**EE3015 – Final Exam – Saturday, May 9th 2020**

*This exam is open book and open notes. Communication with other people is not permitted. Write your solution to each problem in a separate document / sheet of paper. Please submit your solutions in the order they are given in this document. If your solution relies on proofs from an external source (book or notes), please reference this source in your solution (e.g. page number, table number, lecture date). Submit your solutions as a PDF document to Canvas by 5:00pm. If you are registered with the DRC, please ask for additional details regarding submission.*

**Problem 1 (25pts)**

**Consider a causal LTI system with transfer function**

$$H(s) = \frac{-10s + 10}{(s + 10)(s + 1)}$$

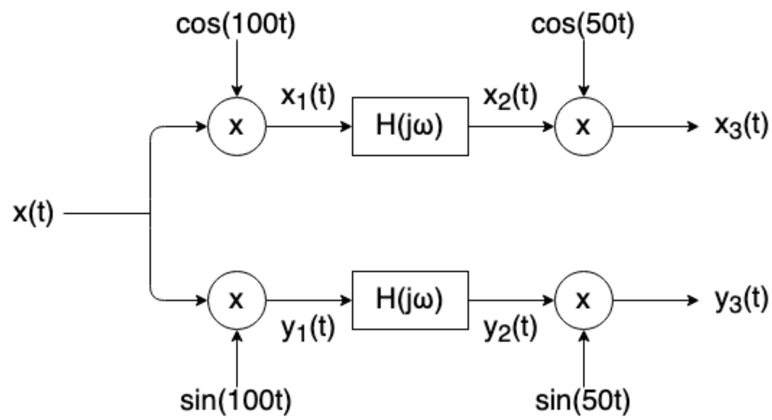
**A.** **(5pts) Write a differential equation in terms of $x(t), y(t)$ (and their derivatives) realizing this system**

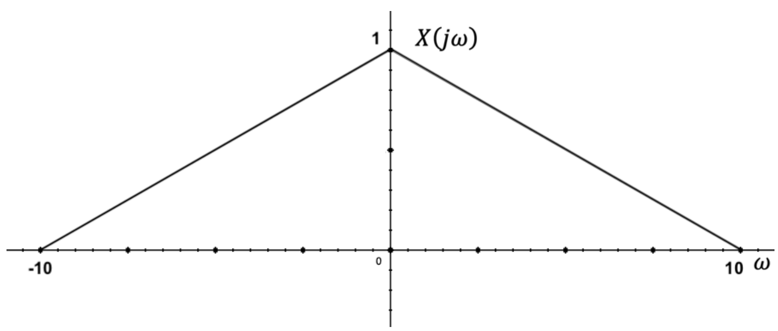
**B.** **(10pts) Find the impulse response $h(t)$ of this system. Determine the ROC of the systems transfer function.**

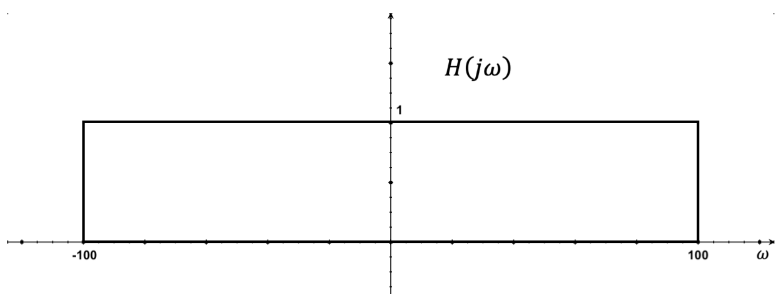
**C.** **(10pts) Find the unit step response of the system, i.e. compute $y(t)$ for $x(t) = u(t)$**

**Problem 2 (30pts)**

**Consider the following block diagram representing a modulating system**



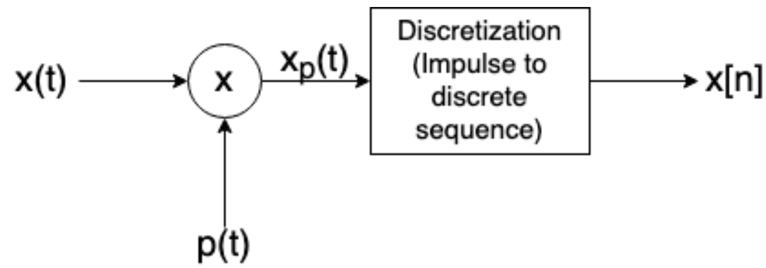



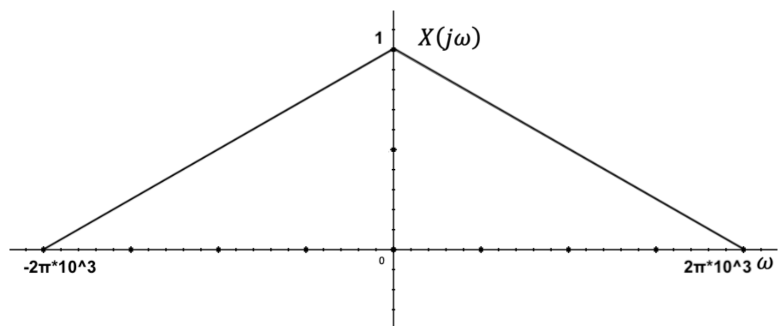

**A.** (15pts) Sketch and label $X_1(j\omega), X_2(j\omega), X_3(j\omega)$

**B.** (15pts) Sketch and label $Y_1(j\omega), Y_2(j\omega), Y_3(j\omega)$

**Problem 3 (30pts)**

**Consider the following Analog / Digital conversion system**




**where**

$$\boldsymbol{p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT), \qquad T = 0.5 \times 10^{-3}}$$

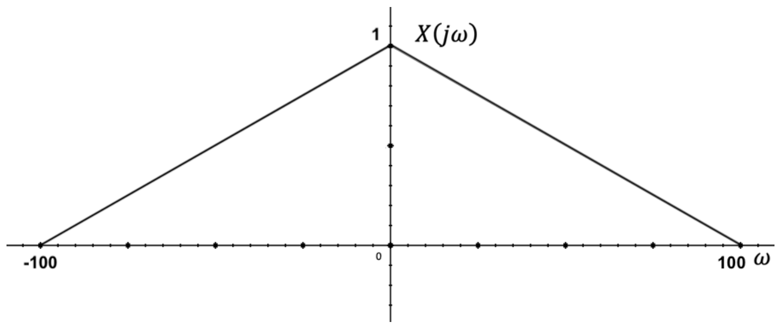
**A. (10pts) Find an expression for $X(j\omega)$, the Fourier transform of $x(t)$. Sketch and label it.**

**B. (10pts) Find an expression for $X_p(j\omega)$, the Fourier transform of $x_p(t)$. Sketch and label it.**

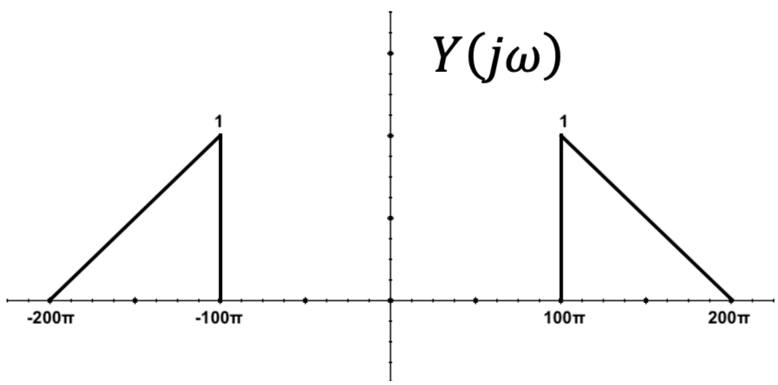
**C. (10pts) Find an expression for $X(e^{j\omega})$, the Fourier transform of $x[n]$. Sketch and label it.**

**Problem 4 (25pts)**

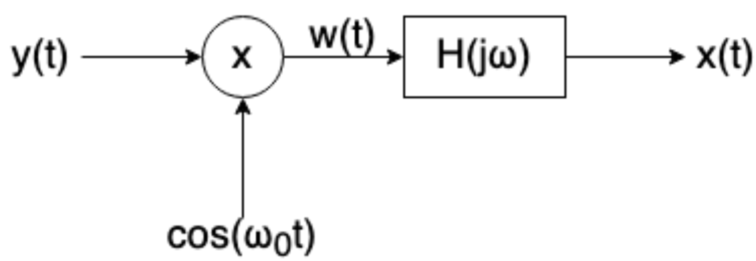
**Consider a signal with Fourier domain representation $X(j\omega)$ of the form**




**We seek to broadcast this signal to a receiver. To achieve this, we filter and modulate $X(j\omega)$ into the signal $Y(j\omega)$, which is then transmitted. $Y(j\omega)$ is of the form**




**We seek to design a receiver that can recover the signal $X(j\omega)$ from the transmitted signal $Y(j\omega)$. This receiver takes the form of the following block diagram.**

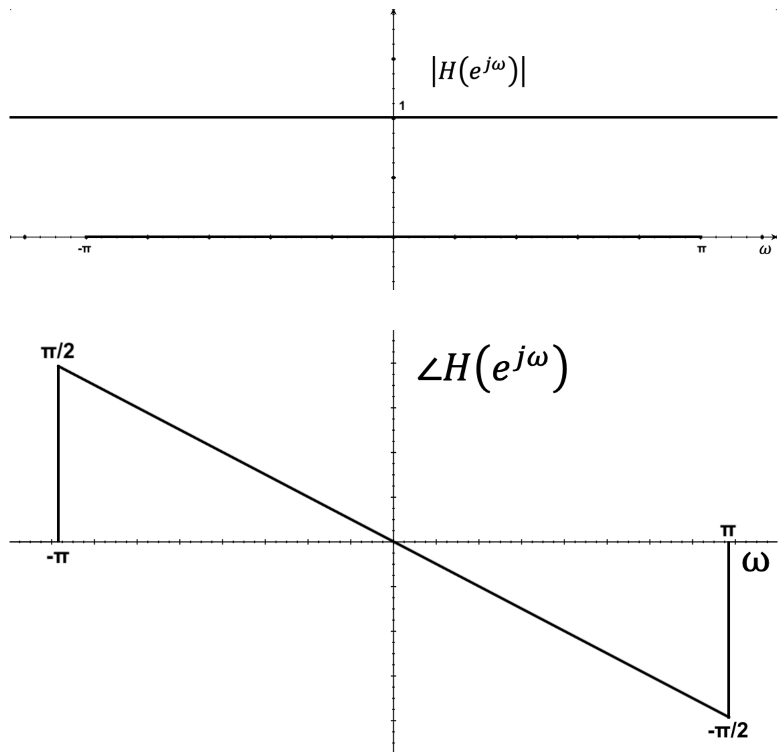



**A. (15pts) Determine $\omega_0$. Sketch and label $W(j\omega)$.**

**B. (10pts) Assume that $H(j\omega)$ is an ideal filter (i.e. it's Fourier Transform $H(j\omega)$ is a sum of rectangle waves). Sketch and label $H(j\omega)$. Furthermore, specify what type of filter this is, and the cutoff frequency and gain of the filter.**

**Problem 5 (30pts)**

**Consider the causal LTI system with transfer function $H(e^{j\omega})$, characterized by**




A. **(10pts) For $x[n] = cos\left(\frac{5\pi}{2}n - \frac{\pi}{4}\right)$, compute $X(e^{j\omega})$, the Fourier transform of $x[n]$**

B. **(20pts) Assume that $x[n]$ from part (A) is fed as an input to the LTI system characterized by $H(e^{j\omega})$ to produce an output signal $y[n]$. Compute $Y(e^{j\omega})$ and $y[n]$.**

**Problem 6 (20pts)**

**Consider the difference equation**

$$\boldsymbol{y[n] = \frac{1}{4}y[n-1] + \frac{1}{8}y[n-2] + x[n] - 2x[n-1]}$$

**A.** **(5pts) Find the Z-domain transfer function of this system, $\boldsymbol{H(z) = \frac{Y(z)}{X(z)}}$.**

**B.** **(5pts) Identify the poles, zeroes, and ROC of the transfer function you obtained in (A).**

**C.** **(10pts) Find the impulse response of this system, $\boldsymbol{h[n]}$**