat{\imath} - \omega_{AB}L\cos\theta\hat{\jmath} - \omega_{AB}L\sin\theta\hat{\imath} \\ &= \hat{\imath}\left(-6.25 - \omega_{AB}L\sin\theta\right) + \hat{\jmath}\left(-\omega_{AB}L\cos\theta\right) \end{aligned} \tag{1}$$

Since point A can only move in vertical direction, then its $\hat{\imath}$ component above must be zero. Therefore

$$\begin{aligned} -6.25 - \omega_{AB}L\sin\theta &= 0 \\ \omega_{AB} &= \frac{-6.25}{L\sin\theta} \end{aligned}$$

Numerically $\omega_{AB} = \frac{-6.25}{\left(\frac{33}{12}\right)\sin\left(48\left(\frac{\pi}{180}\right)\right)} = -3.058$ rad/sec.

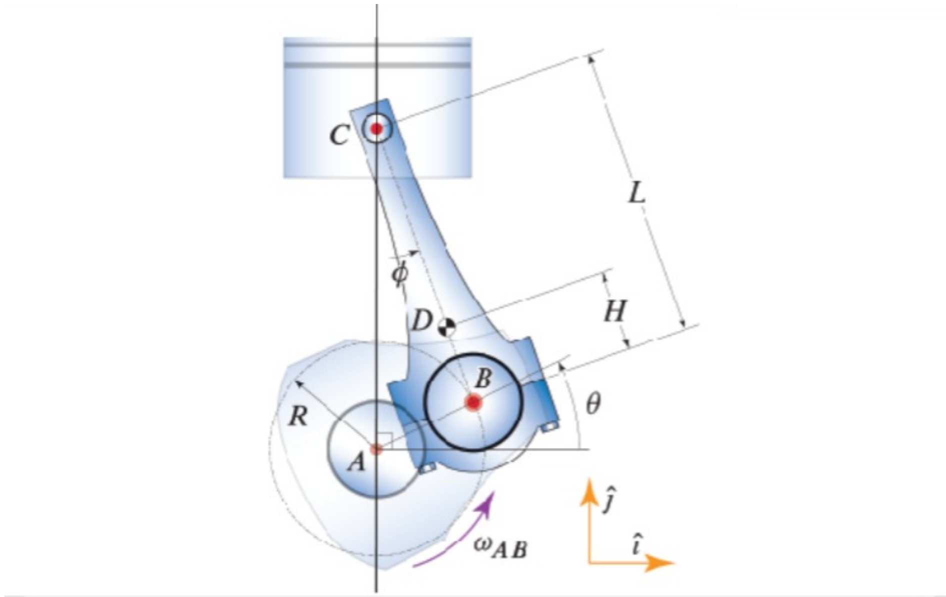
Now from (1) we find $\vec{V}_A$ since now we know ω_{AB}

$$\begin{aligned} \vec{V}_A &= \hat{\jmath}\left(-\omega_{AB}L\cos\theta\right) \\ &= \hat{\jmath}\left(\frac{6.25}{L\sin\theta}L\cos\theta\right) \\ &= \hat{\jmath}\left(\frac{6.25}{\tan\theta}\right) \end{aligned}$$

Since $\theta = 48^0$ then the above becomes

$$\begin{aligned} \vec{V}_A &= \frac{6.25}{\tan\left(48\left(\frac{\pi}{180}\right)\right)}\hat{\jmath} \\ &= 5.627525\hat{\jmath} \\ &= 5.628\hat{\jmath} \text{ ft/sec} \end{aligned}$$

0.2 Problem 2

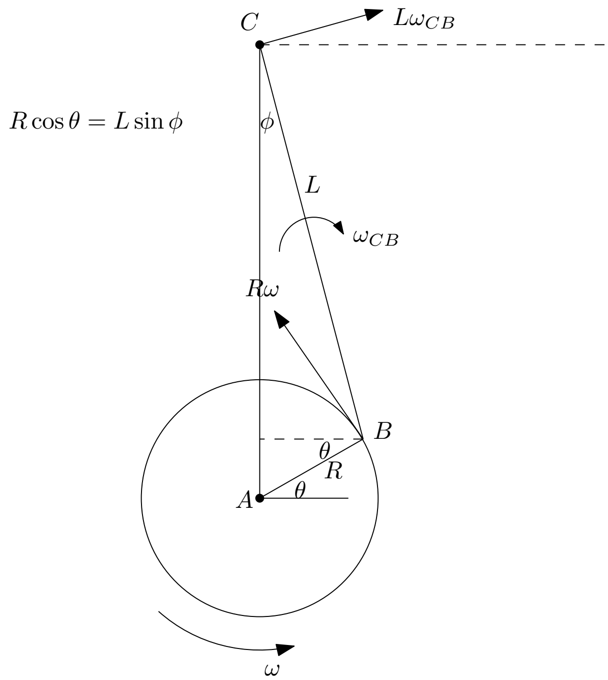


For the slider–crank mechanism shown, let R = 2.3 in., L = 5.3 in., and H = 1.5 in. Also, at the instant shown, let θ = 26° and ω_{AB} = 4,890 rpm.

Determine the velocity of the piston at the instant shown.

v_C = [ ] $\hat{j}$ ft/s

The first step is to find the vector velocities of point B and C and then resolve them along the x, y directions as follows



step 1: Find the vector velocites

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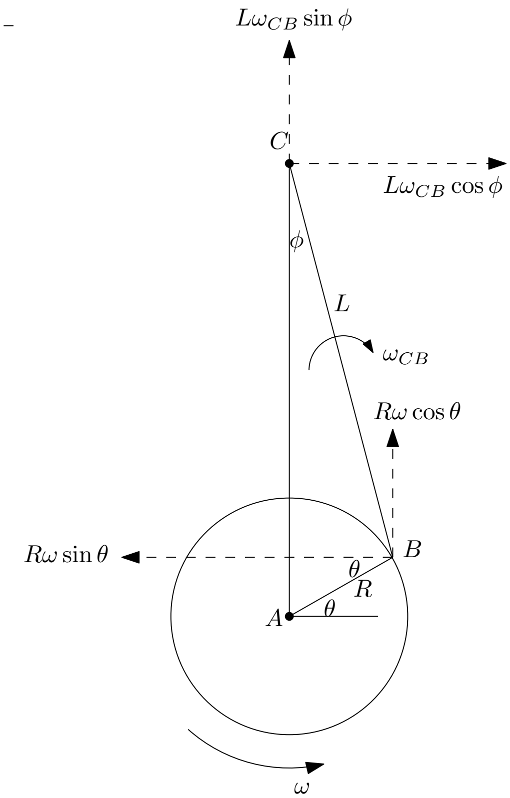


step 2: Resolve along x and y directions

Now we look at point C. We see that its x component of the velocity is

$$Vc_x = L\omega_{CB}\cos\phi - R\omega\sin\theta$$

This is just read from the diagram. In other words, the x component of the velocity of B is added. Since C can only move in the vertical direction, then $Vc_x = 0$. We use this to solve for ω_{CB}

$$\omega_{CB} = \frac{R\omega\sin\theta}{L\cos\phi} \tag{1}$$

Everything on the right above is known. We find ϕ using $R\cos\theta = L\sin\phi$, hence

$$\begin{aligned}\phi &= \arcsin\left(\frac{R\cos\theta}{L}\right)\\ &= \arcsin\left(\frac{(2.3)\cos\left(26\frac{\pi}{180}\right)}{5.3}\right)\\ &= 22.957^0\end{aligned}$$

And $\omega = 4890\left(\frac{2\pi}{60}\right) = 512.0796$ rad/sec. Hence from (1)

$$\omega_{CB} = \frac{(2.3)(512.0796)\sin\left(26\frac{\pi}{180}\right)}{(5.3)\cos\left(22.957\left(\frac{\pi}{180}\right)\right)}$$
$$= 105.7955 \text{ rad/sec}$$

In vector form

$$\vec{\omega}_{CB} = 105.7955\hat{k}$$

From the diagram, we see that the vertical component of the velocity of point C is

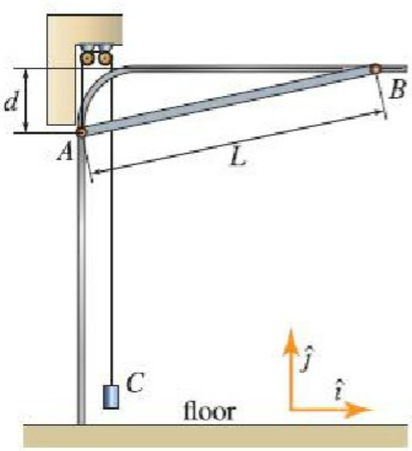
$$\begin{aligned} Vc_y &= L\omega_{CB}\sin\phi + R\omega\cos\theta \\ &= (5.3)(105.7955)\sin\left(22.957\left(\frac{\pi}{180}\right)\right) + (2.3)(512.0796)\cos\left(26\frac{\pi}{180}\right) \\ &= 1277.286 \text{ in/sec} \\ &= 106.441 \text{ ft/sec} \end{aligned}$$

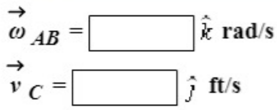
In vector form

$$\vec{V}c = 106.441\hat{j}$$

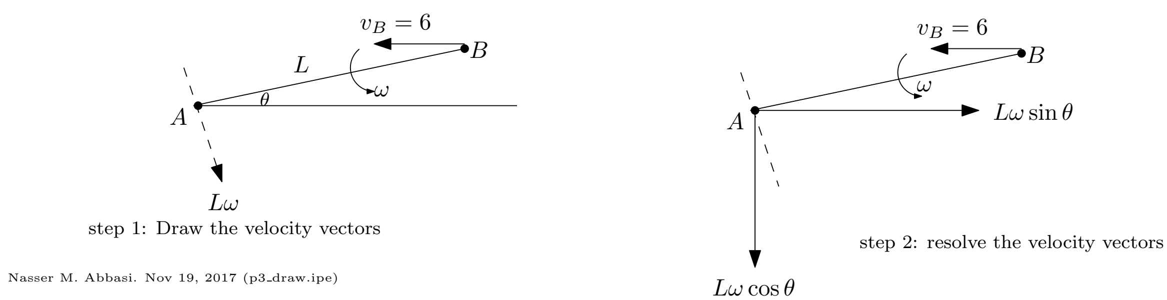
0.3 Problem 3

At the instant shown, an overhead garage door is being shut with point B moving to the left within the horizontal part of the door guide at a speed of 6 ft/s, while point is moving vertically downward. Determine the angular velocity of the door and the velocity of the counterweight C at this instant if $L = 6$ ft and $d = 1.8$ ft.





The first step is to find the vector velocities of point A and B and then resolve them along the x, y directions as follows



Point A will have velocity in x direction of

$$V_{A,x} = L\omega \sin\theta - V_{Bx}$$

But $\sin\theta = \frac{d}{L} = \frac{1.8}{6} = 0.3$, hence $\theta = \arcsin(0.3) = 17.458^0$. Since A can only move in vertical direction, then the above is zero. We use this to find ω

$$\begin{aligned} L\omega \sin\theta - V_{Bx} &= 0 \\ \omega &= \frac{V_{Bx}}{L\sin\theta} \\ &= \frac{6}{6\sin\left(17.458\left(\frac{\pi}{180}\right)\right)} \\ &= 3.333 \text{ rad/sec} \end{aligned}$$

In vector format $\vec{\omega} = 3.333\hat{k}$ rad/sec. Hence the velocity of A in vertical direction is

$$\begin{aligned} V_{Ay} &= -L\omega\cos\theta \\ &= -6\,(3.333)\cos 17.458\left(\frac{\pi}{180}\right) \\ &= -19.076\,83 \text{ ft/sec} \end{aligned}$$

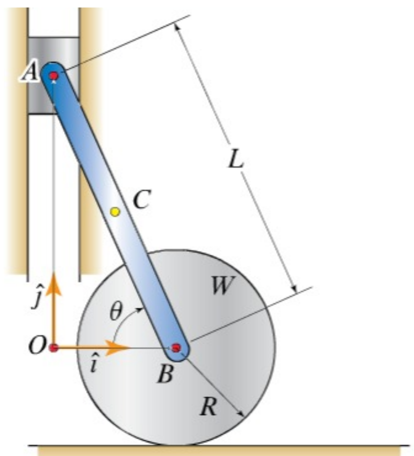
In vector format

$$\vec{V}_A = -19.077\hat{\jmath}$$

This is the same velocity as weight C but C will be going up. Hence

$$\vec{V}_C = 19.077\hat{\jmath}$$

0.4 Problem 4

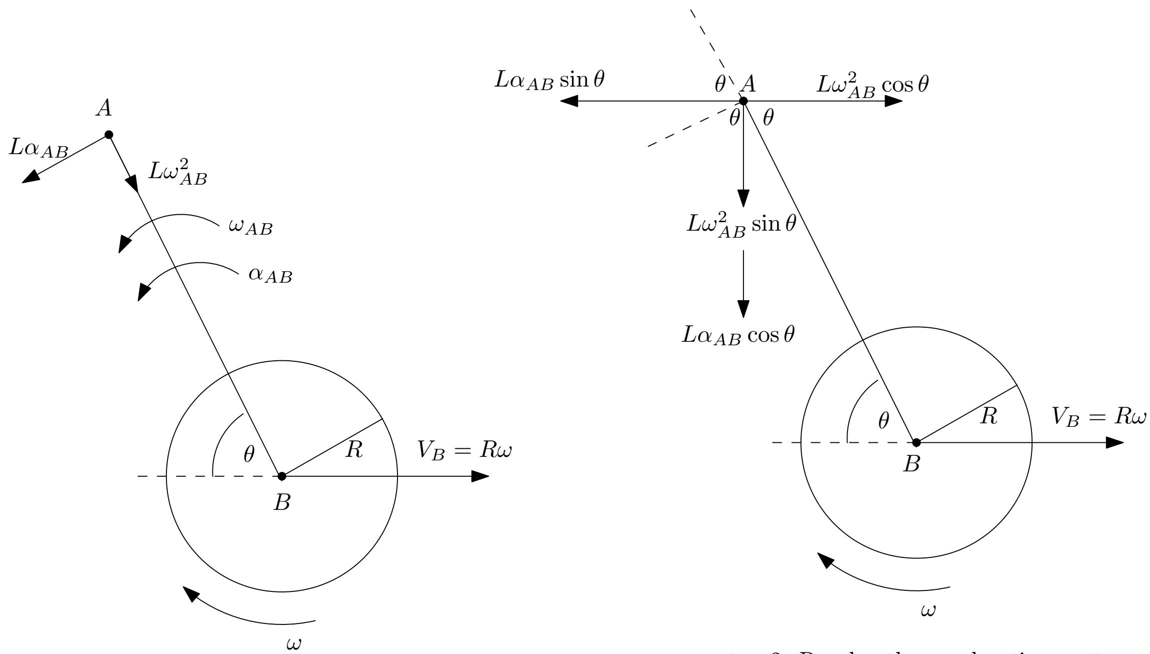


The system shown consists of a wheel of radius R = 1.58 m rolling without slip on a horizontal surface. A bar, AB, of length L = 3.43 m is pin−connected to the center of the wheel and to a slider, A, constrained to move along a vertical guide. Point C is the bar's midpoint.

If the wheel is rolling clockwise with a constant angular speed of 2.1 rad/s, determine the angular acceleration of the bar when θ = 69°.

$\vec{\alpha}$ = [ ] $\frac{\text{rad}}{\text{s}^2}$

The first step is to find the acceleration vectors of point A and B and then resolve them along the x, y directions as follows



step 1: Find the acceleration vectors

step 2: Resolve the acceleration vectors

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step 2: Resolve along x and y directions

To find ω_{AB} we need to resolve velocity vectors and set the x component of the velocity of A to zero to solve for ω_{AB}. If we do that as before, we get

$$V_{B_x} - L\omega_{AB}\sin\theta = 0 \tag{1}$$

The above is just the x component of $\vec{V}_A$. We know V_B which is velocity of center of wheel. It is

$$\begin{aligned} V_{B_x} &= R\omega_{disk} \\ &= 1.58\,(2.1) \\ &= 3.318 \text{ m/s} \end{aligned}$$

And to the right. Hence $\vec{V}_B = 3.318\hat{\imath}$. Now we use (1) to solve for ω_{AB}

$$\begin{aligned} \omega_{AB} &= \frac{V_{Bx}}{L\sin\theta} = \frac{3.318}{(3.43)\sin\left(69\frac{\pi}{180}\right)} \\ &= 1.0362 \text{ rad/sec} \end{aligned}$$

Hence $\vec{\omega}_{AB} = 1.0362\hat{k}$. Now we have all the information to solve for α_{AB}. The x component of $\vec{a}_A$ is zero, since A does not move in x direction. Hence from the figure, we see that

$$L\omega_{AB}^2\cos\theta - L\alpha_{AB}\sin\theta = 0$$

There is no acceleration to transfer from point B since B is not accelerating. Solving the above for α_{AB} gives

$$\begin{aligned}
\alpha_{AB} &= \frac{L\omega_{AB}^2 \cos\theta}{L\sin\theta} \\
&= \frac{\omega_{AB}^2}{\tan\theta} \\
&= \frac{1.0362^2}{\tan\left(69\frac{\pi}{180}\right)} \\
&= 0.41216 \text{ rad/sec}^2
\end{aligned}$$

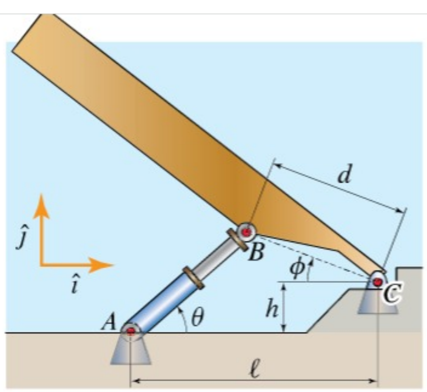
In vector format $\vec{\alpha}_{AB} = 0.41216\hat{k}$. Hence the vertical component of the acceleration $\vec{a}_A$ is (from the diagram)

$$\begin{aligned}
a_{Ay} &= -L\omega_{AB}^2 \sin\theta - L\alpha_{AB}\cos\theta \\
&= -(3.43)\left(1.0362^2\right)\sin\left(69\frac{\pi}{180}\right) - (3.43)(0.41216)\cos\left(69\frac{\pi}{180}\right) \\
&= -3.945 \text{ m/s}^2
\end{aligned}$$

In vector format

$$\vec{a}_A = 0\hat{\imath} - 3.945\hat{\jmath}$$

0.5 Problem 5



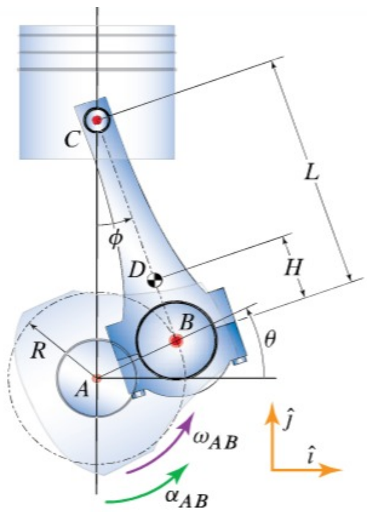
A flood gate is controlled via the hydraulic cylinder AB. If the length of the cylinder is increased with a constant time rate of 2.4 ft/s, determine the angular acceleration of the gate when $\varphi = 0°$. Let $\ell = 10.0$ ft, $h = 2.2$ ft, and $d = 4.7$ ft.

$\vec{\alpha}_{gate}$ = [ ] [ ] rad/s^2

$$\vec{\alpha}_{BC} = 7.507\hat{k} \text{ rad/sec}^2$$

Need to type the solution. This uses constraints method.

0.6 Problem 6



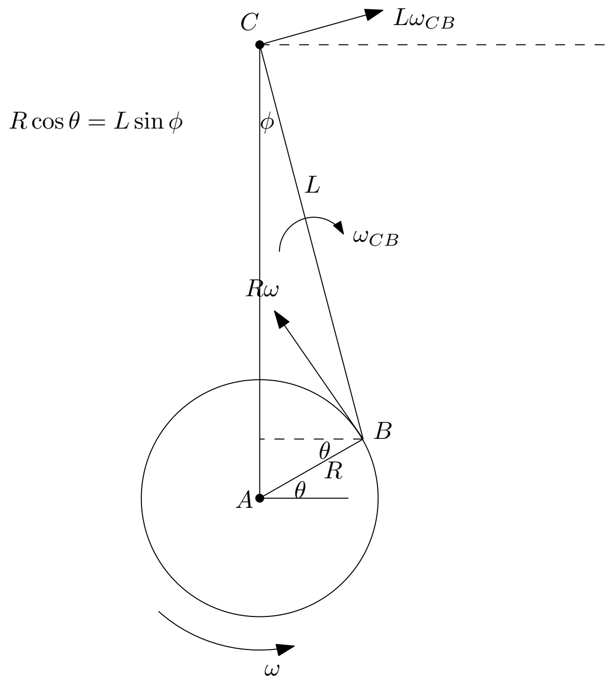
For the slider–crank mechanism shown above, let $R = 2.1$ in., $L = 5.8$ in., and $H = 1.3$ in. Assuming that $\omega_{AB} = 5{,}030$ rpm and is constant, determine the angular acceleration of the

connecting rod, BC, and the acceleration of point C at the instant when $\theta = 28°$.

$\vec{a}_C$ = [ ] [ ] ft/s^2

$\vec{\alpha}_{BC}$ = [ ] [ ] rad/s^2

We need to first find ω_{BC}. This follows similar approach to problem 2. The first step is to find the vector velocities of point B and C and then resolve them along the x, y directions as follows



step 1: Find the vector velocites

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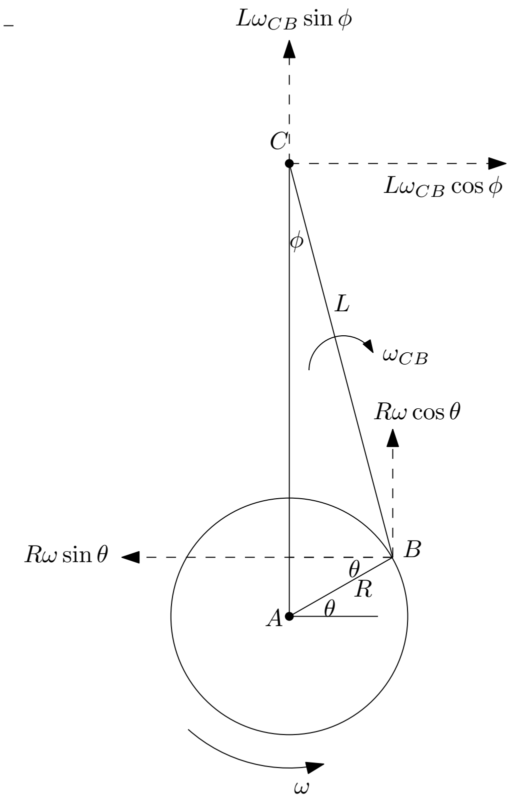


step 2: Resolve along x and y directions

Now we look at point C. We see that its x component of the velocity is

$$Vc_x = L\omega_{CB}\cos\phi - R\omega\sin\theta$$

This is just read from the diagram. In other words, the x component of the velocity of B is added. Since C can only move in the vertical direction, then $Vc_x = 0$. We use this to solve for ω_{CB}

$$\omega_{CB} = \frac{R\omega\sin\theta}{L\cos\phi} \tag{1}$$

Everything on the right above is known. We find ϕ using $R\cos\theta = L\sin\phi$, hence

$$\begin{aligned}
\phi &= \arcsin\left(\frac{R\cos\theta}{L}\right)\\
&= \arcsin\left(\frac{(2.1)\cos\left(28\frac{\pi}{180}\right)}{5.8}\right)\\
&= 0.325\,4 \text{ radians}\\
&= 18.6441^0
\end{aligned}$$

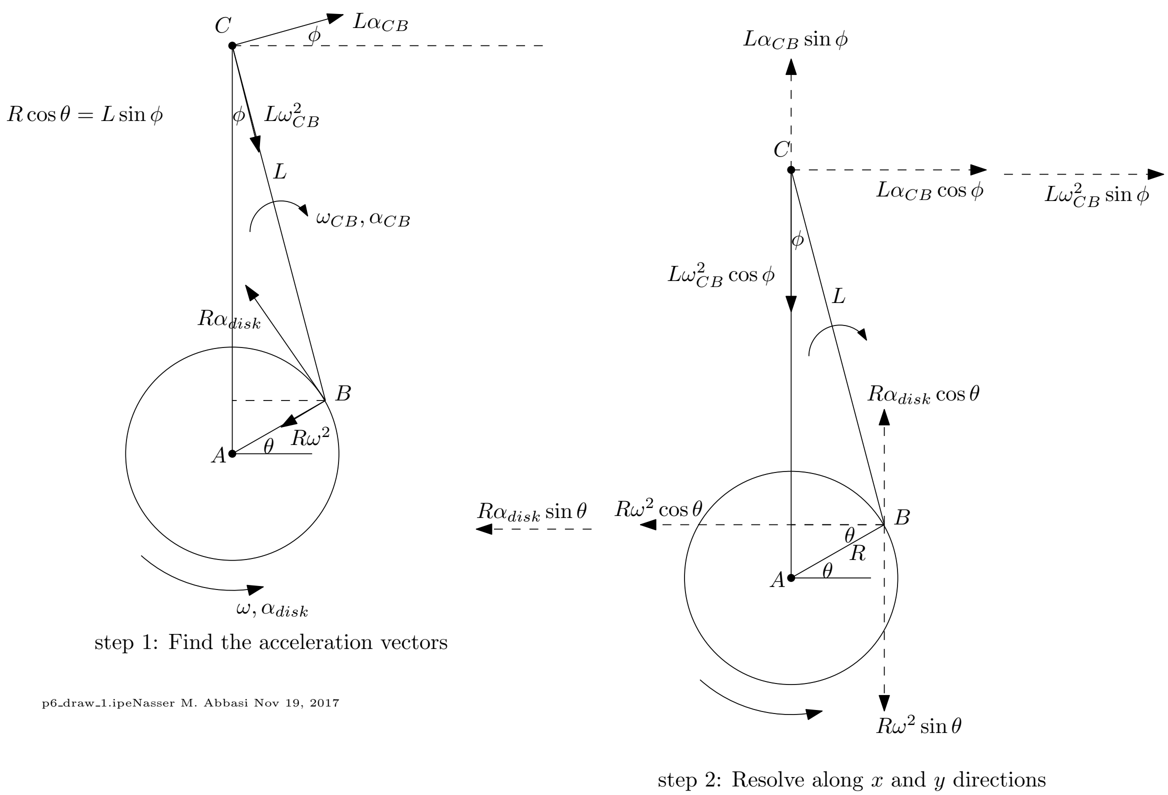
And $\omega = 5030\left(\frac{2\pi}{60}\right) = 526.740\,4$ rad/sec. Hence from (1)

$$\begin{aligned}\omega_{CB} &= \frac{(2.1)(526.740\,4)\sin\left(28\frac{\pi}{180}\right)}{(5.8)\cos(0.325\,4)}\\ &= 94.495 \text{ rad/sec}\end{aligned}$$

In vector form

$$\vec{\omega}_{CB} = 94.495\hat{k}$$

Now we draw the acceleration vectors and resolve them



The x component of the acceleration of point C is zero. Hence from the diagram

$$L\alpha_{CB}\cos\phi + L\omega_{CB}^2\sin\phi - R\alpha_{disk}\sin\theta - R\omega^2\cos\theta = 0$$

Solving for α_{CB}

$$\alpha_{CB} = \frac{R\alpha_{disk}\sin\theta + R\omega^2\cos\theta - L\omega_{CB}^2\sin\phi}{L\cos\phi}$$

Since $\alpha_{disk} = 0$ since we are told ω is constant, then the above simplifies to

$$\alpha_{CB} = \frac{R\omega^2\cos\theta - L\omega_{CB}^2\sin\phi}{L\cos\phi}$$

Using numerical values gives

$$\begin{aligned}\alpha_{CB} &= \frac{(2.1)\,(526.740\,4)^2\cos\left(28\frac{\pi}{180}\right)-(5.8)\,(94.494\,71)^2\sin(0.325\,4)}{(5.8)\cos(0.325\,4)}\\ &= 90598.94 \text{ rad/sec}^2\end{aligned}$$

In vector form

$$\vec{\alpha}_{CB} = -90598.94\hat{k}$$

The acceleration of point C is only in vertical direction. From diagram

$$\begin{aligned}a_{C,y} &= L\alpha_{CB}\sin\phi - L\omega_{CB}^2\cos\phi - R\omega^2\sin\theta\\ &= (5.8)\,(90598.95)\sin(0.325\,4) - (5.8)\,(94.495)^2\cos(0.325\,4) - (2.1)\,(526.740\,4)^2\sin\left(28\frac{\pi}{180}\right)\\ &= -154624.9 \text{ in/sec}^2\\ &= -12885.41 \text{ ft/sec}^2\end{aligned}$$

Hence in vector form

$$\vec{a}_C = -12885.37\hat{\jmath} \text{ ft/sec}^2$$