EE3015 – Midterm 2 – Friday April 3rd 2020

This exam is open book and open notes. Communication with other people is not permitted. Write your solution to each problem in a separate document / sheet of paper. Please submit your solutions in the order they are given in this document. If your solution relies on proofs from an external source (book or notes), please reference this source in your solution (e.g. page number, table number, lecture date). Submit your solutions as a PDF document to Canvas by 12:30pm. Students who took their first midterm through DRC can submit their solutions via email (mahmo006@umn.edu) until 1:30pm.

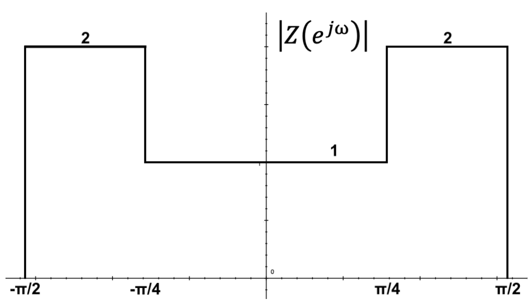
Problem 1 (25pts)

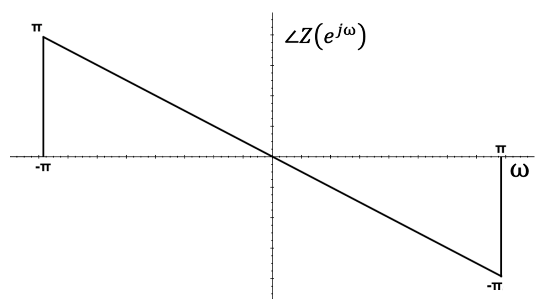
A. **(10pts) Compute the discrete Fourier transform $X\left(e^{j\omega}\right)$ of the signal**

$$x[n] = a^n(u[n] - u[n-5])$$

where $|a| < 1$

B. **(15pts) Given the magnitude $\left|Z\left(e^{j\omega}\right)\right|$ and phase $\angle Z\left(e^{j\omega}\right)$ of the discrete time signal $z[n]$ as shown below, find an expression for the signal $z[n]$. Simplify your answer as much as possible.**





$$\left|Z\left(e^{j\omega}\right)\right| = \begin{cases} 1, & |\omega| < \frac{\pi}{4} \\ 2, & \frac{\pi}{4} < |\omega| < \frac{\pi}{2} \\ 0, & \text{else} \end{cases}$$

(5pts Extra Credit) Solve 1B without explicitly evaluating any integrals. Your final answer must be correct to receive these points.

Problem 2 (25pts)

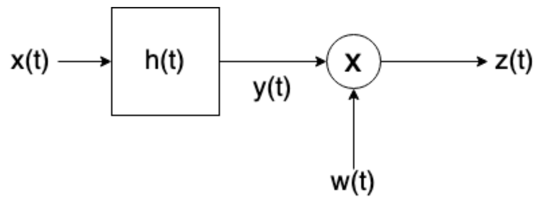
Let the signal $x(t)$ have Fourier transform

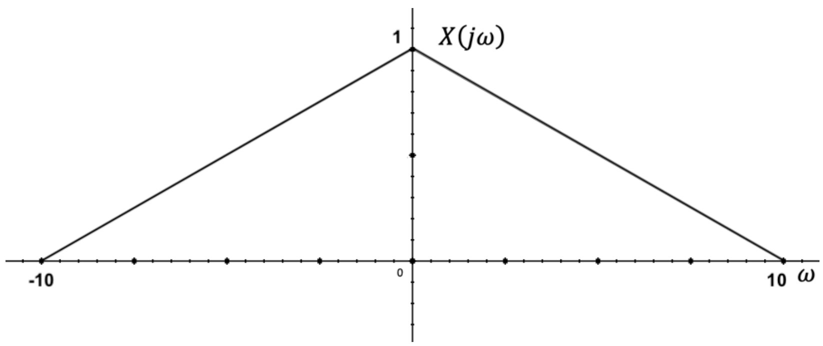
$$X(j\omega) = \left(\frac{4}{9+\omega^2}\right) * \left(e^{-j2\omega}\frac{sin(2\omega)}{\omega}\right)$$

where $*$ represents convolution. Use the inverse Fourier transform to determine the original signal $x(t)$. List any properties you use from notes or from the textbook.

Problem 3 (25pts)

Consider the following filtering and modulation system.





$$h(t) = \frac{sin(5t)}{\pi t} \qquad w(t) = cos(5t) + 2\,cos(15t)$$

A. **(10pts) Find the Fourier transform** $Y(j\omega)$ **of signal** $y(t)$**. Sketch** $Y(j\omega)$**.**

B. **(5pts) Find the Fourier transform** $W(j\omega)$ **of signal** $w(t)$. **Sketch** $W(j\omega)$.

C. **(10pts) Find the Fourier transform** $Z(j\omega)$ **of signal** $z(t)$**. Sketch** $Z(j\omega)$.

Problem 4 (25pts)

Consider the signals

$$x_1(t) = 3\,cos(\pi t)$$
$$x_2(t) = 2\,cos(3\pi t)$$
$$x_3(t) = 2\,sin(5\pi t)$$

and

$$x(t) = x_1(t) + x_2(t) + x_3(t)$$

A. **(5pts) Let the signals $x_1(t), x_2(t), x_3(t)$ be sampled with sampling period $T = 0.4$ to obtain sampled signals $x_1[n], x_2[n], x_3[n]$. For each of these signals, determine if the sampled signal can be used to recover the original signal without aliasing. Can $x(t)$ be recovered from these sampled signals? (e.g. can $x[n] = x_1[n] + x_2[n] + x_3[n]$ be used to recover $x(t)$ without aliasing?)**

B. **(10pts) Find the Nyquist frequency of $x(t)$.**

C. **(10pts) Let the signal $c(t)$ be given by**

$$c(t) = cos(20\pi t)$$

Now suppose the signal $x(t)$ is used to modulate the signal $c(t)$ to produce a signal $y(t)$, where

$$y(t) = x(t)c(t)$$

Find the highest frequency present in the signal $y(t)$. In other words, find the largest frequency ω where $Y(j\omega) \neq 0$.