HW for ECE 3343 EM, Northeastern Univ. Boston

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1993 Compiled on November 16, 2018 at 11:24am [public]

Contents

1 problem 3-11

suppose there exists within the rectangular cavity of fig 2-19 a field

$$E_x = E_o \sin\frac{\pi y}{b} \sinh \gamma z$$

where $\gamma = \sqrt{\left(\frac{\pi}{b}\right)^2 - k^2}$ and k is complex (lossy dielectric). show that this field can be supported by source

$$\mathbf{M}_s = -\mathbf{u}_y E_o \sin\frac{\pi y}{b} \sinh \gamma c$$

at the wall $z = c$. Show that for low loss dielectric, $\mathbf{M}_s$ almost vanishes at the resonant frequency $f_r = \frac{1}{2bc}\sqrt{\frac{b^2+c^2}{\epsilon\mu}}$, that is, a small $\mathbf{M}_s$ produces a large $\mathbf{E}$

1.1 Solution

Using the equivalence theorem, find a current $\mathbf{M}_s$ that will generate the E field given.

first find the E field at the boundary of the region. i.e at $z = c$

$$\mathbf{E}_{z=c} = \mathbf{u}_x E_o \sin\frac{\pi y}{b} \sinh \gamma c$$

so,

$$\mathbf{M_s} = \mathbf{E}_{z=c} \times \mathbf{n}$$

where $\mathbf{n}$ is unit vector pointing out of the region where the original sources were. so $\mathbf{n} = \mathbf{u_z}$

$$\mathbf{M_s} = \begin{vmatrix} u_x & u_y & u_z \\ E_o \sin\frac{\pi y}{b} \sinh \gamma c & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -\mathbf{u_y} E_o \sin\frac{\pi y}{b} \sinh \gamma c$$

now,

$$k = k^{'} - jk^{''}$$

so,

$$\gamma = \sqrt{\left(\frac{\pi}{b}\right)^2 - (k^{'} - jk^{''})^2} \qquad (1)$$

but,

$$k^{'} = \omega\sqrt{\mu\epsilon} \text{ and } k^{''} = \frac{\omega\epsilon^{''}}{2}\sqrt{\frac{\mu}{\epsilon^{'}}} \qquad (2)$$

so, substitute (2) in (1)

$$\gamma = \sqrt{\left(\frac{\pi}{b}\right)^2 - \left(\omega\sqrt{\mu\epsilon} - j\frac{\omega\epsilon^{''}}{2}\sqrt{\frac{\mu}{\epsilon^{'}}}\right)^2} \quad (3)$$

but

$$\epsilon = \epsilon^{'} - j\epsilon^{''}$$

so, from (3) we have

$$\begin{aligned}
\gamma &= \sqrt{\left(\frac{\pi}{b}\right)^2 - \left(\omega\sqrt{\mu\left(\epsilon' - j\epsilon''\right)} - j\frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}}\right)^2} \\
&= \sqrt{\left(\frac{\pi}{b}\right)^2 - \left(\omega^2\mu\left(\epsilon' - j\epsilon''\right) - \frac{\omega^2\left(\epsilon''\right)^2}{4}\frac{\mu}{\epsilon^{'}} - j\omega^2\epsilon''\ \sqrt{\mu\left(\epsilon' - j\epsilon''\right)}\sqrt{\frac{\mu}{\epsilon^{'}}}\right)} \\
&= \sqrt{\left(\frac{\pi}{b}\right)^2 - \left(\omega^2\mu\left(\epsilon' - j\epsilon''\right) - \frac{\omega^2\left(\epsilon''\right)^2}{4}\frac{\mu}{\epsilon^{'}} - j\omega^2\epsilon''\ \sqrt{\mu^2\left(1 - j\frac{\epsilon''}{\epsilon'}\right)}\right)} \\
&= \sqrt{\left(\frac{\pi}{b}\right)^2 - \left(\omega^2\mu\epsilon' - j\omega^2\mu\epsilon'' - \frac{\omega^2\mu\left(\epsilon''\right)^2}{4\epsilon'} - j\omega^2\mu\epsilon^{''}\ \sqrt{1 - j\frac{\epsilon''}{\epsilon'}}\right)} \\
&= \sqrt{\left(\frac{\pi}{b}\right)^2 - \omega^2\mu\epsilon' + j\omega^2\mu\epsilon'' + \frac{\omega^2\mu\left(\epsilon''\right)^2}{4\epsilon'} + j\omega^2\mu\epsilon''\sqrt{1 - j\frac{\epsilon^{''}}{\epsilon'}}}
\end{aligned}$$

when low loss dielectric, $\epsilon^{''} \ll \epsilon^{'}$ so the above value for γ can be simplified to

$$\gamma = \sqrt{\left(\frac{\pi}{b}\right)^2 - \omega^2\mu\epsilon' + 2j\omega^2\mu\epsilon''} \quad (4)$$

but at resonant frequency, $\omega = 2\pi f_r = 2\pi\left(\frac{1}{2bc}\sqrt{\frac{b^2+c^2}{\epsilon\mu}}\right) = 2\pi\left(\frac{1}{2bc}\sqrt{\frac{b^2+c^2}{\mu(\epsilon'-j\epsilon'')}}\right)$

so from (4) we get

$$
\begin{aligned}
\gamma &= \sqrt{\left(\frac{\pi}{b}\right)^2 - \omega^2\mu\left(\epsilon' - 2j\epsilon''\right)} \\
&= \sqrt{\left(\frac{\pi}{b}\right)^2 - \left(\frac{2\pi}{2bc}\sqrt{\frac{b^2+c^2}{\left(\epsilon' - j\epsilon''\right)\mu}}\right)^2 \mu\left(\epsilon' - 2j\epsilon''\right)} \\
&= \sqrt{\left(\frac{\pi}{b}\right)^2 - \frac{4\pi^2}{4b^2c^2}\left(\frac{b^2+c^2}{\left(\epsilon' - j\epsilon''\right)\mu}\right)\mu\left(\epsilon' - 2j\epsilon''\right)} \\
&= \sqrt{\left(\frac{\pi}{b}\right)^2 - \frac{4\pi^2}{4b^2c^2}\left(\frac{b^2+c^2}{\left(\epsilon' - j\epsilon''\right)}\right)\left(\epsilon' - 2j\epsilon''\right)} \\
&= \sqrt{\left(\frac{\pi}{b}\right)^2 - \left(\frac{\pi^2}{c^2\left(\epsilon' - j\epsilon''\right)} + \frac{\pi^2}{b^2\left(\epsilon' - j\epsilon''\right)}\right)\left(\epsilon' - 2j\epsilon''\right)}
\end{aligned}
$$

so, simplifying γ further gives

$$
\begin{aligned}
\gamma &= \sqrt{\left(\frac{\pi}{b}\right)^2 - \left(\frac{\pi^2\left(\epsilon' - 2j\epsilon''\right)}{c^2\left(\epsilon' - j\epsilon''\right)} + \frac{\pi^2\left(\epsilon' - 2j\epsilon''\right)}{b^2\left(\epsilon' - j\epsilon''\right)}\right)} \\
&= \sqrt{\frac{\pi^2}{b^2} - \pi^2\left(\frac{\left(\epsilon' - 2j\epsilon''\right)}{c^2\left(\epsilon' - j\epsilon''\right)} + \frac{\left(\epsilon' - 2j\epsilon''\right)}{b^2\left(\epsilon' - j\epsilon''\right)}\right)} \\
&= \sqrt{\frac{\pi^2}{b^2} - \pi^2\left(\frac{\epsilon'}{c^2\epsilon' - jc^2\epsilon''} - \frac{2j\epsilon''}{c^2\epsilon' - jc^2\epsilon''} + \frac{\epsilon'}{b^2\epsilon' - jb^2\epsilon''} - \frac{2j\epsilon''}{b^2\epsilon' - jb^2\epsilon''}\right)}
\end{aligned}
$$

sinceϵ'' is very small, terms with ϵ'' in denominator can be removed, resulting in

$$
\begin{aligned}
\gamma &= \sqrt{\frac{\pi^2}{b^2} - \pi^2\left(\frac{\epsilon'}{c^2\epsilon'} - \frac{2j\epsilon''}{c^2\epsilon'} + \frac{\epsilon'}{b^2\epsilon'} - \frac{2j\epsilon''}{b^2\epsilon'}\right)} \\
&= \pi\sqrt{\frac{1}{b^2} - \frac{1}{c^2} - \frac{1}{b^2} + \frac{2j\epsilon''}{\epsilon'}\left(\frac{1}{c^2} + \frac{1}{b^2}\right)} \\
&= \pi\sqrt{-\frac{1}{c^2} + \frac{2j\epsilon''}{\epsilon'}\left(\frac{1}{c^2} + \frac{1}{b^2}\right)}
\end{aligned}
$$

so, from equation (5), we see that as

$$
\epsilon'' \Longrightarrow 0, \gamma \Longrightarrow j\frac{\pi}{c}
$$

but $M_y = E_o \sin\frac{\pi y}{b}\sinh\left(\gamma c\right)$ so

$$
\text{as } \epsilon'' \Longrightarrow 0\ , M_y \Longrightarrow E_o \sin\frac{\pi y}{b}\sinh\left(j\pi\right)
$$

but $\sinh(j\pi) = j\sin(\pi) = 0$, so

$$\text{as } \epsilon'' \Longrightarrow 0\ , M_y \Longrightarrow 0$$

QED

2 problem 3-13

in fig 3-6a , suppose we have a small loop of electric current with z-directed moment IS , instead of te current element, show that radiation field is given by

$$E_\phi = \frac{j\eta 2\pi IS}{\lambda^2 r} e^{-jkr} \sin(kd\ \cos\theta)\sin\theta$$

and $\eta H_\theta = -E_\phi$. find the power radiated and show that the radiation resistance referred to I is

$$R_r = 2\pi\eta\left(\frac{KS}{\lambda}\right)^2\left[\frac{1}{3} + \frac{\cos 2kd}{(2kd)^2} - \frac{\sin 2kd}{(2kd)^2}\right]$$

2.1 Solution

I'll use duality to solve the first part.

first replace the small current loop by the equivelant magnetic current element Kl

next, we know what is the solution (field) due to electric current element Il, it is:

for the object current element Il at distance d from the ground and pointing **away** from the ground is given by equation 2-114 at page 79 as

$$H_\phi = \frac{jIl}{2\lambda r} e^{-jkr} \sin\theta \qquad (1)$$

Now, I put an image current element at distance d below the ground, but make the current image element point also **away** from the ground (not towars the ground as is normal), this is so I can use duality later on to get the field due to magnetic current, since the image for mahnetic current is pointing to opposit direction from the object magnetic current.

so, for the image current element Il at distance d from the ground and pointing **away** from the ground is given by

$$H_\phi = -\frac{jIl}{2\lambda r} e^{-jkr} \sin\theta \qquad (2)$$

notice the minus sign in equation (2), this is since current is flowing in opposit direction, and magnetic field generated by electric current going in opposit directions will be in oppsoiti directions also (right hand rule).

so, now use duality on equation (1) and (2) to find the field due to magetic current Kl

note that

$$r_o = r - d\cos\theta$$
$$r_i = r + d\cos\theta$$

equation (1) dual becomes

$$-E_{\phi_o} = \frac{jKl}{2\lambda r_o} e^{-jkr_o} \sin\theta = \frac{jKl}{2\lambda\left(r - d\cos\theta\right)} e^{-jk(r-d\cos\theta)} \sin\theta \qquad (3)$$

equation (2) dual becomes

$$-E_{\phi_i} = -\frac{jKl}{2\lambda r_i} e^{-jkr_i} \sin\theta = -\frac{jKl}{2\lambda\left(r + d\cos\theta\right)} e^{-jk(r+d\cos\theta)} \sin\theta \qquad (4)$$

so total field due to the magnetic current Kl is given by superposition of equation (3) and (4)

$$\begin{aligned}
-E_\phi &= \frac{jKl}{2\lambda}\left(\frac{e^{-jk(r-d\cos\theta)}}{(r - d\cos\theta)} - \frac{e^{-jk(r+d\cos\theta)}}{(r + d\cos\theta)}\right)\sin\theta \\
&= \frac{jKl}{2\lambda}\left(\frac{e^{-jkr} e^{jkd\cos\theta}}{(r - d\cos\theta)} - \frac{e^{-jkr} e^{-jkd\cos\theta}}{(r + d\cos\theta)}\right)\sin\theta \\
&= \frac{jKl}{2\lambda}\left(\frac{e^{-jkr} e^{jkd\cos\theta}}{r} - \frac{e^{-jkr} e^{-jkd\cos\theta}}{r}\right)\sin\theta \quad \text{when } r \gg d \\
&= \frac{jKl}{2\lambda r} e^{-jkr}\left(e^{jkd\cos\theta} - e^{-jkd\cos\theta}\right)\sin\theta \\
&= \frac{jKl}{2\lambda r} e^{-jkr}\left((\cos\alpha + j\sin\alpha) - (\cos\alpha - j\sin\alpha)\right)\sin\theta \quad \text{where } \alpha = kd\cos\theta \\
&= \frac{jKl}{2\lambda r} e^{-jkr}\left(2j\sin\alpha\right)\sin\theta
\end{aligned}$$

so, this means that

$$\begin{aligned}
-E_\phi &= -\frac{Kl}{\lambda r} e^{-jkr} \sin\left(kd\cos\theta\right)\sin\theta \\
E_\phi &= \frac{Kl}{\lambda r} e^{-jkr} \sin\left(kd\cos\theta\right)\sin\theta \qquad (5)
\end{aligned}$$

now, this equation (5) is used for the duality, replace Kl by $j\omega\mu IS$ in (5) we get

$$E_\phi = \frac{j\omega\mu IS}{\lambda r} e^{-jkr} \sin\left(kd\cos\theta\right)\sin\theta \qquad (6)$$

but

$$c = \lambda f$$

and $c = \frac{1}{\sqrt{\epsilon\mu}}$ so

$$\frac{1}{\sqrt{\epsilon\mu}} = \lambda f$$
$$f = \frac{1}{\lambda\sqrt{\epsilon\mu}}$$

and

$$\omega = 2\pi f = 2\pi\frac{1}{\lambda\sqrt{\epsilon\mu}}$$

so, from (6) we get

$$\begin{aligned} E_\phi &= \frac{j\left(2\pi\frac{1}{\lambda\sqrt{\epsilon\mu}}\right)\mu IS}{\lambda r}e^{-jkr}\sin\left(kd\cos\theta\right)\sin\theta \\ &= \frac{j2\pi\mu IS}{\lambda^2 r\sqrt{\epsilon\mu}}e^{-jkr}\sin\left(kd\cos\theta\right)\sin\theta \\ &= \frac{j2\pi\eta IS}{\lambda^2 r}e^{-jkr}\sin\left(kd\cos\theta\right)\sin\theta \quad \text{since } \eta = \sqrt{\frac{\mu}{\epsilon}} \end{aligned}$$

QED.

to find the power the power radiated is

$$p_f = \iint\limits_{\substack{hemi- \\ sphere}} Re\left(\mathbf{E}\times\mathbf{H}^*\right)\ ds$$

so

$$\begin{aligned} p_f &= \iint\limits_{\substack{hemi- \\ sphere}} E_\phi H_\theta^*\ ds \\ &= 2\pi\eta\int_0^{\frac{\pi}{2}} |H_\theta|^2 r^2 \sin\theta\ d\theta \end{aligned} \tag{7}$$

but

$$\begin{aligned} H_\theta &= -\frac{E_\phi}{\eta} \\ &= -\frac{\frac{j2\pi\eta IS}{\lambda^2 r}e^{-jkr}\sin\left(kd\cos\theta\right)\sin\theta}{\eta} \\ &= -\frac{j2\pi IS}{\lambda^2 r}e^{-jkr}\sin\left(kd\cos\theta\right)\sin\theta \end{aligned}$$

Hence

$$
\begin{aligned}
|H_\theta|^2 &= \left|\frac{2\pi IS}{\lambda^2 r}\sin\left(kd\cos\theta\right)\sin\theta\right|^2 \\
&= \left|\frac{2\pi IS}{\lambda^2 r}\right|^2 \sin^2\left(kd\cos\theta\right)\sin^2\theta
\end{aligned}
\tag{8}
$$

so, from (7) and (8)

$$
\begin{aligned}
p_f &= 2\pi\eta\int_0^{\frac{\pi}{2}}\left|\frac{2\pi IS}{\lambda^2 r}\right|^2 r^2\sin^2\left(kd\cos\theta\right)\sin^3\theta d\theta \\
&= 2\pi\eta\left|\frac{2\pi IS}{\lambda^2}\right|^2\int_0^{\frac{\pi}{2}}\sin^2\left(kd\cos\theta\right)\sin^3\theta\ d\theta
\end{aligned}
$$

3 problem 3-17

solution:

the magnetic current density $\mathbf{M_s}$ that will generate the same field as the impressed voltage source ,outside the region where the sources are, is given by

$$\mathbf{M_s} = 2\mathbf{E}_{\substack{boundary \\ surface}} \times \mathbf{n}$$

where the boundary surface is the surface that separates the region where the sources are from the outside region.

To find the boundary surface, we see that the E field propagates in the x direction, so construct the plane along $y=\delta$,where δ is a small distance away from the plane $y=0$, this will make n point in the y^+ direction, i.e.

$$\mathbf{n} = \mathbf{u}_y$$

later on, to find the field in the $y<0$ region, we make $\mathbf{n}=-\mathbf{u}_y$ and place the mathematical plane at $y=-\delta$.

now, since $\mathbf{E}=\mathbf{u_x}\frac{V_m}{w}\sin\left[k\left(\frac{L}{2}-|z|\right)\right]$, where k is the wave number of the medium, given by $k=\sqrt{-\widehat{zy}}$, then this $\mathbf{E}$ is the tangential field along the plan $y=\delta$, so

$$\mathbf{M_s} = 2\left(\mathbf{u_x}\frac{V_m}{w}\sin\left[k\left(\frac{L}{2}-|z|\right)\right]\times\mathbf{u_y}\right)$$

$$M_s = 2\begin{vmatrix} u_x & u_y & u_z \\ \frac{V_m}{w}\sin\left[k\left(\frac{L}{2}-|z|\right)\right] & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = u_z\, 2\frac{V_m}{w}\sin\left[k\left(\frac{L}{2}-|z|\right)\right]$$

now, to find the magnetic current K from $\mathbf{M_s}$ above, since the width is w then total magnetic current is given by

$$K = |\mathbf{M_s}|\, w = 2V_m \sin\left[k\left(\frac{L}{2}-|z|\right)\right] \tag{1}$$

now, the problem of sec 2-10 is that of the linear antenna, in that case we were given a current source

$$I = I_m \sin\left[k\left(\frac{L}{2}-|z|\right)\right] \tag{2}$$

the above was source of the field in that problem. from duality, $K \equiv I$, compare (1) and (2) , we see that

$$I_m \equiv 2V_m \tag{3}$$

now, use this substitution in the solution for the dipole antenna problem, and other dual substitution, to find the field due to the magnetic current density $\mathbf{M}_s$ in the original problem. Since we have solved the field for the dipole antenna problem, and the solution is given by equation 2-125 page 82 as

$$E_\theta = \frac{j\eta I_m e^{-jkr}}{2\pi r}\left[\frac{\cos\left(k\frac{L}{2}\cos\theta\right) - \cos\left(k\frac{L}{2}\right)}{\sin\theta}\right] \tag{4}$$

so, apply duality to (4) using $E_\theta \equiv H_\theta$, $\eta \equiv \frac{1}{\eta}$ and using $I_m \equiv 2V_m$, apply these replacements into (4) , results in

$$H_\theta = \frac{jV_m e^{-jkr}}{\eta\pi r}\left[\frac{\cos\left(k\frac{L}{2}\cos\theta\right) - \cos\left(k\frac{L}{2}\right)}{\sin\theta}\right] \tag{5}$$

note that the above expression was derived with $\mathbf{n} = \mathbf{u_y}$, this was done to find the field in the $y > 0$ region, since the $\mathbf{n}$ vector in always in the direction pointing away from the region where the sources are and into the region where we are interested to find the field due to these sources.

so, to find the field in $y < 0$ region, we put our mathematical plane that divides the region where the sources are and the region where we want to find the field, we put this plane at $y = -\delta$ where δ is small distance from the origin, this makes $\mathbf{n}$ to be $-\mathbf{u_y}$ and this will result in a minus sign added to equation (5) . so

$$H_\theta = -\frac{jV_m e^{-jkr}}{\eta\pi r}\left[\frac{\cos\left(k\frac{L}{2}\cos\theta\right) - \cos\left(k\frac{L}{2}\right)}{\sin\theta}\right]$$

when $y < 0$

second part

now, $G_r = \frac{\overline{p}_{f_{slot-antenna}}}{|V_m|^2}$, but we can use duality from the current dipole antenna to find $\overline{p}_f$ from equation 2-127, we see that for the linear antenna

$$\overline{p}_{f_{wire-antenna}} = \frac{\eta\,|I_m|^2}{2\pi} \int \left[\frac{\cos\left(k\frac{L}{2}\cos\theta\right) - \cos\left(k\frac{L}{2}\right)}{\sin\theta} \right]^2 d\theta$$

applying duality substitution on this we get

$$\overline{p}_{f_{slot-antenna}} = \frac{|2V_m|^2}{\eta 2\pi} \int_0^{\pi} \left[\frac{\cos\left(k\frac{L}{2}\cos\theta\right) - \cos\left(k\frac{L}{2}\right)}{\sin\theta} \right]^2 d\theta$$

so

$$\begin{aligned} G_r &= \frac{\overline{p}_{f_{slot-antenna}}}{|V_m|^2} \\ &= \frac{\frac{|2V_m|^2}{\eta 2\pi} \int_0^{\pi} \left[\frac{\cos\left(k\frac{L}{2}\cos\theta\right) - \cos\left(k\frac{L}{2}\right)}{\sin\theta} \right]^2 d\theta}{|V_m|^2} \\ &= \frac{2}{\eta\pi} \int_0^{\pi} \left[\frac{\cos\left(k\frac{L}{2}\cos\theta\right) - \cos\left(k\frac{L}{2}\right)}{\sin\theta} \right]^2 d\theta \end{aligned} \tag{6}$$

but from equation 2-129 in book, we see that

$$R_{r_{wire-dipole}} = \frac{\eta}{2\pi} \int_0^{\pi} \left[\frac{\cos\left(k\frac{L}{2}\cos\theta\right) - \cos\left(k\frac{L}{2}\right)}{\sin\theta} \right]^2 d\theta \tag{7}$$

so, from (7) we see that

$$\frac{2\pi R_{r_{wire-dipole}}}{\eta} = \int_0^{\pi} \left[\frac{\cos\left(k\frac{L}{2}\cos\theta\right) - \cos\left(k\frac{L}{2}\right)}{\sin\theta} \right]^2 d\theta$$

substitute this expression in (6) results in

$$\begin{aligned} G_r &= \frac{2}{\eta\pi} \frac{2\pi R_{r_{wire-dipole}}}{\eta} \\ &= \frac{4R_{r_{wire-dipole}}}{\eta^2} \end{aligned}$$

finally, since

$$G_{i_{slote-antenna}} = \frac{\overline{p}_{f_{slot-antenna}}}{|V_i|^2} = \frac{\overline{p}_{f_{slot-antenna}}}{\left|V_m \sin\left(\frac{KL}{2}\right)\right|^2} \tag{8}$$

but $G_r = \frac{\overline{p}_{f_{slot-antenna}}}{|V_m|^2}$ so (8) becomes

$$G_{i_{slote-antenna}} = \frac{G_r}{\sin^2\left(\frac{KL}{2}\right)}$$

Q.E.D.

4 problem 3-19

Dr., I solved this using the approach of finding the **F** vector (magnetic vector potential) and from that finding **H**, I know from talking to you on the phone you probably wanted us to use this equation instead:

$$\mathbf{E}(r) = -\nabla \times \iint_{surface} \frac{e^{-jk\left|\mathbf{r}-\mathbf{r}'\right|}}{|\mathbf{r}-\mathbf{r}'|}\mathbf{E}(r') \times \mathbf{ds}$$

but this is how I ended up solving this:

Since E_x inside the wave guide is that of mode TE_{01} then this means that in the relation

$$k_c = \frac{n\pi}{b} \qquad n = 1, 2, 3, \ldots$$

we choose $n = 1$. now , notice that the wave propagates in the y-direction, and the field inside the wave guide has these components: E_x, H_z, H_y and the plane of the wave moves in the y-axis direction. for the filed to be zero at $z = \frac{b}{2}$ and zero at $z = -\frac{b}{2}$ we must have

$$E_x = E_o \cos\left(\frac{\pi}{b}z\right) e^{-\gamma y}$$

also, but since opening of the wave guide is at $y = 0$

$$E_x = E_o \cos\left(\frac{\pi}{b}z\right) \tag{1}$$

now, applying the equivalence principle, we have that the surface magnetic current density is

$$\mathbf{M}_s = 2\mathbf{E} \times \mathbf{n} = \mathbf{u_x}\ 2E_o \sin\left(\frac{\pi}{b}z\right) \times \mathbf{u_y} = \mathbf{u_z}\ 2E_o \cos\left(\frac{\pi}{b}z\right)$$

so

$$M_s = u_z\ 2E_o \cos\left(\frac{\pi}{b}z\right) \tag{2}$$

but we know that for the far field,

$$\mathbf{F} = \frac{\epsilon e^{-jkr}}{4\pi r} \iint_s \mathbf{M}_s e^{jkr' \cos\varpi} \, ds'$$

where ϖ is the angle between $\mathbf{r}$ and $\mathbf{r}'$.

so, for the far field, and since in for this configuration of having the plane parallel to xz plane, we have

$$r' \cos\varpi = x' \sin\theta \cos\phi + z' \cos\theta$$

then

$$\mathbf{F} = \frac{\epsilon e^{-jkr}}{4\pi r} \int_{x'=-a/2}^{a/2} \int_{z'=-b/2}^{b/2} \mathbf{M_s} e^{jk\left(x' \sin\theta\cos\phi + z' \cos\theta\right)} \, ds'$$

now, $\mathbf{M}_s$ is in the z-direction where

$$\mathbf{u}_z = \mathbf{u_r} \cos\theta + \mathbf{u}_\theta \left(-\sin\theta\right)$$

radial components the magnetic current density is zero, so $\mathbf{M}_s$ becomes

$$\mathbf{M}_s = -\mathbf{u}_\theta \sin\theta \left(2E_o \cos\left(\frac{\pi}{b} z'\right)\right)$$

so

$$F_\theta = -b/2^b/2 - 2E_o \cos\left(\frac{\pi}{b} z'\right) \sin\theta \; e^j k \left(x' \sin\theta\cos\phi + z' \cos\theta\right) \; ds'$$

and

$$H_\theta = -j\omega F_\theta$$

let

$$L_\theta = \int_{x'=-a/2}^{a/2} \int_{z'=-b/2}^{b/2} -2E_o \cos\left(\frac{\pi}{b} z'\right) \sin\theta \; e^{jk\left(x' \sin\theta\cos\phi + z' \cos\theta\right)} \; ds'$$

so

$$F_\theta = \frac{\epsilon \; e^- jkr}{4\pi r} L_\theta$$

$$
\begin{aligned}
L_\theta &= \int\limits_{x'=-a/2}^{a/2} \int\limits_{z'=-b/2}^{b/2} \left[-2E_o \cos\left(\frac{\pi}{b} z'\right) \sin\theta\right] e^{jk\left(x' \sin\theta\cos\phi + z' \cos\theta\right)} \, dx' dz' \\
&= -2E_o \sin\theta \int\limits_{x'=-a/2}^{a/2} \int\limits_{z'=-b/2}^{b/2} \cos\left(\frac{\pi}{b} z'\right) e^{jk\left(x' \sin\theta\cos\phi + z' \cos\theta\right)} \, dx' dz' \\
&= -2E_o \sin\theta \int\limits_{z'=-b/2}^{b/2} \cos\left(\frac{\pi}{b} z'\right) e^{jkz' \cos\theta} \left[\int\limits_{x'=-a/2}^{a/2} e^{jk\left(x' \sin\theta\cos\phi\right)} \, dx' \right] dz' \\
&= -2E_o \sin\theta \int\limits_{z'=-b/2}^{b/2} \cos\left(\frac{\pi}{b} z'\right) e^{jkz' \cos\theta} \left[\frac{1}{jk\sin\theta\cos\phi} \left| e^{jk\left(x' \sin\theta\cos\phi\right)} \right|_{-a/2}^{a/2} \right] dz' \\
&= -2E_o \sin\theta \int\limits_{z'=-b/2}^{b/2} \cos\left(\frac{\pi}{b} z'\right) e^{jkz' \cos\theta} \left[\frac{1}{jk\sin\theta\cos\phi} \left| e^{jk\left(x' \sin\theta\cos\phi\right)} \right|_{-a/2}^{a/2} \right] dz' \\
&= -2E_o \sin\theta \int\limits_{z'=-b/2}^{b/2} \cos\left(\frac{\pi}{b} z'\right) e^{jkz' \cos\theta} \left[\frac{1}{jk\sin\theta\cos\phi} \left(e^{jk\left(\frac{a}{2}\sin\theta\cos\phi\right)} - e^{-jk\left(\frac{a}{2}\sin\theta\cos\phi\right)} \right) \right] dz' \\
&= -2E_o \sin\theta \left[\frac{1}{jk\sin\theta\cos\phi} \left(e^{jk\left(\frac{a}{2}\sin\theta\cos\phi\right)} - e^{-jk\left(\frac{a}{2}\sin\theta\cos\phi\right)} \right) \right] \int\limits_{z'=-b/2}^{b/2} \cos\left(\frac{\pi}{b} z'\right) e^{jkz' \cos\theta} dz' \\
&= -2E_o \sin\theta \left[\frac{2\sin\left(k\frac{a}{2}\sin\theta\cos\phi\right)}{k\sin\theta\cos\phi} \right] \overbrace{\int\limits_{z'=-b/2}^{b/2} \cos\left(\frac{\pi}{b} z'\right) e^{jkz' \cos\theta} dz'}^{INT}
\end{aligned}
$$

applying integration by parts to the second integral:

$$
I = \int\limits_{z'=-b/2}^{b/2} \cos\left(\frac{\pi}{b} z'\right) e^{jkz' \cos\theta} dz' \tag{4}
$$

where $\int f dv = fv - \int v df$

let $f = \cos\left(\frac{\pi}{b} z'\right) \Rightarrow df = -\frac{\pi}{b} \sin\left(\frac{\pi}{b} z'\right)$

let $dv = e^{jkz' \cos\theta} \Rightarrow v = \frac{1}{jk\cos\theta} e^{jkz' \cos\theta}$

so

$$I = \cos\left(\frac{\pi}{b}z'\right)\frac{1}{jk\cos\theta}e^{jkz'\cos\theta}\Big|_{-b/2}^{b/2} + \int\limits_{z'=-b/2}^{b/2}\frac{1}{jk\cos\theta}e^{jkz'\cos\theta}\,\frac{b}{\pi}\sin\left(\frac{\pi}{b}z'\right)dz'$$

$$= \cos\left(\frac{\pi}{b}z'\right)\frac{1}{jk\cos\theta}e^{jkz'\cos\theta}\Big|_{-b/2}^{b/2} + \frac{\pi}{b}\frac{1}{jk\cos\theta}\overbrace{\int\limits_{z'=-b/2}^{b/2}e^{jkz'\cos\theta}\sin\left(\frac{\pi}{b}z'\right)dz'}^{\text{apply integration by parts again}}$$

apply the integration by parts rule to the second part of the above where

$f = \sin\left(\frac{\pi}{b}z'\right) \Rightarrow df = \frac{\pi}{b}\cos\left(\frac{\pi}{b}z'\right)$

$dv = e^{jkz'\cos\theta} \Rightarrow v = \frac{1}{jk\cos\theta}e^{jkz'\cos\theta}$

so

$$I = \cos\left(\frac{\pi}{b}z'\right)\frac{1}{jk\cos\theta}e^{jkz'\cos\theta}\Big|_{-b/2}^{b/2} + \frac{\pi}{b}\frac{1}{kj\cos\theta}\left\{\sin\left(\frac{\pi}{b}z'\right)\frac{1}{jk\cos\theta}e^{jkz'\cos\theta}\Big|_{-b/2}^{b/2} - \int\limits_{z'=-b/2}^{b/2}\frac{1}{jk\cos\theta}e^{jkz'\cos\theta}\left(\frac{\pi}{b}\right.\right.$$

$$= \cos\left(\frac{\pi}{b}z'\right)\frac{1}{jk\cos\theta}e^{jkz'\cos\theta}\Big|_{-b/2}^{b/2} + \frac{\pi}{b}\frac{1}{kj\cos\theta}\left\{\sin\left(\frac{\pi}{b}z'\right)\frac{1}{jk\cos\theta}e^{jkz'\cos\theta}\Big|_{-b/2}^{b/2} - \frac{\pi}{b}\frac{1}{jk\cos\theta}\int\limits_{z'=-b/2}^{b/2}e^{jkz'\cos\theta}\right.$$

$$= \cos\left(\frac{\pi}{b}z'\right)\frac{1}{jk\cos\theta}e^{jkz'\cos\theta}\Big|_{-b/2}^{b/2} + \frac{\pi}{b}\frac{1}{jk\cos\theta}\left\{\sin\left(\frac{\pi}{b}z'\right)\frac{1}{jk\cos\theta}e^{jkz'\cos\theta}\Big|_{-b/2}^{b/2} - \frac{\pi}{b}\frac{1}{jk\cos\theta}(I)\right\}$$

$$= \cos\left(\frac{\pi}{b}z'\right)\frac{1}{jk\cos\theta}e^{jkz'\cos\theta}\Big|_{-b/2}^{b/2} + \frac{\pi}{b}\frac{1}{jk\cos\theta}\left(\sin\left(\frac{\pi}{b}z'\right)\frac{1}{jk\cos\theta}e^{jkz'\cos\theta}\Big|_{-b/2}^{b/2}\right) - \left(\frac{\pi}{b}\frac{1}{jk\cos\theta}\right)^2(I)$$

Hence

$$I\left(1+\left(\frac{\pi}{b}\frac{1}{jk\cos\theta}\right)^2\right) = \overbrace{\cos\left(\frac{\pi}{b}z'\right)\frac{1}{jk\cos\theta}e^{jkz'\cos\theta}\Big|_{-b/2}^{b/2}}^{\text{first limit to evaluate}} + \frac{\pi}{b}\frac{1}{jk\cos\theta}\left(\overbrace{\sin\left(\frac{\pi}{b}z'\right)\frac{1}{jk\cos\theta}e^{jkz'\cos\theta}\Big|_{-b/2}^{b/2}}^{\text{second limit to evaluate}}\right) \tag{5}$$

now, evaluate the limits as show above

$$\text{first limit} = \cos\left(\tfrac{\pi}{b}z'\right)\tfrac{1}{jk\cos\theta}e^{jkz'\cos\theta}\Big|_{-b/2}^{b/2} = \cos\left(\tfrac{\pi}{b}\tfrac{b}{2}\right)\tfrac{1}{jk\cos\theta}e^{jk\frac{b}{2}\cos\theta} - \cos\left(-\tfrac{\pi}{b}\tfrac{b}{2}\right)\tfrac{1}{jk\cos\theta}e^{-jk\frac{b}{2}\cos\theta} = 0$$

and the second limit term:

$$\text{second limit} = \frac{\pi}{b}\frac{1}{jk\cos\theta}\left(\sin\left(\frac{\pi}{b}z'\right)\frac{1}{jk\cos\theta}e^{jkz'\cos\theta}\Big|_{-b/2}^{b/2}\right) = \frac{\pi}{b}\frac{1}{jk\cos\theta}\left\{\sin\left(\frac{\pi}{b}\frac{b}{2}\right)\frac{1}{jk\cos\theta}e^{jk\frac{b}{2}\cos\theta} - \sin\left(-\right.\right.$$

$$= \frac{\pi}{b}\frac{1}{jk\cos\theta}\left(\left[\frac{1}{jk\cos\theta}\right]\left(e^{jk\frac{b}{2}\cos\theta} + e^{-jk\frac{b}{2}\cos\theta}\right)\right)$$

$$= \frac{\pi}{b}\frac{1}{jk\cos\theta}\left(\left[\frac{1}{jk\cos\theta}\right]2\cos\left(k\frac{b}{2}\cos\theta\right)\right)$$

$$= \frac{2\pi\ \cos\left(k\frac{b}{2}\cos\theta\right)}{-bk^2\cos^2\theta}$$

so, equation (5) becomes

$$I = \frac{\frac{2\pi\ \cos\left(k\frac{b}{2}\cos\theta\right)}{-bk^2\cos^2\theta}}{1+\left(\frac{\pi}{b}\frac{1}{jk\cos\theta}\right)^2} = \frac{2\pi b\cos\left(k\frac{b}{2}\cos\theta\right)}{\pi^2 - [bk\cos\theta]^2}$$

substitute the above equation into equation (3) we get

$$L_\theta = -2E_o\sin\theta\left[\frac{2\sin\left(k\frac{a}{2}\sin\theta\cos\phi\right)}{k\sin\theta\cos\phi}\right](I)$$

$$= -2E_o\sin\theta\left[\frac{2\sin\left(k\frac{a}{2}\sin\theta\cos\phi\right)}{k\sin\theta\cos\phi}\right]\frac{2\pi b\cos\left(k\frac{b}{2}\cos\theta\right)}{\pi^2-[bk\cos\theta]^2}$$

$$= -2E_o\left(\frac{2\sin\left(k\frac{a}{2}\sin\theta\cos\phi\right)}{k\cos\phi}\right)\frac{2\pi b\cos\left(k\frac{b}{2}\cos\theta\right)}{\pi^2-[bk\cos\theta]^2}$$

$$= \frac{-8E_o\ \sin\left(k\frac{a}{2}\sin\theta\cos\phi\right)}{k\cos\phi}\frac{\pi b\cos\left(k\frac{b}{2}\cos\theta\right)}{\pi^2-[bk\cos\theta]^2}$$

$$= \frac{-8\pi bE_o\ \sin\left(k\frac{a}{2}\sin\theta\cos\phi\right)\cos\left(k\frac{b}{2}\cos\theta\right)}{k\cos\phi\ \left(\pi^2-[bk\cos\theta]^2\right)}$$

now, since from above we already said that

$$F_\theta = \frac{\epsilon\ e^{-jkr}}{4\pi r}L_\theta$$

so

$$F_\theta = -\frac{\epsilon e^{-jkr}}{4\pi r}\left(\frac{8\pi bE_o\ \sin\left(k\frac{a}{2}\sin\theta\cos\phi\right)\cos\left(k\frac{b}{2}\cos\theta\right)}{k\cos\phi\ \left(\pi^2-[bk\cos\theta]^2\right)}\right)$$

but

$$\begin{aligned} H_\theta &= -j\omega F_\theta \\ &= -j\frac{k}{\sqrt{\epsilon\mu}}F_\theta \end{aligned}$$

so using the relation $\eta = \sqrt{\frac{\mu}{\epsilon}}$ we get

$$\begin{aligned} H_\theta &= \left(-j\frac{k}{\sqrt{\epsilon\mu}}\right)\left[-\frac{\epsilon\ e^{-jkr}}{4\pi r}\right]\left(\frac{8\pi bE_o\ \sin\left(k\frac{a}{2}\sin\theta\cos\phi\right)\cos\left(k\frac{b}{2}\cos\theta\right)}{k\cos\phi\ \left(\pi^2-[bk\cos\theta]^2\right)}\right) \\ &= \frac{2jbE_o e^{-jkr}}{\eta r}\frac{\sin\left(k\frac{a}{2}\sin\theta\cos\phi\right)\cos\left(k\frac{b}{2}\cos\theta\right)}{\cos\phi\ \left(\pi^2-[bk\cos\theta]^2\right)} \end{aligned} \tag{6}$$

Dr., as I mentioned to you on the phone, I have an extra "2" factor in the numerator, while the solution in the book has the "2" in denominator, I went over this solution many times, and cant see where I did the math error if I did.

5 problem 3-20

$$\mathbf{E}(r) = -\nabla\times\iint_{area}\frac{e^{-jk\left|\mathbf{r}-\mathbf{r}'\right|}}{2\pi\left|\mathbf{r}-\mathbf{r}'\right|}\mathbf{E}\left(r'\right)\times ds' \tag{1}$$

the incident field is given by

$$\mathbf{E^i} = \mathbf{u}_z E_o e^{jk(x\cos\phi_o + y\sin\phi_o)}$$

assume the plate is perfect conductor, and use approximation that assumes the plate is perfect conductor to be able to use image theory and say that tangential component of the field outside $\mathbf{E}$,can by approximated to be the value of the field $\mathbf{E}^s$ (the scattered field from the plate).

now, since we evaluate the $\mathbf{E_z^i}$ at the boundary (tangential) , which is at the plate, which has $x=0$, then

$$\mathbf{E^i} = \mathbf{u}_z E_o e^{jk(y\sin\phi_o)}$$

$$\mathbf{M}_s = 2\mathbf{E^i}\times\mathbf{n} = 2\mathbf{u_z}E_o e^{jk(y\sin\phi_o)}\times\mathbf{u}_x = -\mathbf{u}_y\left(2E_o e^{jk(y\sin\phi_o)}\right)$$

for the plane yz the differential path from origin to the unit area ds' is given by

$$r'\cos\varpi = y'\sin\theta\sin\phi + z'\cos\theta$$

where ϖ is the angle between $\mathbf{r}^{'}$ and $\mathbf{r}$

so

$$\mathbf{F} = -\mathbf{u}_y \frac{\epsilon e^{-jkr}}{4\pi r} \int_{y=-a/2}^{a/2} \int_{z'=-b/2}^{b/2} \overbrace{e^{jk\left(y^{'} \sin\theta \sin\phi + z^{'} \cos\theta\right)}}^{e^{jk\left|r^{'}-r\right|}} \overbrace{2E_o e^{jk\left(y^{'} \sin\phi_o\right)}}^{M_s} dz^{'} dy^{'}$$

since we want to find $\mathbf{F}$ at $\theta = \pi/2$,(xy plane) then $\sin\theta = 1, \cos\theta = 0$ so

$$F_y = -\frac{\epsilon e^{-jkr}}{4\pi r} \int_{y=-a/2}^{a/2} \int_{z'=-b/2}^{b/2} e^{jk\left(y^{'} \sin\phi\right)} 2E_o e^{jk\left(y^{'} \sin\phi_o\right)} \, dz^{'} dy^{'}$$

Hence

$$\begin{aligned}
F_y &= -\frac{e^{-jkr}}{2\pi r} \int_{y'=-a/2}^{a/2} \int_{z'=-b/2}^{b/2} e^{jk\left(y^{'} \sin\phi\right)} E_o e^{jk\left(y^{'} \sin\phi_o\right)} \, dz^{'} dy^{'} \\
&= -\frac{E_o e^{-jkr}}{2\pi r} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} e^{jk\left(y^{'} \sin\phi\right) + jk\left(y^{'} \sin\phi_o\right)} dz^{'} dy^{'} \\
&= -\frac{E_o e^{-jkr}}{2\pi r} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} e^{jk\left(y^{'} (\sin\phi + \sin\phi_o)\right)} dz^{'} dy^{'} \\
&= -\frac{E_o e^{-jkr}}{2\pi r} \int_{-a/2}^{a/2} e^{jk\left(y^{'} (\sin\phi + \sin\phi_o)\right)} \left[\int_{-b/2}^{b/2} dz \right] dy \\
&= -\frac{E_o e^{-jkr}}{2\pi r} \int_{-a/2}^{a/2} e^{jk\left(y^{'} (\sin\phi + \sin\phi_o)\right)} \, [b] \, dy \\
&= -\frac{E_o e^{-jkr} b}{2\pi r} \int_{-a/2}^{a/2} e^{jk\left(y^{'} (\sin\phi + \sin\phi_o)\right)} dy
\end{aligned}$$

so, now we integrate the dy part

$$
\begin{aligned}
F_y &= -\frac{bE_o e^{-jkr}}{2\pi r} \int\limits_{y=-a/2}^{a/2} e^{jk\left(y'(\sin\phi+\sin\phi_o)\right)} dy \\
&= -\frac{2bE_o e^{-jkr}}{2\pi r} \left\{ \frac{1}{jk(\sin\phi+\sin\phi)} \left| e^{jk\left(y'(\sin\phi+\sin\phi_o)\right)} \right|_{-a/2}^{a/2} \right\} \\
&= -\frac{2bE_o e^{-jkr}}{2\pi r} \left\{ \frac{e^{jk\left(\frac{a}{2}(\sin\phi+\sin\phi)\right)} - e^{-jk\left(\frac{a}{2}(\sin\phi+\sin\phi_o)\right)}}{jk(\sin\phi+\sin\phi_o)} \right\} \\
&= -\frac{2bE_o e^{-jkr}}{2\pi r} \left\{ \frac{2j\sin\left(k\frac{a}{2}(\sin\phi+\sin\phi_o)\right)}{jk(\sin\phi+\sin\phi_o)} \right\} \\
&= -\frac{2bE_o e^{-jkr}}{2\pi r} \left\{ \frac{2\sin\left(k\frac{a}{2}(\sin\phi+\sin\phi_o)\right)}{k(\sin\phi+\sin\phi_o)} \right\}
\end{aligned}
$$

so

$$
\begin{aligned}
F_y &= -\frac{2E_o e^{-jkr}}{2\pi r} \left\{ \frac{2\sin\left(k\frac{a}{2}(\sin\phi+\sin\phi_o)\right)}{k(\sin\phi+\sin\phi_o)} \right\} \\
&= -\frac{2E_o e^{-jkr}}{2\pi r} \left\{ \frac{2\sin\left(k\frac{a}{2}(\sin\phi+\sin\phi_o)\right)}{k(\sin\phi+\sin\phi_o)} \right\}
\end{aligned}
$$

so

$$
\mathbf{E}^{\mathrm{s}} = -\nabla \times \mathbf{u}_y F
$$

well, I cant see what I did wrong Dr. Drane, there is something here I dont see right know and I must have done a mistake. The second part of this problem is completed using the answer given in the book for the first part. Given

$$
E_z^s \simeq \frac{kE_o ab e^{-jkr}}{j2\pi r} \frac{\sin\left(k\left(\frac{a}{2}\right)(\sin\phi+\sin\phi_o)\right)}{k\left(\frac{a}{2}\right)(\sin\phi+\sin\phi_o)} \cos\phi
$$

we need to find the echo area. the echo area is defined as

$$
A_e = \lim_{r\to\infty} \left(4\pi r^2 \frac{\overline{S}^s}{\overline{S}^i} \right)
$$

where $\overline{S}^i$ is the incident power density and $\overline{S}^s$ is the scattered power density for the incident field

$$\overline{S} = \text{Re}(EH^{\#}) = \text{Re}\left(E\left(\frac{E^{\#}}{\eta}\right)\right) = \frac{|E|^2}{\eta}$$

assuming η is real(as in case for air).

$$\overline{S}^i = \text{Re}\left(E_o e^{jk(x\cos\phi_o + y\sin\phi)}\left(\eta E_o e^{jk(x\cos\phi_o + y\sin\phi)}\right)^{\#}\right) = \eta\left|E_o^2\right|$$

since $\left|e^{(*)}\right| = 1$ and

$$\begin{aligned}
\overline{S}^s &= Re\left(\frac{kE_o abe^{-jkr}}{j2\pi r}\frac{\sin\left(k\left(\frac{a}{2}\right)(\sin\phi + \sin\phi_o)\right)}{k\left(\frac{a}{2}\right)(\sin\phi + \sin\phi_o)}\cos\phi\left[\eta\frac{kE_o abe^{-jkr}}{j2\pi r}\frac{\sin\left(k\left(\frac{a}{2}\right)(\sin\phi + \sin\phi_o)\right)}{k\left(\frac{a}{2}\right)(\sin\phi + \sin\phi_o)}\cos\phi\right]^{\#}\right) \\
&= \frac{kE_o ab}{2\pi r}\frac{\sin\left(k\left(\frac{a}{2}\right)(\sin\phi + \sin\phi_o)\right)}{k\left(\frac{a}{2}\right)(\sin\phi + \sin\phi_o)}\cos\phi\left[\eta\frac{kE_o ab}{2\pi r}\frac{\sin\left(k\left(\frac{a}{2}\right)(\sin\phi + \sin\phi_o)\right)}{k\left(\frac{a}{2}\right)(\sin\phi + \sin\phi_o)}\cos\phi\right] \\
&= \eta\left(\frac{E_o ab}{2\pi r}\frac{\sin\left(k\left(\frac{a}{2}\right)(\sin\phi + \sin\phi_o)\right)}{\left(\frac{a}{2}\right)(\sin\phi + \sin\phi_o)}\cos\phi\right)^2
\end{aligned}$$

so

$$\begin{aligned}
A_e &= \lim_{r\to\infty}\left(4\pi r^2\frac{\left[\eta\frac{E_o ab}{2\pi r}\frac{\sin\left(k\left(\frac{a}{2}\right)(\sin\phi+\sin\phi_o)\right)}{\left(\frac{a}{2}\right)(\sin\phi+\sin\phi_o)}\cos\phi\right]^2}{\eta\left|E_o^2\right|}\right) \\
&= \left(ab\frac{\sin\left(k\left(\frac{a}{2}\right)(\sin\phi + \sin\phi_o)\right)}{\pi\left(\frac{a}{2}\right)(\sin\phi + \sin\phi_o)}\cos\phi\right)^2
\end{aligned}$$

That is all i can do on this...

6 problem 3-24

looking at induced currents on the obstacle by each antenna in turn.

when unit current applied at antenna 1 then

$$\begin{aligned}
\mathbf{J}_1^s &= \mathbf{n}\times\left(\mathbf{H}_1^s - \mathbf{H}_1\right) \\
\mathbf{M}_1^s &= \left(\mathbf{E}_1^s - \mathbf{E}_1\right)\times\mathbf{n}
\end{aligned} \tag{1}$$

but

$$\mathbf{E}_1^s = \mathbf{E}_1 - \mathbf{E}_1^i$$
$$\mathbf{H}_1^s = \mathbf{H}_1 - \mathbf{H}_1^i$$

so equation (1) becomes

$$\mathbf{J}_1^s = \mathbf{H}_1^{\mathbf{i}} \times \mathbf{n} \qquad (2)$$
$$\mathbf{M}_1^s = \mathbf{n} \times \mathbf{E}_1^i$$

similarly for antenna 2 we get

$$\mathbf{J}_2^s = \mathbf{H}_2^{\mathbf{i}} \times \mathbf{n} \qquad (3)$$
$$\mathbf{M}_2^s = \mathbf{n} \times \mathbf{E}_2^i$$

now, from reciprocity, we know that $V_1^i = V_2^i$ or in other words $\mathbf{E}_1^i = \mathbf{E}_2^i$

using this and from equation (2) and (3) we get

$$\mathbf{M}_1^s = \mathbf{M}_2^s$$

i.e. the magnetic currents induced on the obstacle by antenna 1 are equal to the magnetic current induced on antenna 2. this is when a unit current source is placed at each antenna respectively.

so, since $\mathbf{M}_1^s = \mathbf{M}_2^s$ then the electric fields generated by theses currents must be the same. i.e. $\mathbf{E}_1^s = \mathbf{E}_2^s$, and this in turn means that $V_1^s = V_2^s$.

Q.E.D.

7 problem 3-29

Derive the left hand term of Eq. 3-50, that is show that

$$\overbrace{\iint\limits_{surface} \mathbf{E} \times \nabla \times \mathbf{G}_1 - \mathbf{G}_1 \times \nabla \times \mathbf{E} + \mathbf{E} \left(\nabla \cdot \mathbf{G}_1 \right) \cdot \mathbf{ds}}^{LHS} \underset{\left|\mathbf{r}-\mathbf{r}'\right| \to 0}{\longrightarrow} 4\pi \mathbf{c} \cdot \mathbf{E} \qquad (1)$$

where surface integration is over a surface of the small sphere, if we let the field point at the surface of the sphere, and the source point, where the unit current source , to be located at the center of the sphere, then the radius of the sphere is

$$R = \left| \mathbf{r} - \mathbf{r}^{'} \right|$$

first note that

$$\mathbf{E}\times\nabla\times\mathbf{G}_1=\nabla\left(\mathbf{E}\cdot\mathbf{G}_1\right)-\left(\mathbf{E}\cdot\nabla\right)\mathbf{G}_1$$

and

$$\mathbf{G}_1\times\nabla\times\mathbf{E}=\nabla\left(\mathbf{G}_1\cdot\mathbf{E}\right)-\left(\mathbf{G}_1\cdot\nabla\right)\mathbf{E}$$

so LHS of equation (1) becomes

$$\begin{aligned}LHS&=\iint\limits_{surface}\overbrace{\nabla\left(\mathbf{E}\cdot\mathbf{G}_1\right)}-\left(\mathbf{E}\cdot\nabla\right)\mathbf{G}_1\overbrace{-\nabla\left(\mathbf{G}_1\cdot\mathbf{E}\right)}+\left(\mathbf{G}_1\cdot\nabla\right)\mathbf{E}+\mathbf{E}\left(\nabla\cdot\mathbf{G}_1\right)\cdot\mathbf{ds} \qquad (2)\\&=\iint\limits_{surface}-\left(\mathbf{E}\cdot\nabla\right)\mathbf{G}_1+\left(\mathbf{G}_1\cdot\nabla\right)\mathbf{E}+\mathbf{E}\left(\nabla\cdot\mathbf{G}_1\right)\cdot\mathbf{ds}\end{aligned}$$

where the terms marked above has been canceled with each others.

now, another cancellation is made by observing $\mathbf{G}_1=\phi\mathbf{c}$, where $\mathbf{c}$ is a *constant vector,* this means that

$$\left(\mathbf{E}\cdot\nabla\right)\mathbf{G}_1=0$$

and

$$\mathbf{E}\left(\nabla\cdot\mathbf{G}_1\right)=0$$

so, equation (2) above becomes

$$LHS=\iint\limits_{surface}\left(\mathbf{G}_1\cdot\nabla\right)\mathbf{E}\cdot\mathbf{ds}$$

this is the equation we need to show it goes to $4\pi\mathbf{c}\cdot\mathbf{E}$ as $\left|\mathbf{r}-\mathbf{r}'\right|\rightarrow 0$, or in other hands, as the radius of the small sphere goes to zero.

since

$$\mathbf{ds}=\mathbf{n}\cdot ds$$

to find $\mathbf{n}$, we note that the equation of the sphere is

$$x^2+y^2+z^2=R^2$$

where R is the radius. a normal vector to this locus is given by $\nabla\left(x^2+y^2+z^2\right)=\mathbf{u}_x2x+\mathbf{u}_y2y+\mathbf{u}_z2z$

so a unit vector becomes

$$\mathbf{n}=\frac{\mathbf{u}_x2x+\mathbf{u}_y2y+\mathbf{u}_z2z}{\left|\mathbf{u}_x2x+\mathbf{u}_y2y+\mathbf{u}_z2z\right|}=\frac{\mathbf{u}_x2x+\mathbf{u}_y2y+\mathbf{u}_z2z}{\sqrt{4x^2+4y^2+4z^2}}=\frac{\mathbf{u}_x2x+\mathbf{u}_y2y+\mathbf{u}_z2z}{2R}$$

where the relation $x^2 + y^2 + z^2 = R^2$ was used to simplify last step above.

now, the projection of the unit area ds into the xy plane is

$$ds = \frac{dxdy}{|\mathbf{n} \cdot \mathbf{u}_z|}$$

so LHS can now be written as

$$\begin{aligned}
LHS &= \iint_{surface} \mathbf{E}(\nabla \cdot \mathbf{G}_1) \cdot \mathbf{ds} \\
&= \iint_{xy-plan} \mathbf{E}(\nabla \cdot \mathbf{G}_1) \cdot \mathbf{n}\frac{dxdy}{|\mathbf{n} \cdot \mathbf{u}_z|} \\
&= \iint_{xy-plane} \mathbf{E}(\nabla \cdot \mathbf{G}_1) \cdot \left(\frac{\mathbf{u}_x 2x + \mathbf{u}_y 2y + \mathbf{u}_z 2z}{2R}\right) \frac{dxdy}{\left|\frac{2z}{2R}\right|} \\
&= \iint_{xy-plane} \mathbf{E}(\nabla \cdot \mathbf{G}_1) \cdot \left(\frac{\mathbf{u}_x x + \mathbf{u}_y y + \mathbf{u}_z z}{R}\right) \frac{dxdy}{z/R} \\
&= \iint_{xy-plane} \mathbf{E}(\nabla \cdot \mathbf{G}_1) \cdot (\mathbf{u}_x x + \mathbf{u}_y y + \mathbf{u}_z z) \frac{dxdy}{z}
\end{aligned}$$

now, since $x^2 + y^2 + z^2 = R$, then

$$z = \sqrt{R^2 - x^2 - y^2}$$

let $\mathbf{E}(\nabla \cdot \mathbf{G}_1) =$

$$\begin{aligned}
LHS &= \iint_{surface} \mathbf{E}(\nabla \cdot \mathbf{G}_1) \cdot (\mathbf{u}_x x + \mathbf{u}_y y + \mathbf{u}_z z) \frac{dxdy}{z} \\
&= \iint_{surface} \cdot (\mathbf{u}_x x + \mathbf{u}_y y + \mathbf{u}_z z) \frac{dxdy}{\sqrt{R^2 - x^2 - y^2}} \\
&= \iint_{surface} (\Phi_x x + \Phi_y y + \Phi_z z) \frac{dxdy}{\sqrt{R^2 - x^2 - y^2}} \\
&= \iint_{surface} \left(\Phi_x x + \Phi_y y + \Phi_z \sqrt{R^2 - x^2 - y^2}\right) \frac{dxdy}{\sqrt{R^2 - x^2 - y^2}} \\
&= \iint_{surface} \Phi_x x \frac{dxdy}{\sqrt{R^2 - x^2 - y^2}} + \iint_{surface} \Phi_y y \frac{dxdy}{\sqrt{R^2 - x^2 - y^2}} + \iint_{surface} \Phi_z \sqrt{R^2 - x^2 - y^2} \frac{dxdy}{\sqrt{R^2 - x^2 - y^2}}
\end{aligned}$$

8 problem 3-27

In the vector Green's theorm [eqn 3-46], let $\mathbf{A} = \mathbf{E}^a$ and $\mathbf{B} = \mathbf{E}^b$ in a homogenouse isotropic region, and show that it reduces to quation 3-35.

solution

we want to show that, but letting $\mathbf{A} = \mathbf{E}^a$ and $\mathbf{B} = \mathbf{E}^b$ in

$$\iint_{surface} (\mathbf{A}\times\nabla\times\mathbf{B} - \mathbf{B}\times\nabla\times\mathbf{A})\cdot\mathbf{ds} = \iiint_{\tau} (\mathbf{B}\cdot\nabla\times\nabla\times\mathbf{A} - \mathbf{A}\cdot\nabla\times\nabla\times\mathbf{B})\ d\tau \tag{1}$$

it reduces to

$$-\iint_{surface} \left(\mathbf{E}^a\times\mathbf{H}^b - \mathbf{E}^b\times\mathbf{H}^a\right)\cdot ds = \iiint_{\tau} \left(\mathbf{E}^a\cdot\mathbf{J}^b - \mathbf{H}^a\cdot\mathbf{M}^b - \mathbf{E}^b\cdot\mathbf{J}^a + \mathbf{H}^b\cdot\mathbf{M}^a\right) d\tau$$

in (1), let $\mathbf{A} = \mathbf{E}^a$ and $\mathbf{B} = \mathbf{E}^b$ we get

$$\overbrace{\iint_{surface} \left(\mathbf{E}^a\times\nabla\times\mathbf{E}^b - \mathbf{E}^b\times\nabla\times\mathbf{E}^a\right)\cdot\mathbf{ds}}^{LHS} = \overbrace{\iiint_{\tau} \left(\mathbf{E}^b\cdot\nabla\times\nabla\times\mathbf{E}^a - \mathbf{E}^a\cdot\nabla\times\nabla\times\mathbf{E}^b\right) d\tau}^{RHS} \tag{2}$$

looking at the left hand side of equation (2) for now, and using maxwell equation where $\nabla\times\mathbf{E} = (-\widehat{z}\mathbf{H} - \mathbf{M})$

$$\begin{aligned} LHS &= \iint_{surface} \left(\mathbf{E}^a\times\overbrace{\nabla\times\mathbf{E}^b}^{-\widehat{z}\mathbf{H}^b-\mathbf{M}^b} - \mathbf{E}^b\times\overbrace{\nabla\times\mathbf{E}^a}^{-\widehat{z}\mathbf{H}^a-\mathbf{M}^a}\right)\cdot\mathbf{ds} \\ &= \iint_{surface} \left(\mathbf{E}^a\times\left(-\widehat{z}\mathbf{H}^b - \mathbf{M}^b\right) - \mathbf{E}^b\times(-\widehat{z}\mathbf{H}^a - \mathbf{M}^a)\right)\cdot\mathbf{ds} \end{aligned}$$

now, apply the following 2 relations into the above equation

$$\mathbf{E}^a\times\left(-\widehat{z}\mathbf{H}^b - \mathbf{M}^b\right) = \left(-\mathbf{E}^a\times\widehat{z}\mathbf{H}^b\right) + \left(-\mathbf{E}^a\times\mathbf{M}^b\right)$$

$$\mathbf{E}^b\times(-\widehat{z}\mathbf{H}^a - \mathbf{M}^a) = \left(-\mathbf{E}^b\times\widehat{z}\mathbf{H}^a\right) + \left(-\mathbf{E}^b\times\mathbf{M}^a\right)$$

this leads to

$$\begin{aligned} LHS &= \iint\limits_{surface} \left\{\left(-\mathbf{E}^a \times \widehat{z}\mathbf{H}^b\right) + \left(-\mathbf{E}^a \times \mathbf{M}^b\right)\right\} - \\ &\quad \left\{\left(-\mathbf{E}^b \times \widehat{z}\mathbf{H}^a\right) + \left(-\mathbf{E}^b \times \mathbf{M}^a\right)\right\} \cdot \mathbf{ds} \\ &= \iint\limits_{surface} \left(-\mathbf{E}^a \times \widehat{z}\mathbf{H}^b\right) - \left(-\mathbf{E}^b \times \widehat{z}\mathbf{H}^a\right) + \\ &\quad \left(-\mathbf{E}^a \times \mathbf{M}^b\right) - \left(-\mathbf{E}^b \times \mathbf{M}^a\right) \cdot \mathbf{ds} \\ &= \iint\limits_{surface} \left(-\mathbf{E}^a \times \widehat{z}\mathbf{H}^b\right) - \left(-\mathbf{E}^b \times \widehat{z}\mathbf{H}^a\right) \cdot\mathbf{ds} + \\ &\quad \iint\limits_{surface} \left(-\mathbf{E}^a \times \mathbf{M}^b\right) - \left(-\mathbf{E}^b \times \mathbf{M}^a\right) \cdot \mathbf{ds} \\ &= -\widehat{z} \iint\limits_{surface} \left(\mathbf{E}^a \times \mathbf{H}^b\right) - \left(\mathbf{E}^b \times \mathbf{H}^a\right) \cdot\mathbf{ds} + \\ &\quad \overbrace{\iint\limits_{surface} \left(-\mathbf{E}^a \times \mathbf{M}^b\right) + \left(\mathbf{E}^b \times \mathbf{M}^a\right) \cdot \mathbf{ds}}^{\text{apply divergence theorm on this}} \end{aligned}$$

so

$$LHS = -\widehat{z} \iint\limits_{surface} \left(\mathbf{E}^a \times \mathbf{H}^b\right) - \left(\mathbf{E}^b \times \mathbf{H}^a\right) \cdot\mathbf{ds} \tag{3}$$

$$+ \iiint\limits_{\tau} \overbrace{-\nabla\cdot \left(\mathbf{E}^a \times \mathbf{M}^b\right) + \nabla\cdot \left(\mathbf{E}^b \times \mathbf{M}^a\right)}^{\text{let this be called } \Theta} d\tau$$

but the above expression Θ by simplifed further by noting that

$$\nabla\cdot \left(\mathbf{E}^a \times \mathbf{M}^b\right) = \mathbf{M}^b \cdot (\nabla \times \mathbf{E}^a) - \mathbf{E}^a \cdot \left(\nabla \times \mathbf{M}^b\right)$$

$$\nabla\cdot \left(\mathbf{E}^b \times \mathbf{M}^a\right) = \mathbf{M}^a \cdot \left(\nabla \times \mathbf{E}^b\right) - \mathbf{E}^b \cdot (\nabla \times \mathbf{M}^a)$$

subtitute the above 2 equations into the volume integral of equation (3) leads to

$$\begin{aligned} \Theta &= -\nabla\cdot \left(\mathbf{E}^a \times \mathbf{M}^b\right) + \nabla\cdot \left(\mathbf{E}^b \times \mathbf{M}^a\right) \\ &= \begin{matrix} -\mathbf{M}^b \cdot (\nabla \times \mathbf{E}^a) + \mathbf{E}^a \cdot \left(\nabla \times \mathbf{M}^b\right) \\ +\mathbf{M}^a \cdot \left(\nabla \times \mathbf{E}^b\right) - \mathbf{E}^b \cdot (\nabla \times \mathbf{M}^a) \end{matrix} \end{aligned}$$

so, the LHS of equation (2) has been simplified to this

$$-\widehat{z}\iint_{surface}\left(\mathbf{E}^{a}\times\mathbf{H}^{b}\right)-\left(\mathbf{E}^{b}\times\mathbf{H}^{a}\right)\ \cdot\mathbf{ds}+\iiint_{\tau}\Theta\ d\tau \tag{4}$$

now we work on the RHS of equation (2), and we see cancelations with equation (4) above , that will lead to the final answer.

the RHS of equation (2) becomes, noting that $\nabla\times\nabla\times\mathbf{E}=\nabla\times(-\widehat{z}\mathbf{H}-\mathbf{M})=-\nabla\times\widehat{z}\mathbf{H}-\nabla\times\mathbf{M}$

so

$$\begin{aligned}
RHS &= \iiint_{\tau}\left(\mathbf{E}^{b}\cdot\nabla\times\nabla\times\mathbf{E}^{a}-\mathbf{E}^{a}\cdot\nabla\times\nabla\times\mathbf{E}^{b}\right)\ d\tau\\
&= \begin{array}{l}\iiint_{\tau}\mathbf{E}^{b}\cdot(-\nabla\times\widehat{z}\mathbf{H}^{a}-\nabla\times\mathbf{M}^{a})\\ -\mathbf{E}^{a}\cdot\left(-\nabla\times\widehat{z}\mathbf{H}^{b}-\nabla\times\mathbf{M}^{b}\right)\ d\tau\end{array}\\
&= \begin{array}{l}\iiint_{\tau}\mathbf{E}^{b}\cdot(-\nabla\times\widehat{z}\mathbf{H}^{a})-\mathbf{E}^{b}\cdot(\nabla\times\mathbf{M}^{a})\\ -\mathbf{E}^{a}\cdot\left(-\nabla\times\widehat{z}\mathbf{H}^{b}\right)+\mathbf{E}^{a}\cdot\left(\nabla\times\mathbf{M}^{b}\right)\ d\tau\end{array}
\end{aligned}$$

looking at the last equation above, and at Θ in the LHS of equation (4), we see that $\mathbf{E}^{b}\cdot(\nabla\times\mathbf{M}^{a})$ and $\mathbf{E}^{a}\cdot\left(\nabla\times\mathbf{M}^{b}\right)$ cancels out with each others, this means that the whole equation can now be written as

$$\begin{aligned}
LHS &= RHS\\
\begin{array}{l}-\widehat{z}\iint_{area}\left(\mathbf{E}^{a}\times\mathbf{H}^{b}\right)-\left(\mathbf{E}^{b}\times\mathbf{H}^{a}\right)\ \cdot\mathbf{ds}+\\ \iiint-\mathbf{M}^{b}\cdot(\nabla\times\mathbf{E}^{a})+\mathbf{M}^{a}\cdot\left(\nabla\times\mathbf{E}^{b}\right)d\tau\end{array} &= \begin{array}{l}\iiint_{\tau}\mathbf{E}^{b}\cdot(-\nabla\times\widehat{z}\mathbf{H}^{a})-\\ \mathbf{E}^{a}\cdot\left(-\nabla\times\widehat{z}\mathbf{H}^{b}\right)\ d\tau\end{array}
\end{aligned} \tag{5}$$

now,

$$\mathbf{M}^{b}\cdot(\nabla\times\mathbf{E}^{a})=\mathbf{M}^{b}\cdot(-\widehat{z}\mathbf{H}^{a}+\mathbf{M}^{a})=-\mathbf{M}^{b}\cdot\widehat{z}\mathbf{H}^{a}+\mathbf{M}^{b}\cdot\mathbf{M}^{a}$$

and

$$\mathbf{M}^{a}\cdot\left(\nabla\times\mathbf{E}^{b}\right)=\mathbf{M}^{a}\cdot\left(-\widehat{z}\mathbf{H}^{b}+\mathbf{M}^{b}\right)=-\mathbf{M}^{a}\cdot\widehat{z}\mathbf{H}^{b}+\mathbf{M}^{a}\cdot\mathbf{M}^{b}$$

subtitute the above into the LHS of equation (5) we get

$$\begin{aligned}
LHS &= -\widehat{z}\iint_{surface}\left(\mathbf{E}^{a}\times\mathbf{H}^{b}\right)-\left(\mathbf{E}^{b}\times\mathbf{H}^{a}\right)\ \cdot\mathbf{ds}+\\
&\iiint+\mathbf{M}^{b}\cdot\widehat{z}\mathbf{H}^{a}-\mathbf{M}^{b}\cdot\mathbf{M}^{a}-\mathbf{M}^{a}\cdot\widehat{z}\mathbf{H}^{b}+\mathbf{M}^{a}\cdot\mathbf{M}^{b}\\
&= -\widehat{z}\iint_{surface}\left(\mathbf{E}^{a}\times\mathbf{H}^{b}\right)-\left(\mathbf{E}^{b}\times\mathbf{H}^{a}\right)\ \cdot\mathbf{ds}+\\
&\iiint+\mathbf{M}^{b}\cdot\widehat{z}\mathbf{H}^{a}-\mathbf{M}^{a}\cdot\widehat{z}\mathbf{H}^{b}
\end{aligned} \tag{6}$$

now, looking at the RHS of equation (5) and using relation that $\nabla\times\mathbf{H} = \widehat{y}\mathbf{E} + \mathbf{J}$, that is, $\nabla\times\widehat{z}\mathbf{H} = \widehat{z}\left(\widehat{y}\mathbf{E} + \mathbf{J}\right)$

$$\begin{aligned}\mathbf{E}^b\cdot\left(-\nabla\times\widehat{z}\mathbf{H}^a\right) &= -\mathbf{E}^b\cdot\left(\nabla\times\widehat{z}\mathbf{H}^a\right) = -\widehat{z}\mathbf{E}^b\cdot\left(\widehat{y}\mathbf{E}^a + \mathbf{J}^a\right)\\ &= -\widehat{z}\mathbf{E}^b\cdot\widehat{y}\mathbf{E}^a - \widehat{z}\mathbf{E}^b\cdot\mathbf{J}^a\end{aligned}$$

and

$$\begin{aligned}\mathbf{E}^a\cdot\left(-\nabla\times\widehat{z}\mathbf{H}^b\right) &= -\mathbf{E}^a\cdot\left(\nabla\times\widehat{z}\mathbf{H}^b\right) = -\widehat{z}\mathbf{E}^a\cdot\left(\widehat{y}\mathbf{E}^b + \mathbf{J}^b\right)\\ &= -\widehat{z}\mathbf{E}^a\cdot\widehat{y}\mathbf{E}^b - \widehat{z}\mathbf{E}^a\cdot\mathbf{J}^b\end{aligned}$$

so, the RHS of equation (5) becomes

$$\begin{aligned}&RHS = \iiint -\widehat{z}\mathbf{E}^b\cdot\left(\widehat{y}\mathbf{E}^a + \mathbf{J}^a\right) =\\ &-\widehat{z}\mathbf{E}^b\cdot\widehat{y}\mathbf{E}^a - \widehat{z}\mathbf{E}^b\cdot\mathbf{J}^a - \left(-\widehat{z}\mathbf{E}^a\cdot\widehat{y}\mathbf{E}^b - \widehat{z}\mathbf{E}^a\cdot\mathbf{J}^b\right)\\ &\quad = \iiint -\widehat{z}\mathbf{E}^b\cdot\widehat{y}\mathbf{E}^a - \widehat{z}\mathbf{E}^b\cdot\mathbf{J}^a + \widehat{z}\mathbf{E}^a\cdot\widehat{y}\mathbf{E}^b + \widehat{z}\mathbf{E}^a\cdot\mathbf{J}^b\, d\tau\\ &\quad = \iiint -\widehat{z}\mathbf{E}^b\cdot\mathbf{J}^a + \widehat{z}\mathbf{E}^a\cdot\mathbf{J}^b\, d\tau\end{aligned} \tag{7}$$

so, from (6) and (7), we see that our equation finaly looks like

$$\begin{aligned}LHS &= RHS\\ -\widehat{z}\iint_{surface}\left(\mathbf{E}^a\times\mathbf{H}^b\right) - \left(\mathbf{E}^b\times\mathbf{H}^a\right)\cdot\mathbf{ds} + \iiint \mathbf{M}^b\cdot\widehat{z}\mathbf{H}^a - \mathbf{M}^a\cdot\widehat{z}\mathbf{H}^b d\tau &= \iiint -\widehat{z}\mathbf{E}^b\cdot\mathbf{J}^a + \widehat{z}\mathbf{E}^a\cdot\mathbf{J}^b\, d\tau\\ -\iint_{surface}\left(\mathbf{E}^a\times\mathbf{H}^b\right) - \left(\mathbf{E}^b\times\mathbf{H}^a\right)\cdot\mathbf{ds} + \iiint \mathbf{M}^b\cdot\mathbf{H}^a - \mathbf{M}^a\cdot\mathbf{H}^b\, d\tau &= \iiint -\mathbf{E}^b\cdot\mathbf{J}^a + \mathbf{E}^a\cdot\mathbf{J}^b\, d\tau\\ -\iint_{surface}\left(\mathbf{E}^a\times\mathbf{H}^b\right) - \left(\mathbf{E}^b\times\mathbf{H}^a\right)\cdot\mathbf{ds} &= \iiint\left(\mathbf{E}^a\cdot\mathbf{J}^b - \mathbf{M}^b\cdot\mathbf{H}^a - \mathbf{E}^b\cdot\mathbf{J}^a + \mathbf{M}^b\cdot\mathbf{H}^a\right) d\tau\end{aligned}$$

QED

9 problem 3-29it

Derive the left hand term of Eq. 3-50, that is show that

$$\overbrace{\iint_{surface}\mathbf{E}\times\nabla\times\mathbf{G}_1 - \mathbf{G}_1\times\nabla\times\mathbf{E} + \mathbf{E}\left(\nabla\cdot\mathbf{G}_1\right)\cdot\mathbf{ds}}^{LHS}\ \underset{\left|\mathbf{r}-\mathbf{r}'\right|\to 0}{\longrightarrow}\ 4\pi\mathbf{c}\cdot\mathbf{E} \tag{1}$$

where surface integration is over a surface of the small sphere, if we let the field point at the surface of the sphere, and the source point, where the unit current source , to be located at the center of the sphere, then the radius of the sphere is

$$R = \left|\mathbf{r} - \mathbf{r}^{'}\right|$$

first note that

$$\mathbf{E} \times \nabla \times \mathbf{G}_1 = \nabla\left(\mathbf{E} \cdot \mathbf{G}_1\right) - \left(\mathbf{E} \cdot \nabla\right)\mathbf{G}_1$$

and

$$\mathbf{G}_1 \times \nabla \times \mathbf{E} = \nabla\left(\mathbf{G}_1 \cdot \mathbf{E}\right) - \left(\mathbf{G}_1 \cdot \nabla\right)\mathbf{E}$$

so LHS of equation (1) becomes

$$\begin{aligned} LHS &= \iint\limits_{surface} \overbrace{\nabla\left(\mathbf{E} \cdot \mathbf{G}_1\right)} - \left(\mathbf{E} \cdot \nabla\right)\mathbf{G}_1 \overbrace{-\nabla\left(\mathbf{G}_1 \cdot \mathbf{E}\right)} + \left(\mathbf{G}_1 \cdot \nabla\right)\mathbf{E} + \mathbf{E}\left(\nabla \cdot \mathbf{G}_1\right) \cdot \mathbf{ds} \\ &= \iint\limits_{surface} -\left(\mathbf{E} \cdot \nabla\right)\mathbf{G}_1 + \left(\mathbf{G}_1 \cdot \nabla\right)\mathbf{E} + \mathbf{E}\left(\nabla \cdot \mathbf{G}_1\right) \cdot \mathbf{ds} \end{aligned} \tag{2}$$

where the terms marked above has been canceled with each others.

now, another cancellation is made by observing $\mathbf{G}_1 = \phi\mathbf{c}$, where $\mathbf{c}$ is a *constant vector,* this means that

$$\left(\mathbf{E} \cdot \nabla\right)\mathbf{G}_1 = 0$$

and

$$\mathbf{E}\left(\nabla \cdot \mathbf{G}_1\right) = 0$$

so, equation (2) above becomes

$$LHS = \iint\limits_{surface} \left(\mathbf{G}_1 \cdot \nabla\right)\mathbf{E} \cdot \mathbf{ds} \tag{3}$$

this is the equation we need to show it goes to $4\pi\mathbf{c} \cdot \mathbf{E}$ as $\left|\mathbf{r} - \mathbf{r}^{'}\right| \rightarrow 0$, or in other hands, as the radius of the small sphere goes to zero.

$$\left(\mathbf{G}_1 \cdot \nabla\right)\mathbf{E} = \mathbf{u}_x G_{1x}\frac{\partial E_x}{\partial x} + \mathbf{u}_y G_{1y}\frac{\partial E_y}{\partial y} + \mathbf{u}_z G_{1z}\frac{\partial E_z}{\partial z}$$

but $\mathbf{G}_1 = \phi\mathbf{c}$, where $\mathbf{c}$ is a *constant vector,* so the above becomes

$$\left(\mathbf{G}_1 \cdot \nabla\right)\mathbf{E} = \mathbf{u}_x\frac{\partial E_x G_{1x}}{\partial x} + \mathbf{u}_y\frac{\partial E_y G_{1y}}{\partial y} + \mathbf{u}_z\frac{\partial E_z G_{1z}}{\partial z} = \nabla\left(E_x G_{1x} + E_y G_{1y} + E_z G_{1z}\right) = \nabla\left(\mathbf{E} \cdot \mathbf{G}_1\right)$$

so from equation (3) we get

$$LHS = \iint\limits_{surface} \nabla \left(\mathbf{E} \cdot \mathbf{G}_1\right) \cdot \mathbf{ds} \tag{4}$$

now, $\int \frac{d}{dt} f\left(t\right) = f\left(t\right)$, so apply this rule to the above, so equation (4) becomes

$$LHS = \left(\mathbf{E} \cdot \mathbf{G}_1\right) \iint\limits_{surface} \left(\mathbf{u_x} + \mathbf{u_y} + \mathbf{u_z}\right) \cdot \mathbf{ds} \tag{5}$$

now, $\mathbf{ds} = \mathbf{n} \cdot ds$

where

$$\mathbf{l} = \nabla \left(x^2 + y^2 + z^2\right) = \mathbf{u}_x 2x + \mathbf{u}_y 2y + \mathbf{u}_z 2z$$

since the equation of the sphere is $x^2 + y^2 + z^2 = R^2$ where R is the radius. and $\mathbf{l}$ is vector normal to the surface of the sphere.

so

$$\mathbf{n} = \frac{\mathbf{l}}{|\mathbf{l}|} = \frac{\mathbf{u}_x 2x + \mathbf{u}_y 2y + \mathbf{u}_z 2z}{\sqrt{4x^2 + 4y^2 + 4z^2}} = \frac{\mathbf{u}_x 2x + \mathbf{u}_y 2y + \mathbf{u}_z 2z}{2R}$$

now, the projection of the unit area ds into the xy plane is

$$ds = \frac{dxdy}{|\mathbf{n} \cdot \mathbf{u}_z|} = \frac{dxdy}{z/R}$$

but

$$z = \sqrt{R^2 - x^2 - y^2}$$

so equation (5) becomes

$$LHS = \left(\mathbf{E} \cdot \mathbf{G}_1\right) \iint\limits_{surface} \left(\mathbf{u_x} + \mathbf{u_y} + \mathbf{u_z}\right) \cdot ds$$

or

$$LHS = \left(\mathbf{E} \cdot \mathbf{G}_1\right) \iint\limits_{surface} \left(\mathbf{u_x} + \mathbf{u_y} + \mathbf{u_z}\right) \cdot \frac{\mathbf{u}_x 2x + \mathbf{u}_y 2y + \mathbf{u}_z 2z}{2} \frac{dxdy}{\sqrt{R^2 - x^2 - y^2}}$$

Hence

$$\begin{aligned} LHS &= (\mathbf{E}\cdot\mathbf{G}_1)\iint\limits_{surface}\left(\mathbf{u_x}+\mathbf{u_y}+\mathbf{u_z}\right)\cdot\frac{\mathbf{u}_x 2x+\mathbf{u}_y 2y+\mathbf{u}\ _z 2z}{2}\frac{dxdy}{\sqrt{R^2-x^2-y^2}} \\ &= (\mathbf{E}\cdot\mathbf{G}_1)\iint\limits_{surface}\left(\mathbf{u_x}+\mathbf{u_y}+\mathbf{u_z}\right)\cdot\mathbf{u_x}x+\mathbf{u}_y y+\mathbf{u}\ _z z\frac{dxdy}{\sqrt{R^2-x^2-y^2}} \\ &= (\mathbf{E}\cdot\mathbf{G}_1)\iint\limits_{surface} x+y+\left(\sqrt{R^2-x^2-y^2}\right)\frac{dxdy}{\sqrt{R^2-x^2-y^2}} \\ &= (\mathbf{E}\cdot\mathbf{G}_1)\int\limits_{x=-R}^{x=R}\int\limits_{y=-R}^{R} x+y+\left(\sqrt{R^2-x^2-y^2}\right)\frac{dxdy}{\sqrt{R^2-x^2-y^2}} \end{aligned}$$

Dr., I have not managed to integrate the above within time, if the integration above results in a value of $4\pi R$ the final results will follow:

$$LHS = (\mathbf{E}\cdot\mathbf{G}_1)\,4\pi R = \phi\mathbf{c}\cdot\mathbf{E}\ 4\pi R \tag{6}$$

but

$$\phi = \frac{e^{-jkR}}{R}$$

so equation (6) becomes, as $R \longrightarrow 0$

$$\phi\mathbf{c}\cdot\mathbf{E}\ 4\pi R = \frac{e^{-jkR}}{R}\mathbf{c}\cdot\mathbf{E}\ 4\pi R = 4\pi\mathbf{c}\cdot\mathbf{E}$$

Q.E.D.

Dr., I did not carry the integration above completely, so my result for the integration can be wrong. I am not sure if there is a more direct approach without doing this long integration.

10 problem 3.3

Suppose that the two current sheets

$$\begin{aligned} \mathbf{J}_s &= \mathbf{u_x}\tfrac{A}{Z_o}\sin\tfrac{\pi y}{b} \\ \mathbf{M_s} &= \mathbf{u_y}A\sin\tfrac{\pi y}{b} \end{aligned}$$

exist simulaneously over the cross section z=0 of fig 3-2. show that these produce a field

$$E_x = \begin{cases} -A\sin\frac{\pi y}{b}e^{-j\beta z} & z>0 \\ \\ 0 & z<0 \end{cases}$$

Solution

Since each sheet alone will produce an solution for E_x and since these solutions are linear, then we will need to add the electric field due to the electric current sheet, to the electric field due to the maganetic current sheet to get the total electric field for both.

we know from page 97 in text that when

$$\mathbf{J}_s = \mathbf{u_x}\, J_o \sin\frac{\pi y}{b} \Rightarrow E_x = \begin{cases} -\frac{J_o Z_o}{2}\sin\frac{\pi y}{b}e^{-j\beta z} & z>0 \\ \\ -\frac{J_o Z_o}{2}\sin\frac{\pi y}{b}e^{j\beta z} & z<0 \end{cases}$$

then, we can conclude by comparison and replacing J_o by $\frac{A}{Z_o}$ we get:

$$\mathbf{J}_s = \mathbf{u_x}\, \frac{A}{Z_o} \sin\frac{\pi y}{b} \Rightarrow E_x = \begin{cases} -\frac{A}{2}\sin\frac{\pi y}{b}e^{-j\beta z} & z>0 \\ \\ -\frac{A}{2}\sin\frac{\pi y}{b}e^{j\beta z} & z<0 \end{cases} \tag{1}$$

also, we know from problem 3-2, that when

$$\mathbf{M}_s = \mathbf{u_y}\, M_o \sin\frac{\pi y}{b} \Rightarrow E_x = \begin{cases} -\frac{M_o}{2}\sin\frac{\pi y}{b}e^{-j\beta z} & z>0 \\ \\ \frac{M_o}{2}\sin\frac{\pi y}{b}e^{j\beta z} & z<0 \end{cases}$$

then,we can conclude by comparison and replacing M_o by A we get:

$$\mathbf{M}_s = \mathbf{u_y}\, A \sin\frac{\pi y}{b} \Rightarrow E_x = \begin{cases} -\frac{A}{2}\sin\frac{\pi y}{b}e^{-j\beta z} & z>0 \\ \\ \frac{A}{2}\sin\frac{\pi y}{b}e^{j\beta z} & z<0 \end{cases} \tag{2}$$

so, the electric field due to $\mathbf{J}_s = \mathbf{u_x}\frac{A}{Z_o}\sin\frac{\pi y}{b}$ and $\mathbf{M_s} = \mathbf{u_y} A\sin\frac{\pi y}{b}$ is given by adding equation (1) and equation (2):

$$E_x = \begin{cases} -A\sin\frac{\pi y}{b}e^{-j\beta z} & z>0 \\ \\ 0 & z<0 \end{cases}$$

QED.

11 problem 3.6

Obtain the field of an infintsmall loop of magnetic current having z-directed moment KS. Show that this produces the same field as the electric current element of fig 2-21 if

$$Il = j\omega\epsilon KS$$

<u>Solution</u>

the field due to infintsmall loop of electric current is given by problem 2-42 as

$$\begin{aligned} H_r &= \frac{IS}{2\pi} e^{-jkr} \left(\frac{jk}{r^2} + \frac{1}{r^3}\right) \cos\theta \\ H_\theta &= \frac{IS}{4\pi} e^{-jkr} \left(-\frac{k^2}{r} + \frac{jk}{r^2} + \frac{1}{r^3}\right) \sin\theta \\ E_\phi &= \frac{\eta IS}{4\pi} e^{-jkr} \left(\frac{k^2}{r} - \frac{jk}{r^2}\right) \sin\theta \end{aligned}$$

apply duality substitution to the above equation yield

$$\begin{aligned} E_r &= \frac{\mathrm{KS}}{2\pi} e^{-jkr} \left(\frac{jk}{r^2} + \frac{1}{r^3}\right) \cos\theta \\ E_\theta &= \frac{\mathrm{KS}}{4\pi} e^{-jkr} \left(-\frac{k^2}{r} + \frac{jk}{r^2} + \frac{1}{r^3}\right) \sin\theta \\ H_\phi &= \frac{\mathrm{KS}}{\eta 4\pi} e^{-jkr} \left(\frac{k^2}{r} - \frac{jk}{r^2}\right) \sin\theta \end{aligned} \tag{1}$$

but, the field due to electric current element of fig 2-21 is

$$\begin{aligned} E_r &= \frac{Il}{2\pi} e^{-jkr} \left(\frac{\eta}{r^2} + \frac{1}{j\omega\epsilon r^3}\right) \cos\theta \\ E_\theta &= \frac{Il}{4\pi} e^{-jkr} \left(\frac{j\omega\mu}{r} + \frac{\eta}{r^2} + \frac{1}{j\omega\epsilon r^3}\right) \sin\theta \\ H_\phi &= \frac{Il}{4\pi} e^{-jkr} \left(\frac{jk}{r} - \frac{}{r^2}\right) \sin\theta \end{aligned} \tag{2}$$

from the above equations (2) and (2), we see that if we substitute $Il = j\omega\epsilon KS$ in the (2), we will get (1).

QED.

12 problem 3.4

in fig 3-2, suppose that a "shorting plate" (conductor) in placed over the cross section $z = -d$.

show that the current sheet of eq. 3-2 now produces a field

$$E_x = \begin{cases} -\frac{J_o Z_o}{2}\left(1 - e^{-j2\beta d}\right)\sin\frac{\pi y}{b}e^{-j\beta z} & z > 0 \\ \\ -jJ_o Z_o e^{-j\beta d}\sin\frac{\pi y}{b}\sin\left[\beta\left(d+z\right)\right] & -d < z < 0 \end{cases}$$

Note that when d is an odd number of guide quarter-wavelengths, E_x for $z > 0$ is twice that for the current sheet alone [see equation 3-3], but when d is an integral number of guide half-wavelengths, no E_x exists for $z > 0$.

solution

this is a problem of scattering, the idea is that equivalent magnetic current densities are introduced to replace the physical obstacles, in this case the shorting plate.

solution steps:

1) Find the field due to the current sheet $\mathbf{J} = \mathbf{u_x} J_o \sin\frac{\pi y}{b}$ when the shorting plate is removed. Call this field $\mathbf{E^i}$.

2) Find the induced maganetic current $\mathbf{M_s}$ on the shorting plate from $\mathbf{E^i}$ by the relation

$$\mathbf{M_s} = 2\,\mathbf{n} \times \mathbf{E^i}\Big|_{z=-d}$$

3) Find the perturbation electric field due to $\mathbf{M_s}$,call this field $\mathbf{E^s}$.(usually called the scattered field)

4) The field due to the current sheet $\mathbf{J} = \mathbf{u_x} J_o \sin\frac{\pi y}{b}$ and due to $\mathbf{M_s}$ is then found by adding $\mathbf{E^i} + \mathbf{E^s}$.

Assumptions

the dominant part of the magnetic current $\mathbf{M_s}$ resides only in the front fact of the shorting plate, the face facing the incident wave, and that the image theory holds for finite plate.

step 1 from equation 3-3 page 98

$$E_x^i = \begin{cases} -\frac{J_o Z_o}{2}\sin\frac{\pi y}{b}e^{-j\beta z} & z > 0 \\ \\ -\frac{J_o Z_o}{2}\sin\frac{\pi y}{b}e^{j\beta z} & z < 0 \end{cases} \tag{1}$$

step 2

$$\mathbf{M_s} = 2\,\mathbf{n} \times \mathbf{E^i}\Big|_{z=-d}$$

where **n** is the unit vector normal to the plane where current $\mathbf{M_s}$ lies in, that will be the x-y plane.

So $\mathbf{n} = -\mathbf{u_z}$, the negative sign used since the normal vector is in the direction of propgration of the field, which in negtive directed in the negtaive z-axis.

For $\mathbf{E^i}$, since the shorting sheet is located in the negative z-axis, use this component:

$$E_x = -\frac{J_o Z_o}{2} \sin \frac{\pi y}{b} e^{j\beta z}$$

so

$$\overline{M}_s = 2 \begin{vmatrix} \overline{u_x} & \overline{u_y} & \overline{u_z} \\ 0 & 0 & -1 \\ -\frac{J_o Z_o}{2} \sin \frac{\pi y}{b} e^{j\beta z} & 0 & 0 \end{vmatrix}_{z=-d} = -\overline{u_y}\left(-J_o Z_o \sin \frac{\pi y}{b} e^{j\beta z}\right)\Big|_{z=-d}$$

$$\overline{M}_s = \overline{u_y}\left(J_o Z_o \sin \frac{\pi y}{b} e^{-j\beta d}\right)$$

<u>step 3</u>

Find the electric field due to $\mathbf{M_s}$.

$$\mathbf{E^s} = -\nabla \times \mathbf{F}$$

where

$$\mathbf{F} = \frac{1}{4\pi} \iint\limits_{xy\ plane} \frac{\mathbf{M_s}\, e^{-jk\left|\mathbf{r}-\mathbf{r}'\right|}}{|\mathbf{r}-\mathbf{r}'|} ds'$$

for the far field, **F** becomes

$$\mathbf{F} = \frac{e^{-j\beta r}}{4\pi} \iint\limits_{xy\ plane} \mathbf{M_s}\left(\mathbf{r}'\right) e^{j\beta r' \cos\psi}\, ds' \tag{2}$$

where **r** is the position vector of the field point (where observation of the electric potional vector is made from) and $\mathbf{r'}$ is the position vector of the source element, and ψ is the angle betwen the vectors **r** and $\mathbf{r'}$.

$$r' \cos\psi = x' \cos\phi \sin\theta + y' \sin\phi \sin\theta + z' \cos\theta \tag{3}$$

so, from equation (4), we get, noting that $z' = -d$:

$$\mathbf{F} = \frac{e^{-j\beta r}}{4\pi} \iint_{xy\ plane} \mathbf{M_s}\, e^{j\beta\left(x' \cos\phi \sin\theta + y' \sin\phi \sin\theta + z' \cos\theta\right)}\, dx' dy'$$

$$= \frac{e^{-j\beta r}}{4\pi} \int_{x'=0}^{a} \int_{y'=0}^{b} \overline{u_y} \left(J_o Z_o \sin \tfrac{\pi y}{b} e^{-j\beta d}\right) e^{j\beta\left(x' \cos\phi \sin\theta + y' \sin\phi \sin\theta + z' \cos\theta\right)}\, dx' dy'$$

$$F_y = \frac{e^{-j\beta r}}{4\pi} J_o Z_o \sin \tfrac{\pi y}{b} e^{-j\beta d} \int_{x'=0}^{a} \int_{y'=0}^{b} e^{j\beta\left(x' \cos\phi \sin\theta + y' \sin\phi \sin\theta + (-d)\cos\theta\right)}\, dy' dx'$$

$$= \frac{e^{-j\beta r}}{4\pi} J_o Z_o \sin \tfrac{\pi y}{b} e^{-(j\beta d)} e^{-(j\beta d \cos\theta)} \int_{x'=0}^{a} \int_{y'=0}^{b} e^{j\beta\left(x' \cos\phi \sin\theta + y' \sin\phi \sin\theta\right)}\, dy' dx'$$

$$= \frac{e^{-j\beta r}}{4\pi} J_o Z_o \sin \tfrac{\pi y}{b} e^{-j\beta d} e^{-(j\beta d \cos\theta)} \int_{x'=0}^{a} e^{j\beta\left(x' \cos\phi \sin\theta\right)} \left(\int_{y'=0}^{b} e^{j\beta\left(y' \sin\phi \sin\theta\right)} dy' \right) dx'$$

$$= \frac{e^{-j\beta r}}{4\pi} J_o Z_o \sin \tfrac{\pi y}{b} e^{-j\beta d} e^{-(j\beta d \cos\theta)} \int_{x'=0}^{a} e^{j\beta\left(x' \cos\phi \sin\theta\right)} \left(\frac{e^{j\beta b \sin\phi \sin\theta} - 1}{j\beta \sin\phi \sin\theta} \right) dx'$$

$$= \frac{e^{-j\beta r}}{4\pi} J_o Z_o \sin \tfrac{\pi y}{b} e^{-j\beta d} e^{-(j\beta d \cos\theta)} \left(\frac{e^{j\beta b \sin\phi \sin\theta} - 1}{j\beta \sin\phi \sin\theta} \right) \left(\frac{e^{j\beta a \cos\phi \sin\theta} - 1}{j\beta \cos\phi \sin\theta} \right)$$

so

$$F_y = \frac{e^{-j\beta r}}{4\pi} J_o Z_o \sin \tfrac{\pi y}{b} e^{-j\beta d} e^{-(j\beta d \cos\theta)} \left(\frac{e^{j\beta b \sin\phi \sin\theta} - 1}{j\beta \sin\phi \sin\theta} \right) \left(\frac{e^{j\beta a \cos\phi \sin\theta} - 1}{j\beta \cos\phi \sin\theta} \right)$$

$$= \Lambda \frac{e^{-j\beta r}}{4\pi} J_o Z_o \sin \tfrac{\pi y}{b} e^{-j\beta d} e^{-(j\beta d \cos\theta)}$$

$$\text{where } \Lambda = \left(\frac{e^{j\beta b \sin\phi \sin\theta} - 1}{j\beta \sin\phi \sin\theta} \right) \left(\frac{e^{j\beta a \cos\phi \sin\theta} - 1}{j\beta \cos\phi \sin\theta} \right)$$

$$= \frac{\left(e^{j\beta b \sin\phi \sin\theta} - 1\right)\left(e^{j\beta a \cos\phi \sin\theta} - 1\right)}{-\beta^2 \sin^2\theta \sin\phi \cos\phi}$$

$$F_y = \Lambda \frac{e^{-j\beta r}}{4\pi} J_o Z_o \sin \frac{\pi y}{b} e^{-j\beta d} e^{-(j\beta d \cos\theta)}$$

$$
\begin{aligned}
\mathbf{E^s} &= -\nabla \times \mathbf{F} \\
&= -\begin{vmatrix} \mathbf{u_x} & \mathbf{u_y} & \mathbf{u_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & \Lambda \frac{e^{-j\beta r}}{4\pi} J_o Z_o \sin \frac{\pi y}{b} e^{-j\beta d} e^{-(j\beta d \cos\theta)} & 0 \end{vmatrix} \\
&= \mathbf{u_x}\left(\partial/\partial z\left(\Lambda \frac{e^{-j\beta r}}{4\pi} J_o Z_o \sin \frac{\pi y}{b} e^{-j\beta d} e^{-(j\beta d \cos\theta)}\right)\right) \\
&\quad -\mathbf{u_z}\left(\partial/\partial x\left(\Lambda \frac{e^{-j\beta r}}{4\pi} J_o Z_o \sin \frac{\pi y}{b} e^{-j\beta d} e^{-(j\beta d \cos\theta)}\right)\right) \\
E_x^s &= \partial/\partial z\left(\Lambda \frac{e^{-j\beta r}}{4\pi} J_o Z_o \sin \frac{\pi y}{b} e^{-j\beta d - j\beta d \cos\theta}\right) \\
&= \partial/\partial z\left(\Lambda \frac{e^{-j\beta r}}{4\pi} J_o Z_o \sin \frac{\pi y}{b} e^{-j\beta(d + d\cos\theta)}\right)
\end{aligned}
$$

final step is to do $\mathbf{E^i} + \mathbf{E^s}$.

$$
E_x^i = \begin{cases} -\frac{J_o Z_o}{2} \sin \frac{\pi y}{b} e^{-j\beta z} & z > 0 \\ \\ -\frac{J_o Z_o}{2} \sin \frac{\pi y}{b} e^{j\beta z} & z < 0 \end{cases} + E_x^s
$$

Dr, that is the result i could get using a directo approach, I think may be I should have used duality to help solve this problem, since we know what is the field due to electric current sheet, we can replace the terms of this field by those for the dua; term for maganetic sheet. I dont know if I can solve it this way . I dont have more time to look at this, I have an exam tommorrow i need to study for. sorry about that.

to with the time i have left, I have a mid term exam tommorrow and

I tried my best to get the answer in the book, this is the closest I got.