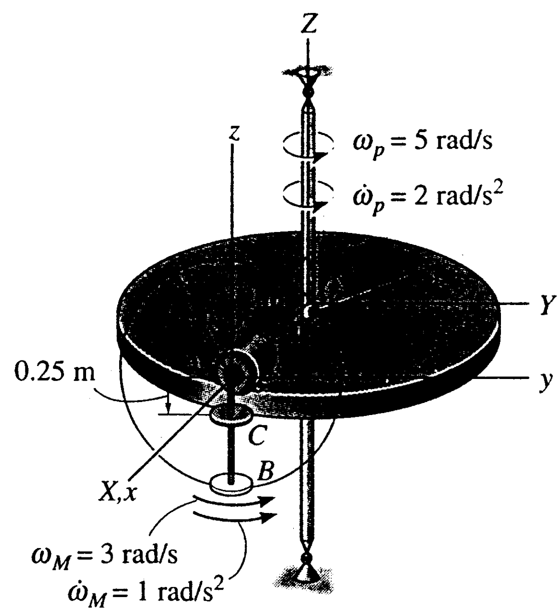
EMA 542

Home Work to be Handed In

3) A motor and attached rod *AB* have the angular motion shown in the figure below. A collar *C* on the rod is located 0.25 m from *A*, and is moving downward with a velocity of 3 m/s and an acceleration of 2 m/s^2. Determine the velocity and acceleration of *C* at this instant.



SOLUTION TO 3

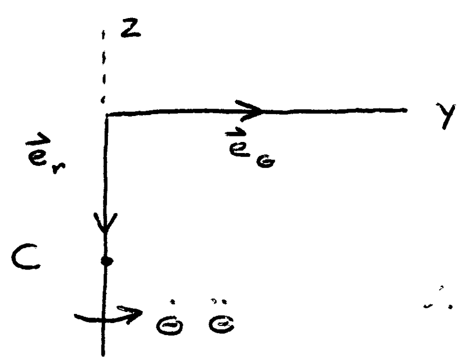
ATTACH XYZ TO PLATFORM AT A

$\vec{\omega} = 5\bar{k}$ $\qquad \underline{\dot{\vec{R}} = 10\bar{j}}$ $\qquad \vec{\rho} = -.25\bar{k}$

$\vec{V}_c = \dot{\vec{R}} + \vec{\omega}\times\vec{\rho} + \dot{\vec{\rho}}_r$

$\underline{\vec{\omega}\times\vec{\rho}} = 5\bar{k} \times -.25\bar{k} = 0$

USE POLAR COORDS.:



$\dot{\vec{\rho}}_r = \dot{r}\,\vec{e}_r + r\dot{\theta}\,\vec{e}_\theta$

$= -3\bar{k} + (.25)3\bar{j}$

$\therefore \underline{\dot{\vec{\rho}}_r} = .75\bar{j} - 3\bar{k}$

$\therefore \boxed{\vec{V}_c = 10.75\bar{j} - 3\bar{k}}$

$\vec{a}_c = \ddot{\vec{R}} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho}) + \dot{\vec{\omega}}\times\vec{\rho} + 2\vec{\omega}\times\dot{\vec{\rho}}_r + \ddot{\vec{\rho}}_r$

$\ddot{\vec{R}} = 2(2)\bar{j} - 2(5)^2\bar{i}$

$\underline{\ddot{\vec{R}}} = -50\bar{i} + 4\bar{j}$

$$\underline{\vec{\omega} \times (\vec{\omega} \times \vec{\rho})} = 5\bar{k} \times 0 = 0$$

$$\dot{\vec{\omega}} = 2\bar{k} \qquad \therefore \quad \underline{\dot{\vec{\omega}} \times \vec{\rho}} = 2\bar{k} \times .25\bar{k} = 0$$

$$2\vec{\omega} \times \dot{\vec{\rho}}_r = 2(5)\bar{k} \times [.75\bar{j} - 3\bar{k}]$$

$$\underline{2\vec{\omega} \times \dot{\vec{\rho}}_r} = -7.5\bar{i}$$

$$\ddot{\vec{\rho}}_r = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$$

$$= -[2 - (.25)(3)^2]\bar{k} + [(.25)(1) + 2(3)(3)]\bar{j}$$

$$\Rightarrow \underline{\ddot{\vec{\rho}}_r} = .25\bar{k} + 18.25\bar{j}$$

$$\therefore \quad \vec{a}_c = [-50 - 7.5]\bar{i} + [4 + 18.25]\bar{j} + .25\bar{k}$$

$$\therefore \quad \boxed{\vec{a}_c = -57.5\bar{i} + 22.5\bar{j} + .25\bar{k}}$$

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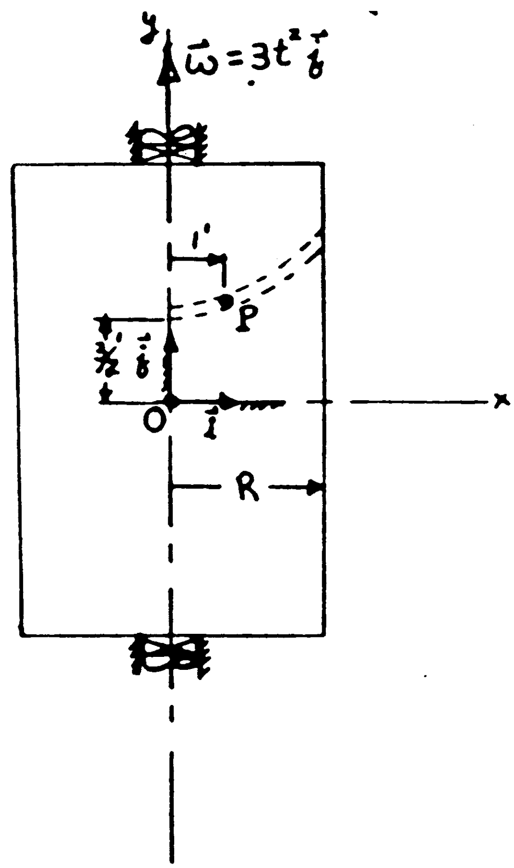
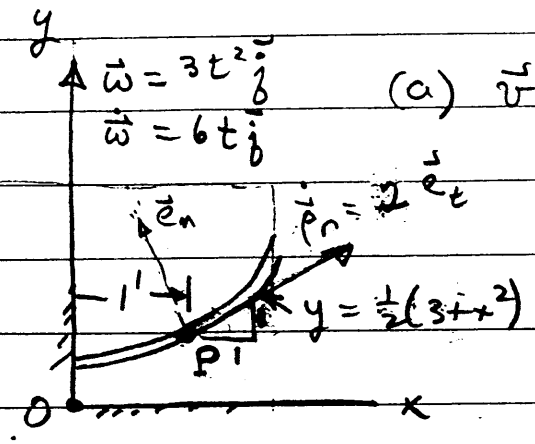
3A) The circular cylindrical shell (shown) of radius R rotates about a vertical axis at the angular velocity $\omega = 3t^2$. The shape of an oil line going from the axis of rotation (y axis) to the outer surface of the shell is given by $y = \frac{1}{2}\left(3 + x^2\right)$ where the xyz axes are body axes described by the rotating $\vec{i}, \vec{j}, \vec{k}$ unit vectors as shown. Oil flows outward along the oil line at a constant speed of $\dot{s} = 2.0$ ft/sec. relative to the oil line. Determine the total velocity of the oil particle P that is instantaneously located at 1.0 ft. radially outward from the y axis at time 2.0 sec. Give answers in terms of $\vec{i}, \vec{j}, \vec{k}$ components. Use the equation $\dot{\vec{A}}_R = \dot{\vec{A}}_r + \vec{\omega}_{cs} \times \vec{A}$ to get your answer.



Homework

Solution to Problem 7 using Eqs. (1-63) and (1-66)



(a) $\vec{v}_P = \vec{v}_o + \vec{\omega}_{cs} \times \vec{\rho} + \dot{\vec{\rho}}_r$

where $\vec{v}_o = 0$

$\left.\begin{array}{l}\vec{\omega}_{cs} = 12\vec{j} \\ \vec{\rho} = \vec{i} + 2\vec{j}\end{array}\right\} \vec{\omega}_{cs} \times \vec{\rho} = -12\vec{k}$

$\dot{\vec{\rho}}_r = 2\vec{e}_t = \sqrt{2}\,\vec{i} + \sqrt{2}\,\vec{j}$

$\therefore \boxed{\vec{v}_P = \sqrt{2}\,\vec{i} + \sqrt{2}\,\vec{j} - 12\vec{k}}$

Important Data

$y = \frac{3}{2} + \frac{x^2}{2}$; $y\big|_{x=1} = 2$

$\frac{dy}{dx} = x$; $\frac{dy}{dx}\big|_{x=1} = 1$

$\frac{d^2y}{dx^2} = 1$; $\frac{d^2y}{dx^2}\big|_{x=1} = 1$

$\vec{e}_t = \frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$; $\vec{e}_n = -\frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$

(b) $\vec{a}_P = \vec{a}_o + \vec{\omega}_{cs} \times (\vec{\omega}_{cs} \times \vec{\rho}) + \dot{\vec{\omega}} \times \vec{\rho} + \ddot{\vec{\rho}}_r + 2\vec{\omega} \times \dot{\vec{\rho}}_r$

where

$\vec{a}_o = 0$

$\vec{\omega}_{cs} \times (\vec{\omega}_{cs} \times \vec{\rho}) = 12\vec{j} \times (-12\vec{k}) = -144\vec{i}$

$\dot{\vec{\omega}}_{cs} \times \vec{\rho} = 12\vec{j} \times (\vec{i} + 2\vec{j}) = -12\vec{k}$

$\ddot{\vec{\rho}}_r = \frac{\dot{s}^2}{\rho}\vec{e}_n = \frac{4}{2\sqrt{2}}\vec{e}_n = \frac{2}{\sqrt{2}}\vec{e}_n = \sqrt{2}\,\vec{e}_n = -\vec{i} + \vec{j}$

where $\rho_{x=1} = \dfrac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \dfrac{[1+1]^{3/2}}{1} = 2\sqrt{2} = 2.828$

$2\vec{\omega} \times \dot{\vec{\rho}}_r = 2(12\vec{j}) \times [\sqrt{2}\,\vec{i} + \sqrt{2}\,\vec{j}] = -24\sqrt{2}\,\vec{k} = -33.9\vec{k}$

$\boxed{\vec{a}_P = -145\vec{i} + \vec{j} - 45.9\vec{k}}$