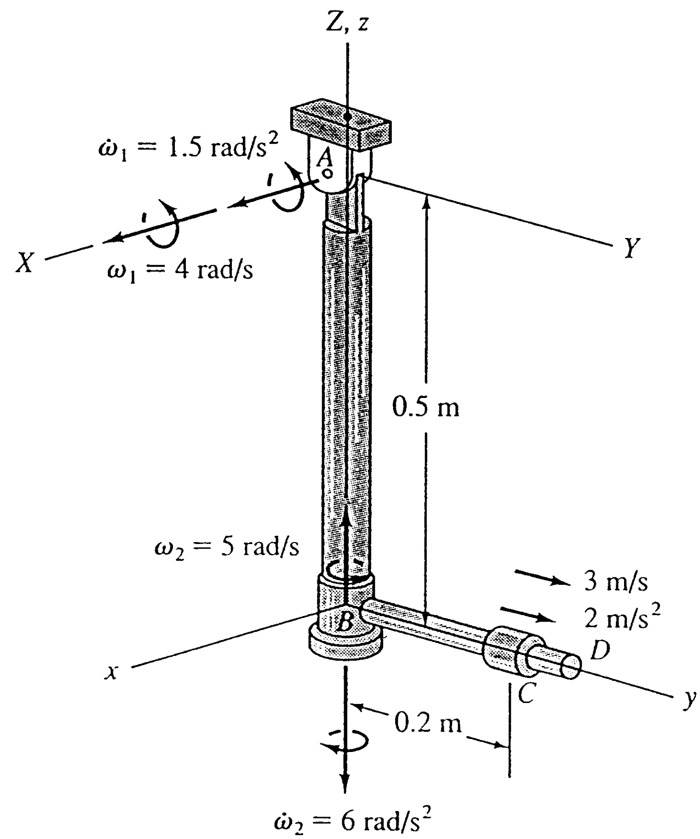
EMA 542

Home Work to be Handed In

4) The pendulum shown in the figure consists of two rods. *AB* is pin-supported at *A* and swings only in the Y-Z plane, whereas a bearing at *B* allows the attached rod *BD* to spin about rod *AB*. At a given instant, the rods have the angular motions shown. If a collar *C* is located 0.2 m from *B*, has a velocity of 3.0 m/s and an acceleration of 2.0 m/s^2 along the rod, determine the velocity and acceleration of the collar at this instant.



use a single coord. syst. attached to O,
xyz

$\dot{\vec{\rho}}_r = 3\bar{j} \qquad \ddot{\vec{\rho}}_r = 2\bar{j} \qquad \vec{\rho} = .2\bar{j}$

$\vec{\omega} = 4\bar{i} + 5\bar{k} = \omega_1\bar{i} + \omega_2\bar{k}$

$\dot{\vec{\omega}} = \dot{\omega}_1\bar{i} + (-\omega_2\bar{k} \times \omega_1\bar{i}) + \dot{\omega}_2\bar{k}$

note that a observer in the xyz frame sees $\vec{\omega}_1$ rotate with a angular velocity $-\vec{\omega}_2$

$\therefore \quad \dot{\vec{\omega}} = 1.5\bar{i} - 20\bar{j} - 6\bar{k}$

$\vec{R}_o = -.5\bar{k} \qquad \dot{\vec{R}}_o = 2\bar{j} \qquad \ddot{\vec{R}}_o = .75\bar{j} + 8\bar{k}$

Velocity $- \quad \vec{V}_c = \dot{\vec{R}}_o + \vec{\omega}\times\vec{\rho}_r + \dot{\vec{\rho}}_r$

$\therefore \quad \vec{V}_c = 2\bar{j} + (4\bar{i} + 5\bar{k}) \times .2\bar{j} + 3\bar{j}$

$$\boxed{\vec{V}_c = -1.0\bar{i} + 5\bar{j} + .8\bar{k}}$$

Acc $\quad \vec{a}_c = \ddot{\vec{R}}_o + \vec{\omega}\times(\vec{\omega}\times\vec{\rho}_r) + 2\vec{\omega}\times\dot{\vec{\rho}}_r + \ddot{\vec{\rho}}_r + \dot{\vec{\omega}}\times\vec{\rho}_r$

$= .75\bar{j} + 8\bar{k} + (4\bar{i} + 5\bar{k}) \times (.8\bar{k} - 1\bar{i})$

$+ 2(4\bar{i} + 5\bar{k}) \times 3\bar{j} + 2\bar{j} + (1.5\bar{i} - 20\bar{j} - 6\bar{k}) \times .2\bar{j}$

$$\vec{a}_c = .75\bar{j} + \cancel{8\bar{k}} - \cancel{3.2\bar{j}} - \cancel{5\bar{j}} + \cancel{24\bar{k}}$$

$$- \cancel{30\bar{i}} + \cancel{2\bar{j}} + .3\bar{k} + \cancel{1.2\bar{i}}$$

$$\boxed{\vec{a}_c = -28.8\bar{i} - 5.45\bar{j} + 32.3\bar{k}}$$

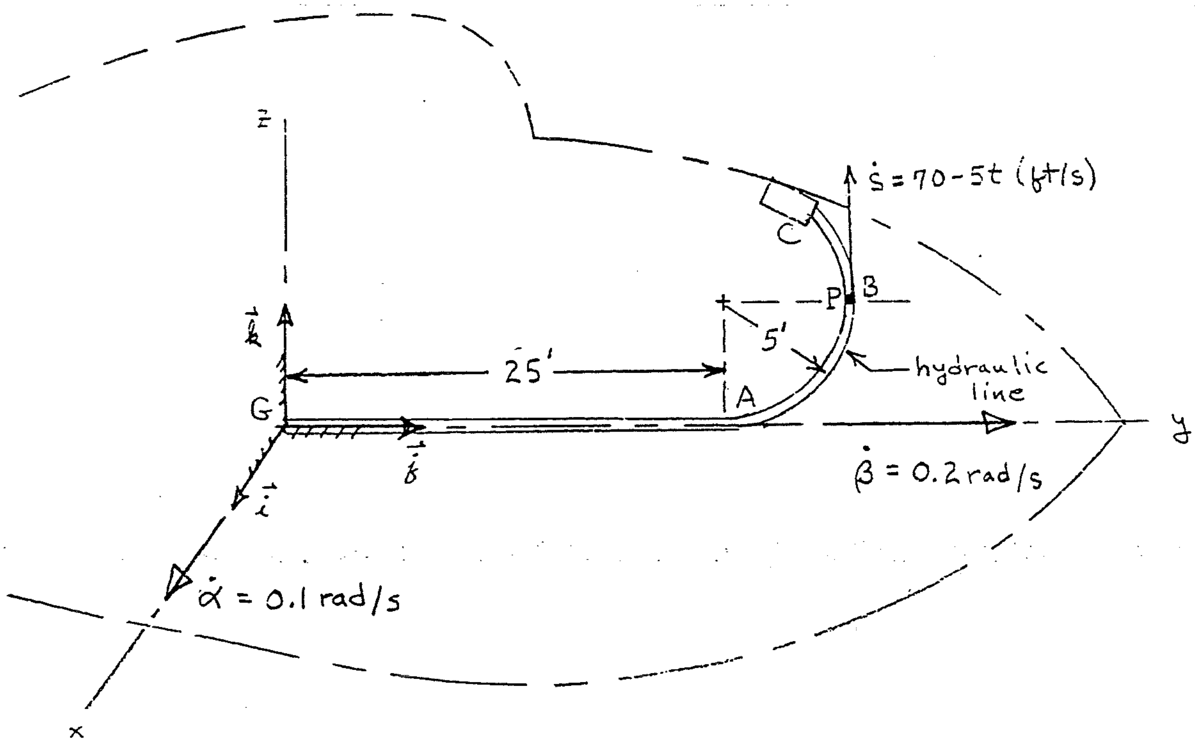
4B

The mass center G of an airplane has its velocity vector given as a function of time electronically as $\vec{v}_G = 200\vec{j} + 3t\,\vec{k}$ (ft/s) where the body axes $\vec{i}, \vec{j}, \vec{k}$ are shown. Also rate gyros indicate that its pitch rate $\dot{\alpha}$ is constant at $\dot{\alpha} = 0.1$ rad/s, its roll rate $\dot{\beta}$ is constant at $\dot{\beta} = 0.2$ rad/s and its yaw rate $\dot{\gamma}$ remains zero. Fluid is flowing along the hydraulic line G,A,B,C shown, where the portion A,B,C is a circular path of radius 5 ft in the yz body plane. The speed of all fluid particles relative to the hydraulic line is given by $\dot{s} = 70 - 5t$ (ft/s). Determine at the time t = 10 seconds:

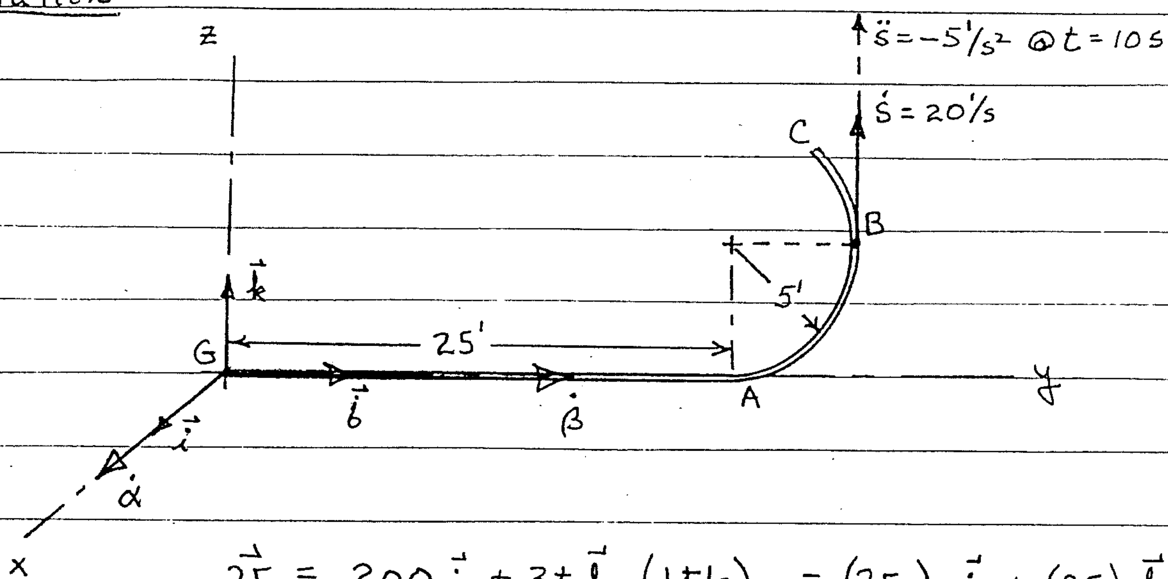
(a) the inertial velocity $\vec{v}_P$ of the fluid particle instantaneously at B, and

(b) its inertial acceleration $\vec{a}_P$.

Note: Give all answers in terms of components along the rotating body axes $\vec{i}, \vec{j}, \vec{k}$. Please report all terms because they will be graded individually.



Ⓘ Solution



$$\vec{v}_G = 200\vec{j} + 3t\vec{k} \ (ft/s) = (v_G)_y\vec{j} + (v_G)_z\vec{k}$$

$$\dot{\alpha} = 0.1 \text{ rad/s}$$

$$\dot{\beta} = 0.2 \text{ rad/s}$$

$$\therefore \vec{a}_G = (\dot{v}_G)_y\vec{j} + (\dot{v}_G)_z\vec{k} + \vec{\omega}_{cs} \times \vec{v}_G$$
$$= 3\vec{k} + (0.1\vec{i} + 0.2\vec{j}) \times (200\vec{j} + 3t\vec{k})$$
$$= 3\vec{k} + \{20\vec{k} - .3t\vec{j} + .6t\vec{i}\}$$

In general: $\vec{a}_G = 0.6t\vec{i} - 0.3t\vec{j} + 23\vec{k}$

$\therefore$ @ $t = 10s$; $\quad \vec{v}_G = 200\vec{j} + 30\vec{k} \ (ft/s)$

$\quad \vec{a}_G = 6\vec{i} - 3\vec{j} + 23\vec{k} \ (ft/s^2)$

Also $\left.\begin{array}{l} \dot{s} = 70 - 5t \\ \ddot{s} = -5 \end{array}\right\}$ @ $t = 10s$; $\quad \begin{array}{l} \dot{s} = 20 \\ \ddot{s} = -5 \end{array}$

$$\vec{v}_P = \vec{v}_G + \vec{\omega}_{cs} \times \vec{\rho} + \dot{\vec{\rho}}_r$$

where: $\vec{v}_G = \underline{200\vec{j} + 30\vec{k}}$ at $t = 10s$

$\left.\begin{array}{l} \vec{\omega}_{cs} = .1\vec{i} + .2\vec{j} \\ \vec{\rho} = 30\vec{j} + 5\vec{k} \end{array}\right\} \vec{\omega}_{cs} \times \vec{\rho} = \underline{\vec{i} - .5\vec{j} + 3\vec{k}}$

$\dot{\vec{\rho}}_r = \underline{20\vec{k}}$

$$\therefore \boxed{\vec{v}_P = \vec{i} + 199.5\vec{j} + 53\vec{k}} \quad (ft/s)$$

$$\vec{a}_P = \vec{a}_G + \vec{\omega} \times (\vec{\omega} \times \vec{\rho}) + \dot{\vec{\omega}} \times \vec{\rho} + 2\vec{\omega} \times \dot{\vec{\rho}}_r + \ddot{\vec{\rho}}_r$$

where: $\vec{a}_G = \underline{6\vec{i} - 3\vec{j} + 23\vec{k}}$ @ $t = 10s$

$\vec{\omega} \times (\vec{\omega} \times \vec{\rho}) = (.1\vec{i} + .2\vec{j}) \times (\vec{i} - .5\vec{j} + 3\vec{k})$

$= -.05\vec{k} - .3\vec{j} - .2\vec{k} + .6\vec{i}$

$= \underline{.6\vec{i} - .3\vec{j} - .25\vec{k}}$

$\dot{\vec{\omega}} \times \vec{\rho} = 0$ since $\dot{\vec{\omega}} = 0$

$2\vec{\omega} \times \dot{\vec{\rho}}_r = 2(.1\vec{i} + .2\vec{j}) \times (20\vec{k}) = \underline{8\vec{i} - 4\vec{j}}$

$\ddot{\vec{\rho}}_r = -5\vec{k} - (20)^2/5\,\vec{j} = \underline{-80\vec{j} - 5\vec{k}}$

$$\therefore \boxed{\vec{a}_P = 14.6\vec{i} - 87.3\vec{j} + 17.75\vec{k}}$$