9.3. Using an analysis similar to that used in Example 9.3, we know that the given signal has a Laplace transform of the form

$$X(s) = \frac{1}{s+5} + \frac{1}{s+\beta}.$$

The corresponding ROC is $\mathcal{R}e\{s\} > max(-5, \mathcal{R}e\{\beta\})$. Since we are given that the ROC is $\mathcal{R}e\{s\} > -3$, we know that $\mathcal{R}e\{\beta\} = 3$. There are no constraints on the imaginary part of β.

9.9. Using partial fraction expansion

$$X(s) = \frac{4}{s+4} - \frac{2}{s+3}.$$

Taking the inverse Laplace transform,

$$x(t) = 4e^{-4t}u(t) - 2e^{-3t}u(t).$$

9.15. Taking the Laplace transforms of both sides of the two differential equations, we have

$$sX(s) = -2Y(s) + 1 \quad \text{and} \quad sY(s) = 2X(s).$$

Solving for $X(s)$ and $Y(s)$, we obtain

$$X(s) = \frac{s}{s^2+4} \quad \text{and} \quad Y(s) = 2s^2 + 4.$$

The region of convergence for both $X(s)$ and $Y(s)$ is $\mathcal{R}e\{s\} > 0$ because both are right-sided signals.

9.28. (a) The possible ROCs are
- (i) $\mathcal{R}e\{s\} < -2$.
- (ii) $-2 < \mathcal{R}e\{s\} < -1$.
- (iii) $-1 < \mathcal{R}e\{s\} < 1$.
- (iv) $\mathcal{R}e\{s\} > 1$.

(b)
- (i) Unstable and anticausal.
- (ii) Unstable and non causal.
- (iii) Stable and non causal.
- (iv) Unstable and causal.

9.40. Taking the unilateral Laplace transform of both sides of the given differential equation, we get

$$\begin{aligned} s^3\mathcal{Y}(s) \; - \; & s^2y(0^-) - sy'(0^-) - y''(0^-) + 6s^2\mathcal{Y}(s) - 6sy(0^-) \\ & -6y(0^-) + 11s\mathcal{Y}(s) - 11y(0^-) + 6\mathcal{Y}(s) = \mathcal{X}(s). \end{aligned} \qquad \text{(S9.40-1)}$$

(a) For the zero state response, assume that all the initial conditions are zero. Furthermore, from the given $x(t)$ we may determine

$$\mathcal{X}(s) = \frac{1}{s+4}, \qquad \mathcal{R}e\{s\} > -4.$$

From eq. (S9.40-1), we get

$$\mathcal{Y}(s)[s^3 + 6s^2 + 11s + 6] = \frac{1}{s+4}.$$

Therefore,

$$\mathcal{Y}(s) = \frac{1}{(s+4)(s^3 + 6s^2 + 11s + 6)}.$$

Taking the inverse unilateral Laplace transform of the partial fraction expansion of the above equation, we get

$$y(t) = \frac{1}{6}e^{-t}u(t) - \frac{1}{6}e^{-4t}u(t) + \frac{1}{2}e^{-2t}u(t) - \frac{1}{2}e^{-3t}u(t).$$

(b) For the zero-input response, we assume that $\mathcal{X}(s) = 0$. Assuming that the initial conditions are as given, we obtain from (S9.40-1)

$$\mathcal{Y}(s) = \frac{s^2 + 5s + 6}{s^3 + 6s^2 + 11s + 6} = \frac{1}{s+1}.$$

Taking the inverse unilateral Laplace transform of the above equation, we get

$$y(t) = e^{-t}u(t).$$

(c) The total response is the sum of the zero-state and zero-input responses.

$$y(t) = \frac{7}{6}e^{-t}u(t) - \frac{1}{6}e^{-4t}u(t) + \frac{1}{2}e^{-2t}u(t) - \frac{1}{2}e^{-3t}u(t).$$