1. Problem 1.6.1. (20 points)

2. Consider $f(x) = (1 + x)^p$ for (a) $p = 1/3$ and (b) $p = -2$, respectively.

(1) Find the Taylor series of $f(x)$ around $x = 0$. (10 points)

(2) From the form of the general term, find the interval of convergence for the series. (10 points)

(3) How many terms in the series do you need to estimate $f(0.1)$ to within 1%? Check that the difference between your estimate and the actual result has approximately the same magnitude as the next term in the series. (10 points)

3. Expand $f(x) = \tan x^2$ to oder x^6 using (a) direct Taylor expansion of $\tan x$ with a substitution (20 points), and (b) the Taylor series of $\sin x$ and $\cos x$ along with appropriate substitutions (20 points).

4. A particle of mass m moves along the $+x$-axis (i.e., $x > 0$) with a potential energy

$$V(x) = \frac{a}{2x^2} - \frac{b}{x},$$

where a and b are positive parameters.

(a) Find the equilibrium position x_0. (3 points)

(b) Show that the particle executes harmonic oscillations near $x = x_0$. (5 points)

(c) Find the angular frequency of oscillations. (2 points)