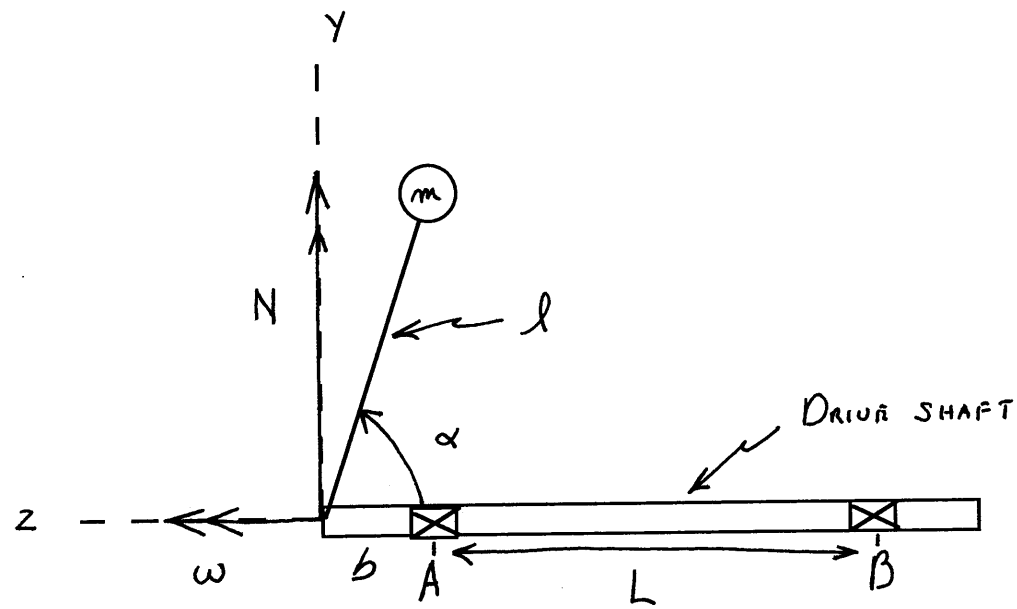
EMA 542

Home Work to be Handed In

6A) A mass is mounted on a rigid weightless rod of length l. The rod is inclined at an angle α with respect to the shaft AB as shown. The shaft spins with a constant angular velocity ω and precesses about a fixed vertical axis with constant angular velocity N. Determine the bearing forces on the shaft at A and B due to the prescribed motion. Neglect the effect of gravity.



EMA 542 Solution to Hwk 6A

ω = const α = const. XYZ rotating with angular velocity $\vec{\omega}_{cs} = N\bar{j}$

$$\vec{a}_p = \vec{a}_o + \vec{\omega}\times(\vec{\omega}\times\vec{\rho}) + \dot{\vec{\omega}}\times\vec{\rho} + 2\vec{\omega}\times\dot{\vec{\rho}}_r + \ddot{\vec{\rho}}_r$$

$$\vec{\rho} = -l\cos\alpha\,\bar{k} + l\sin\alpha\,\bar{j} \qquad \underline{\dot{\vec{\rho}}_r} = -l\sin\alpha\,\omega\,\bar{i}$$

$$\underline{\ddot{\vec{\rho}}_r} = -l\sin\alpha\,\omega^2\,\bar{j} \qquad \vec{\omega}\times\vec{\rho} = N\bar{j}\times(-l\cos\alpha\,\bar{k} + l\sin\alpha\,\bar{j})$$

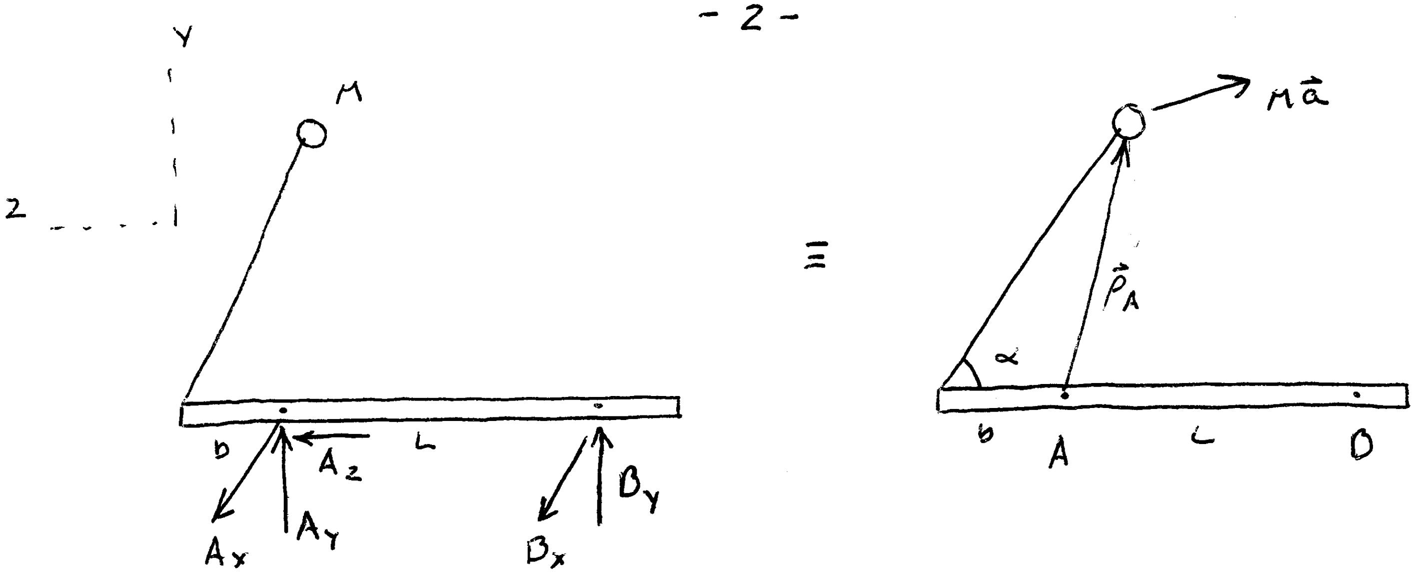
$$\vec{\omega}\times\vec{\rho} = -Nl\cos\alpha\,\bar{i} \qquad \vec{\omega}\times(\vec{\omega}\times\vec{\rho}) = N\bar{j}\times(-Nl\cos\alpha\,\bar{i})$$

$$\underline{\vec{\omega}\times(\vec{\omega}\times\vec{\rho})} = N^2 l\cos\alpha\,\bar{k} \qquad \dot{\vec{\omega}} = 0$$

$$\therefore\ \dot{\vec{\omega}}\times\vec{\rho} = 0 \qquad 2\vec{\omega}\times\dot{\vec{\rho}}_r = 2N\bar{j}\times(-l\sin\alpha\,\omega\,\bar{i})$$

$$\therefore\ \underline{2\vec{\omega}\times\dot{\vec{\rho}}_r} = 2N\omega\,l\sin\alpha\,\bar{k}$$

$$\therefore\ \vec{a} = -l\sin\alpha\,\omega^2\,\bar{j} + (N\cos\alpha + 2\omega\sin\alpha)Nl\,\bar{k}$$



ASSUME ONLY A HAS A THRUST BEARING

$$\Sigma F_x \Rightarrow A_x + B_x = 0 \qquad (1)$$

$$\Sigma F_y \Rightarrow A_y + B_y = -M l \sin\alpha\, \omega^2 \qquad (2)$$

$$\Sigma F_z \Rightarrow A_z = M N l \left(N \cos\alpha + 2 \omega \sin\alpha \right) \qquad (3)$$

$$\Sigma M_A \Rightarrow$$

$$B_y L \bar{i} - B_x L \bar{j} = \vec{\rho}_A \times M\vec{a} \qquad \vec{\rho}_A = l \sin\alpha\, \bar{j} - (l\cos\alpha - b)\, \bar{k}$$

$$\vec{\rho}_A \times M\vec{a} = M\left[l \sin\alpha\, \bar{j} - (l\cos\alpha - b)\, \bar{k} \right] \times \left[-l \sin\alpha\, \omega^2 \bar{j} + N l \left(N\cos\alpha + 2\omega \sin\alpha \right) \bar{k} \right]$$

$$= M N l^2 \sin\alpha \left(N\cos\alpha + 2\omega\sin\alpha \right) \bar{i} - l \sin\alpha\, \omega^2 \left(l\cos\alpha - b \right) \bar{i}$$

$$\Rightarrow \quad \vec{\rho}_A \times M\vec{a} = M\left[N^2 l^2 \sin\alpha\cos\alpha + 2N\omega l^2 \sin^2\alpha - l^2\omega^2 \sin\alpha\cos\alpha + lb\sin\alpha\,\omega^2\right]\bar{i}$$

$$\vec{\rho}_A \times M\vec{a} = Ml\sin\alpha\left[2N\omega l\sin\alpha + l(N^2-\omega^2)\cos\alpha + b\omega^2\right]\bar{i}$$

$$\Sigma M_{Ax} \Rightarrow \quad B_y L = \left[\vec{\rho}\times M\vec{a}\right]_x$$

or
$$B_y = M\frac{l}{L}\sin\alpha\left[2N\omega l\sin\alpha + l(N^2-\omega^2)\cos\alpha + b\omega^2\right] \qquad (4)$$

$$\Sigma M_{Ay} \Rightarrow \quad -B_x L = 0$$

$$\Rightarrow \quad B_x = 0 \qquad (5)$$

$$(1) \Rightarrow \quad A_x = 0$$

$$(2) \Rightarrow \quad A_y = -Ml\sin\alpha\,\omega^2 - \frac{Ml}{L}\sin\alpha\left[2N\omega l\sin\alpha + l(N^2-\omega^2)\cos\alpha + b\omega^2\right]$$

$$\therefore \quad A_x = 0 \qquad\qquad B_x = 0$$

$$A_y = -\frac{Ml}{L}\sin\alpha\left[2N\omega l\sin\alpha + l(N^2-\omega^2)\cos\alpha + (b+L)\omega^2\right]$$

$$A_z = MNl\left[N\cos\alpha + 2\omega\sin\alpha\right] \qquad\qquad B_y = M\frac{l}{L}\sin\alpha\left[2N\omega l\sin\alpha + (N^2-\omega^2)\,l\cos\alpha + b\omega^2\right]$$

EMA 542

Home Work to be Handed In

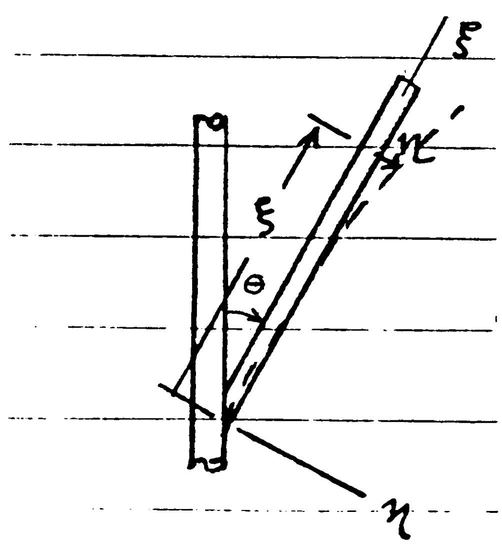
7) Shown below is a simple model of a oil delivery system. The vertical drive shaft spins with a constant angular velocity ω. The oil delivery tube is modeled as a slender flexible beam of length L, total mass m, elastic modulus E, and cross sectional moment of inertia I. For preliminary design purposes you can neglect the effects of the fluid within the tube.

The oiling system must not strike the side of its housing as it rotates, therefore, your boss asks you to determine the following:

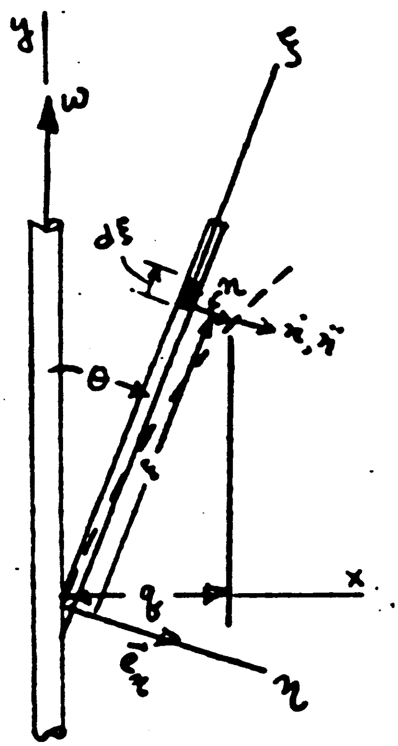
a) The steady state moment, M_ς, at a general distance, ς, from point A along the tube.

b) The steady state deflection, η_s, at the tip of the tube.

Assume for this design iteration that $\eta \cos\theta << \varsigma \sin\theta$



EMA 542 - SOLUTION HWK 7



<u>"Steady State"</u> shape of a rotating rod

1) $q = \xi\sin\theta + \eta\cos\theta$

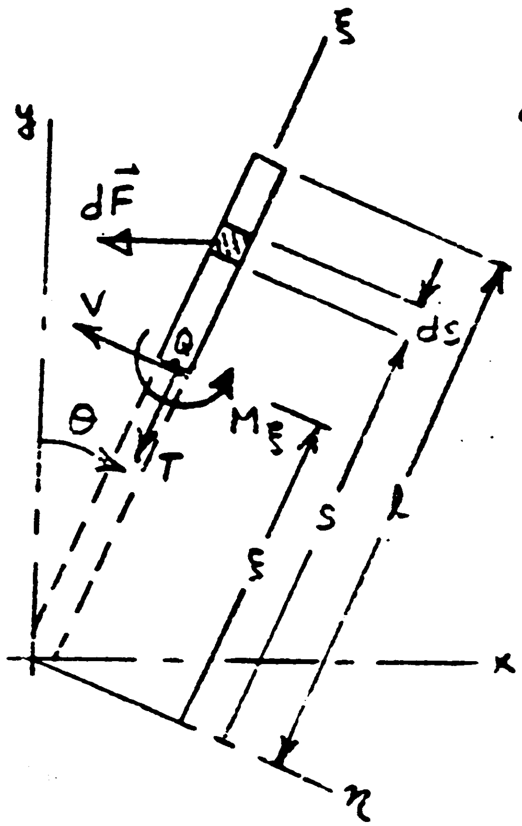
2) $d\vec{F} = \vec{a}\,dm = -q\omega^2\frac{m}{\ell}d\xi\,\vec{i}$ (dynamic load)

or

$d\vec{F} = -\{\xi\sin\theta + \eta\cos\theta\}\omega^2\frac{m}{\ell}d\xi\,\vec{i}$ (dynamic load)

*It is a reasonably good assumption that $\eta\cos\theta << \xi\sin\theta$

Assume $\eta\cos\theta << \xi\sin\theta$



$$d\vec{F} = \vec{a}\,dm = -(s\sin\theta)\omega^2\frac{m}{\ell}ds\,\vec{i}$$

$$\therefore d\vec{M}_Q = |dF|(s-\xi)\cos\theta\,\vec{k}$$

or $d\vec{M}_Q = \frac{m}{\ell}\omega^2\sin\theta\cos\theta\, s(s-\xi)\,ds\,\vec{k}$

$$\therefore \vec{M}_Q = \frac{m}{\ell}\omega^2\sin\theta\cos\theta\int_\xi^\ell s(s-\xi)\,ds\,\vec{k}$$

or $\vec{M}_Q = k_1\int_\xi^\ell s(s-\xi)\,ds\,\vec{k}$

where $k_1 = \frac{m}{l}\omega^2 \sin\theta\cos\theta$

$$\therefore\ \vec{M}_Q = k_1\left[\frac{1}{3}(l^3-\xi^3) - \frac{1}{2}\xi(l^2-\xi^2)\right]\bar{k}$$

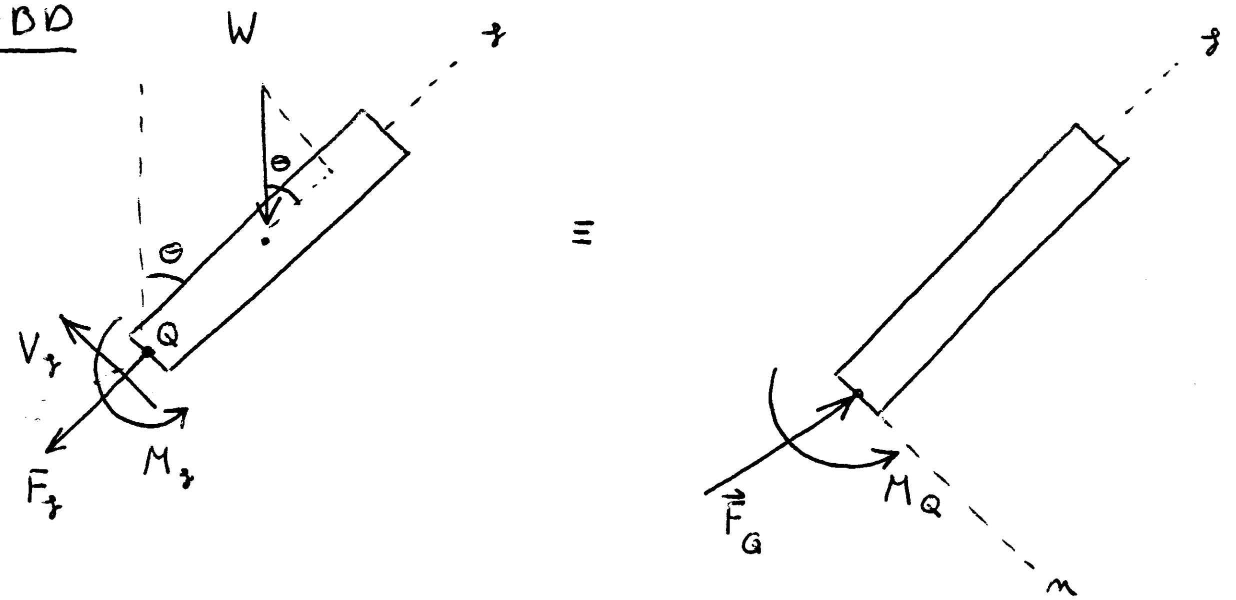
$$= k_1\left[\frac{1}{3}(l-\xi)^3 + \frac{1}{2}\xi(l-\xi)^2\right]\bar{k} = (\text{dynamic moment})_Q$$

NOTE THAT FOR $\xi = 0$:

$$\vec{M}_Q = \vec{M}_A = \frac{1}{3}\omega^2 M l^2 \sin\theta\cos\theta\ \bar{k}$$

THE DYNAMIC MOMENT FOR THE ENTIRE BEAM FROM PREVIOUS ANALYSIS

FDD



$$W = \frac{M}{l}g(l-\xi)$$

$$\therefore \quad M_\xi - \frac{M}{l} g (l-\xi)\left(\frac{l-\xi}{2}\right) \sin\theta = M_Q$$

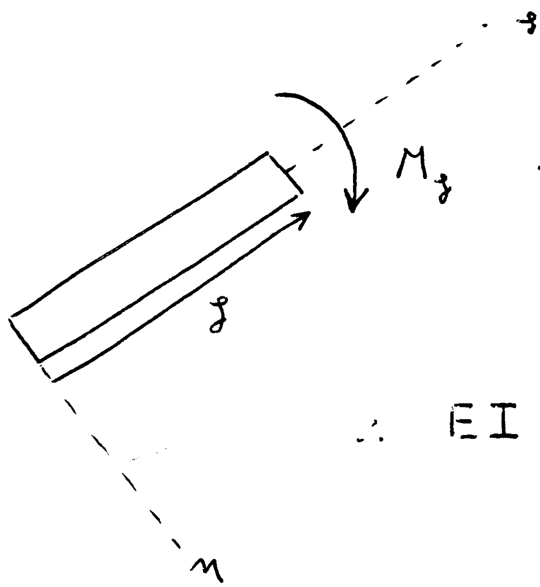
$$\Rightarrow \quad M_\xi = k_1 \left[\tfrac{1}{3}(l-\xi)^3 + \tfrac{1}{2}\xi(l-\xi)^2 \right] + \frac{Mg}{2l}\sin\theta\,(l-\xi)^2$$

OR $$M_\xi = k_2 (l-\xi)^2 + k_1 \left[\tfrac{1}{3}(l-\xi)^3 + \tfrac{1}{2}\xi(l-\xi)^2 \right]$$

WHERE: $k_2 = \dfrac{Mg}{2l}\sin\theta \qquad k_1 = \dfrac{M}{l}\omega^2 \sin\theta\cos\theta$

NOTE: $M_\xi = 0$ @ $\xi = l$ END OF BEAM

LOOK AT OTHER PIECE



THIS IS POSITIVE FOR COORD. SYSTEM SHOWN

$$\therefore \quad EI\eta'' = M_\xi$$

INTEGRATE TWICE $\Rightarrow$ TWO CONSTANTS OF INTEGRATION C_1 + C_2

IMPOSE: $\eta'(\zeta = 0) = 0 \Rightarrow C_1 = 0$

$\eta(\zeta = 0) = 0 \Rightarrow C_2 = 0$

$$\Rightarrow \eta(\zeta) = \frac{1}{EI}\left[k_2\left(\frac{1}{2} l^2 \zeta^2 - \frac{1}{3} l \zeta^3 + \frac{1}{12}\zeta^4\right) + k_1\left(\frac{1}{6} l^3 \zeta^2 - \frac{1}{12} l^2 \zeta^3 + \frac{1}{120}\zeta^5\right)\right]$$

∴ TIP DEFLECTION:

$$\eta_s = \frac{l^4}{EI}\left[\frac{1}{4} k_2 + \frac{k_1 l}{120}(11)\right]$$

OR $$\eta_s = \frac{l^4}{EI}\left[\frac{1}{4}\frac{Mg}{2l}\sin\theta + \frac{l(11)}{120}\frac{M}{l}\omega^2 \sin\theta\cos\theta\right]$$

OR $$\eta_s = \frac{Ml^3}{2EI}\sin\theta\left[\frac{1}{4} g + \frac{11}{60} l\omega^2\cos\theta\right]$$