samples. Therefore, $X(j\omega) = 0$ for $|\omega| > 5000\pi$.

7.2. From the Nyquist theorem, we know that the sampling frequency in this case must be at least $\omega_s = 2000\pi$. In other words, the sampling period should be at most $T = 2\pi/(\omega_s) = 1 \times 10^{-3}$. Clearly, only (a) and (c) satisfy this condition.

where $(\omega_0/2) < \omega_c$ ...

7.6. Consider the signal $w(t) = x_1(t)x_2(t)$. The Fourier transform $W(j\omega)$ of $w(t)$ is given by

$$W(j\omega) = \frac{1}{2\pi}[X_1(j\omega) * X_2(j\omega)].$$

Since $X_1(j\omega) = 0$ for $|\omega| \geq \omega_1$ and $X_2(j\omega) = 0$ for $|\omega| \geq \omega_2$, we may conclude that $W(j\omega) = 0$ for $|\omega| \geq \omega_1 + \omega_2$. Consequently, the Nyquist rate for $w(t)$ is $\omega_s = 2(\omega_1 + \omega_2)$. Therefore, the *maximum* sampling period which would still allow $w(t)$ to be recovered is $T = 2\pi/(\omega_s) = \pi/(\omega_1 + \omega_2)$.

Therefore, the given statement ...

7.11. We know from Section 7.4 that

$$X_d(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c(j(\omega - 2\pi k)/T).$$

(a) Since $X_d(e^{j\omega})$ is just formed by shifting and summing replicas of $X(j\omega)$, we may as[sume] that if $X_d(e^{j\omega})$ is real, then $X(j\omega)$ must also be real.

(b) $X_d(e^{j\omega})$ consists of replicas of $X(j\omega)$ which are scaled by $1/T$. Therefore, if $X_d($... has a maximum of 1, then $X(j\omega)$ will have a maximum of $T = 0.5 \times 10^{-3}$.

(c) The region $3\pi/4 \leq |\omega| \leq \pi$ in the discrete-time domain corresponds to the re[gion] $3\pi/(4T) \leq |\omega| \leq \pi/T$ in the continuous-time domain. Therefore, if $X_d(e^{j\omega}) =$ ... $3\pi/4 \leq |\omega| \leq \pi$, then $X(j\omega) = 0$ for $1500\pi \leq |\omega| \leq 2000\pi$. But since we already [have] $X(j\omega) = 0$ for $|\omega| \geq 2000\pi$, we have $X(j\omega) = 0$ for $|\omega| \geq 1500\pi$.

(d) In this case, since π in discrete-time frequency domain corresponds to 2000π in the continuous-time frequency domain, this condition translates to $X(j\omega) = (j(\omega - 2000\pi))$.

... continuous-time frequencies Ω and ω are ...

7.15. In this problem we are interested in the lowest rate which $x[n]$ may be sampled without the possibility of aliasing. We use the approach used in Example 7.4 to solve this problem. To find the lowest rate at which $x[n]$ may be sampled while avoiding the possibility of aliasing, we must find an N such that

$$\frac{2\pi}{N} \geq 2\left(\frac{3\pi}{7}\right) \Rightarrow N \leq \frac{7}{3}.$$

Therefore, N can at most be 2.

... the signal $x_1[n] = 2\sin(\pi n/2)/(\pi n)$ satisfies the first two conditions, it does ... Fourier transform $X_1(e^{j\omega})$ of this factor of 2. Therefore, in this problem ... to an ideal lowpass filter with cutoff frequency $\pi/$ and a passband gain ...

7.19. The Fourier transform of $x[n]$ is given by

$$X(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \omega_1 \\ 0, & \text{otherwise} \end{cases}.$$

This is as shown in Figure S7.19.

(a) When $\omega_1 \le 3\pi/5$, the Fourier transform $X_1(e^{j\omega})$ of the output of the zero-insertion system is as shown in Figure S7.19. The output $W(e^{j\omega})$ of the lowpass filter is as shown in Figure S7.19. The Fourier transform of the output of the decimation system $Y(e^{j\omega})$ is an expanded or stretched out version of $W(e^{j\omega})$. This is as shown in Figure S7.19.

Therefore,

$$y[n] = \frac{1}{5}\frac{\sin(5\omega_1 n/3)}{\pi n}.$$

(b) When $\omega_1 > 3\pi/5$, the Fourier transform $X_1(e^{j\omega})$ of the output of the zero-insertion system is as shown in Figure S7.19. The output $W(e^{j\omega})$ of the lowpass filter is as shown in Figure S7.19.

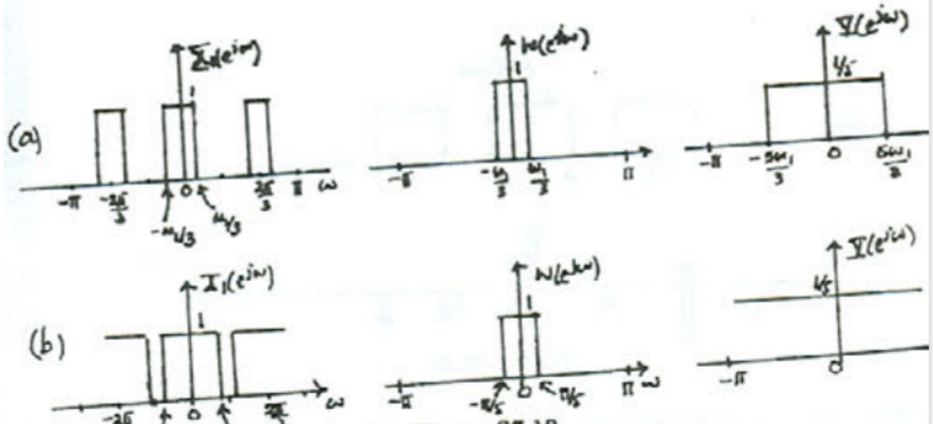

Figure S7.19

The Fourier transform of the output of the decimation system $Y(e^{j\omega})$ is an expa... or stretched out version of $W(e^{j\omega})$. This is as shown in Figure S7.19. Therefore,

$$y[n] = \frac{1}{5}\delta[n].$$

7.24. We may express $s(t)$ as $s(t) = \hat{s}(t) - 1$, where $\hat{s}(t)$ is as shown in Figure S7.24.

We may easily show that

$$\hat{S}(j\omega) = \sum_{k=-\infty}^{\infty} \frac{4\sin(2\pi k\Delta/T)}{k}\delta(\omega - k2\pi/T).$$

From this, we obtain

$$S(j\omega) = \hat{S}(j\omega) - 2\pi\delta(\omega) = \sum_{k=-\infty}^{\infty} \frac{4\sin(2\pi k\Delta/T)}{k}\delta(\omega - k2\pi/T) - 2\pi\delta(\omega).$$

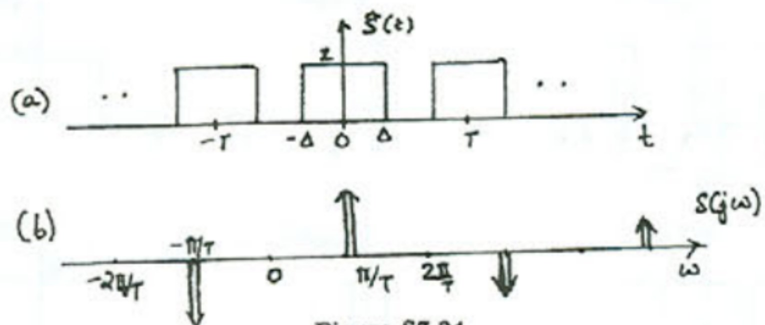

Figure S7.24

Clearly, $S(j\omega)$ consists of impulses spaced every $2\pi/T$.

(a) If $\Delta = T/3$, then

$$S(j\omega) = \sum_{k=-\infty}^{\infty} \frac{4\sin(2\pi k/3)}{k}\delta(\omega - k2\pi/T) - 2\pi\delta(\omega).$$

Now, since $w(t) = s(t)x(t)$,

$$W(j\omega) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} \frac{4\sin(2\pi k/3)}{k}X(j(\omega - k2\pi/T)) - 2\pi X(j\omega).$$

Therefore, $W(j\omega)$ consists of replicas of $X(j\omega)$ which are spaced $2\pi/T$ apart. In order to avoid aliasing, ω_M should be less that π/T. Therefore, $T_{max} = \pi/\omega_M$.

(b) If $\Delta = T/3$, then

$$S(j\omega) = \sum_{k=-\infty}^{\infty} \frac{4\sin(2\pi k/4)}{k}\delta(\omega - k2\pi/T) - 2\pi\delta(\omega).$$

We note that $S(j\omega) = 0$ for $k = 0, \pm 2, \pm 4, \cdots$. This is as sketched in Figure S7.24

Therefore, the replicas of $X(j\omega)$ in $W(j\omega)$ are now spaced $4\pi/T$ apart. In order to avoid aliasing, ω_M should be less that $2\pi/T$. Therefore, $T_{max} = 2\pi/\omega_M$.