Physics 3041 (Spring 2021) Homework Set 10 **(Due 4/30)**

1. (10 points) Given

$$\int_{-\infty}^{\infty} \exp(-x^2)dx = \sqrt{\pi}\,,$$

make a 3D integral and use the transformation from Cartesian to spherical coordinates to evaluate

$$\int_{0}^{\infty} x^2 \exp(-x^2)dx.$$

2. Follow the lecture example of deriving the gravitational field of a thin shell and calculate the gravitational potential of such a shell over all space. (10 points)

3. Follow the lecture example of deriving the gas pressure and calculate the number of gas particles hitting the container per unit area per unit time. Give your answer in terms of the net number density and the average speed of these particles. (10 points)

4. Derive the expressions of the quantum mechanical orbital angular momentum operators L_x, L_y, L_z in spherical coordinates. Show that

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} - \frac{\vec{L}\cdot\vec{L}}{\hbar^2 r^2}$$

in spherical coordinates. (40 points)

5. Consider $\psi(x,t)$ for $0 \le x \le L$. Given that $\psi(0,t) = \psi(L,t) = 0$ and

$$\psi(x,0) = \begin{cases} A\sin(2\pi x/L), & 0 \le x \le L/2, \\ 0, & L/2 < x \le L, \end{cases}$$

find $\psi(x,t)$ that satisfies the following partial differential equation:

$$i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2\mu}\frac{\partial^2 \psi}{\partial x^2},$$

where A, L, $\hbar$, and μ are positive constants. (30 points)