HW4 ECE 332 Feedback Control

Fall 2015

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November 28, 2019

Contents

0.1 Problem 1 3
0.1.1 Part (a) 3
0.1.2 Part(b) 5
0.1.3 Part (c) 7

List of Figures

List of Tables

0.1 Problem 1

Problem 2: Consider the block diagram set-up for the disturbance attenuation problem formulated in lecture with

$$G_1(s) = \frac{s+1}{s^2+10s+100}$$

and

$$G_2(s) = \frac{1}{s+2}.$$

(a) Design control blocks $H_1(s)$ and $H_2(s)$ such that the following two specifications are satisfied: First,

$$|\frac{Y(j\omega)}{N(j\omega)}| \leq 0.01$$

for all frequencies $\omega \geq 0$. Second, the output $y(t)$ should respond to command to $r(t)$ in approximately the same manner in the closed loop as in the open loop; i.e. for the closed loop, we desire

$$Y(s) \approx G_1(s)G_2(s)R(s).$$

Note: In class, we did not fully solve for H_1; i.e., we never found the constant α. In this homework, a specific solution is sought.

(b) To make your solution "proper," introduce a second order lowpass filter as appropriate and solve for the filter parameter ϵ.

(c) For the compensated system resulting from (b), generate a frequency response plot for the closed loop transfer function $|Y(j\omega)/R(j\omega)|$ and compare it to the target transfer function $|G_(j\omega)G_2(j\omega)|$. Plot the error between these two frequency responses as a function of the frequency $\omega \geq 0$.

SOLUTION:

0.1.1 Part (a)

The second condition which says that the closed loops should approximate the open loop response, implies that we should use $H_2(s) = \frac{1}{G_1G_2}$, i.e. to apply the inversion. This is because $\frac{Y(s)}{R(s)} = \left.\frac{H_1G_1G_2}{1+H_1G_1G_2H_2}\right|_{N=0}$ and this becomes $\frac{Y(s)}{R(s)} \approx G_1G_2$ when we set $H_2 = \frac{1}{G_1G_2}$ and also by making $H_1 = \alpha$ where α is a large gain. So now we just need to worry about finding α s.t. $\left|\frac{Y(j\omega)}{N(j\omega)}\right| \leq 0.01$ for all $\omega > 0$.

We know that $\frac{Y(s)}{N(s)} = \frac{G_2}{1+G_1G_2H_2H_1}$, but since we are using the inversion, this reduces to

$$\frac{Y(s)}{N(s)} = \frac{G_2}{1+H_1}$$

By setting $H_1(s) = \alpha$ and using $G_2 = \frac{1}{s+2}$ and moving to the frequency domain, the above becomes

$$\frac{Y(j\omega)}{N(j\omega)} = \frac{\frac{1}{j\omega+2}}{1+\alpha} = \frac{1}{(j\omega+2)(1+\alpha)} = \frac{1}{(1+\alpha)j\omega + 2(1+\alpha)}$$

Taking the magnitude

$$\left|\frac{Y(j\omega)}{N(j\omega)}\right| = \frac{1}{\sqrt{(1+\alpha)^2\omega^2 + 4(1+\alpha)^2}}$$

We want the above to be smaller than 0.01 for all ω, which implies

$$\begin{aligned}
\frac{1}{\sqrt{(1+\alpha)^2\omega^2 + 4(1+\alpha)^2}} &\leq 0.01 \\
\frac{1}{(1+\alpha)^2\omega^2 + 4(1+\alpha)^2} &\leq 0.01^2 \\
(1+\alpha)^2\omega^2 + 4(1+\alpha)^2 &\geq 10000 \\
\omega^2 &\geq \frac{10000 - 4(1+\alpha)^2}{(1+\alpha)^2} \\
\omega^2 &\geq \frac{10000}{(1+\alpha)^2} - 4 \\
\omega &\geq \sqrt{\frac{10000}{(1+\alpha)^2} - 4}
\end{aligned}$$

The smallest α to allow the above is when $\omega = 0$, hence we need to solve for α from

$$\begin{aligned}
\sqrt{\frac{10000}{(1+\alpha)^2} - 4} &= 0 \\
\frac{10000}{(1+\alpha)^2} - 4 &= 0 \\
\frac{1}{(1+\alpha)^2} &= \frac{4}{10000} \\
(1+\alpha)^2 &= \frac{10000}{4} = 2500 \\
1+\alpha &= 50
\end{aligned}$$

Hence

$$\alpha \geq 49$$

Therefore $H_1(s) = \alpha$ where $\alpha \geq 49$ and $H_1(s) = \frac{1}{G_1(s)G_2(s)}$. This complete this part.

0.1.2 Part(b)

One problem with the above inversion method for finding $H_2(s) = \frac{1}{G_1 G_2}$ is that $H_2(s)$ becomes improper:

$$\begin{aligned} H_2(s) &= \frac{1}{G_1 G_2} = \frac{1}{\frac{s+1}{s^2+10s+100}\frac{1}{s+2}} \\ &= \frac{\left(s^2+10s+100\right)(s+2)}{s+1} \\ &= \frac{s^3+12s^2+120s+200}{s+1} \end{aligned}$$

$H_2(s)$ is improper, since the numerator has a degree larger than the denominator. This introduces differentiator in the feedback loop which is something we do not like to have.

We will now replace $H_2 = \frac{1}{G_1 G_2}$ by $\left(\frac{1}{G_1 G_2}\right) H_{LP}(s)$ where $H_{LP}(s) = \frac{1}{(\varepsilon s+1)^k}$ is a low pass filter where k is an integer and ε is some parameter, both are positive. The goal is to block high frequency noise content and also make $\left(\frac{1}{G_1 G_2}\right) H_{LP}(s)$ become a proper transfer function. We also want to make sure $\left|\frac{Y(j\omega)}{N(j\omega)}\right|$ remain less than 0.01.

Let

$$\begin{aligned} H_2(s) &= \frac{1}{G_1 G_2}\frac{1}{(\varepsilon s+1)^k} \\ &= \frac{s^3+12s^2+120s+200}{(s+1)}\frac{1}{(\varepsilon s+1)^k} \end{aligned}$$

The degree of the numerator is 3. So we want k to be at least 2 (it can be more), so that the denominator has at least degree 3 as well. If we want strict proper, then we make $k = 3$. Using $k = 2$ we now have

$$H_2(s) = \frac{1}{G_1 G_2}\frac{1}{(\varepsilon s+1)^2}$$

Therefore, $\frac{Y(j\omega)}{N(j\omega)}$ now becomes

$$\begin{aligned} \frac{Y(s)}{N(s)} &= \frac{G_2}{1+G_1 G_2 H_1\left(\frac{1}{G_1 G_2}\frac{1}{(\varepsilon s+1)^2}\right)} \\ &= \frac{\frac{1}{s+2}}{1+\frac{\alpha}{(\varepsilon s+1)^2}} \end{aligned} \tag{1}$$

Where we used α for H_1. We now move to the frequency domain and take the magnitude in order to solve for ε. We will use the same α found in part (1), otherwise, there will be two free parameters to adjust at the same time, which would make this a hard problem, and the problem seems to indicate we are to use same α value found in part (1) although it did not say that explicitly. Therefore (1) becomes (using $\alpha = 49$)

$$\frac{Y(s)}{N(s)} = \frac{\frac{(\varepsilon s+1)^2}{s+2}}{(\varepsilon s + 1)^2 + 49}$$

Hence

$$\left|\frac{Y(j\omega)}{N(j\omega)}\right| = \frac{\left|\frac{(\varepsilon j\omega+1)^2}{j\omega+2}\right|}{\left|(\varepsilon j\omega + 1)^2 + 49\right|}$$

$$= \frac{\frac{\varepsilon^2\omega^2+1}{\sqrt{\omega^2+4}}}{\left|-\varepsilon^2\omega^2 + 1 + 2\varepsilon j\omega + 49\right|}$$

$$= \frac{\frac{\varepsilon^2\omega^2+1}{\sqrt{\omega^2+4}}}{\sqrt{4\varepsilon^2\omega^2 + \left(50 - \varepsilon^2\omega^2\right)^2}}$$

Hence

$$\left|\frac{Y(j\omega)}{N(j\omega)}\right|^2 = \frac{\left(\varepsilon^2\omega^2 + 1\right)^2}{\left(\omega^2 + 4\right)\left(4\varepsilon^2\omega^2 + \left(50 - \varepsilon^2\omega^2\right)^2\right)}$$

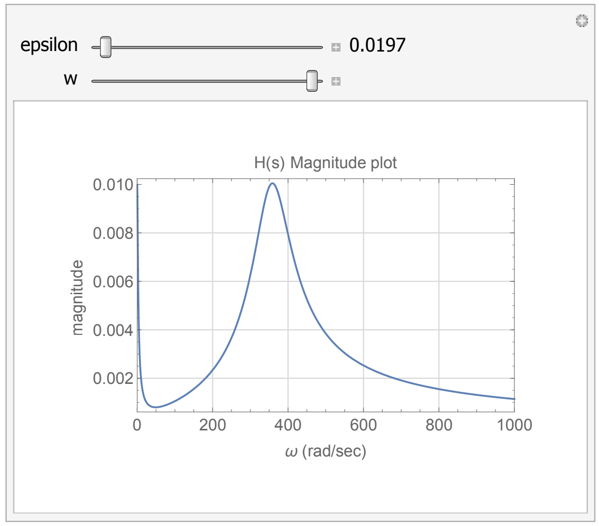
We now find ω where $\left|\frac{Y(j\omega)}{N(j\omega)}\right|$ is maximum, which is the same as where the above is maximum. The above is maximum when the denominator is minimum. Hence

$$\frac{d}{d\omega}\left(\omega^2 + 4\right)\left(4\varepsilon^2\omega^2 + \left(50 - \varepsilon^2\omega^2\right)^2\right) = 0$$

Solving for ω from the above using computer algebra (the algebra is too complicated to do by hand. May be there is a short cut) in terms of ε, and plugging the solution $\omega_{\max}$ back to $\left|\frac{Y(j\omega)}{N(j\omega)}\right|$ and setting the result to 0.01 and solving numerically for ε that satisfy the equation gives

$$\varepsilon = 0.0197$$

To verify this, a small demo was made to plot $\left|\frac{Y(j\omega)}{N(j\omega)}\right|$ for different ε values. The following plot shows $\left|\frac{Y(j\omega)}{N(j\omega)}\right|$ using $k = 2, \varepsilon = 0.0197$ and the maximum magnitude was checked to be just less than 0.01



0.1.3 Part (c)

We will now use

$$H_1 = 49$$
$$H_2 = \frac{1}{G_1G_2}\frac{1}{(0.0197s+1)^2}$$

And plot $\left|\frac{Y(s)}{R(s)}\right| = \left|\frac{H_1G_1G_2}{1+H_1G_1G_2H_2}\right|$ against the $|G_1G_2|$ to see how good the choice of H_1 and H_2 are.

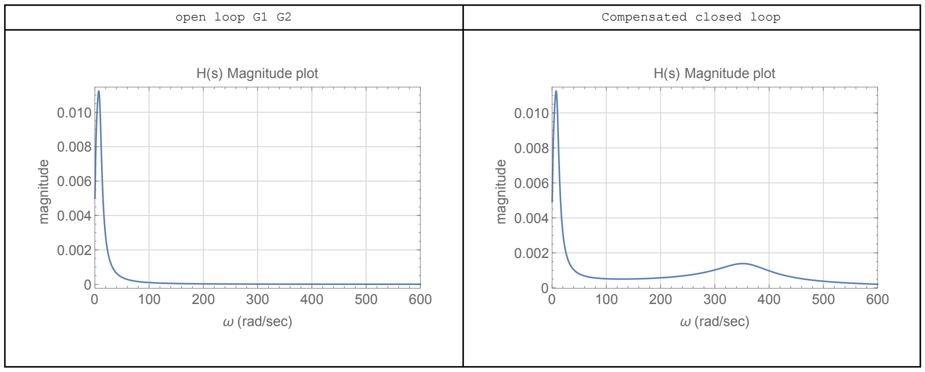
$$\begin{aligned}\frac{Y(s)}{R(s)} &= \frac{H_1G_1G_2}{1+H_1G_1G_2H_2}\\ &= \frac{49G_1G_2}{1+49G_1G_2\frac{1}{G_1G_2}\frac{1}{(0.0197s+1)^2}}\\ &= \frac{49\frac{s+1}{s^2+10s+100}\frac{1}{s+2}}{1+49\frac{1}{(0.0197s+1)^2}}\\ &= \frac{49\frac{s+1}{s^2+10s+100}\frac{1}{s+2}(0.0197s+1)^2}{(0.0197s+1)^2+49}\end{aligned}$$

While

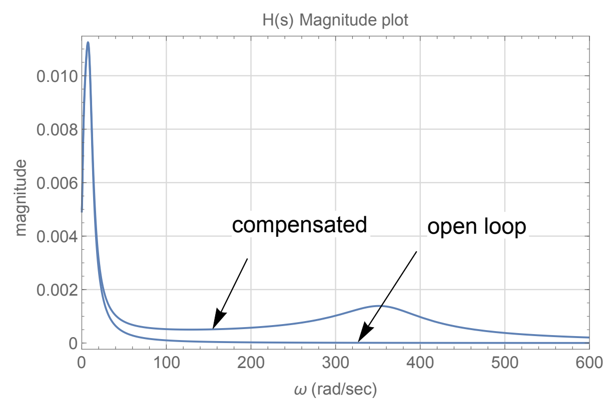
$$G_1G_2 = \frac{s+1}{s^2+10s+100}\frac{1}{s+2}$$

The following plot shows $\left|\frac{Y(s)}{R(s)}\right|$ vs. $|G_1G_2|$ side by side

Out[348]=

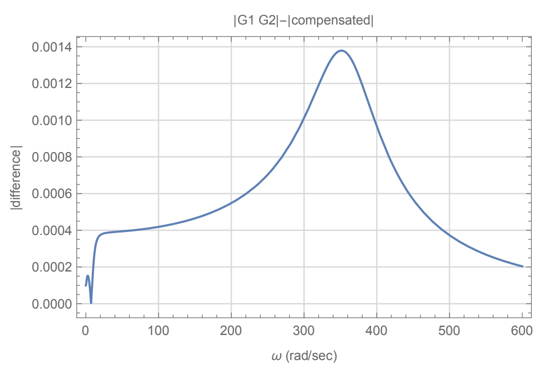


The following plot shows both on the same plot



The following plot show difference between the magnitudes

Out[386]=



Observations:

From the above difference plot, we see that the maximum difference between $|G_1 G_2|$ and the compensated $\left|\frac{H_1 G_1 G_2}{1+H_1 G_1 G_2 H_2}\right|$ occurred at around $\omega = 350$ and had value of about 0.0014. This value seems relatively small, and seems to indicate that H_1 and H_2 used for compensation were a good choice.