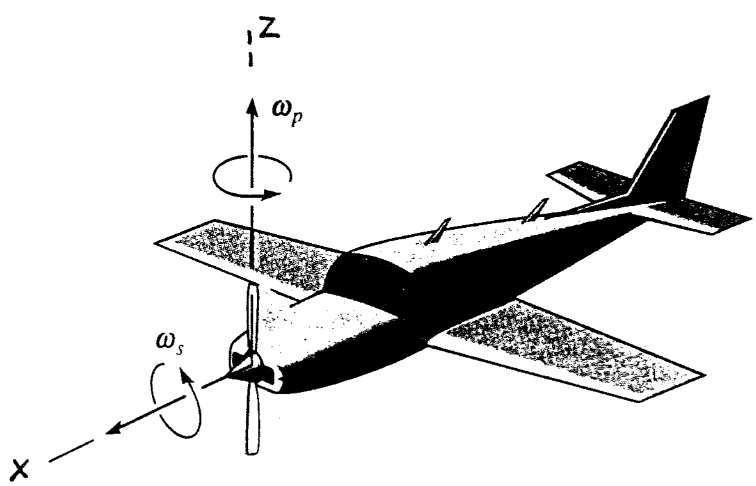
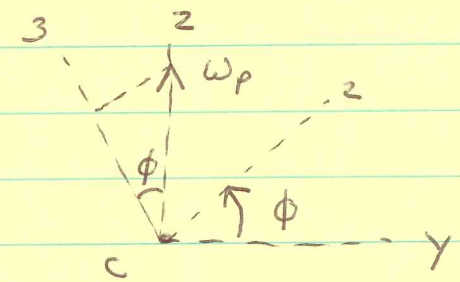
EMA 542

Home Work to be Handed In

11) The airplane shown in the figure below is in the process of making a steady horizontal turn at the rate ω_p. During this motion, the airplane's propeller is spinning at the rate of ω_s. If the propeller has two blades, determine the moments which the propeller shaft exerts on the propeller when the blades are in the vertical position. For simplicity, assume the propeller to be a uniform slender bar with total mass m AND LENGTH ℓ.



ATTACH 123 TO PROPELLOR



$$\vec{\omega} = \omega_s \vec{e}_1 + \omega_p \sin\phi \, \vec{e}_2 + \omega_p \cos\phi \, \vec{e}_3$$

$$I_1 = \frac{1}{12} M l^2 = I_2$$

$$I_3 = 0$$

$$\Rightarrow \vec{M}_c = \dot{\vec{h}}_c$$

$$\vec{h}_c = \frac{1}{12} M l^2 \omega_s \vec{e}_1 + \frac{1}{12} M l^2 \omega_p \sin\phi \, \vec{e}_2$$

$$\dot{\vec{h}}_c = \dot{\vec{h}}_{cr} + \vec{\omega} \times \vec{h}_c$$

$$\dot{\vec{h}}_{cr} = \frac{1}{12} M l^2 \omega_p \dot{\phi} \cos\phi \, \vec{e}_2 = \frac{1}{12} M l^2 \omega_p \omega_s \vec{e}_2 \ @ \ \phi = 0$$

$$\vec{\omega} \times \vec{h}_c = \left(\omega_s \vec{e}_1 + \omega_p \sin\phi \, \vec{e}_2 + \omega_p \cos\phi \, \vec{e}_3 \right) \times \left(\frac{1}{12} M l^2 \omega_s \vec{e}_1 + \frac{1}{12} M l^2 \omega_p \sin\phi \, \vec{e}_2 \right)$$

or @ $\phi = 0$, $\vec{\omega} \times \vec{h}_c = \left(\omega_s \vec{e}_1 + \omega_p \vec{e}_3 \right) \times \frac{1}{12} M l^2 \omega_s \vec{e}_1$

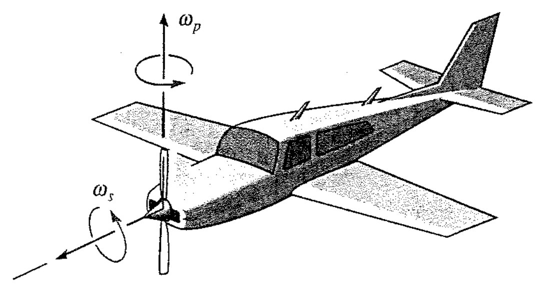
$$= \frac{1}{12} M l^2 \omega_p \omega_s \vec{e}_2$$

$$\Rightarrow \vec{M}_c = \dot{\vec{h}}_c = \frac{1}{6} M l^2 \omega_p \omega_s \vec{e}_2$$

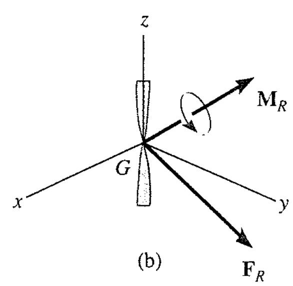
or $M_x = 0$ $\quad M_y = \frac{1}{6} M l^2 \omega_p \omega_s$ $\quad M_z = 0$

Example 21–5

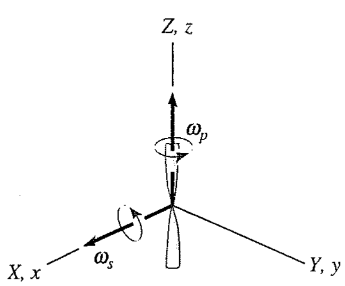
The airplane shown in Fig. 21–13a is in the process of making a steady *horizontal* turn at the rate of ω_p. During this motion, the airplane's propeller is spinning at the rate of ω_s. If the propeller has two blades, determine the moments which the propeller shaft exerts on the propeller when the blades are in the vertical position. For simplicity, assume the blades to be a uniform slender bar having a moment of inertia I about an axis perpendicular to the blades and passing through their center, and having zero moment of inertia about a longitudinal axis.



(a)



(b)



(c)

Fig. 21–13

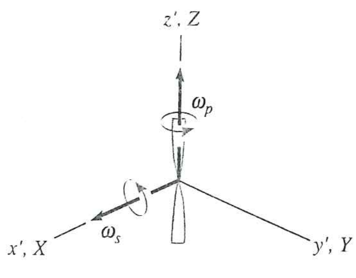
SOLUTION

Free-Body Diagram. Fig. 21–13b.The effect of the connecting shaft on the propeller is indicated by the resultants $\mathbf{F}_R$ and $\mathbf{M}_R$. (The propeller's weight is assumed to be negligible.) The x, y, z axes will be taken fixed to the propeller, since these axes always represent the principal axes of inertia for the propeller. Thus, $\boldsymbol{\Omega} = \boldsymbol{\omega}$. The moments of inertia I_x and I_y are equal ($I_x = I_y = I$) and $I_z = 0$.

Kinematics. The angular velocity of the x, y, z axes observed from the X, Y, Z axes, coincident with the x, y, z axes, Fig. 21–13c, is $\boldsymbol{\omega} = \boldsymbol{\omega}_s + \boldsymbol{\omega}_p = \omega_s\mathbf{i} + \omega_p\mathbf{k}$, so that the x, y, z components of $\boldsymbol{\omega}$ are

$$\omega_x = \omega_s \qquad \omega_y = 0 \qquad \omega_z = \omega_p$$

Since $\boldsymbol{\Omega} = \boldsymbol{\omega}$, then $\dot{\omega} = (\dot{\omega})_{xyz}$. Hence, like Example 21–4, the time derivative of $\boldsymbol{\omega}$ will be computed with respect to the fixed X, Y, Z axes and then $\dot{\boldsymbol{\omega}}$ will be resolved into components along the moving x, y, z axes to obtain $(\dot{\omega})_{xyz}$. To do this, Eq. 20–6 must be used since $\boldsymbol{\omega}$ is changing direction relative to X, Y, Z. (Note that this was unnecessary for the case in Example 21–4.) Since $\boldsymbol{\omega} = \boldsymbol{\omega}_s + \boldsymbol{\omega}_p$, then $\dot{\boldsymbol{\omega}} = \dot{\boldsymbol{\omega}}_s + \dot{\boldsymbol{\omega}}_p$. Similar to Example 20–1, the time rate of change of each of these components relative to the X, Y, Z axes can be obtained by using a third coordinate system x', y', z', which has an angular velocity $\boldsymbol{\Omega}' = \boldsymbol{\omega}_p$ and is coincident with the X, Y, Z axes at the instant shown. Thus



(d)

$$\dot{\boldsymbol{\omega}} = (\dot{\boldsymbol{\omega}})_{x'y'z'} + \boldsymbol{\Omega}' \times \boldsymbol{\omega}$$
$$= (\dot{\boldsymbol{\omega}}_s)_{x'y'z'} + (\dot{\boldsymbol{\omega}}_p)_{x'y'z'} + \boldsymbol{\omega}_p \times (\boldsymbol{\omega}_s + \boldsymbol{\omega}_p)$$
$$= \mathbf{0} + \mathbf{0} + \boldsymbol{\omega}_p \times \boldsymbol{\omega}_s + \boldsymbol{\omega}_p \times \boldsymbol{\omega}_p$$
$$= \mathbf{0} + \mathbf{0} + \omega_p\mathbf{k} \times \omega_s\mathbf{i} + \mathbf{0} = \omega_p\omega_s\mathbf{j}$$

Since the X, Y, Z axes are also coincident with the x, y, z axes at the instant shown, Fig. 21–13d, the components of $\dot{\boldsymbol{\omega}}$ along these axes are

$$\dot{\omega}_x = 0 \qquad \dot{\omega}_y = \omega_p\omega_s \qquad \dot{\omega}_z = 0$$

These same results can, of course, also be determined by direct calculation of $(\dot{\omega})_{xyz}$. To do this, it will be necessary to view the propeller in some general position such as shown in Fig. 21–13e. Here the plane has turned through an angle ϕ and the propeller has turned through an angle ψ relative to the plane. Notice that $\boldsymbol{\omega}_p$ is always directed along the fixed Z axis and $\boldsymbol{\omega}_s$ follows the x axis. Thus the components of $\boldsymbol{\omega}$ are

$$\omega_x = \omega_s \qquad \omega_y = -\omega_p \sin\psi \qquad \omega_z = \omega_p \cos\psi$$

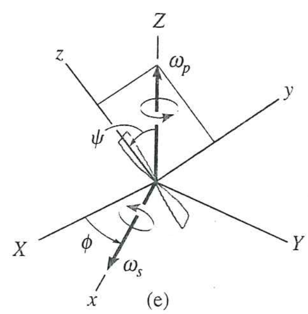
Since ω_s and ω_p are constant, the time derivatives of these components become

$$\dot{\omega}_x = 0 \qquad \dot{\omega}_y = \omega_p \cos\psi\,\dot{\psi} \qquad \omega_z = \omega_p \sin\psi\,\dot{\psi}$$

but $\psi = 0°$ and $\dot{\psi} = \omega_s$ at the instant considered. Thus,

$$\dot{\omega}_x = 0 \qquad \dot{\omega}_y = \omega_p\omega_s \qquad \dot{\omega}_z = 0$$

which are the same results as those computed above.



(e)

Fig. 21–13 *(cont'd)*

Equations of Motion. Using Eqs. 21–25, we have

$$\Sigma M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z = I(0) - (I - 0)(0)\omega_p$$
$$M_x = 0 \qquad \textit{Ans.}$$
$$\Sigma M_y = I_y\dot{\omega}_y - (I_z - I_x)\omega_z\omega_x = I(\omega_p\omega_s) - (0 - I)\omega_p\omega_s$$
$$M_y = 2I\omega_p\omega_s \qquad \textit{Ans.}$$
$$\Sigma M_z = I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y = 0(0) - (I - I)\omega_s(0)$$
$$M_z = 0 \qquad \textit{Ans.}$$

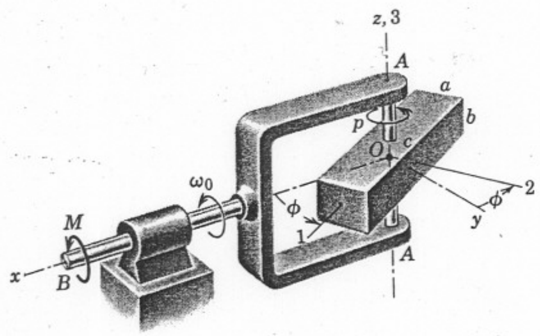
$M_y = \frac{1}{6} M l^2 \omega_p \omega_s$

$I = \frac{1}{12} M l^2$

HWK TO HAND IN

12. As shown below, the homogeneous rectangular block of mass m is centrally mounted on the shaft $A-A$ about which it rotates with a constant speed $\dot{\phi} = p$. Meanwhile the yoke is forced to rotate about the x-axis with a constant speed ω_o. Find the magnitude of the torque $\mathbf{M}$ as a function of ϕ. The center O of the block is the origin of the $x-y-z$ coordinates. Principal axes 1-2-3 are attached to the block as shown, and with respect to these axes:

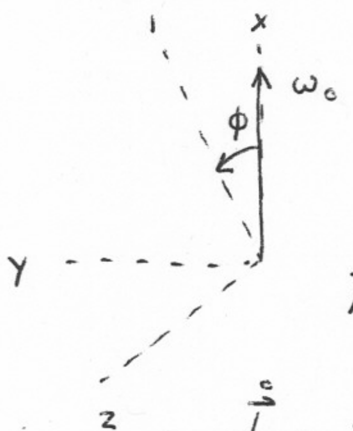
$$
\begin{aligned}
I_{11} &= m(a^2+b^2)/12 \\
I_{22} &= m(b^2+c^2)/12 \\
I_{33} &= m(a^2+c^2)/12
\end{aligned}
$$



SOLUTION FOR 2

$\dot{\phi} = \rho = \text{CONST.}$ $\qquad\qquad \omega_0 = \text{CONST.}$

USE 123 BODY COORD. SYSTEM



$$\vec{\omega} = \omega_0 \cos\phi \, \vec{e}_1 - \omega_0 \sin\phi \, \vec{e}_2 + \rho \vec{e}_3$$

$$\Rightarrow \quad \vec{h} = I_{11} \omega_0 \cos\phi \, \vec{e}_1 - I_{22} \omega_0 \sin\phi \, \vec{e}_2 + I_{33} \rho \vec{e}_3$$

$$\dot{\vec{h}} = \overset{\circ}{\vec{h}} + \vec{\omega} \times \vec{h}$$

$$\overset{\circ}{\vec{h}} = -I_{11} \omega_0 \dot{\phi} \sin\phi \, \vec{e}_1 - I_{22} \omega_0 \dot{\phi} \cos\phi \, \vec{e}_2$$

$$\vec{\omega} \times \vec{h} = \left[\omega_0 \cos\phi \, \vec{e}_1 - \omega_0 \sin\phi \, \vec{e}_2 + \rho \vec{e}_3 \right] \times$$

$$\left[I_{11} \omega_0 \cos\phi \, \vec{e}_1 - I_{22} \omega_0 \sin\phi \, \vec{e}_2 + I_{33} \rho \vec{e}_3 \right]$$

$$= -I_{22} \omega_0^2 \cos\phi \sin\phi \, \vec{e}_3 - I_{33} \rho \omega_0 \cos\phi \, \vec{e}_2$$

$$+ I_{11} \omega_0^2 \sin\phi \cos\phi \, \vec{e}_3 - I_{33} \omega_0 \rho \sin\phi \, \vec{e}_1$$

$$+ I_{11} \omega_0 \rho \cos\phi \, \vec{e}_2 + I_{22} \omega_0 \rho \sin\phi \, \vec{e}_1$$

$$\therefore \quad \dot{\vec{h}} = \left[-I_{11}\,\omega_0\,p\sin\phi - I_{33}\,\omega_0\,p\sin\phi + I_{22}\,\omega_0\,p\sin\phi\right]\vec{e}_1$$

$$+ \left[-I_{22}\,\omega_0\,p\cos\phi - I_{33}\,\omega_0\,p\cos\phi + I_{11}\,\omega_0\,p\cos\phi\right]\vec{e}_2$$

$$+ \left[I_{11} - I_{22}\right]\omega_0^{\,2}\sin\phi\cos\phi\;\vec{e}_3$$

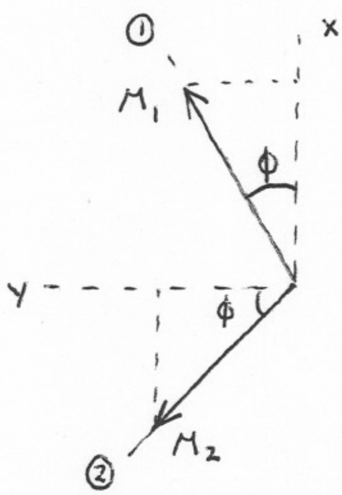
or

$$\dot{\vec{h}} = \left(I_{22} - I_{11} - I_{33}\right)\omega_0\,p\sin\phi\;\vec{e}_1$$

$$+ \left(I_{11} - I_{22} - I_{33}\right)\omega_0\,p\cos\phi\;\vec{e}_2$$

$$+ \left(I_{11} - I_{22}\right)\omega_0^{\,2}\sin\phi\cos\phi\;\vec{e}_3 \qquad = \vec{M}$$

Convert to xyz:



$$M_x = M_1\cos\phi - M_2\sin\phi$$

$$\Rightarrow \quad M_x = \left(I_{22} - I_{11} - I_{33}\right)\omega_0\,p\sin\phi\cos\phi$$

$$- \left(I_{11} - I_{22} - I_{33}\right)\omega_0\,p\sin\phi\cos\phi$$

$$M_x = 2\left(I_{22} - I_{11}\right)\omega_0\,p\sin\phi\cos\phi$$

$$I_{22} - I_{11} = \frac{M}{12}\left(b^2 + c^2 - a^2 - b^2\right) = \frac{M}{12}\left(c^2 - a^2\right)$$

$$\therefore \quad \boxed{M_x = \frac{M}{6}\left(c^2 - a^2\right)\omega_0\,p\sin\phi\cos\phi}$$