work 6 solutions

2, 6.5, 6.7, 6.17, 6.22, 6.27 a, b, c

Discrete time LTI system has frequency response $H(e^{j\omega}) = |H(e^{j\omega})| e^{j\measuredangle H(e^{j\omega})}$ & real impulse response $h[n]$. We apply input $x[n] = \sin(\omega_0 n + \phi_0)$. Resulting output can be shown to be of the form

$$y[n] = |H(e^{j\omega_0})| x[n - n_0]$$

provided $\measuredangle H(e^{j\omega})$ & ω_0 are related in a particular way.

$$x[n] = \sin(\omega_0 n + \phi_0) = \frac{e^{j(\omega_0 n + \phi_0)} - e^{-j(\omega_0 n + \phi_0)}}{2j}$$

let $x_1[n] = \frac{1}{2j} e^{j(\omega_0 n + \phi_0)}$ & $x_2[n] = \frac{1}{2j} e^{-j(\omega_0 n + \phi_0)}$

Then $x[n] = x_1[n] + x_2[n]$

Using linearity $y[n] = y_1[n] + y_2[n]$, where $y_1[n]$ is the response to $x_1[n]$ & $y_2[n]$ is the response to $x_2[n]$.

x_1 & x_2 are complex exponentials, so the response

to x_1 is $y_1[n] = \frac{1}{2j} |H(e^{j\omega_0})| e^{j(\omega_0 n + \phi_0 + \measuredangle H(e^{j\omega_0}))}$

to x_2 is $y_2[n] = \frac{-1}{2j} |H(e^{-j\omega_0})| e^{-j(\omega_0 n + \phi_0 - \measuredangle H(e^{-j\omega_0}))}$

The fact that $h[n]$ is real tells us

$$|H(e^{j\omega_0})| = |H(e^{-j\omega_0})| \quad \& \quad \measuredangle H(e^{j\omega_0}) = -\measuredangle H(e^{-j\omega_0})$$

Thus $y_1[n] = \frac{1}{2j} |H(e^{j\omega_0})| e^{j(\omega_0 n + \phi_0 + \measuredangle H(e^{j\omega_0}))}$

$y_2[n] = \frac{1}{2j} |H(e^{j\omega_0})| e^{-j(\omega_0 n + \phi_0 + \measuredangle H(e^{j\omega_0}))}$

Then $y[n] = y_1[n] + y_2[n] = |H(e^{j\omega_0})| \sin(\omega_0 n + \phi_0 + \measuredangle H(e^{j\omega_0}))$

$= |H(e^{j\omega_0})| x[n - n_0]$

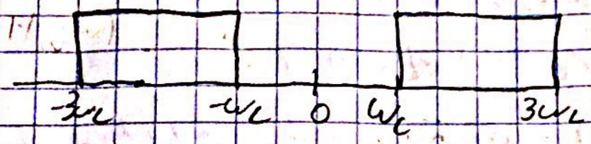
where $n_0 = \frac{-\measuredangle H(e^{j\omega_0})}{\omega_0}$

So $\measuredangle H(e^{j\omega_0})$ must be $-n_0\omega_0 + 2\pi k$ for $k \in \mathbb{Z}$.

5. $H(j\omega) = \begin{cases} 1 & \omega_c \le |\omega| \le 3\omega_c \\ 0 & \text{elsewhere} \end{cases}$

a) find $g(t)$ s.t. $h(t) = \frac{\sin(\omega_c t)}{\pi t} g(t)$.

$H(j\omega)$



Note that $H(j\omega)$ consists of two box functions. One is shifted by $+2\omega_c$ & 1 by $-2\omega_c$.

Then $H(j\omega)$ can be written as
$$H(j\omega) = F(j(\omega + 2\omega_c)) + F(j(\omega - 2\omega_c))$$
$$\text{where } F(j\omega) = \begin{cases} 1 & |\omega| \le \omega_c \\ 0 & \text{Otherwise} \end{cases}$$

The inverse transform of $F(j\omega)$ can be found in the transform tables to be
$$\mathcal{F}^{-1}(F(j\omega)) = \frac{\sin(\omega_c t)}{\pi (t)}$$

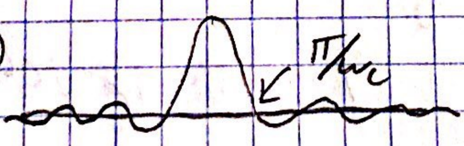
So, $$\mathcal{F}^{-1}(H(j\omega)) = \left(e^{2\omega_c t} + e^{-2\omega_c t}\right) \frac{\sin(\omega_c t)}{\pi t}$$

$$= 2\cos(2\omega_c t) \frac{\sin(\omega_c t)}{\pi t}.$$

Then $g(t) = 2\cos(2\omega_c t)$

b) as ω_c increases, does $h(t)$ become more or less concentrated about the origin.

$\frac{\sin(\omega_c t)}{\pi t}$



Increasing ω_c causes this to become <u>more</u> concentrated about the origin.

Then $h(t)$ also becomes more concentrated.

.7] LPF has passband of 1000Hz & stopband of 1200Hz w/ passband ripple of 0.1 & stopband ripple of 0.05.

The impulse response of this filter is denoted $h(t)$.

we want a filter with impulse response

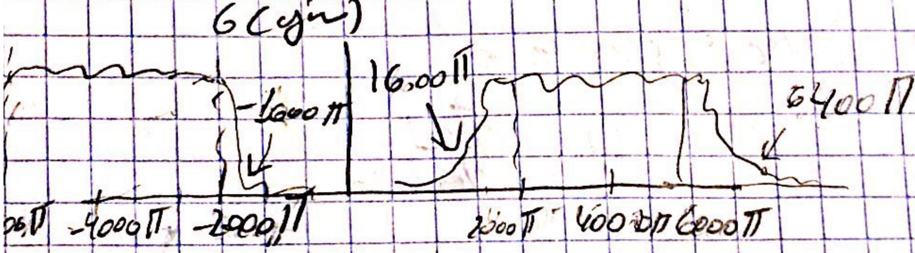
$g(t) = 2h(t)\cos(4000\pi t)$

Note that the $\mathcal{F}\{\cos(4000\pi t)\} = \pi[\delta(w-4000\pi) + \delta(w+4000\pi)]$

Then $G(jw) = 2H(jw) * [\pi\delta(w-4000\pi) + \pi\delta(w+4000\pi)]$

$= 2\pi[H(j(w-4000\pi)) + H(j(w+4000\pi))]$

So $G(jw)$ just consists of 2 $H(jw)$'s, scaled & shifted in opposite directions.



$\Rightarrow$ passband is $\frac{2000\pi}{2\pi}$ to $\frac{6000\pi}{2\pi}$ = 1000 to 3000 Hz

stopband is $\frac{1600\pi}{2\pi} = 800$ Hz

or $\frac{6400}{2\pi} = 3200$ Hz.

Scanned with CamScanner

.17 For each of the following stable LTI systems, determine whether the step response is oscillatory.

a) $y[n] + y[n-1] + \frac{1}{4} y[n-2] = x[n]$

Take DTFT of both sides

$$Y(e^{j\omega}) + e^{-j\omega} Y(e^{j\omega}) + \frac{1}{4} e^{-2j\omega} Y(e^{j\omega}) = X(e^{j\omega})$$

$$\Rightarrow \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1+e^{-j\omega}+\frac{1}{4}e^{-2j\omega}} = \frac{1}{(1+\frac{1}{2}e^{-j\omega})^2}$$

$\Rightarrow$ response of $(n+1)\underbrace{(-\tfrac{1}{2})^n}_{\text{oscillatory}} u[n]$

b) $y[n] - y[n-1] + \frac{1}{4} y[n-2] = x[n]$

$$Y(e^{j\omega}) - e^{-j\omega} Y(e^{-j\omega}) + \frac{1}{4} e^{-2j\omega} Y(e^{j\omega}) = X(e^{j\omega})$$

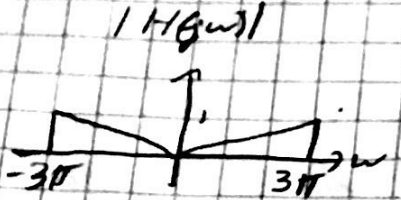
$$\Rightarrow \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1-e^{-j\omega}+\frac{1}{4}e^{-2j\omega}} = \frac{1}{(1-\frac{1}{2}e^{-j\omega})^2}$$

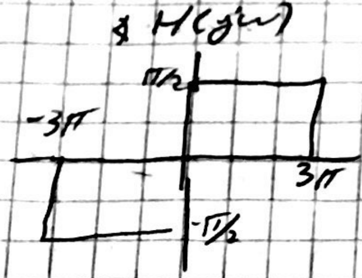
$\Rightarrow$ response of

$(n+1)(\tfrac{1}{2})^n u[n]$

not oscillatory

22.





determine filtered output

a) $x(t) = \cos(2\pi t + \theta)$

$$y(t) = |H(j\omega)|\Big|_{\omega=2\pi} \cos(2\pi t + \theta + \measuredangle H(j\omega)|_{\omega=2\pi})$$

$$\Rightarrow y(t) = \frac{2}{3}\cos(2\pi t + \theta + \pi/2)$$

b) $x(t) = \cos(4\pi t + \theta)$

$$y(t) = |H(j\omega)|\Big|_{\omega=4\pi} \cos(2\pi t + \theta + \measuredangle H(j\omega)|_{\omega=4\pi})$$

$$= 0.$$

c) $x(t) = \begin{cases} \sin(2\pi t) & n < t \leq n + 1/2 \\ 0 & n + 1/2 < t < n \end{cases}$ for $n \in \mathbb{Z}$

$$y(t) = \begin{cases} |H(j\omega)|_{\omega=2\pi} \sin(2\pi t + \measuredangle H(j\omega))|_{\omega=2\pi} \\ 0 \end{cases}$$

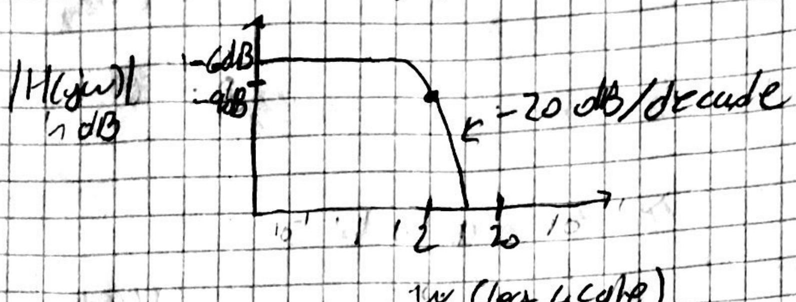
$$= \begin{cases} 2/3 \sin(2\pi t + \pi/2) & n \leq t \leq n + 1/2 \\ 0 & n + 1/2 < t \leq n \end{cases} \quad n \in \mathbb{Z}.$$

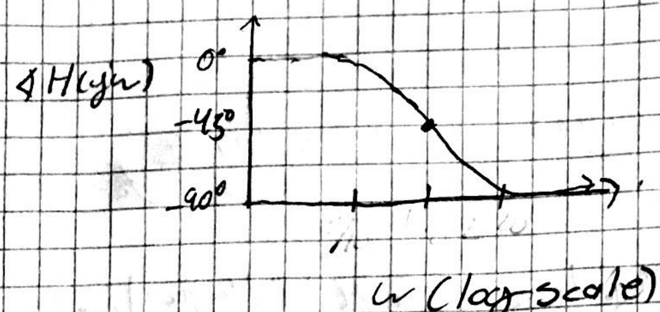
27. $\frac{dy(t)}{dt} + 2y(t) = x(t)$

Take the fourier transform at both sides to get

$j\omega Y(j\omega) + 2Y(j\omega) = X(j\omega)$

a) $\Rightarrow \frac{Y(j\omega)}{X(j\omega)} = \frac{1}{j\omega+2}$





b) group delay $= -\frac{d}{d\omega}(\measuredangle H(j\omega)) = -\frac{d}{d\omega}\left(-\tan^{-1}\left(\frac{\omega}{2}\right)\right)$

$= -\frac{1}{2} \cdot \frac{-1}{1+(\omega/2)^2}$

$= \frac{1}{2+\omega^2/2} = \frac{2}{4+\omega^2}$

c) $x(t) = e^{-t}u(t)$, $X(j\omega) = \mathcal{F}(x(t)) = \frac{1}{1+j\omega}$

$\Rightarrow Y(j\omega) = \frac{1}{(j\omega+2)(j\omega+1)}$