ECE 3341 formulas (Stochastic processes)
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1 Statistical averages

expectation, the expected or mean

$$E[X] \equiv \mu_x = \int_{-\infty}^{\infty} x \; f_x(x) \; dx$$

if x is discrete then

$$E[X] \equiv \mu_x = \sum_n x_n P_x(x_n)$$

for a normalized system, i.e. total weight = 1, then μ_x can be considered to be the center of gravity.

expected value of $Y = G(x)$

$$E[g(x)] \equiv \mu_y = \int_{-\infty}^{\infty} y \; f_Y(y) \; dy = \int_{-\infty}^{\infty} g(x) f_Y(y) \; dy$$

$$\mu_y = \int_{-\infty}^{\infty} g(x) f_X(x) \; dx$$

theorm

$$f_Y\left(y\right)=\frac{f_X\left(x=g^{-1}\left(y\right)\right)}{\left|g^{'}\left(x\right)\right|}$$

$$dy=\left|g^{'}\left(x\right)\right|\ dx$$

conditonal expectation

$$E\left[Y|B\right]\equiv\int_{-\infty}^{\infty}y\ f_{Y|B}\left(y|b\right)\ dy$$

moments, nth moment

n^{th} moment of X denoted by ϵ_n

$$\epsilon_n\equiv\int_{-\infty}^{\infty}x^n\ f_X\left(x\right)\ dx$$

$$\begin{aligned}
\epsilon_0 &= 1 \\
\epsilon_1 &= \mu_x && \text{mean value} \\
\epsilon_2 &= E\left[X^2\right] && \text{mean squared value}
\end{aligned}$$

central moments

$$m_n\equiv\int_{-\infty}^{\infty}\left(u-\mu_x\right)^n\ f_X\left(x\right)\ dx$$

$$\begin{aligned}
m_0 &= 1 \\
m_1 &= 0 \\
m_2 &= E\left[\left\{X-\mu_X\right\}^2\right] && \text{spread or variance} =\sigma_x^2 \\
m_3 &= E\left[\left\{X-\mu_X\right\}^3\right] && \text{skew}
\end{aligned}$$

standard deviation

$$\sigma_x = \sqrt{m_2}$$

realtionships between moments

$$\sigma^2 = m_2 = \epsilon_2 - \epsilon_1^2 = E\left[X^2\right] - \{E[X]\}^2$$

2 Random sequences

mean sequence

$$\mu_X(n) \equiv E[X_n] = \int_{-\infty}^{\infty} x_n \ f(x_n) \ dx_n$$

autocorrelation Bisequence

$$R_X(m,n) \equiv E\left[X_m X_n^*\right] = \int\int x_m x_n^* \ f(x_m, x_n) \ dx_m dx_n$$

Auto covariance Bisequence

$$K_X(m,n) \equiv E\left[\{X_m - \mu_X(m)\}\left\{X_m^* - \mu_X^*(m)\right\}\right]$$

relation

$$K_X(m,n) = R_X(m,n) - \mu_X(m)\,\mu_X^*(n)$$

definitions

uncorrelated random sequence

$$\begin{aligned}
&if \\
&K_x(m,n) = 0 \qquad \forall m,n \quad m \neq n \\
&\qquad\qquad = \sigma_x^2 \qquad m = n
\end{aligned}$$

or

$$\mathbf{R_X(m,n) = \mu_X(m)\,\mu_X^*(n) \qquad \forall m,n \quad m \neq n}$$

then the sequence is called uncorrelated random sequence

orthogonal random sequence

if

$$R_X(m,n) = 0 \qquad \forall m,n \quad m \neq n$$
$$= E\left[x_n^2\right] \qquad m = n$$

then the sequence is called an orthogonal random sequence

Gausian random sequence

if all kth order distributions of a random sequence X_n are jointly Gaussian then it is called a Gaussian random seq.

strict sense stationary SSS

if the kth order probability functions do not depend on the index n, then it is SSS.

Wide sense stationary WSS

if the mean function is constant and the autocorrelation (covariance) is shift-invariant then it is WSS.

i.e.

$$\mu_x(n) = \mu_x$$

and

$$R_X(m,n) = R_X(m-n)$$

usefull identity

$$\int_0^\infty x^n e^{-x}\, dx = n!$$

when adding 2 i.i.d R.V., their f's convolve and their characterstic functions is multiplied

if X is an i.i.d, then at each n it is the same RV, and they are independent RV's

convolution

for a discrete, linear time invariant

$$y\left(n\right) = \sum_{m=-\infty}^{\infty} h(n-m)u(m)$$

$$y = h * u$$

for a continouse liner time invariant

$$y\left(t\right) = \int_{-\infty}^{\infty} h\left(t-\tau\right) u\left(\tau\right)\ d\tau$$

$$y = h * u$$