Physics 3041 (Spring 2021) **Final Exam**

1. The electron spin is represented by the operator $\vec{s} = (\hbar/2)\vec{\sigma}$, where $\vec{\sigma}$ corresponds to the Pauli matrices. Consider the operator $s_n = \vec{s} \cdot \hat{n}$, where $\hat{n}$ is the unit vector with polar angle θ and azimuthal angle ϕ.

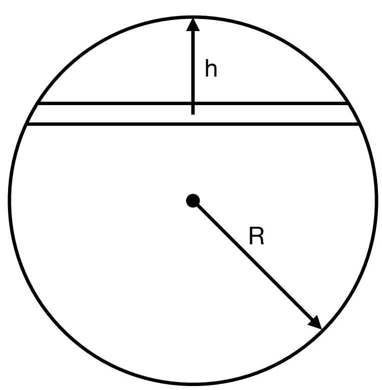
(1) Find the eigenvalues and eigenvectors of s_n. (20 points)

(2) If an electron is in the spin state of $s_z = \hbar/2$, what are the possible results and the corresponding probabilities when s_n is measured? (5 points)

2. A straight tunnel is dug between two cities on a planet of uniform mass density ρ (see cross sectional view below). The effect of the tunnel on the planet's gravity can be ignored. A train with no engine moves on the frictionless rail in the tunnel.

(1) Derive a differential equation that governs the position of the train as a function of time. There is no need to solve the equation for this part. (15 points)

(2) Assume that the train starts from rest at one city. Find the time required for the train to complete a round trip between the two cities. (10 points)



3. In a photon gas, the number density of photons with momentum between $\vec{p}$ and $\vec{p} + d\vec{p}$ is

$$dn = \frac{2}{(2\pi\hbar)^3} \frac{dp_x dp_y dp_z}{\exp[pc/(kT)] - 1},$$

where c is the speed of light, k is the Boltzmann constant, and T is the temperature of the photon gas.

(1) Use the force law $\vec{F} = d\vec{p}/dt$ to derive an expression of the pressure exerted by the photon gas on its container. Your result should be in terms of a dimensional factor multiplied by a dimensionless integral. (20 points)

(2) Evaluate the dimensionless integral in part (1) in terms of a numerical series. (5 points)

4. Two identical pendulums are hung from the same height and coupled with a spring. Each pendulum consists of a string of length l and a bob of mass m. The entire system is in a fixed vertical plane (see figure below). The spring has a spring constant k and is relaxed when $\theta_1 = \theta_2 = 0$. The masses of the strings and the spring can be ignored. The acceleration of gravity is g.

(1) Derive the Lagrangian of the system to the second order in θ_1 and θ_2 (i.e., including terms up to θ_1^2, θ_2^2, and $\theta_1\theta_2$). (10 points)

(2) Find the normal modes and the corresponding oscillation frequencies. (15 points)