Mathematical Tables

Trigonometric Identities

$\tan(\alpha) = [\sin(\alpha)]/\cos(\alpha)$
$\mathrm{cosec}\,(\alpha) = 1/\sin(\alpha)$
$\sec(\alpha) = 1/\cos(\alpha)$
$\cot(\alpha) = 1/\tan(\alpha)$
$\sin(\alpha) = \cos(90^o - \alpha) = \sin(180^o - \alpha)$
$\cos(\alpha) = \sin(90^o - \alpha) = - \cos(180^o - \alpha)$
$\tan(\alpha) = \cot(90^o - \alpha) = - \tan(180^o - \alpha)$

$\sin(\alpha + \beta) = \sin(\alpha) \cos(\beta) + \cos(\alpha) \sin(\beta)$
$\sin(\alpha - \beta) = \sin(\alpha) \cos(\beta) - \cos(\alpha) \sin(\beta)$
$\cos(\alpha + \beta) = \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta)$
$\cos(\alpha - \beta) = \cos(\alpha) \cos(\beta) + \sin(\alpha) \sin(\beta)$
$\tan(\alpha + \beta) = [\tan(\alpha) + \tan(\beta)]/[1 - \tan(\alpha) \tan(\beta)]$
$\tan(\alpha - \beta) = [\tan(\alpha) - \tan(\beta)]/[1 + \tan(\alpha) \tan(\beta)]$

$\sin(\alpha) \cos(\beta) = (1/2) [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$
$\sin(\alpha) \sin(\beta) = (1/2) [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
$\cos(\alpha) \cos(\beta) = (1/2) [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$
$\cos(\alpha) \sin(\beta) = (1/2) [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$

$\sin (2\alpha) = 2 \sin(\alpha) \cos(\alpha) = [2 \tan(\alpha)]/[1 + \tan^2(\alpha)]$
$\cos(2\alpha) = 2 \cos^2(\alpha) - 1 = 1 - 2 \sin^2(\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$
$= [1 - \tan^2(\alpha)]/[1 + \tan^2(\alpha)]$
$\tan(2\alpha) = [2 \tan(\alpha)]/[1 - \tan^2(\alpha)]$
$\sin(3\alpha) = 3 \sin(\alpha) - 4 \sin^3(\alpha)$
$\cos(3\alpha) = 4 \cos^3(\alpha) - 3 \cos(\alpha)$
$\tan(3\alpha) = [3 \tan(\alpha) - \tan^3(\alpha)]/[1 - 3 \tan^2(\alpha)]$
$\sin(4\alpha) = 4 \sin(\alpha) \cos(\alpha) - 8 \sin^3(\alpha) \cos(\alpha)$
$\cos(4\alpha) = 8 \cos^4(\alpha) - 8 \cos^2(\alpha) + 1$
$\tan(4\alpha) = [4 \tan(\alpha) - 4 \tan^3(\alpha)]/[1 - 6 \tan^2(\alpha) + \tan^4(\alpha)]$

$\sin^2(\alpha) = (1/2) [1 - \cos(2\alpha)] = 1 - \cos^2(\alpha)$
$\cos^2(\alpha) = (1/2) [1 + \cos(2\alpha)] = 1 - \sin^2(\alpha)$
$\tan^2(\alpha) = [1 - \cos(2\alpha)]/[1 + \cos(2\alpha)]$
$\sin^3(\alpha) = (1/4) [3 \sin(\alpha) - \sin(3\alpha)]$
$\cos^3(\alpha) = (1/4) [3 \cos(\alpha) + \cos(3\alpha)]$
$\sin^4(\alpha) = (1/8) [3 - 4 \cos(2\alpha) + \cos(4\alpha)]$
$\cos^4(\alpha) = (1/8) [3 + 4 \cos(2\alpha) + \cos(4\alpha)]$

$\cos(\alpha) = [e^{j\alpha} + e^{-j\alpha}]/2$

$\sin(\alpha) = [e^{j\alpha} - e^{-j\alpha}]/2j$
$\tan(\alpha) = (-j) [e^{j\alpha} - e^{-j\alpha}]/[e^{j\alpha} + e^{-j\alpha}]$
$e^{j\alpha} = \cos(\alpha) + j \sin(\alpha)$
$e^{-j\alpha} = \cos(\alpha) - j \sin(\alpha)$

$\sin^2(\alpha) + \cos^2(\alpha) = 1$
$1 + \tan^2(\alpha) = \sec^2(\alpha)$
$1 + \cot^2(\alpha) = \mathrm{cosec}^2(\alpha)$

$\sin(\alpha) + \sin(\beta) = 2 \sin[(1/2) (\alpha + \beta)] \cos[(1/2) (\alpha - \beta)]$
$\sin(\alpha) - \sin(\beta) = 2 \cos[(1/2) (\alpha + \beta)] \sin[(1/2) (\alpha - \beta)]$
$\cos(\alpha) + \cos(\beta) = 2 \cos[(1/2) (\alpha + \beta)] \cos[(1/2) (\alpha - \beta)]$
$\cos(\alpha) - \cos(\beta) = -2 \sin[(1/2) (\alpha + \beta)] \sin[(1/2) (\alpha - \beta)]$
$\tan(\alpha) + \tan(\beta) = [\sin(\alpha + \beta)]/[\cos(\alpha) \cos(\beta)]$
$\tan(\alpha) - \tan(\beta) = [\sin(\alpha - \beta)]/[\cos(\alpha) \cos(\beta)]$

Indefinite Integrals

$$\int sin(ax)dx = -\frac{1}{a}cos(ax)$$

$$\int cos(ax)dx = \frac{1}{a}sin(ax)$$

$$\int sin^2(ax)dx = \frac{x}{2} - \frac{sin(2ax)}{4a}$$

$$\int cos^2(ax)dx = \frac{x}{2} + \frac{sin(2ax)}{4a}$$

$$\int sin(ax)cos(ax)dx = \frac{1}{2a}sin^2(ax)$$

$$\int xsin(ax)dx = \frac{1}{a^2}[sin(ax) - axcos(ax)]$$

$$\int xcos(ax)dx = \frac{1}{a^2}[cos(ax) + axsin(ax)]$$

$$\int x^2 sin(ax)dx = \frac{1}{a^3}[2axsin(ax) + 2cos(ax) - a^2x^2 cos(ax)]$$

$$\int x^2 cos(ax)dx = \frac{1}{a^3}[2axcos(ax) - 2sin(ax) + a^2x^2 sin(ax)]$$

$$\int e^{ax}\,dx=\frac{1}{a}e^{ax}$$

$$\int xe^{ax}\,dx=\frac{1}{a^2}e^{ax}(ax-1)$$

$$\int x^2e^{ax}\,dx=\frac{1}{a^3}e^{ax}(a^2x^2-2ax+2)$$

$$\int x^ne^{ax}\,dx=\frac{x^n}{a}e^{ax}-\frac{n}{a}\int x^{n-1}e^{ax}\,dx$$

$$\int e^{ax}\sin(bx)dx=\frac{1}{a^2+b^2}e^{ax}[a\sin(bx)-b\cos(bx)]$$

$$\int e^{ax}\cos(bx)dx=\frac{1}{a^2+b^2}e^{ax}[a\cos(bx)+b\sin(bx)]$$

$$\int\frac{dx}{a^2+b^2x^2}=\frac{1}{ab}\tan^{-1}\left(\frac{bx}{a}\right)$$

$$\int\frac{x^2dx}{a^2+b^2x^2}=\frac{x}{b^2}-\frac{a}{b^3}\tan^{-1}\left(\frac{bx}{a}\right)$$

Sums of Powers of the First n Integers

$$\sum_{k=1}^{n} k = 1+2+3+\ldots\ldots\ldots+n = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = 1^2+2^2+3^2+\ldots\ldots\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=1}^{n} k^4 = \frac{n}{30}(n+1)(2n+1)(3n^2+3n-1)$$

$$\sum_{k=1}^{n} k^5 = \frac{n^2}{12}(n+1)^2(2n^2+2n-1)$$

If

$$\sum_{k=1}^{n} k^p = a_1n^{p+1}+a_2n^p+a_3n^{p-1}+\ldots\ldots\ldots+a_{p+1}n$$

then

$$\sum_{k=1}^{n} k^{p+1} = \frac{p+1}{p+2} a_1 n^{p+2} + \frac{p+1}{p+1} a_2 n^{p+1} + \frac{p+1}{p} a_3 n^{p} + \dots\dots\dots + \frac{p+1}{2} a_{p+1} n^{2}$$

$$+ \left[1 - (p+1) \sum_{k=1}^{p+1} \frac{a_k}{(p+3-k)} \right] n$$

Series Expansion

$$tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \dots\dots\dots$$

$$sin^{-1}(x) = x + \frac{1}{2}\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7} + \frac{1\cdot 3\cdot 5\cdot 7}{2\cdot 4\cdot 6\cdot 8}\frac{x^9}{9} + \dots\dots\dots$$

$$cos^{-1}(x) = \frac{\pi}{2} - x - \frac{x^3}{6} - \frac{3x^5}{40} - \dots\dots\dots$$

$$sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \dots\dots\dots$$

$$cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots\dots\dots$$