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20/20

ECE 405 QUIZ #6 20 POINTS

1. A sinusoidal message $x(t) = 2\cos(2\pi 2000t)$ is sampled at a rate of 10000 samples per second ($f_s = 10000$, $T_s = 1/10000$). The sampling signal p(t) is a rectangular pulse train with period 1/10000 seconds, amplitude h = 1V, and duty cycle d = 1/3.

(a) Plot x(t) in the time domain for two periods.
(b) Plot the spectrum X(f) in the frequency domain.
(c) Find the expression of P(f) and plot the spectrum P(f) of the pulse train in the frequency domain.
(d) Find expression of the sampled waveform $x_s(t)$, and plot $x_s(t)$ in the time domain for two periods of x(t).
(e) Find the expression of the spectrum $X_s(f)$ of the sampled waveform, and plot $X_s(f)$ in the frequency domain for $-6f_s \le f \le 6f_s$.
(f) The sampled signal is applied to an ideal lowpass filter with bandwidth $f_s/2$. Find the expression of the spectrum Y(f) of the output signal, and plot Y(f) in the frequency domain.
(g) The sampled signal is applied to an ideal lowpass filter with bandwidth $f_s/2$. Find the expression y(t) of the output signal, and plot y(t) in the time domain for two periods.

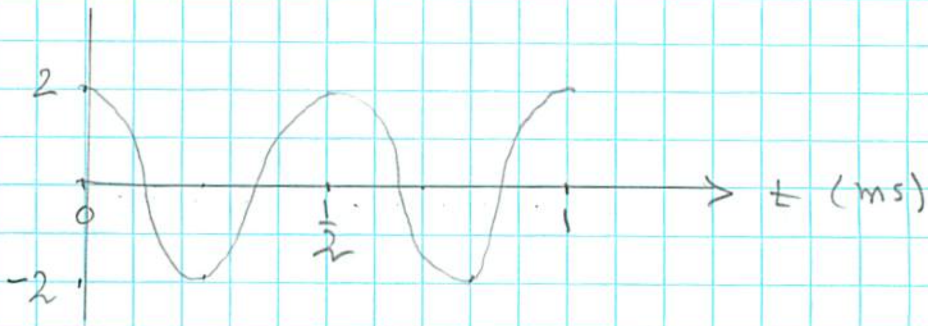
$\left(\frac{t}{\tau} - n\frac{T_s}{\tau}\right) = \left(\frac{t}{\frac{1}{30{,}000}} - 3n\right) = (30{,}000t - 3n)$

(1)

$x(t) = 2\cos(2\pi\, 2000 t) \Rightarrow f_1 = 2000\,\text{hz},\ T_1 = \frac{1}{2}\,\text{ms}.$

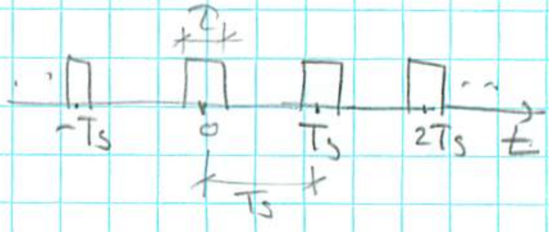
$f_s = 10{,}000\ \text{HZ}.$

(a)



(b)

(c) $p(t) = \sum_{n=-\infty}^{\infty} h\, \text{rect}\left(\frac{t - nT_s}{\tau}\right)$



$= \sum \text{rect}\left(\frac{t - nT_s}{\tau}\right)$

but $\frac{\tau}{T_s} = d = \frac{1}{3}$, and $T_s = \frac{1}{10000}$ so $10{,}000\,\tau = \frac{1}{3}$ ∴ $\tau = \frac{1}{3}\frac{1}{10{,}000}$

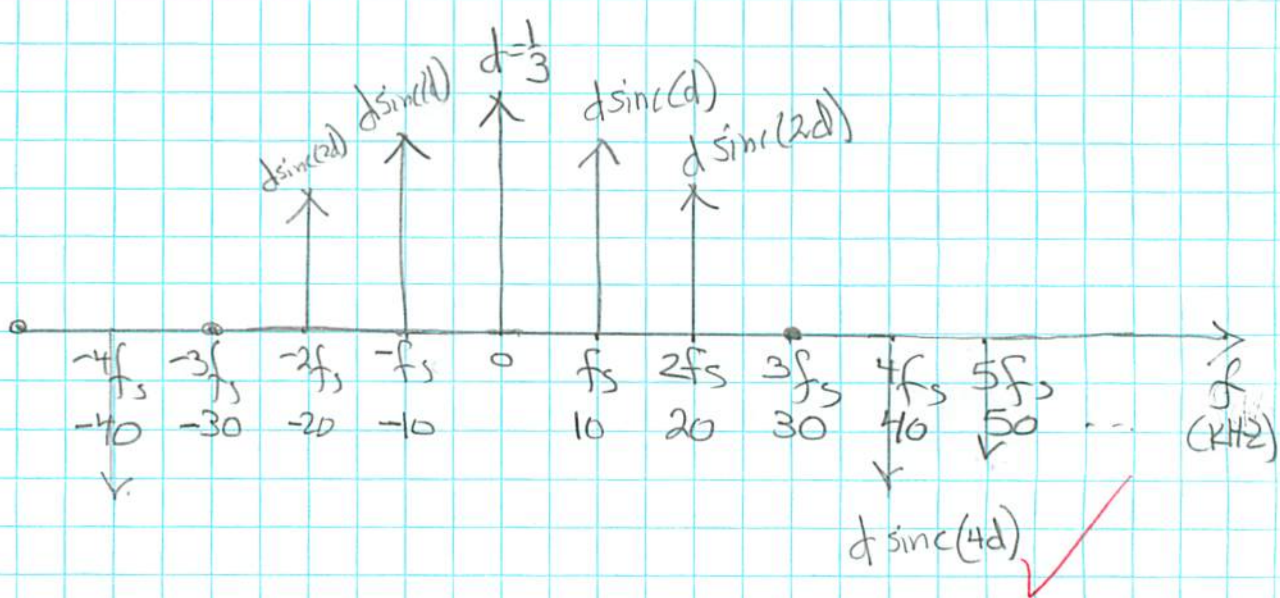
so $\tilde{p}(t) = \sum_{n=-\infty}^{\infty} \frac{\tau}{T_s} \text{sinc}(nd)\, e^{j\frac{2\pi}{T_s} nt}$

so $$P(f) = \frac{\tau}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}(nd)\, \delta(f - nf_s)$$

or $$P(f) = \frac{1}{3} \sum_{n=-\infty}^{\infty} \text{sinc}\left(\frac{n}{3}\right) \delta(f - n\,10000)$$

→

plot $P(f)$:

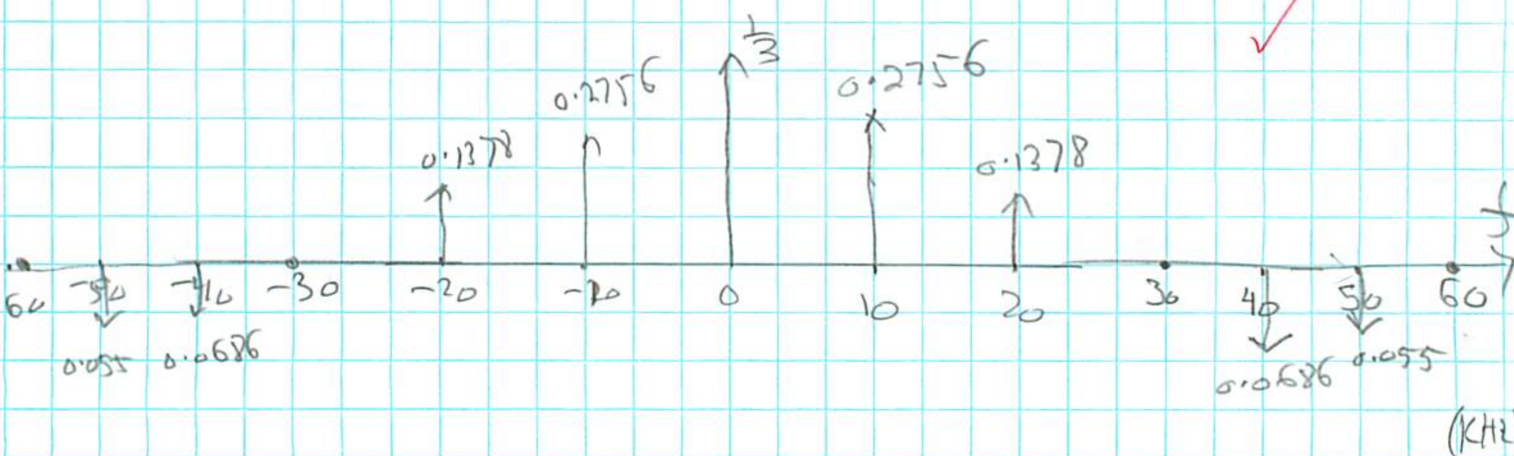


$$\mathrm{sinc}(d) = \frac{\sin(\pi \frac{1}{3})}{\pi \frac{1}{3}} = 0.8269 \left(\tfrac{1}{3}\right) = 0.2756$$

$$\mathrm{sinc}(2d) = \frac{\sin(\pi \frac{2}{3})}{\pi \frac{2}{3}} = 0.4134 \left(\tfrac{1}{3}\right) = 0.1378$$

$$\mathrm{sinc}(4d) = \frac{\sin(\pi \cdot \frac{4}{3})}{\pi \frac{4}{3}} = -0.206 \left(\tfrac{1}{3}\right) = -0.0686$$

$$\mathrm{sinc}(5d) = \frac{\sin(\pi \frac{5}{3})}{\pi \frac{5}{3}} = -0.165 \left(\tfrac{1}{3}\right) = -0.055$$



(d) $x_s(t) = x(t)\left(\sum h \, \text{rect}\left(\frac{t-nT_s}{\tau}\right)\right)$

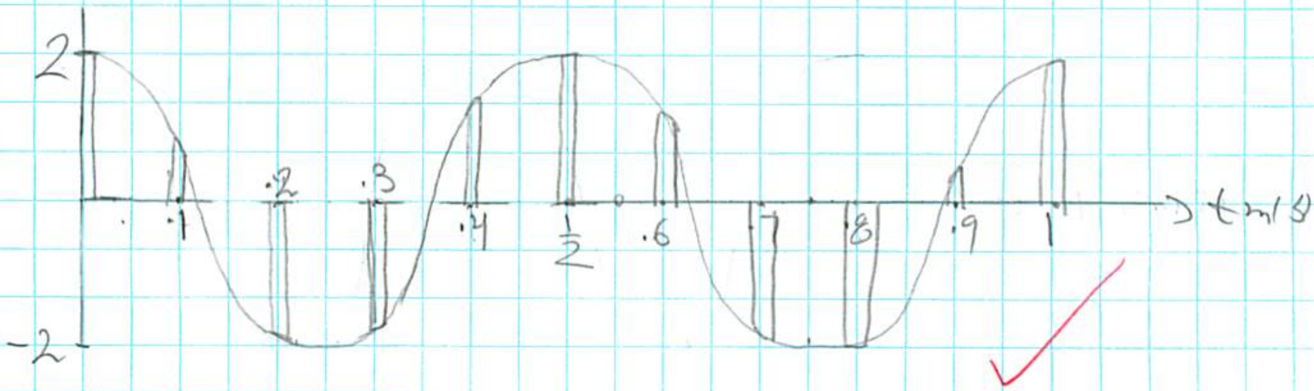
$= x(t)\left(d\sum \text{sinc}(nd)\, e^{j\frac{2\pi}{T_s}nt}\right)$

$= x(t)\left(\frac{1}{3}\sum \text{sinc}\left(\frac{n}{3}\right) e^{j2\pi(10{,}000)nt}\right)$

or $x_s(t) = \frac{1}{3}\sum 2\cos(2\pi 2000t)\,\text{sinc}\left(\frac{n}{3}\right) e^{j2\pi 10000nt}$

$$x_s(t) = \frac{2}{3}\sum \cos(2\pi 2000\, nT_s)\,\text{sinc}\left(\frac{n}{3}\right) e^{j2\pi 10000nt}$$

$T_s = 0.1$ (ms), $T_1 = \frac{1}{2}$ ms, $d = \frac{1}{3} = \frac{\tau}{T_s}$ so $\tau = \frac{1}{3}T_s$.



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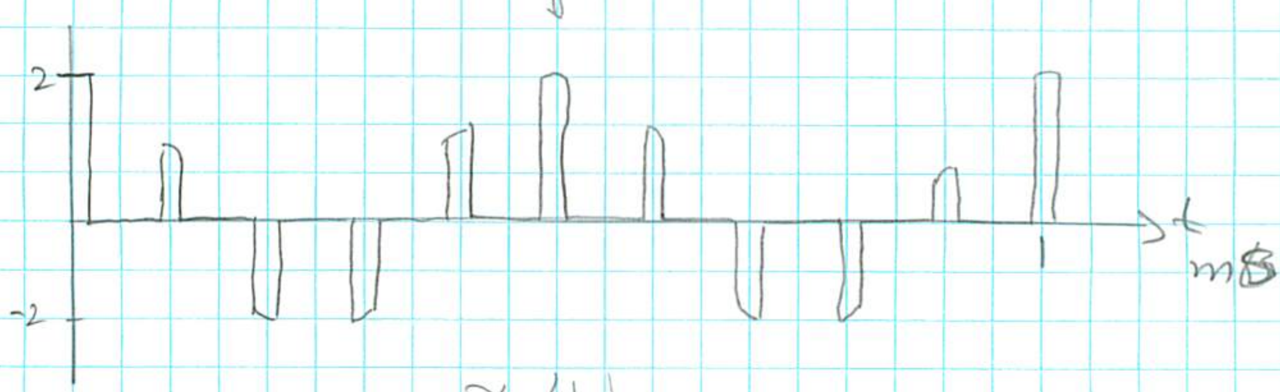

$x_s(t)$.

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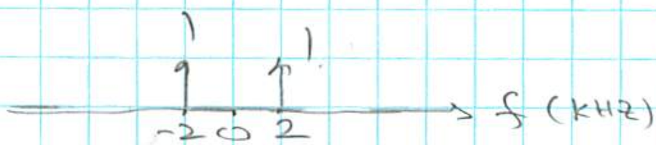
(e)

spectrum of $x(t)$ — spectrum of practical puls train

$$X_s(f) = X(f) \circledast G(f)$$

$$= X(f) \circledast d\sum \text{sinc}(nd)\,\delta(f-nf_s)$$

$$\boxed{X_s(f) = d\sum \text{sinc}(nd)\, X(f-nf_s)}$$



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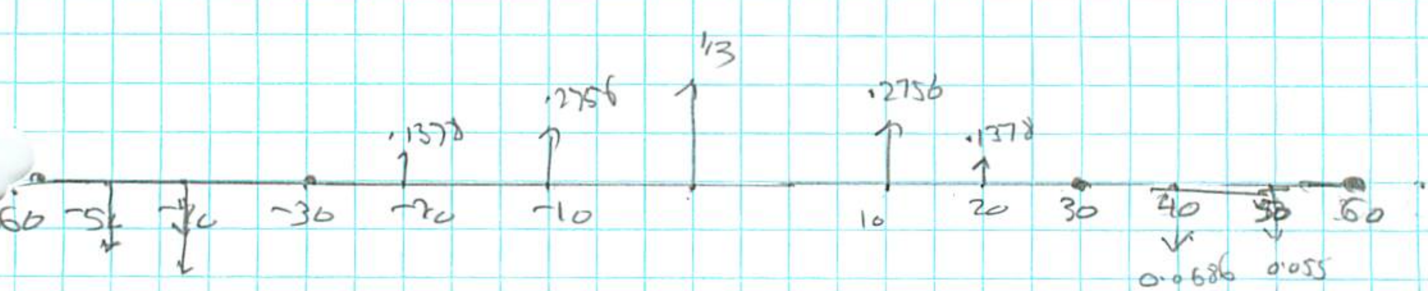
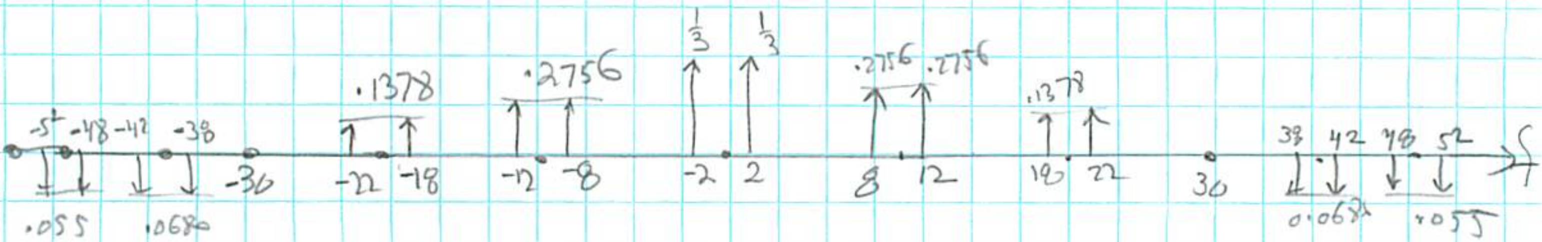
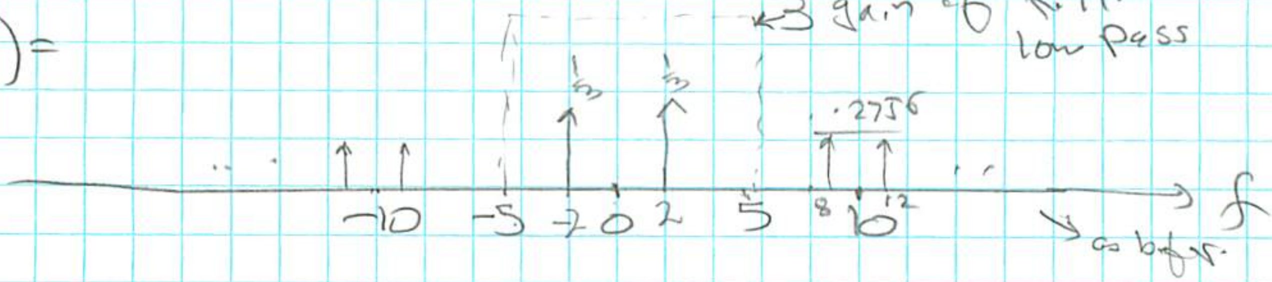




$X_s(f)$

$Y(f) =$



$$Y(f) = X_s(f) \cdot H(f)$$

where $H(f) = \frac{1}{d} \text{Rect}\left(\frac{f}{2B}\right)$

where $d = \frac{1}{3}$, $B = 2\,\text{KHZ}$.

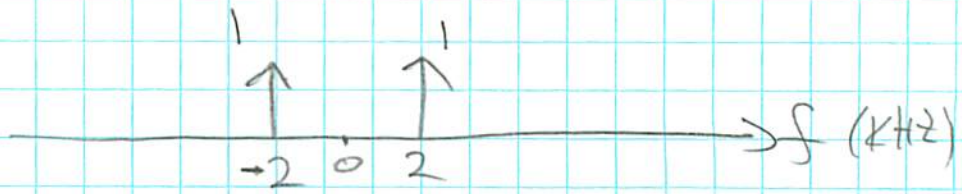
so $$\boxed{H(f) = 3\,\text{Rect}\left(\frac{f}{4000}\right)}$$

so $$Y(f) = \left(d \sum \text{sinc}(nd)\, X(f - nf_s)\right) \frac{1}{d} \text{Rect}\left(\frac{f}{2B}\right)$$

$$\boxed{Y(f) = \sum_{n=-\infty}^{\infty} \text{Rect}\left(\frac{f}{4000}\right) \text{sinc}\left(\frac{n}{3}\right) X(f - n6000)}$$

$Y(f) =$

after filter applied.



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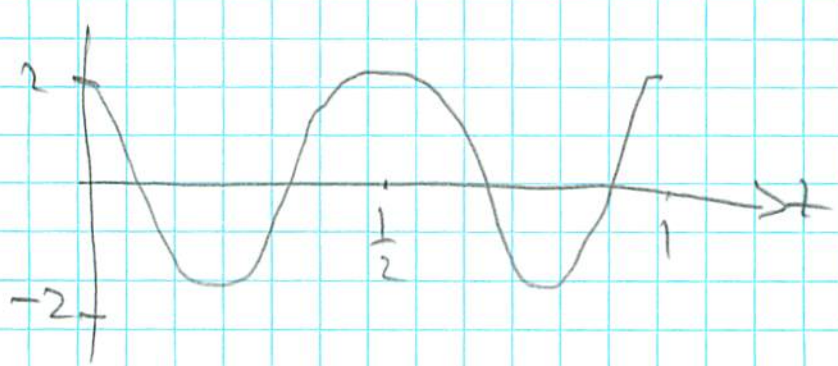
(g) so $y(t) = 2\cos(2\pi\, 2000t)$

$y(t)$ is same as $x(t)$!

so same plot as (a) !



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