discussion week 3

EE 3015
Signals and Systems

Spring 2020
University of Minnesota, Twin Cities

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Contents

1 Questions

Discussion 4 - practice problems for Midterm 1. Wed Oct 10

problem 1. Consider the convolution $y(t) = x(t) * h(t)$ with

$x(t) = \cos(\pi t) \cdot [u(t+1) - u(t-1)]$

$h(t) = u(t+1) - u(t-1)$

Compute $y(t)$ for $t < 0$

problem 2. Calculate all Fourier Series Coeff. of Signal $x(t)$

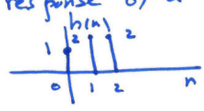
$x(t) = \sin\left(\frac{3\pi t}{2}\right) + \cos(7\pi t)$

identify all frequencies? - what is the fundamental frequency ω_0 ?

problem 3. obtain Discrete Convolution of $y(n) = x(n) * h(n)$ where $x(n) = a^n u[n-5]$, $h(n) = u(-n)$

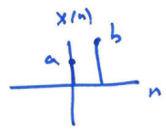
Assuming $|a| < 1$

problem 4. The impulse response of a discrete LTI system is

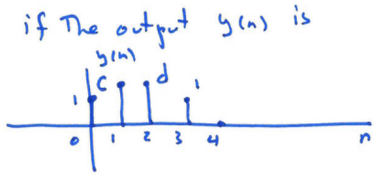


$h(n) = [1 \underset{\uparrow}{2} 2]$, $n=0$

when input $x(n)$ is $x(n) = [a \underset{\uparrow}{b}]$, $n=0$



if the output $y(n)$ is



Find $\begin{bmatrix} a = ? & b = ? \\ c = ? & d = ? \end{bmatrix}$

2 Problem 1

Solution

Folding $h(\tau)$ to becomes $h(-\tau)$. Therefore, when $1 + t < -1$ or $t < -2$, then $y(t) = 0$ since there is no overlap.

When $-1 < 1 + t < 1$, or $-2 < t < 0$, then there is partial overlap. In this case

$$\begin{aligned} y(t) &= \int_{-1}^{1+t} \cos(\pi\tau)\, d\tau \qquad -2 < t < 0 \\ &= \frac{1}{\pi} [\sin(\pi\tau)]_{-1}^{1+t} \\ &= \frac{1}{\pi} [\sin(\pi(1+t)) - \sin(-\pi)] \\ &= \frac{1}{\pi} \sin(\pi(1+t)) \end{aligned}$$

When $1 < 1 + t < 3$, or $0 < t < 2$, then there is partial overlap. In this case

$$\begin{aligned} y(t) &= \int_{t-1}^{1} \cos(\pi\tau)\, d\tau \qquad 0 < t < 2 \\ &= \frac{1}{\pi} \left[\sin(\pi\tau) \right]_{t-1}^{1} \\ &= \frac{1}{\pi} \left[\sin(\pi) - \sin(\pi(t-1)) \right] \\ &= \frac{-1}{\pi} \sin(\pi(t-1)) \end{aligned}$$

When $3 < 1 + t$ or $t > 2$ then $y(t) = 0$ since there is no overlap any more. Hence solution is

$$y(t) = \begin{cases} 0 & t \le -2 \\ \frac{1}{\pi} \sin(\pi(1+t)) & -2 < t \le 0 \\ \frac{-1}{\pi} \sin(\pi(t-1)) & 0 < t \le 2 \\ 0 & t > 2 \end{cases}$$

The following is a plot of $y(t)$

```
In[•]:= mySolution[t_] := Piecewise[{
          {0, t < -2},
          {1 / Pi Sin[Pi (1 + t)], -2 < t < 0},
          {-1 / Pi Sin[Pi (t - 1)], 0 < t < 2}, {0, True}}]
        Plot[mySolution[t], {t, -3, 3}, PlotStyle → Red, GridLines → Automatic,
          GridLinesStyle → LightGray, AxesLabel → {"t", "y(t)"}]
```

Out[•]=

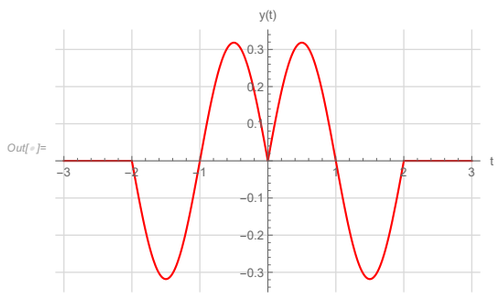


Figure 1: Plot of $y(t)$

3 Key solution

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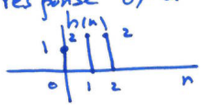

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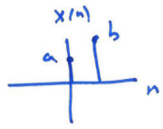

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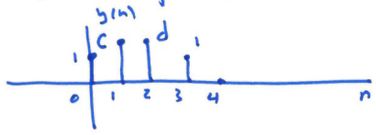

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