HW set for ECE 3386 Characteristics and models of solid state devices I-B, Northeastern Univ. Boston, Massachusetts

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1 problem 1

1) An n-type sample with a donor doping of $\left(n_n = 10^{15} cm^{-3}\right)$ contains $\left(N_t = 10^{15} cm^{-3}\right)$ generation-recombination centers located at the intrinsic fermi level with $\sigma_p = \sigma_n = 10^{-15} cm^2$. Assume $v_{th} = 10^7 cm/\text{sec}$.

(a) Calculate the generation rate if the region is depleted of mobile carries

since there are no mobile carries, then $p_n = n_n = 0$.

but the recombination rate is

$$U = \frac{v_{th}\sigma_n\sigma_p N_t\left(p_n n_n - n_i^2\right)}{\sigma_p\left(p_n + n_i e^{\left(\frac{E_i - E_t}{KT}\right)}\right) + \sigma_n\left(n_n + n_i e^{\left(\frac{E_t - E_i}{KT}\right)}\right)}$$

but since $p_n = n_n = 0$ then U becomes

$$U = \frac{v_{th}\sigma_n\sigma_p N_t\left(-n_i^2\right)}{\sigma_p\left(n_i e^{\left(\frac{E_i - E_t}{KT}\right)}\right) + \sigma_n\left(n_i e^{\left(\frac{E_t - E_i}{KT}\right)}\right)}$$

substituting in the above equation for the values given and assuming room tempreture, results in

$$\begin{aligned} U &= \frac{\left(10^7\right)\left(10^{-15}\right)\left(10^{-15}\right)\left(10^{15}\right)\left(-\left(1.45\times10^{10}\right)^2\right)}{10^{-15}\left(1.45\times10^{10}\ e^{\left(\frac{0}{KT}\right)}\right)+10^{-15}\left(1.45\times10^{10}\ e^{\left(\frac{0}{KT}\right)}\right)} = \frac{-2.1025\times10^{12}}{2.9\times10^{-5}} \\ &= -7.25\times10^{16} cm^{-3}\ \text{sec} \end{aligned}$$

since U is negative, then this is the generation rate.

(b)Calculate the generation rate in the region where only minority carries concentration has been reduced appreciably below its equilibrium rate.

since this is an n-type sampe, then the minority carriers are the holes p, and also we are given that p_n has been reduced below its equilibrium rate p_{no}.

(*side note*: *in thermal equilibrium, the generation rate must be balanced by recombination rate*).

(*side note*: *if no external source , such as light,* $\Rightarrow p_n n_n = n_i^2$ *but if external source exist, then* $p_n n_n > n_i^2$)

$n_n \gg p_n$, and since $E_t = E_i$,then $n_n \gg n_i e^{\left(\frac{E_t - E_i}{KT}\right)}$ .i.e. $n_n \gg n_i$

so, from the expression for U

$$U = \frac{v_{th}\sigma_n\sigma_p N_t\left(p_n n_n - n_i^2\right)}{\sigma_p\left(p_n + n_i e^{\left(\frac{E_i - E_t}{KT}\right)}\right) + \sigma_n\left(n_n + n_i e^{\left(\frac{E_t - E_i}{KT}\right)}\right)}$$

$$= \frac{v_{th}\sigma_n\sigma_p N_t\left(p_n n_n - n_i^2\right)}{\sigma_p(p_n + n_i) + \sigma_n(n_n + n_i)}$$

since only p_n has changed appreciably below its equilibrium, then

$$U = \frac{v_{th}\sigma_n\sigma_p N_t(p_n n_{no} - p_{no} n_{no})}{\sigma_p(small) + \sigma_n(n_{no} + small)}$$

$$= \frac{n_{no} v_{th}\sigma_n\sigma_p N_t(p_n - p_{no})}{\sigma_n n_{no}}$$

$$= v_{th}\sigma_p N_t\left(p_n - p_{no}\right)$$

$$= -v_{th}\sigma_p N_t p_{no}$$

since $p_n \ll p_{no}$.

So U is now given by, after noting that $p_{no} = \frac{n_i^2}{n_n}$

$$U = -\left(10^7\right)\left(10^{-15}\right)\left(10^{15}\right)\left(\frac{\left(1.45 \times 10^{10}\right)^2}{10^{15}}\right) = -2.1025 \times 10^{12} cm^{-3} \text{ sec}$$

since this is negative, then this is the genration rate.

2 problem 2

prove that the recombination rate in the space charge region of the p-n junction in which $\sigma_p = \sigma_n$ is maximized when $p = n = n_i e^{\left(\frac{qv}{KT}\right)}$.

solution

the space charge region of a p-n junction is the depletion region, since outside the depletion region the space is netural .

from the equation for U

$$U = \frac{v_{th}\sigma_n\sigma_p N_t\left(p_n n_n - n_i^2\right)}{\sigma_p\left(p_n + n_i e^{\left(\frac{E_i-E_t}{KT}\right)}\right) + \sigma_n\left(n_n + n_i e^{\left(\frac{E_t-E_i}{KT}\right)}\right)}$$

since $\sigma_p = \sigma_n = \sigma_0$, then U become

$$\begin{aligned} U &= \frac{v_{th}\sigma_0^2 N_t\left(p_n n_n - n_i^2\right)}{\sigma_0\left(p_n + n_i e^{\left(\frac{E_i-E_t}{KT}\right)}\right) + \sigma_0\left(n_n + n_i e^{\left(\frac{E_t-E_i}{KT}\right)}\right)} \\ &= \frac{v_{th}\sigma_0^2 N_t\left(p_n n_n - n_i^2\right)}{\sigma_0\left(p_n + n_n + n_i\left(e^{\left(\frac{E_i-E_t}{KT}\right)} + e^{\left(\frac{E_t-E_i}{KT}\right)}\right)\right)} \\ &= \frac{v_{th}\sigma_0 N_t\left(p_n n_n - n_i^2\right)}{p_n + n_n + 2n_i\cosh\left(\frac{E_t-E_i}{KT}\right)} \end{aligned}$$

now, since $p_n = p_{no}e^{\frac{qV}{KT}}$ and $n_n = n_{no}e^{\frac{qV}{KT}}$ then U becomes

$$U = \frac{v_{th}\sigma_0 N_t\left(p_{no}n_{no}e^{\frac{qV}{KT}} - n_i^2\right)}{p_n + n_n + 2n_i\cosh\left(\frac{E_t-E_i}{KT}\right)}$$

but $p_{no}n_{no} = n_i^2$ so U becomes

$$U = \frac{v_{th}\sigma_0 N_t n_i^2\left(e^{\frac{qV}{KT}} - 1\right)}{p_n + n_n + 2n_i\cosh\left(\frac{E_t-E_i}{KT}\right)}$$

this U is max when the denimonator $p_n + n_n + 2n_i\cosh\left(\frac{E_t-E_i}{KT}\right)$ is minimum, but min value for cosh $(*)$ is 1. so min value of denomiator is $p_n + n_n + 2n_i$

but n_i is a constant, so we need that $p_n + n_n$ be min.

so

$$\begin{aligned} dp_n + dn_n &= 0 \\ dp_n &= -dn_n \end{aligned}$$

but since

$$p_n n_n = n_i^2 e^{\frac{qV}{KT}} = \text{constant} \tag{1}$$

then, differentiat and get

$$p_n dn_n + n_n dp_n = 0$$

so

$$p_n dn_n + n_n(-dn_n) = 0$$

$$p_n = n_n \tag{2}$$

so from equation (1) and from equation (2) we get

$$p_n^2 = n_i^2 e^{\frac{qV}{KT}}$$

so $p_n = n_n = n_i e^{\frac{qV}{2KT}}$ QED.

3 problem 3

if recombination of electrons takes place through donorlike recmobination center, the capture cross section σ_n for the site can be crudely estimated by considering that the free electron enters a region where its thermal energy $\frac{3}{2}KT$ is less than the energy associated with the columbic attraction; this means that the carrier is drawn to the center. The raduis at which this occurse describes an area that may be taken to be equal to σ_n .

a)Obtain an expression for σ_n based upon this model and evaluate it for Si at 300K.

the columbic force is given by

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

note that this is an attraction force, since the electron is negative charged and the center is poistive charged relative to the electron (since it is not occupied). so, the electric field to pull in the elctron is given by

$$V(r) = \int_0^r F\, dr = \frac{1}{4\pi\epsilon_0} q^2 (-1/r)$$

when the electron thermal energy is < the above potional energy, the electron is captuered. so , the largest posible cross section raduis is given by the solution to

$$\frac{1}{4\pi\epsilon_0} q^2 (1/r) = \frac{3}{2}KT$$

$$r = \frac{1}{4\pi\epsilon_0}\frac{2\,q^2}{3KT} = \frac{1}{4\pi\left(8.85418\times10^{-14}\right)}\frac{2\left(1.60218\times10^{-19}\right)^2}{3\,(.0259)\left(1.60218\times10^{-19}\right)} = 3.7064\times10^{-6}\ cm$$

and

$$\sigma_n = \pi r^2 = 4.315 \times 10^{-11} cm^2$$

b)Considering the dependence on temperature of σ_n from this model togother with other temperature dependent terms, how would the electron capture cross section depend on termp? (assum the center exists in extrinsic Si).

as the themral energy of the carrier increases, σ_0 will decrease, due to the inverse relationship between r and thermal energy.so as temp. increases, the capture cross section will become smaller.

4 problem 4

Light in incident on an Si bar doped with $N_D = 10^{16} cm^{-3}$ donors, creating $G_L = 10^{21} cm^{-3}$ sec EH (electron-hole) pairs uniformly throughout the sample. There are $N_t = 10^{15} cm^{-3}$ bulk recombination centers at E_i with electron and hole concentrations with the light turned on. $\sigma_0 = 10^{-15} cm^2$.

a)Calculate the steady state hole electron concentration with the light turned on .

the life time for holes in an n-type semiconductor is given by

$$\tau_p = \frac{1}{v_{th}\sigma_p N_t} \tag{1}$$

note that while the light is turned on, $p_n = p_{no} + \delta p$ where the δp are the additional holes generated by the light effect.

now, $\delta p = \tau_p G_L = \frac{1}{v_{th}\sigma_p N_t} G_L$

$$\delta p = \frac{1}{(10^7)(10^{-15})(10^{15})} 10^{21} = 10^{14} cm^{-3}$$

but also

$$\delta p = \delta n$$

so, the steady state electron concentration is

$$N_D + \delta n = 10^{16} + 10^{14} = 1.01 \times 10^{16} cm^{-3}$$

the steady state hole concentration is $\frac{n_i^2}{N_D} + \delta p = \frac{(1.45 \times 10^{10})^2}{10^{16}} + 10^{14} \simeq 10^{14} cm^{-3}$

b)At time t=0 the light is turned off. Calculate the time response of the total hole density and find the life time. the thermal velosity is $10^7 cm/\text{sec}$ and there is no current flowing.

the govering equation is

$$p_n(t) = p_{no} + \delta p(0) e^{\frac{-t}{\tau_p}}$$

where $\tau_p = \frac{1}{v_{th}\sigma_p N_t} = \frac{1}{(10^7)(10^{-15})(10^{15})} = 10^{-7}$ sec also we know that

$p_{no} = \frac{n_i^2}{N_D} = 21025\ cm^{-3}$

$$p_n(t) = 21025 + \left(10^{14}\right) e^{\frac{-t}{10^{-7}}}$$

$$\frac{(p_n(t) - 21025)}{10^{14}} = e^{\frac{-t}{10^{-7}}}$$

$$\ln(p_n(t) - 21025) - \ln\left(10^{14}\right) = -10^7 t$$

$$\ln(p_n(t) - 21025) - 32.236 = -10^7 t$$

the above equation gives $p_n(t)$ at any time after the light is turned off. plug in a value for t to find p_n

5 QUESTION 1, HW5

Determine the forward current through a $p^+ - n$ junction device at forward bias of .675 V, given the following particulars:

$\sigma_n = \sigma_p = 10^{-14} cm^2,\ v_{th} = 10^7 cm/\text{sec},\ n_i = 10^{10} cm^{-3},\ N_A = 10^{18} cm^{-3},\ N_D = 10^{15} cm^{-3},\ A = 10^{-3} cm^2,\ \tau_p = 10^{-7}\ \text{sec},\ w = deplectionwidth = 1\mu m,\ N_t = 3 \times 10^{13} cm^{-3}$

5.0.1 solution

in a forward biased junction, the process is recombination U, i.e. $pn \gg n_i^2$.

so the forward current (the current entering the n side) is made of 2 components, the recombination current j_{rec} and the junction current j_{jun}

$$J_{forward} = J_{rec} + J_{jun}$$

$$\left.\begin{aligned} J_{jun} &= J_{saturation} e^{\frac{V}{V_T}} \\ J_{saturation} &= q\sqrt{\frac{D_p}{\tau_p}} \frac{n_i^2}{N_D} \end{aligned}\right\} \qquad (1)$$

note, equation (1) above is for low injection, where $p_{no} \gg n_{po}$ and where $V \gg 3\frac{KT}{q}$ this assumption is valid since

$p_{no} = \frac{n_i^2}{N_D} = \frac{\left(1.45 \times 10^{10}\right)^2}{10^{15}} = 210250\ cm^{-3}$, and $n_{po} = \frac{n_i^2}{N_A} = \frac{\left(1.45 \times 10^{10}\right)^2}{10^{18}} = 210\ cm^{-3}$. so $p_{no} \gg n_{po}$

and $V = 0.675\ V$ while $3\frac{KT}{q}$ $=3(0.0259) = 0.0777V$ so $V \gg 3\frac{KT}{q}$. so this is a reasonable approximation.

if this assumption is not valid, one needs to use this equation instead for finding the saturation current:

$$J_{sat} = q\left(\frac{D_p p_{no}}{L_p} + \frac{D_n n_{po}}{L_n}\right)$$

D_p is found from table to be 12 cm^2 sec so from equation (1):

$$J_{jun} = 1.60218 \times 10^{-19}\sqrt{\frac{12}{10^{-7}}}\frac{\left(1.45 \times 10^{10}\right)^2}{10^{15}}\ e^{\frac{0.675}{0.0259}} = \boxed{76.82 \quad \text{Amp per cm}^2}$$

$$J_{rec} = \frac{qWn_i}{2\tau_r}e^{\frac{V}{2V_T}}$$

but

$$\tau_r = \frac{1}{\sigma_o v_{th} N_t} = \frac{1}{(10^{-14})(10^7)(3 \times 10^{13})} = 3.333 \times 10^{-7}\ \text{sec}$$

so

$$J_{rec} = \frac{1.60218 \times 10^{-19}\left(10^{-4}\right)\left(1.45 \times 10^{10}\right)}{2\left(3.333 \times 10^{-7}\right)}e^{\frac{0.675}{2(0.0259)}} = \boxed{0.159 \text{ Amp per cm}^2}$$

so total forward current density is 76.82 + 0.159 = 76.979 $Amp\ per\ cm^2$

but $J = \frac{I}{Area}$

so,

$$\boxed{\text{I=76.979×10}^-\text{3=0.0769 Amp}}$$

6 QUESTION 2, HW5

Determine the forward bias required to sustain a forward current of 10 mA and assume all other particulars are the same as problem 1.

6.0.1 solution

since

$$J = \frac{I}{Area}$$

then

$$I = A\left(J_{rec} + J_{jun}\right) \qquad (1)$$

where

$$\left.\begin{array}{r} J_{jun} = J_{saturation} e^{\frac{V}{V_T}} \\ J_{saturation} = q\sqrt{\frac{D_p}{\tau_p}}\frac{n_i^2}{N_D} \end{array}\right\} \quad (2)$$

D_p is found from table to be 12 cm^2 sec so from equation (2):

$$J_{jun} = 1.60218 \times 10^{-19}\sqrt{\frac{12}{10^{-7}}}\frac{\left(1.45 \times 10^{10}\right)^2}{10^{15}} \quad e^{\frac{V}{0.0259}}$$

$$J_{rec} = \frac{qWn_i}{2\tau_r} e^{\frac{V}{2V_T}}$$

but

$$\tau_r = \frac{1}{\sigma_o v_{th} N_t} = \frac{1}{(10^{-14})(10^7)(3 \times 10^{13})} = 3.333 \times 10^{-7} \text{ sec}$$

so

$$J_{rec} = \frac{1.60218 \times 10^{-19}\left(10^{-4}\right)\left(1.45 \times 10^{10}\right)}{2(3.333 \times 10^{-7})} e^{\frac{V}{2(0.0259)}}$$

so, equation (1) becomes:

$$\begin{array}{c} I = A\left(J_{rec} + J_{jun}\right) \\ \frac{10 \times 10^{-3}}{A} = 1.60218 \times 10^{-19}\sqrt{\frac{12}{10^{-7}}}\frac{\left(1.45 \times 10^{10}\right)^2}{10^{15}} \quad e^{\frac{V}{0.0259}} + \frac{1.60218 \times 10^{-19}\left(10^{-4}\right)\left(1.45 \times 10^{10}\right)}{2(3.333 \times 10^{-7})} e^{\frac{V}{2(0.0259)}} \\ 10 = 3.69 \times 10^{-10} x + 3.485 \times 10^{-7} x^{\frac{1}{2}} \end{array} \quad (3)$$

where $x = \quad e^{\frac{V}{0.0259}}$

so, from equation (3), solve for x

let $y^2 = x$ so equation (3) becomes

$$10 = 3.69 \times 10^{-10} y^2 + 3.485 \times 10^{-7} y$$
$$y = \frac{-4b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $ay^2 + by + c = 0 \Rightarrow a = 3.69 \times 10^{-10}, b = 3.485 \times 10^{-7}, c = -10$

$$y = \frac{-4\left(3.485 \times 10^{-7}\right) \pm \sqrt{\left(3.485 \times 10^{-7}\right)^2 (3.69 \times 10^{-10}) 10}}{2(3.69 \times 10^{-10})} = \frac{-1.394 \times 10^{-6} \pm 2.117 \times 10^{-11}}{7.38 \times 10^{-10}}$$
$$y = -1888.9$$

but $y^2 = x$, so

$$x = 3567943$$
$$e^{\frac{V}{0.0259}} = 3567943$$
$$\frac{V}{0.0259} = \ln(3567943) = 15.087$$
$$\boxed{\text{V=0.390 V}}$$

7 QUESTION 3, HW5

Determine the forward current through a $p^+ - n$ junction device at forward bias of .675 V, given the following particulars:

$\sigma_n = \sigma_p = 10^{-14} cm^2,\ v_{th} = 10^7 cm/\text{sec},\ n_i = 10^{10} cm^{-3},\ N_A = 10^{18} cm^{-3},\ N_D = 10^{15} cm^{-3},\ A = 10^{-3} cm^2,\ \tau_p = 10^{-7}\text{ sec},\ w = deplectionwidth = 1\mu m,\ N_t = 10^{16} cm^{-3}$

7.0.1 solution

in a forward biased junction, the process is recombination U, i.e. $pn \gg n_i^2$.

so the forward current (the current entering the n side) is made of 2 components, the recombination current j_{rec} and the junction current j_{jun}

$$J_{forward} = J_{rec} + J_{jun}$$

$$\left.\begin{aligned} J_{jun} &= J_{saturation} e^{\frac{V}{V_T}} \\ J_{saturation} &= q\sqrt{\frac{D_p}{\tau_p}}\frac{n_i^2}{N_D} \end{aligned}\right\} \tag{1}$$

note, equation (1) above is for low injection, where $p_{no} \gg n_{po}$ and where $V \gg 3\frac{KT}{q}$ this assumption is valid since

$p_{no} = \frac{n_i^2}{N_D} = \frac{\left(1.45\times10^{10}\right)^2}{10^{15}} = 210250\ cm^{-3}$, and $n_{po} = \frac{n_i^2}{N_A} = \frac{\left(1.45\times10^{10}\right)^2}{10^{18}} = 210\ cm^{-3}$. so $p_{no} \gg n_{po}$

and $V = 0.675\ V$ while $3\frac{KT}{q}$ $=3\,(0.0259) = 0.0777V$ so $V \gg 3\frac{KT}{q}$. so this is a reasonable approximation.

if this assumption is not valid, one needs to use this equation instead for finding the saturation current:

$$J_{sat} = q\left(\frac{D_p p_{no}}{L_p} + \frac{D_n n_{po}}{L_n}\right)$$

D_p is found from table to be 12 cm^2 sec so from equation (1):

$$J_{jun} = 1.60218 \times 10^{-19}\sqrt{\frac{12}{10^{-7}}}\frac{\left(1.45\times10^{10}\right)^2}{10^{15}}\ e^{\frac{0.675}{0.0259}} = \boxed{76.82 \quad \text{Amp per cm}^2}$$

$$J_{rec} = \frac{qWn_i}{2\tau_r}e^{\frac{V}{2V_T}}$$

but

$$\tau_r = \frac{1}{\sigma_o v_{th} N_t} = \frac{1}{(10^{-14})(10^7)(10^{16})} = 10^{-9}\sec$$

so

$$J_{rec} = \frac{1.60218 \times 10^{-19}\left(10^{-4}\right)\left(1.45 \times 10^{10}\right)}{2\left(10^{-9}\right)} e^{\frac{0.675}{2(0.0259)}} = \boxed{53 \text{ Amp per cm}^2}$$

so total forward current density is 76.82 + 53 = 129.8 $Amp\ per\ cm^2$

but $J = \frac{I}{Area}$

so,

$$\boxed{\text{I=129.8×10}^{-}\text{3=0.1298 Amp}}$$

notice that when the recombination centers numbers increased, the recombination current component increased, about 1000 times increase in the number of centers caused an increase from 0.159 to 53 $Amp/\sec$ increase.

8 QUESTION 4,HW5

3/1/1993

Determine N_t through a $p^+ - n$ junction device at forward bias of .64 V, and diod current is 35 mA given the following particulars:

$\sigma_n = \sigma_p = 10^{-14} cm^2,\ v_{th} = 10^7 cm/\sec,\ n_i = 10^{10} cm^{-3},\ N_A = 10^{18} cm^{-3},\ N_D = 10^{15} cm^{-3},\ A = 10^{-3} cm^2,\ \tau_p = 10^{-7}\sec,\ w = deplectionwidth = 1\mu m$

8.0.1 solution

in a forward biased junction, the process is recombination U, i.e. $pn \gg n_i^2$.

so the forward current (the current entering the n side) is made of 2 components, the recombination current j_{rec} and the junction current j_{jun}

$$J_{forward} = J_{rec} + J_{jun}$$

$$\left.\begin{aligned} J_{jun} &= J_{saturation} e^{\frac{V}{V_T}} \\ J_{saturation} &= q\sqrt{\frac{D_p}{\tau_p}} \frac{n_i^2}{N_D} \end{aligned}\right\} \quad (1)$$

D_p is found from table to be 12 cm^2 sec so from equation (1):

$$J_{jun} = 1.60218 \times 10^{-19} \sqrt{\frac{12}{10^{-7}}} \frac{\left(1.45 \times 10^{10}\right)^2}{10^{15}} \ e^{\frac{0.64}{0.0259}} = \boxed{19 \quad \text{Amp per cm}^2}$$

now,

$$J_{rec} = \frac{qWn_i}{2\tau_r} e^{\frac{V}{2V_T}}$$

but

$$\tau_r = \frac{1}{\sigma_o v_{th} N_t} = \frac{1}{(10^{-14})(10^7) N_t} = \frac{10^7}{N_t} \text{ sec}$$

so

$$J_{rec} = \frac{N_t \times 1.60218 \times 10^{-19} \left(10^{-4}\right) \left(1.45 \times 10^{10}\right)}{2\left(10^7\right)} e^{\frac{0.675}{2(0.0259)}} = \boxed{5.3 \times 10^-15 \times N_t \text{ Amp per cm}^2}$$

so total forward current density is

$19 + 5.3 \times 10^{-15} \times N_t = 35$ Amp per cm^2

so,

$$N_t = \frac{35 - 19}{5.3 \times 10^{-15}} = \boxed{3.0188 \times 10^15 \text{ cm}^-3}$$

9 problem 1, set 6

3/10/93

$$V_{bi} = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.0259 \ln\left(\frac{10^{18} 10^{16}}{(1.45 \times 10^{10})^2}\right) = 0.8156 \text{ V}$$

depletion junction width

$$W = \sqrt{\frac{2\epsilon_s}{q}} \sqrt{\left(\frac{1}{N_D} + \frac{1}{N_A}\right)(V_{bi} - V_{bias})} = 3604 \sqrt{\left(\frac{1}{10^{18}} + \frac{1}{10^{16}}\right)(0.8156 - V_{bias})}$$

but for p-n abrupt junction

$$E_{\max} = \frac{2\left(V_{bi} - V_{bias}\right)}{W}$$

since break down will occur when $E_{\max}$ reaches $E_{crtitcal}$

so from above equation

$$2 \times 10^5 = \frac{2\left(V_{bi} - V_{bias}\right)}{W} = \frac{2\left(0.8156 - V_{bias}\right)}{3604\sqrt{\left(\frac{1}{10^{18}} + \frac{1}{10^{16}}\right)(0.8156 - V_{bias})}}$$

solve the above for V_{bias}

$$V_{bias} = 0.8156 - 13.11 = -12.3 \text{ V}$$

to the break down voltage is $\boxed{0.8156 - 12.3 = 13.11 \text{ V}}$

depletion width at V_{br}

$$W = 3604\sqrt{\left(\frac{1}{10^{18}} + \frac{1}{10^{16}}\right)(13.11)} = 1.31 \times 10^{-4} cm$$

10 problem 2, set 6

$$V_{bi} = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.0259 \ln\left(\frac{10^{18} 10^{18}}{(1.45 \times 10^{10})^2}\right) = 0.9349 \text{ V}$$

depletion junction width

$$W = \sqrt{\frac{2\epsilon_s}{q}} \sqrt{\left(\frac{1}{N_D} + \frac{1}{N_A}\right)(V_{bi} - V_{bias})} = 3604 \sqrt{\left(\frac{1}{10^{18}} + \frac{1}{10^{18}}\right)(0.9349 - V_{bias})}$$

but for p-n abrupt junction

$$E_{\max} = \frac{2\,(V_{bi} - V_{bias})}{W}$$

since we assume that break down will occur when $E_{\max} = E_{crtitcal}$

so

$$10^6 = \frac{2\,(V_{bi} - V_{bias})}{W} = \frac{2\,(0.9349 - V_{bias})}{3604\sqrt{\left(\frac{1}{10^{18}} + \frac{1}{10^{18}}\right)(0.9349 - V_{bias})}}$$

solve the above for V_{bias}

$$V_{bias} = 0.9349 - 6.49 = -5.559 \text{ V}$$

to the break down voltage is $\boxed{6.49 \text{ V}}$

depletion width at V_{br}

$$W = 3604 \sqrt{\left(\frac{1}{10^{18}} + \frac{1}{10^{18}}\right)(6.49)} = 12.9 \times 10^{-4} cm$$

11 problem 3 set 6

in BJT punch through occurs when the collector depletion region extends too far into the base that it reaches the emitter depletion region, i.e. making the active base width to be zero.

for the emitter-base depletion region

$$V_{bi} = 0.0259 \ln\left(\frac{5\times 10^{18}\times 10^{15}}{(2.1\times 10^{10})^2}\right) = 0.778 \text{ v}$$

the emitter -base width

$$W = \sqrt{\frac{2\epsilon_s}{q}}\sqrt{\left(\frac{1}{N_E}+\frac{1}{N_B}\right)(V_{bi}-V_{bias})}$$

$$W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{5\times 10^{18}}+\frac{1}{10^{15}}\right)(0.778-0.5)} = 6\times 10^{-5}cm$$

emitter depletion region part into the base

$$W_{EB}^{'} = W\left(\frac{N_E}{N_E+N_B}\right) = 6\times 10^{-5}\left(\frac{5\times 10^{18}}{5\times 10^{18}+10^{15}}\right) = 5.99\times 10^{-5}cm$$

for the collector-base

$$V_{bi} = 0.0259 \ln\left(\frac{10^{15}\times 10^{15}}{(2.1\times 10^{10})^2}\right) = 0.558 \text{ v}$$

collector-base depletion width

$$W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{10^{15}}+\frac{1}{10^{15}}\right)(0.558-V_{CB})} \tag{1}$$

collector depeltion region into the base

$$\begin{aligned} W_{CB}^{'} &= \sqrt{\tfrac{2\epsilon_s}{q}\left(\tfrac{1}{10^{15}}+\tfrac{1}{10^{15}}\right)(0.558-V_{CB})}\left(\tfrac{N_c}{N_c+N_B}\right) \\ &= \tfrac{1}{2}\sqrt{\tfrac{2\epsilon_s}{q}\left(\tfrac{1}{10^{15}}+\tfrac{1}{10^{15}}\right)(0.558-V_{CB})} \end{aligned}$$

but total W is given as $5\mu m = 5\times 10^{-4}cm$

so

$$W_{EB}^{'} + W_{CB}^{'} = 5 \times 10^{-4} cm$$

$$5.99 \times 10^{-5} + \frac{1}{2}\sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{10^{15}} + \frac{1}{10^{15}}\right)(0.558 - V_{CB})} = 5 \times 10^{-4} cm$$

so

$$V_{CB_{critical}} = -29.8 \text{ V}$$

so

$$\boxed{\text{V}_b r\text{=30.35 V}}$$

for abrupt junction,

$$30.35 = \frac{1}{2} E_{critical} W$$

but from (1)

$$W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{10^{15}} + \frac{1}{10^{15}}\right)(30.35)} = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{10^{15}} + \frac{1}{10^{15}}\right)(30.35)} = 8.88 \times 10^{-4} cm$$

so

$$E_{critical} = 68370 \text{ V/cm}$$

now, for avalanche to happen

$$\int_0^w \alpha dx = 1$$

where α is from figure 27 page 67 for the $E_{cirtical}$ we just found. so find α and use W = the collector-base depletion width found above $W = 8.88 \times 10^{-4} cm$, and see if the integral goes to 1 or not, if it is, then avalanche will occur first else punchthrough.

also we see that $\text{V}_{br} > \frac{6E_g}{q} = \frac{6(1.12)}{q}$, so avalanche will occure first.