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ECE 409 QUIZ #4 20 POINTS

16/20

1. The loop filter of the second-order analog PLL is given by $F(s)=\frac{1+T_1 s}{s}$. Assume that K = 0.32, T_1 = 15/4.

(a) Find the open loop transfer function G(s) = F(s)K/s.

(b) Find the closed-loop transfer function H(s) = $\frac{\hat{\Phi}(s)}{\Phi(s)}$.

(c) Find and plot the impulse response h(t) of this APLL.

(d) Find and plot the step response $h_s(t)$ of this APLL.

(e) Find and plot the error response $h_e(t)$ to step input of this APLL.

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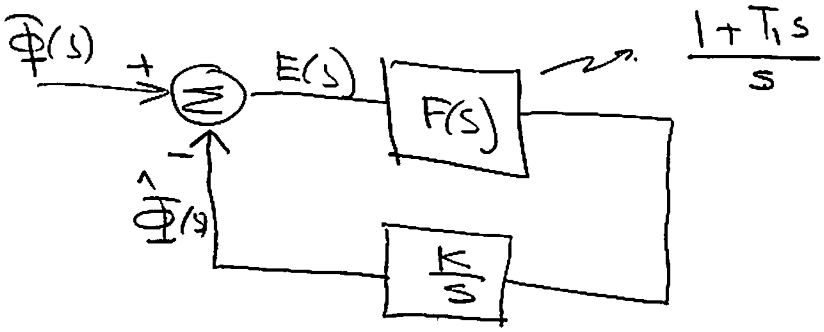


(a) $G(s) = F(s)\frac{K}{s} = \left(\frac{1+T_1 s}{s}\right)\left(\frac{k}{s}\right)$

(b) $H(s) = \frac{\hat{\Phi}(s)}{\Phi(s)} = \frac{E(s)\,G(s)}{\Phi(s)} = \frac{[\Phi(s) - \hat{\Phi}(s)]\,G(s)}{\Phi(s)}$

$= \frac{\Phi(s)\,G(s) - \hat{\Phi}(s)G(s)}{\Phi(s)} = G(s) - \frac{\hat{\Phi}(s)}{\Phi(s)}\,G(s)$

$H(s) = G(s) - H(s)\,G(s)$

Solve for $H(s)$

$$\boxed{H(s) = \frac{G(s)}{1+G(s)}}$$

here $H(s) = \frac{\frac{1+T_1 s}{s}\,\frac{k}{s}}{1 + \frac{1+T_1 s}{s}\,\frac{k}{s}} = \frac{(1+T_1 s)\,k}{s^2 + (1+T_1 s)k}$

$$\boxed{H(s) = \frac{K + KT_1 s}{s^2 + T_1 k s + k}}$$

→

$$: H(s) = \frac{K + K T_1 s}{s^2 + T_1 K s + K}$$

$$K = \frac{32}{100}, \quad T_1 = \frac{15}{4}.$$

$$H(s) = \frac{\frac{32}{100} + \frac{32}{100}\frac{15}{4} s}{s^2 + \frac{15}{4}\frac{32}{100} s + \frac{32}{100}} = \frac{.32 + 1.2 s}{s^2 + 1.2 s + .32}$$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1.2}{2} \pm \frac{1}{2}\sqrt{1.2^2 - 4(.32)}$$

$$= -0.6 \pm \frac{1}{2}\sqrt{1.44 - 1.28} = -0.6 \pm \frac{1}{2}\sqrt{0.16}$$

$$= -0.6 \pm \frac{1}{2}(0.4) = -0.6 + .2$$

$$\text{or} = -0.6 - 0.2$$

$$\text{so } \boxed{S_1 = -0.4, \quad S_2 = -0.8}$$

note the poles < 0 hence stable system

$$\text{so } \boxed{H(s) = \frac{0.32 + 1.2S}{(S+0.4)(S+0.8)}} = \frac{A}{S+0.4} + \frac{B}{S+0.8}$$

$$A = \left.\frac{0.3 + 1.2S}{S+0.8}\right|_{S=-0.4} = \frac{(0.3) + 1.2(-0.4)}{(-0.4) + (0.8)} = \frac{-0.18}{0.4}$$

$$= \boxed{\cancel{-0.45}}$$

$$B = \left.\frac{0.32 + 1.2S}{S+0.4}\right|_{S=-0.8} = \frac{(0.32) + 1.2(-0.8)}{-0.8 + 0.4} = \frac{.32 - 0.96}{-0.4}$$

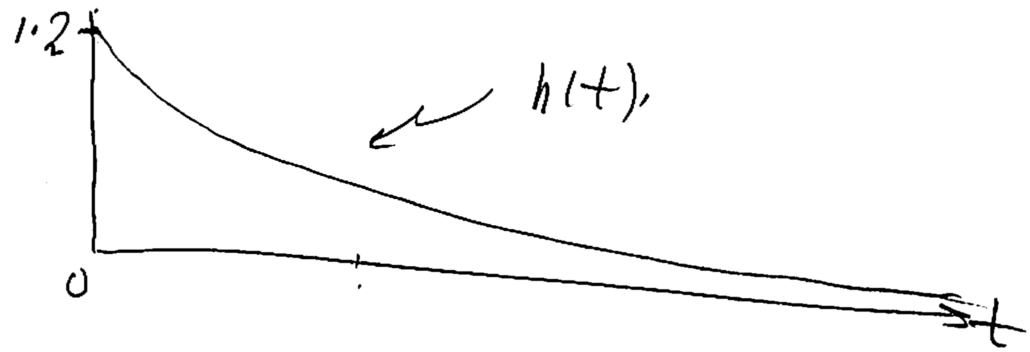
$$= 1.6$$

-0.4

$$\text{so } \boxed{H(s) = \frac{-0.45}{S+0.4} + \frac{1.6}{S+0.8}} \longrightarrow$$

so $\boxed{h(t) = \left[-0.45\, e^{-0.4t} + 1.6\, e^{-0.8t}\right] u(t)}$ (3)

so



(4) step response is when $\Phi(t) = u(t)$.

so $\boxed{\Phi(s) = \frac{1}{s}}$

hence $\hat{\Phi}(s) = \Phi(s)\, H(s)$

$$= \frac{1}{s}\,\frac{0.32 + 1.2s}{s^2 + 1.2s + 0.32}$$

$$= \frac{1}{s}\,\frac{0.32 + 1.2s}{(s+0.4)(s+0.8)}$$

so $\hat{\Phi}(s) = \frac{A}{s} + \frac{B}{s+0.4} + \frac{C}{s+0.8}$

$$A = \left.\frac{0.32 + 1.2s}{(s+0.4)(s+0.8)}\right|_{s=0} = \frac{0.32}{(0.4)(0.8)} = 1$$

$$B = \left.\frac{0.32 + 1.2s}{s(s+0.8)}\right|_{s=-0.4} = \frac{(0.32)+(1.2(-0.4))}{(-0.4)(-0.4+0.8)} = \frac{-0.16}{-0.16} = 1$$

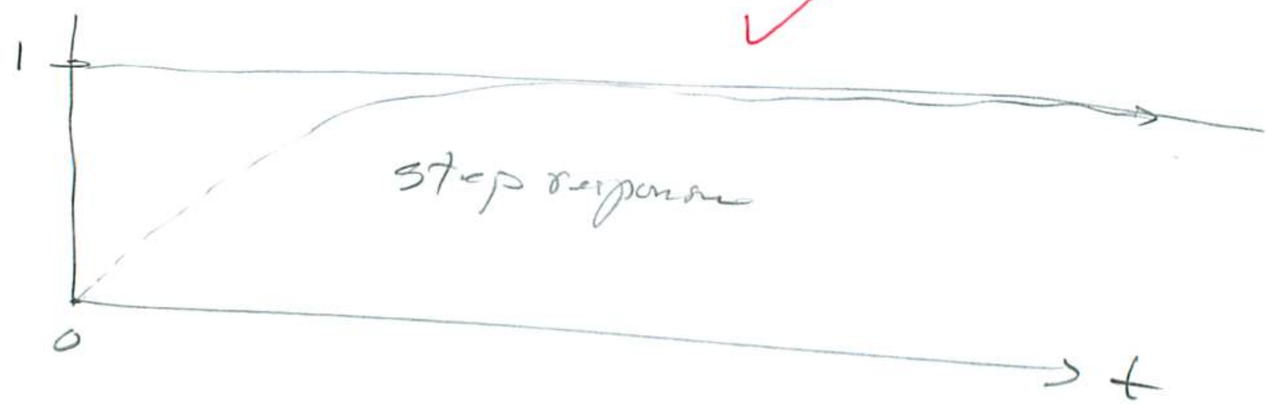
$$C = \left.\frac{0.32 + 1.2s}{s(s+0.4)}\right|_{s=-0.8} = \frac{(.32)-(1.2)(.8)}{(-.8)(-.8+0.4)} = \frac{-.64}{-.32} = 2$$

$\rightarrow$

so $\hat{\Phi}(s) = \frac{1}{s} + \frac{1}{s+0.4} - \frac{2}{s+0.8}$

so $\hat{\Phi}(t) = u(t) + e^{-0.4t}u(t) - 2e^{-0.8t}u(t)$

$= \left(1 + e^{-0.4t} - 2e^{-0.8t}\right)u(t)$



at $t=0$, $1+1-2 = 0$

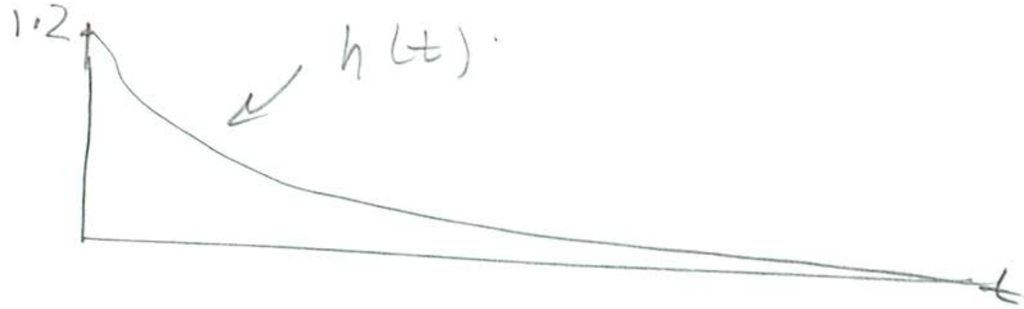
at $t \to \infty$ $\to 1$

at $t=1 \to$ $1 + .67 - 2(.449) = .772$.

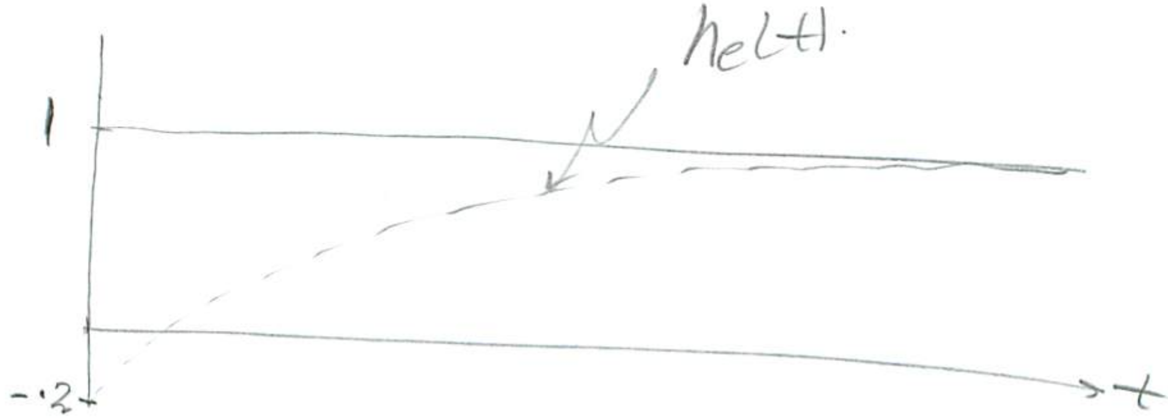
$\underline{h_e(t)}$

since $H_e(s) = 1 - H(s)$

then $h_e(t) = 1 - h(t)$



so $1-h(t)$ is



at $t=0$, $1-1.2=-.2$

at $t=1$, $1-.8=.2$

at $t\to\infty$ $h_e(t)=1-0=1$

$$h_e(t)=\left[\cancel{1}+0.4\cancel{e}^{-0.4t}-1.6\cancel{e}^{-0.8t}\right]u(t)$$

$$\left(2e^{-0.8t}-e^{-0.4t}\right)u(t)$$