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19/20

ECE 405 QUIZ #3 20 POINTS SUMMER 2010

1. The block diagram of USSB generation using phasing method is shown in Figure 1. Let $x(t) = 2\sin(2\pi 3000t)$ and $f_c = 30000$ Hz.

(a) Plot x(t) in the time domain for two periods starting from t = 0.
(b) Plot the spectrum X(f) of x(t) in the frequency domain.
(c) Find the Hilbert transform $\hat{x}(t)$ and plot $\hat{x}(t)$ in the time domain for two periods starting from t = 0.
(d) Plot the spectrum $\hat{X}(f)$ of $\hat{x}(t)$ in the frequency domain.
(e) Find the waveform at (1) and plot it in the time domain.
(f) Find the spectrum at (1) and plot it in the frequency domain.
(g) Find the waveform at (2) and plot it in the time domain.
(h) Find the spectrum at (2) and plot it in the frequency domain.
(i) Find the waveform at (3) and plot it in the time domain.
(j) Find the spectrum at (3) and plot it in the frequency domain.

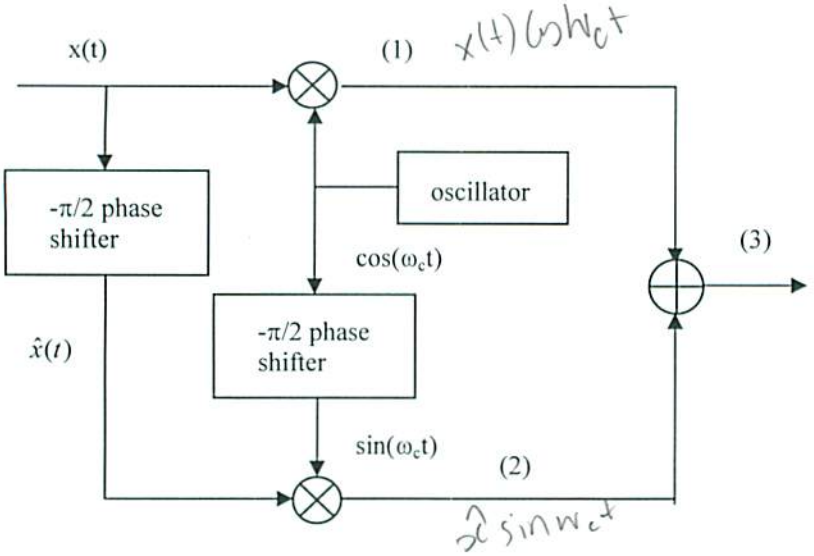


Figure 1

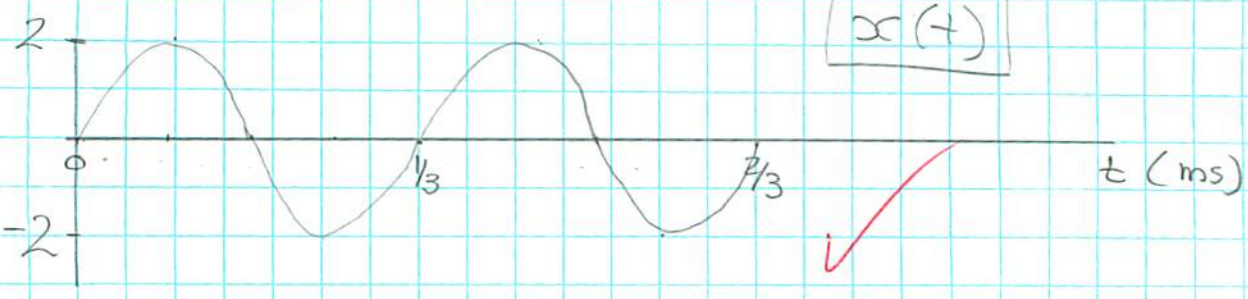
$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$
$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$
$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$
$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$
$\sin(\alpha)\cos(\beta) = (1/2)\,[\sin(\alpha - \beta) + \sin(\alpha + \beta)\,]$
$\sin(\alpha)\sin(\beta) = (1/2)\,[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
$\cos(\alpha)\cos(\beta) = (1/2)\,[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$
$\cos(\alpha)\sin(\beta) = (1/2)\,[-\sin(\alpha - \beta) + \sin(\alpha + \beta)]$

$x(t) = 2\sin(2\pi\, 3000t - \frac{\pi}{2})$

$x(t) = 2\sin(2\pi\, 3000t)$ $\quad f_c = 30{,}000$ Hz

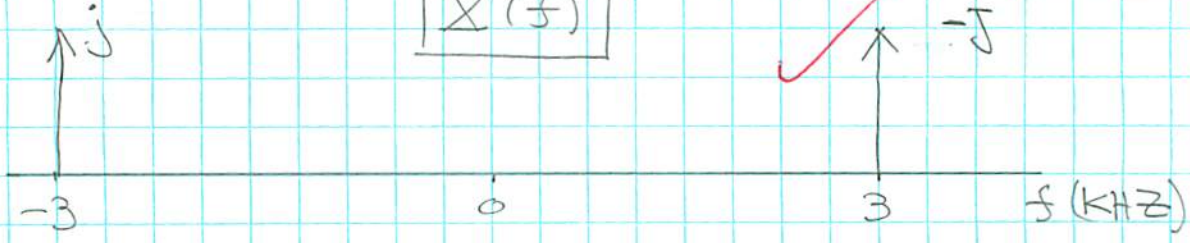
a) $f_m = 3000$ Hz. so $T_0 = \frac{1}{3000} = \boxed{\frac{1}{3}\text{ ms}}$.

$\boxed{x(t)}$



✓

(b) $\boxed{X(f)}$



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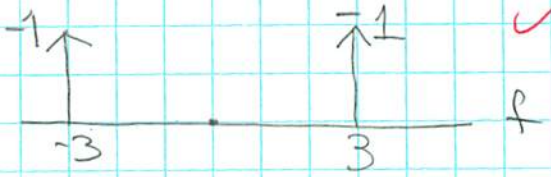
(c) $\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau}\, d\tau$.

or $\hat{X}(f) = -j\,\text{sgn}(f)\,X(f)$.

$=$

$=$



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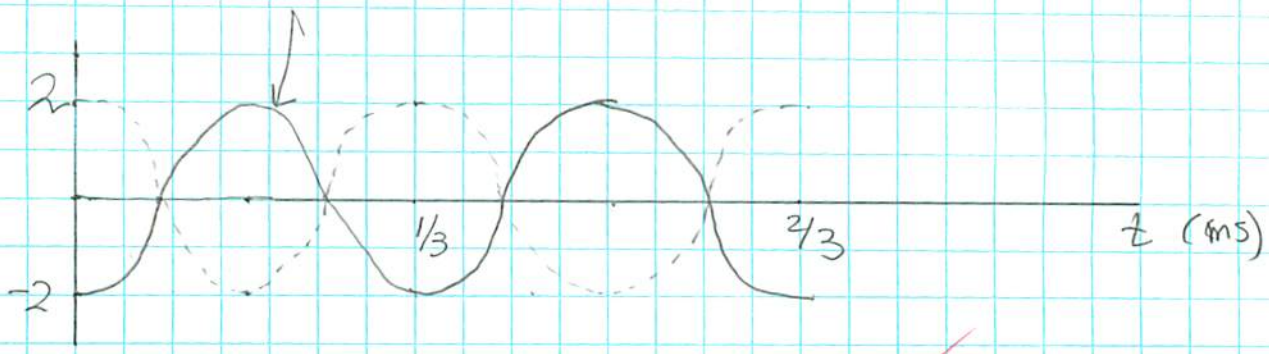
so $\hat{x}(t) = \boxed{-2\cos(2\pi\, 3000t)}$

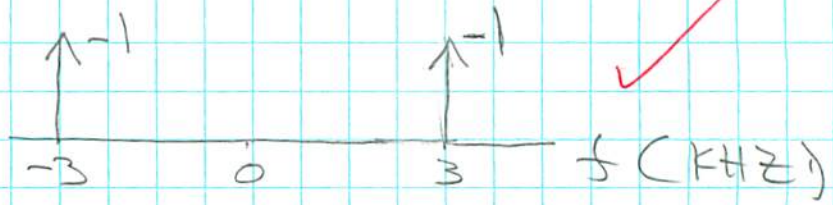
Plot $\hat{x}(t)$ for 2 periods

$$\hat{x}(t) = -2\cos(2\pi\, 3000 t)$$



(d) Plot $\hat{X}(f)$.

I already did this:



(e) Waveform at ① is $\underbrace{x(t)}_{\alpha_1}\underbrace{\cos w_c t}_{\beta}$

$$s_1(t) = 2\sin(2\pi\, 3000 t)\cdot\cos(2\pi\, 30000 t)$$

$$= 2\left[\tfrac{1}{2}\left[\sin(-27000 t\,(2\pi)) + \sin(33000 t\,(2\pi))\right]\right]$$

$$\boxed{s_1(t) = -\sin(2\pi\, 27000 t) + \sin(33000 t + 2\pi)}$$

plot in time domain next page.

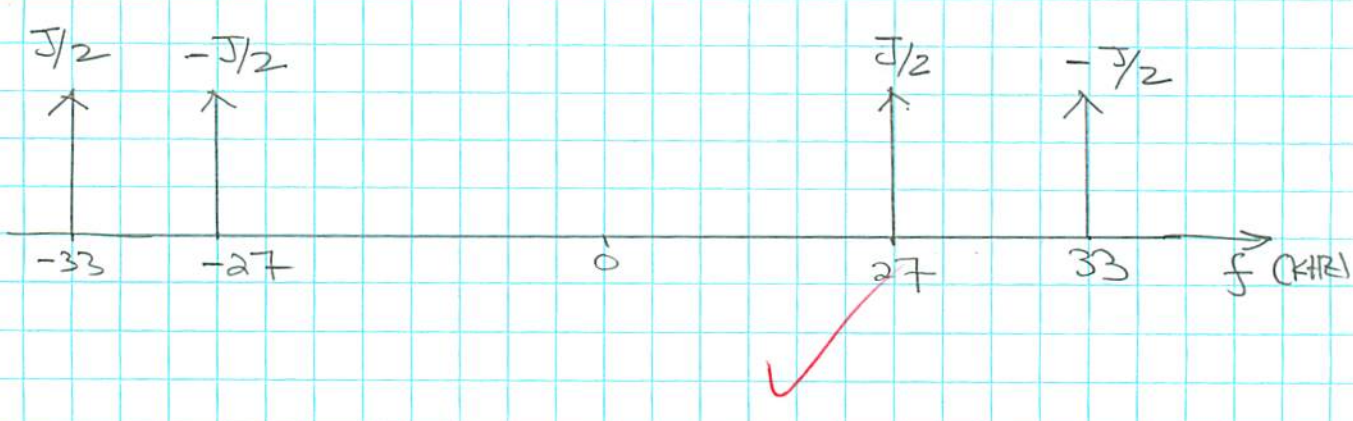
$$\text{so } S_1(f) = -\left[-j/2\,\delta(f-27000) + \tfrac{j}{2}\,\delta(f+27000)\right]$$

$$+ \left[-j/2\,\delta(f-33000) + \tfrac{j}{2}\,\delta(f+33000)\right]$$

$$= j/2\,\delta(f-27^k) - j/2\,\delta(f+27^k) - j/2\,\delta(f-33^k) + j/2\,\delta(f+33^k)$$

→

$$\therefore S_1(f) = \frac{j}{2}\left[\delta(f-27000) - \delta(f+27000) - \delta(f-33000) + \delta(f+33000)\right]$$

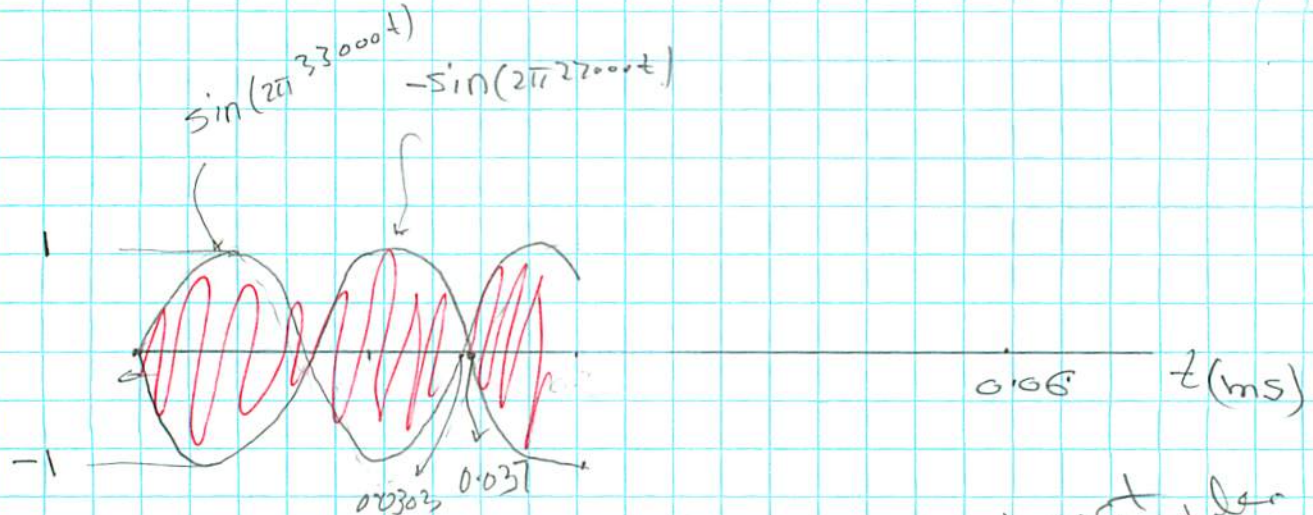


Plot $s_1(t)$ in time domain:

$$s_1(t) = -\sin(2\pi\, 27000t) + \sin(33000t\, 2\pi)$$

$f = 27000 \Rightarrow T = \frac{1}{27000} = 0.037$ ms for part

$f = 33000 \Rightarrow T = \frac{1}{33000} = 0.0303$ ms for part.



notice almost no amplitude

So add

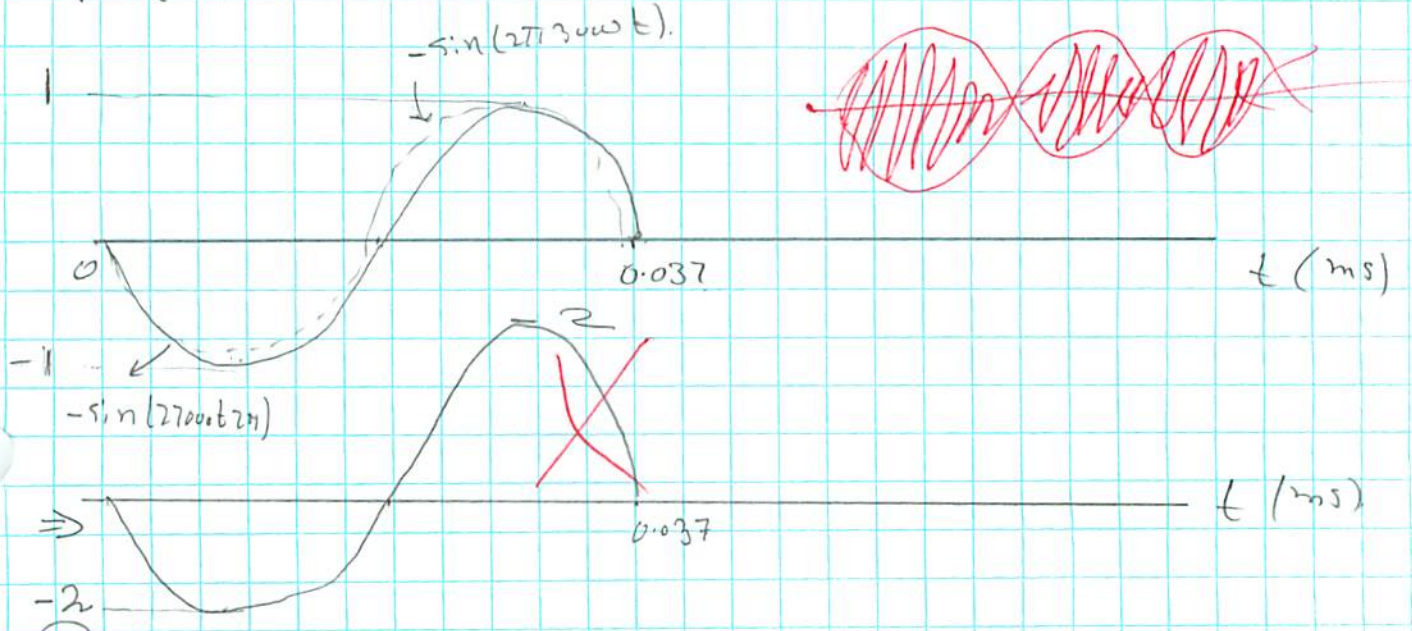
(g) $s_2(t) = \hat{x}(t) \sin(2\pi f_c t)$

$= -2\cos(2\pi 3000t)\sin(2\pi 30000t)$

$= -2\left[\frac{1}{2}\left(-\sin(-27000t + 2\pi) + \sin(2\pi 33000t)\right)\right]$

$= -\sin(27000t + 2\pi) - \sin(2\pi 33000t)$

Plot in time domain.



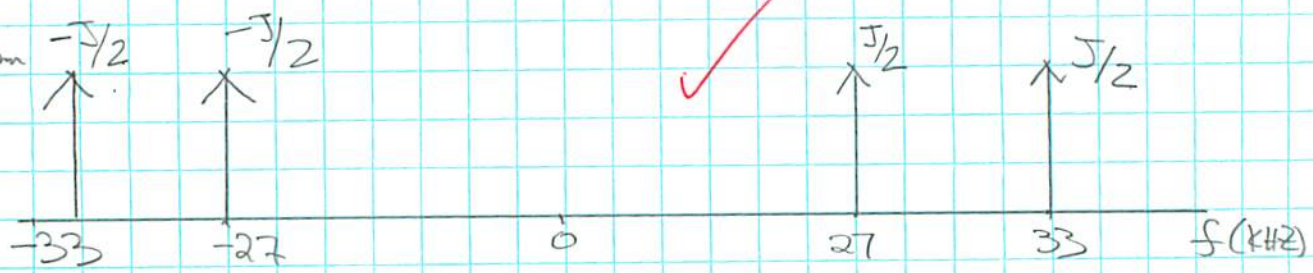
(h) Spectrum is

$S_2(f) = -\left[\frac{-1}{2}\delta(f-27000) + \frac{1}{2}\delta(f+27000)\right]$

$-\left[\frac{-1}{2}\delta(f-33000) + \frac{1}{2}\delta(f+33000)\right]$

$= \frac{1}{2}\delta(f-27000) - \frac{1}{2}\delta(f+27000) + \frac{1}{2}\delta(f-33000) - \frac{1}{2}\delta(f+33000)$

So spectrum



(1)

Since S_{USSB}, Then need to subtract

$S_1(f) - S_2(f)$ to find $S_3(f)$.

First, in time domain.

$$S_3(t) = x(t)\cos\omega_c t - \hat{x}(t)\sin\omega_c t.$$

$$= S_1(t) - S_2(t)$$

$$= \left(-\sin(2\pi 27000t) + \sin(33000t\,2\pi)\right)$$

$$- \left(-\sin(2\pi 27000t) - \sin(2\pi 33000t)\right)$$

$$= -\sin(2\pi 27000t) + \sin(33000t\,2\pi) + \sin(2\pi 27000t) + \sin(2\pi 33$$

$$S_3(t) = 2\sin(33000t\,2\pi)$$

Plot in time domain:

$2\sin(33000t)$

(j) spectrum is

since $S_3(t) = 2 \sin(2\pi\ 33000t)$ then spectrum is

$$S_3(f) = -J\delta(f-33000) + J\delta(f+33000)$$

Which is also $S_1(f) - S_2(f)$ ok verified