MATH 5525- MIDTERM EXAMINATION II

March 30, 2020

Problem 1.

1. State the Bendixon criterion of non-existence of periodic orbits of a differential equation.

2. Consider the differential equation

$$\ddot{x} + f(x)\dot{x} + x = 0,$$

where $f(x) = x^2 + x + a,\ a \in \mathbb{R}$. Determine the range of values of a for which the equation does not have any periodic orbits.

Problem 2.

Consider the system of differential equations that models the growth of two competing species with populations $x \geq 0$ and $y \geq 0$:

$$\dot{x} = x(2 - x - y), \quad \dot{y} = y(3 - 2x - y).$$

1. Find all equilibruium points and determine their stability type.

2. Determine the nullclines of the system.

3. Find the invariant regions of the xy-plane.

4. Draw the phase-plane using your favorite sofware (Matlab, Mathematica, ...).

5. Explain why these equations make it mathematically possible, but extremely unlikely, for both species to survive.

Guidelines:

1. You may use books, notes and internet resources as you wish.
2. The work has to be personal, that is, you may not consult with anyone or receive any help. (You may always email me, if you have questions or difficulties.)
3. The exam should be back tonight, by midnight.
4. Upload the complete work on canvas. If you experience difficulties, please email it directly to me.

Please, sign the following statement:
I hereby certify that I have not received help from anyone in the completion of this test.

Signature:

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Minneaplis, March 30, 2020.