Properties of the Fourier Transform

Property	$f(t)$	$F(\omega)$
Linearity (Superposition)	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Time Shifting	$f(t - t_d)$	$e^{-j\omega t_d} F(\omega)$
Time Scaling	$f(ct)$	$\frac{1}{\lvert c \rvert} F\left(\frac{\omega}{c}\right)$
Symmetry (Duality)	$F(t)$	$2\pi f(-\omega)$
Time Reversal	$f(-t)$	$F(-\omega)$
Frequency Scaling	$f(t) e^{j\omega_c t}$	$F(\omega - \omega_c)$
Modulation	$f(t) cos(\omega_c t)$	$\frac{1}{2} F(\omega - \omega_c) + \frac{1}{2} F(\omega + \omega_c)$
Time Differentiation	$\frac{df(t)}{dt}$	$j\omega F(\omega)$
Frequency Differentiation	$tf(t)$	$j \frac{dF(\omega)}{d\omega}$
Conjugate	$f^*(t)$	$F^*(-\omega)$
Integration	$\int_{-\infty}^{t} f(\lambda) d\lambda$	$\frac{1}{j\omega} F(\omega) + \pi F(0) \delta(\omega)$
Convolution	$\int_{-\infty}^{\infty} h(\lambda) x(t - \lambda) d\lambda$	$H(\omega) X(\omega)$
Multiplication	$f_1(t) f_2(t)$	$\frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(v) F_2(\omega - v) dv$
Parseval's Theorem	$\int_{-\infty}^{\infty} \lvert f(t) \rvert^2 dt$	$\frac{1}{2\pi} \int_{-\infty}^{\infty} \lvert F(\omega) \rvert^2 d\omega$

Table 16.2 Fourier Transform Pairs (a > 0)

$f(t)$	$F(\omega)$	$F(f)$
$\Pi\left(\frac{t}{a}\right) = rect\left(\frac{t}{a}\right)$	$a\,\mathrm{sinc}\left(\frac{\omega a}{2\pi}\right)$	$a\,\mathrm{sinc}(fa)$
$\Lambda\left(\frac{t}{a}\right) = tri\left(\frac{t}{a}\right)$	$a\,\mathrm{sinc}^2\left(\frac{\omega a}{2\pi}\right)$	$a\,\mathrm{sinc}^2(fa)$
$e^{-at}u(t)$	$\frac{1}{j\omega + a}$	$\frac{1}{j2\pi f + a}$
$e^{at}u(-t)$	$\frac{1}{-j\omega + a}$	$\frac{1}{-j2\pi f + a}$
$e^{-a\|t\|}$	$\frac{2a}{\omega^2 + a^2}$	$\frac{2a}{4\pi^2 f^2 + a^2}$
$e^{-at}u(t) - e^{at}u(-t)$	$\frac{-2j\omega}{\omega^2 + a^2}$	$\frac{-j4\pi f}{4\pi^2 f^2 + a^2}$
$\delta(t)$	1	1
1	$2\pi\delta(\omega)$	$\delta(f)$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$tu(t)$	$\frac{\pi}{j\omega}\delta(\omega) + \frac{1}{(j\omega)^2}$	$\frac{1}{j4\pi f}\delta(f) + \frac{1}{(j2\pi f)^2}$
$te^{-at}u(t)$	$\frac{1}{(j\omega + a)^2}$	$\frac{1}{(j2\pi f + a)^2}$
$cos(\omega_c t) = cos(2\pi f_c t)$	$\pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)]$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$

$sin(\omega_c t) = sin(2\pi f_c t)$	$-j\pi[\delta(\omega-\omega_c)-\delta(\omega+\omega_c)]$	$\frac{-j}{2}[\delta(f-f_c)-\delta(f+f_c)]$
$e^{-at}u(t)cos(\omega_c t)$	$\frac{j\omega+a}{(j\omega+a)^2+\omega_c^2}$	$\frac{j2\pi f+a}{(j2\pi f+a)^2+(2\pi f_c)^2}$
$e^{-at}u(t)sin(\omega_c t)$	$\frac{\omega_c}{(j\omega+a)^2+\omega_c^2}$	$\frac{2\pi f_c}{(j2\pi f+a)^2+(2\pi f_c)^2}$
$sgn(t)$	$\frac{2}{j\omega}$	$\frac{1}{j\pi f}$
$\mathrm{sinc}(ct)$	$\frac{1}{c}rect\left(\frac{\omega}{2\pi c}\right)$	$\frac{1}{c}rect\left(\frac{f}{c}\right)$
$\mathrm{sinc}^2(ct)$	$\frac{1}{c}tri\left(\frac{\omega}{2\pi c}\right)$	$\frac{1}{c}tri\left(\frac{f}{c}\right)$
$cos\left(\frac{\pi t}{a}\right)rect\left(\frac{t}{a}\right)$	$\frac{2a}{\pi}\frac{cos\left(\frac{\omega a}{2}\right)}{1-\left(\frac{\omega a}{\pi}\right)^2}$	$\frac{2a}{\pi}\frac{cos(\pi af)}{1-(2af)^2}$
$\frac{1}{2}\left[1+cos\left(\frac{\pi t}{a}\right)\right]rect\left(\frac{t}{2a}\right)$	$a\frac{sin(\omega a)}{\omega a\left[1-\left(\frac{\omega a}{\pi}\right)^2\right]}$	$a\frac{sin(2\pi fa)}{2\pi fa\left[1-(2af)^2\right]}$